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SENSITIVITY ANALYSIS FOR HEAT TRANSFER IN A FIN OF A NUCLEAR FUEL ROD

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ABSTRACT

To carry out sensitivity analysis on a finned surface, the differential perturbative method is applied in a heat conduction problem within a thermal system, made up of a one-dimensional circumferential fin on a nuclear fuel element. The model is described by the temperature distribution equation and the further specific boundary conditions. The adjoint system is used to determine the sensitivity coefficients to the case of interest. Both, the direct model and the perturbative formalism resultant equations are solved. The heat flow rate at a specific point of the fin and the average temperature excess were the functional responses studied. The half thickness, the thermal conductivity, the heat transfer coefficients and the heat flow at the base of the fin were the parameters of interest to the sensitivity analysis. The results obtained through the perturbative method and the direct variation presented, in a general form and within acceptable physical limits, good concordance and excellent representativeness to the analyzed cases. One evidences that the differential formalism is an important tool to the sensitivity analysis, and it validates the application of the methodology in heat transmission problems on extended surfaces. The method proves to be useful and efficient while elaborating thermal engineering projects.

Keywords: sensitivity analysis, perturbative methods, extended surfaces, fins.

I. INTRODUCTION

In thermal engineering, the complete analysis of the heat transmission plays an important role to assess appropriate dimensioning of the equipments and to guarantee their efficiency, economy and security.

When there is a large difference in reference to the heat transfer coefficient on two sides of a surface, the convective heat transport can be increased through the use of fins on the lower coefficient side, say, extending the thermal contact area [1].

The use of extended surfaces is currently very relevant because it enables the transmission of a large heat amount among fluids and surfaces. Those surfaces can be found in many thermal systems, such as: nuclear fuel elements, condensers and heat exchangers used at the industry, and also in vehicle radiators, cryogenics, air conditioners, gas turbines, microcomputers and other applications.

The use of fins on the nuclear fuel elements is a characteristic of gas-cooled nuclear reactors, because this fluid has a low heat transfer coefficient. There are few reactors that have this shape, about 30 units worldwide.

In the optimization of thermal systems is frequently used a computational model that represents the real phenomena. However, the model entry data, called parameters as well, are subject to many uncertainties or imprecisions that can impose important restrictions to the reliability of the results in the output of the model.

Thus, the system analysis (thermal, for example), at the industrial sector or at scientific studies, will include the determination of the resultant influence of the variation, or perturbation, of some parameter of the problem in the system behavior. This technique is known as “sensitivity analysis” [2].

The sensitivity analysis methodology using the direct or conventional method, consists of the variation of one or more control parameters maintaining the others fixed. The calculation is repeated with the parameters of interest, constructing “response surface”. This method has several disadvantages, which make it sometimes an impracticable methodology, because many parameters can cause alterations (or perturbations) in the system. Moreover, some models adopted for the calculations are very complex and time-consuming.

In reference to the perturbative methods, these are applied to the sensitivity analysis mainly when there is not an analytical solution for the model and when its numerical solution is very expensive from a CPU-time stand point. The principal advantage of these formalisms, in a general form, is the sensitivity calculus of the response in reference to the equation parameters without previous choosing of the parameter variation range (the opposite of the direct method, where the previous choosing is compulsory). For the calculation of the new response, for each parameter variation, is used an expression of simple resolution, and it works with an unique additional equation system for each analyzed response. Thus, it makes viable the model solution in complex equations and it reduces the calculation time. The methodology is currently expanding to other engineering areas and industrial applications [3]. Previous research works have had successful results that were obtained by [4], that used the formalism in a solute transfer model through soils; [5], in a steam generator model used in light water cooled nuclear reactors; and [6], in waterhammer problems in hydraulic networks.

The principal objective of this paper is to carry out a sensitivity analysis, using the differential formalism of the perturbation theory to determine the influence of the parameter variations in the functional responses of interest. The method is applied to a problem of heat conduction, made up of a one-dimensional circumferential fin on a nuclear fuel element with specific boundary conditions. The heat flow rate at a specific point of the fin and the average temperature excess were the functional responses studied. The half thickness, the thermal conductivity, the heat transfer coefficients and the heat flow at the base of the fin were the parameters of interest to the sensitivity analysis. The obtained results will be discussed considering the influence of each studied parameter in the thermal system perturbation, for the given conditions. In this way, the more sensible parameters will be determined, targets of special care, while studying and elaborating projects of finned thermal equipments. The advantages and the validity of this application in extended surfaces will be presented.

II. THE UNIFORM THICKNESS CIRCUMFERENTIAL FIN MODEL

General Considerations. The uniform thickness circumferential fin is a common type of transversal extended surface, used in several thermal equipments, mainly in industrial heat exchange pipes, as well as in gas-cooled nuclear reactor fuel elements.

The thermal sensitivity analysis that is carried out through the application of the differential perturbative method is the main focus of this paper. In this section, the model will be described with the considerations and boundary conditions. Next, the fin temperature distribution equation is solved for the temperature excess in the fin and heat flux is obtained.

Description of the Model, Considerations and Boundary Conditions. The cylindrical coordinates and the Bessel functions for the solution of the problem are used to study the fin. The uniform thickness circumferential fin model, fixed on the nuclear fuel element, is shown on Fig. 1, where r_0 is the fin base radius, i.e., the fuel element radius, r_1 is the fin and $2y_0$ is the fin thickness.

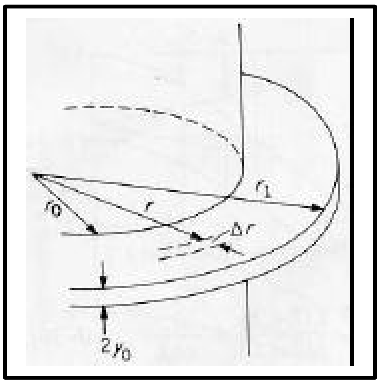

Figure 1. Uniform Thickness Circumferential Fin.

Considerations to the studied model:

- the cooling fluid temperature, t_f, is constant around the fin;
- the heat transfer coefficient, h, along the fin, is an uniform value;
- the fin is made of homogeneous material and it is assumed that the thermal conductivity coefficient k has an uniform value, taken at the mean temperature of the fin;
- the temperature distribution and the heat conduction are one-dimensional for a low Biot number. The temperature is a function of the radius r only, because the thickness is very small if it is compared with the value of $r_1 - r_0$;
- the fin heat generation is negligible, that is, $q''' = 0$.

The fin temperature distribution is obtained through the analytical solution of the Eq. (1) [7],

$$\frac{d^2 q}{dr^2} + \frac{1}{r}\frac{dq}{dr} - m_0^2\, q = 0 \qquad (1)$$

where $m_0^2 = \dfrac{h}{k\, y_0}$ and q is the temperature excess $t - t_f$ (t is the temperature of the fin).

The boundary conditions are obtained with the knowledge of the heat flow in two points of the fin:

$$q'' = q_0 \quad at \quad r = r_0$$

$$\frac{dq}{dr} = 0 \quad at \quad r = r_1 + y_0 = r'$$

The boundary condition at r' is deduced through a process that the fin surface is extended in its length by half of the original thickness, and the new edge is supposed to be isolated. According to [8], the errors in relation to that approximation are less than 8% for a Biot number less than 0.5. The studied case has a Biot number of $h\,2y_0/k = 0.17$, using the Table 1 data. Thus, r' is the fin equivalent radius.

The fin temperature distribution equation is given by:

$$q = \frac{q_0}{m_0\,k}\,\frac{K_1(m_0\,r')\,I_0(m_0\,r) + I_1(m_0\,r')\,K_0(m_0\,r)}{I_1(m_0\,r')\,K_1(m_0\,r_0) - I_1(m_0\,r_0)\,K_1(m_0\,r')} \tag{2}$$

where I_1 and K_1 are first order modified Bessel functions of first and second types, respectively.

The equation for the heat flow is:

$$q'' = -k\,\frac{dq}{dr} \tag{3}$$

Thus,

$$q'' = q_0\,\frac{I_1(m_0\,r')\,K_1(m_0\,r) - K_1(m_0\,r')\,I_1(m_0\,r)}{I_1(m_0\,r')\,K_1(m_0\,r_0) - I_1(m_0\,r_0)\,K_1(m_0\,r')} \tag{4}$$

The fin physical and geometric properties are shown on Table 1.

TABLE 1. One-dimensional Circumferential Fin Data

Material: stainless steel–type 304 (alloy: Cr, Ni, Mn, C, Si)	
Fin thickness:	$2y_0 = 0.005\,m$
Fuel element radius (without fin):	$r_0 = 0.0125\ m$
Fin radius:	$r_1 = 0.0225\,m$
Fin equivalent radius:	$r' = r_1 + y_0 = 0.025\,m$
Fluid temperature:	$t_f = 200\,^{\circ}C$
Heat flow at the base of the fin:	$q_0 = 360{,}000\,W/m^2$
Thermal conductivity coef. at $250^{\circ}C$:	$k = 18\ W/m\,^{\circ}C$
Heat transfer coefficient:	$h = 600\ \ W/m^2\ ^{\circ}C$

III. DETERMINATION OF THE GENERAL SENSITIVITY COEFFICIENT – APPLICATION OF THE DIFFERENTIAL FORMALISM.

The sensitivity coefficient is the basic element of the sensitivity analysis using the perturbative methods. The differential formalism developed by the perturbation theory is one of the most direct and accurate tools among several proposed formulations. The adjoint equations system is obtained by the following procedure [9]:

1 - derivating the equation system;
2 - extracting the source terms $S_{(i)}$ of the derivated system;
3 - obtaining the adjoint operator;
4 - bilinear concomitant calculus;
5 - adjoint system boundary conditions determination.

The sensitivity coefficient is calculated for a two-equation system, equivalent to Eq. (1), because, in this way, the heat flow can be incorporated as a state variable. So, it is possible to deal with direct problems and general functional responses [10]. So, the generalized system is:

$$\begin{bmatrix} \dfrac{dq'}{dr} + \dfrac{1}{r}\,q'' + k\,m_0^2\,q \\ q'' + k\,\dfrac{dq}{dr} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{5}$$

The adjoint equation, the boundary conditions and the bilinear concomitant. Following the procedure above, the adjoint system is found:

$$\begin{bmatrix} -\dfrac{dq'^*}{dr} + q^* \\ -k\,\dfrac{dq^*}{dr} - \dfrac{k}{r}\,q^* + k\,m_0^2\,q''^* \end{bmatrix} = \begin{bmatrix} S_1^+ \\ S_2^+ \end{bmatrix} \tag{6}$$

where S^+, the adjoint equation source term, will be a known function in the phase space coordinate when the functional responses are chosen.

The adjoint equation boundary conditions were chosen to reduce the bilinear concomitant, producing a result that can be easily solved. Thus, the chosen boundary conditions are:

$q^* = 0 \;\; when \;\; r = r_0$ and $q^* = 0 \;\; when \;\; r = r'$

And the bilinear concomitant is:

$$P\,(f^*,\, f_{/i}) = 2p\,\left\{ (q_{0/i}\; q''^*\,r)_{r_0} \right\} \tag{7}$$

where the subscribed /i means the derivation in reference to parameter i.

Chosen Cases for the Sensitivity Analysis.

<u>Studied Functionals.</u>

* Heat flow rate at the point p of the fin - Q_p

$$Q_p = \int_z q''\,d(r-r_p)\,2y_0\,dz = 2y_0 \int_{r_0}^{r'} q''\,d(r-r_p)\,2pr\,dr \tag{8}$$

The known functions in the phase space coordinate are $S_1^+ = d\,(r-r_p)\,2y_0$ and $S_2^+ = 0$, where $d\,(r-r_p)$ is the impulse function (Delta Dirac) concentrated at the point p of the fin, located at $r = r_p = 0.018\ m$ and arbitrarily chosen. The nominal value of the functional Q_p is obtained with q'' from Eq. (4) and Tab. (1) data.

* Fin average temperature excess - $\bar{q}$

$$\bar{q} = \frac{1}{pr'^2 - pr_0^2} \int_{r_0}^{r'} q\ 2pr\ dr \qquad (9)$$

The known functions in the phase space coordinate are $S_1^+ = 0$ and $S_2^+ = \frac{1}{pr'^2 - pr_0^2}$. The nominal value of the functional $\bar{q}$ is obtained with q from Eq. (2) and Table 1 data.

Studied Parameters. The parameters for the sensitivity analysis were chosen in reference to its importance in the thermal engineering projects, given by:

* Fin half thickness - y_0
* Thermal conductivity coefficient - k
* Heat transfer coefficient - h
* Heat flow at the base of the fin - q_0

Calculations and Profile of the Adjoint Functions $q*$ and $q''*$. Regarding the values of S_1^+ and S_2^+ for each studied functional, the Eq. (6) is solved using the Green function and Dirac superposition methods [11]. The result is:

Functional Q_p - Heat Flow at the point p of the fin.

$$q* = 0{,}13015\ I_1(m_0\ r) - 0{,}40058\ K_1(m_0\ r)$$

for $r_0 < r < r_p$ (10)

$$q* = -0{,}021307\ I_1(m_0\ r) + 1{,}65295\ K_1(m_0\ r)$$

for $r_p < r < r'$ (11)

and,

$$q''* = \frac{1}{m_0^2}\left\{\left[0{,}13015\left(m_0\ I_2(m_0\ r) + \frac{1}{r} I_1(m_0\ r)\right) - 0{,}40058\left(-m_0\ K_2(m_0\ r) + \frac{1}{r} K_1(m_0\ r)\right)\right] + \frac{1}{r}\left[0{,}13015\ I_1(m_0\ r) - 0{,}40058\ K_1(m_0\ r)\right]\right\}$$

for $r_0 < r < r_p$ (12)

$$q''* = \frac{1}{m_0^2}\left\{\left[0{,}021307\left(m_0\ I_2(m_0\ r) + \frac{1}{r} I_1(m_0\ r)\right) + 1{,}65295\left(-m_0\ K_2(m_0\ r) + \frac{1}{r} K_1(m_0\ r)\right)\right] + \frac{1}{r}\left[-0{,}021307\ I_1(m_0\ r) + 1{,}65295\ K_1(m_0\ r)\right]\right\}$$

for $r_p < r < r'$ (13)

The profile observed on the Fig. 2 is due to the presence of the impulse source at the point p, where in this example, it is the maximum value of the adjoint function at that point.

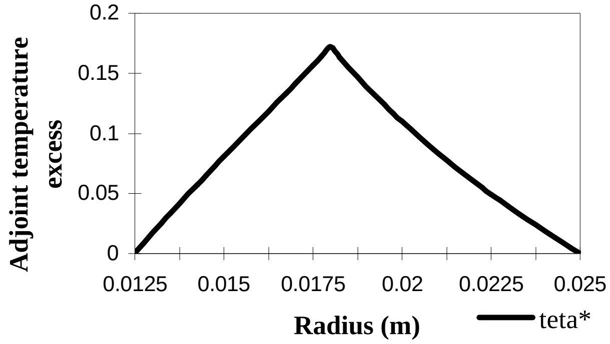

Figure 2. Adjoint Temperature Excess versus Radius.

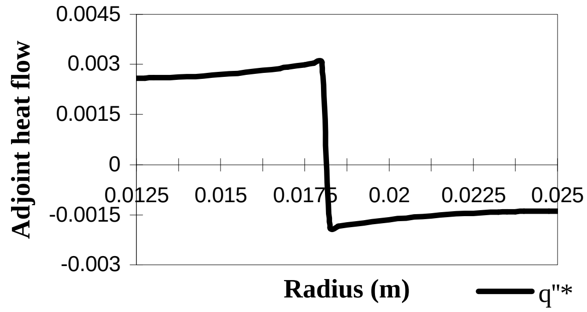

Figure 3. Adjoint Heat Flow versus Radius.

Fig. 3 shows a discontinuity leap at the point p, due to the relation between $q*$ and $q''*$. It also shows the importance of the function around the point p, and the positive and negative contributions at the left and right of that point, respectively.

Functional $\bar{q}$ - Average Temperature Excess of the Fin.

The solution is:

$$q* = 0 \qquad (14)$$

$$q''* = \frac{1}{k\ m_0^2\ (pr'^2 - pr_0^2)} \tag{15}$$

The results show that, in respect to the functional $\bar{q}$, $q''*$ contributes uniformly, but the contribution of $q*$ is null because of the boundary conditions imposed to the problem.

Sensitivity Coefficients Calculations. The general expression for the sensitivity coefficient is [9,10]:

$$\frac{dR}{dp_i} = < f\ S^+_{/i} > + < f * S_{(i)} > + P\ (f*,\ f_{/i}) \tag{16}$$

The expressions for each functional-parameter combination can be found in a detailed way in [12].

IV. RESULTS ANALYSIS

The sensitivity coefficients using Table 1 data and a numerical computational program are:

TABLE 2. Sensitivity Coefficients

	Functionals	
Parameters	$Q_p\ (W)$	$\bar{q}\ \ (^oC)$
$y_0\ (m)$	35,055.6 W/m	32,000 $^oC/m$
$k\ (W/m\,^oC)$	0.768 $m\,^oC$	0 $m\,^oC^2/W$
$h\ (W/m^2\,^oC)$	- 0.0238 $m^2\ ^oC$	-0.133 $m^2\,^oC^2/W$
$q_0\ (W/m^2)$	1 m^2	2.22x10^{-4} $m^2\,^oC/W$

Table 2 shows relative values, where it is not possible to do a comparison of the results in an absolute way. As a consequence, to make the comparison possible, the sensitivity S (or absolute sensitivity coefficient) has the definition:

$$S = \frac{dR}{dp}\frac{p_0}{R_0} \tag{17}$$

where $\frac{dR}{dp}$ is the relative sensitivity coefficient and R_0 and p_0 are the nominal values at the reference point.

The nominal parameters and functional values are presented below, and then the absolute sensitivity coefficients.

TABLE 3. Parameter and Functional Values without Perturbation

Functionals	
$Q_p\ (W)$	$\bar{q}\ \ (^oC)$
73.321	80.0

Parameters			
$y_0\,(m)$	$k\ (W/m\,^oC)$	$h(W/m^2\,^oC)$	$q_0\,(W/m^2)$
0.0025	18	600	360,000

TABLE 4. Absolute Sensitivity Coefficients

	Functionals	
Parameters	$Q_p\ (W)$	$\bar{q}\ \ (^oC)$
$y_0\ (m)$	1.195	1
$k\ (W/m\,^oC)$	0.188	0
$h\ (W/m^2\,^oC)$	- 0.195	-1
$q_0\ (W/m^2)$	1.000363	0.9997

Table 4 shows absolute values, for the comparative effects on the influence of the parameters in the functional response of interest. In a first analysis of the table, y_0 and q_0 are the parameters that cause the major influence in the functional responses. In other words, for each 1% uncertainty in the parameter, the functional has about the same percentage of variation. The parameter k presents, in reference to the functional Q_p, the smaller influence of the absolute values. On the functional $\bar{q}$, its sensitivity coefficient is null, where there is no dependence of it in relation to the parameter k, because it is linked to the kind of admitted boundary condition in the fin extremities. Additionally the parameter h causes a weak influence in the functional Q_p and an influence in the functional $\bar{q}$ comparable to y_0 and q_0.

To verify the precision of the utilized formalism, the nominal values of the parameters were varied with an increase and decrease of 5%, 10% and 15%, respectively. The results of the functionals were obtained through the use of: i) the direct method with Eqs. (8) and (9), and ii) the differential perturbative method with sensitivity coefficients from Table 2. The values were calculated with Table 1 data.

The differential formalism presented the following results:

a) The relative errors were below 2.5% and most of them below 0.2%. Thus, the used method has a good precision. In regard to the functional $\bar{q}$, the calculus is exact to the parameters y_0 and k, as expected.

b) Slopes in the response surface were correctly reproduced. For instance, the functional Q_p has a decreasing behavior as parameter h increases. The negative sign of the calculated sensitivity coefficient (Table 2) confirms that expectation.

c) The parameter k causes a low sensibility in face of 1% uncertainty in the functional Q_p. And, for the functional $\bar{q}$, there is no sensitivity at all to the parameter variation.

V. CONCLUSIONS

Through the obtained results, the first conclusion is that the absolute sensitivity coefficient is essential to the discovery of the more sensible parameters in a functional response. The used method presented a good representativeness for the analyzed cases, where most of the cases had good precision and accuracy.

The most influent parameters on the functional response Q_p were the h (heat transfer coefficient) and the q_0 (heat flow at the base of the fin). About the functional $\bar{q}$, the q_0 was the most influential as well.

Another noticed advantage: repetitions to obtain the perturbated values are eliminated through the determination of the sensitivity coefficient.

The good results achieved with the use of the formalism on the model demonstrated the validity of the application in heat transmission problems of extended surfaces, where it proved to be a very efficient technique to elaborate thermal engineering projects.

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