BR7801500

INSTITUTO DE ENERGIA ATÔMICA - SÃO PAULO

DISCRETE FREQUENCY IDENTIFICATION USING THE HP 5451B FOURIER ANALYSER

L. Holland and P. Berry

INFORMAÇÃO IEA 56 CEN - AFR 17	AGOSTO/1977

INF. IEA 58
CEN - AFR 17

AGOSTO/1977

DISCRETE FREQUENCY IDENTIFICATION USING THE HP 5451B FOURIER ANALYSER

L. Holland and P. Barry

CENTRO DE ENGENHARIA NUCLEAR
Área de Física de Reatores

INSTITUTO DE ENERGIA ATÔMICA
SÃO PAULO – BRASIL

APROVADO PARA PUBLICAÇÃO EM MAIO/1977

INSTITUTO DE ENERGIA ATÔMICA
Caixa Postal 11.049 (Pinheiros)
Cidade Universitária "Armando de Salles Oliveira"
SÃO PAULO — BRASIL

NOTA: Este trabalho foi conferido pelo autor depois de composto e sua redação está conforme o original, sem qualquer correção ou mudança.

DISCRETE FREQUENCY IDENTIFICATION USING THE HP 5451B FOURIER ANALYSER

L. Holland* and P. Barry**

ABSTRACT

This report is directly concerned with frequency analysis by the HP5451B discrete frequency Fourier Analyser. Relevent background theory is given with particular emphasis on the problems associated with discrete frequency analysis. The advantages of cross correlation analysis to identify discrete frequencies in a background noise are discussed in conjunction with the elimination of aliasing and wraparound error. Frequency time windows are illustrated with reference to the unit amplitude and Hanning windows.

Organization and analysis of data by HP5451B is outlined with emphasis being given to the mass storage the system and the analog to digital converter unit. Use of the special keyboard control unit enables frequency analysis problems of widely differing complexity to be undertaken without the need for specialist computing knowledge

Discrete frequency identification is illustrated by a series of graphs giving the results of analysing "electrical" and "acoustical" white noise and sinusoidal signals

INTRODUCTION – NOISE ANALYSIS WITH THE HP 5451B FOURIER ANALYSER

The HP 5451 B Fourier Analyser provides a powerful capability for real time and off line analysis of time varying signals obtained using suitable signal transducers[1]. This report is intended to indicate the type of information which may be obtained from such analysis and is primarily directed at researchers not specifically working in the field of noise analysis. Some brief background theory is given in order to define the terms used in discussing results of analysing time varying signals. A more rigorous treatment can be found in the many excellent texts on Fourier and Noise Analysis[1,3,5,7,8].

Interpretation of analysed non-periodic data is greatly simplified when 'pure' signals are handled. In pratical situations however, useful signal information is almost invariably associated with background interference or "noise". The influence of different background levels is here illustrated by comparing the results of analysing various combinations of sine and white noise signals and of the same signals modified by intermediate acoustic transducers.

1 – BACKGROUND THEORY

1.1 – General Comments

"Noise" may generally be defined as fluctuations of a physical variable of a process which exists when the process is in a steady state condition[4] and in general such noise contains information on the system. Its more specific definition will depend on the phenomenon being considered, examples of which are the acoustic intensity of motor traffic, the voltage across a resistor operating at "constant" power and neutron density fluctuations in a nuclear reactor. Each noise phenomenon will have a characteristic range of frequencies. In nuclear reactors these range from below 10^{-2} Hz (cycle times of reaction coolant circulation) to greater than 10 kHz (acoustic noise and loose parts monitoring). In acoustic measurements frequencies are in the range 20 Hz to 20 kHz.

(*) Instituto de Energia Atômica – Cidade Universitária – São Paulo, SP – Brasil.
(**) Instituto de Pesquisas Tecnologicas – São Paulo – Brasil

In the time domain we may define a phenomenon in terms of amplitude as a function of time; in the frequency domain the defining parameters are amplitude, frequency and phase. A given phenomenon may be defined in either of these reference frames and information is transferred from one domain to the other using the well know Fourier Transform pair.

x(t) ←——→ **Fourier Transform** ←——→ X(ω)

time domain — frequency domain

Invariably the energy of real fluctuations will be contained within a definable frequency bandwidth, and in general energy concentrations or resonances occur within this bandwidth. At one extreme all the "fluctuating energy" is concentrated at a single frequency in a sinusoidal signal. White noise is the other extreme in which all the energy is distributed with equal amplitude at all frequencies from zero to infinity. For practical purposes a noise source is considered to be "white" when the source energy is equally distributed within the bandwidth of interest.

Measurements are made in the time domain and analysed in both the time and frequency domains. The type of analysis will depend on the system being studied. Non-deterministic or random systems may only be defined in terms of average or statistical properties. Deterministic systems may, in theory, be exactly defined. One of the simplest examples of a deterministic phenomenon is a sinusoidal oscillation whose amplitude and phase may be completely defined at any time.

Signals obtained in practical situations will frequently contain both deterministic and non-deterministic information.

1.2 — Fourier Transform

Periodic and Non periodic data may be transferred between the time and frequency domains using the Fourier Transform Pair

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt \quad \text{(Forward Transform)}$$

and

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}d\omega \quad \text{(Inverse Transform)}$$

A periodic time function, x(t), may be analysed into its frequency components to give the well-know Fourier Series

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos \omega_0 nt + b_n \sin\omega_0 nt)$$

where $\omega_o = 2\pi/T$, and T is the period of the signal.

x(t) may alternatively be expressed in complex form

$$x(t) = \sum_{n=-\infty}^{\infty} X(\omega_n) \; e^{j\omega_o n t}$$

where $X(\omega_n)$ is the amplitude of the n^{th} term given by

$$X(\omega_n) = \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-j\omega_o n t} \, dt$$

The complex form does not contain different information, but is sometimes more convenient to manipulate. The real and imaginary parts give the cosine and sine terms or alternatively the magnitude and phase. The function is also two sided, that is it includes negative frequencies.

1.3 – Auto Correlation and Power Spectral Density

The power of a signal, rather than its amplitude, is usually of interest. In the time domain power is defined as the average squared value of the signal; that is

$$P(t) = \lim_{T \to \infty} \frac{1}{T} \int_0^T [\, x(t) \,]^2 \, dt$$

As $T \to \infty$ this expression approaches a finite value termed the expected value $E[\, x(t) \,.\, x(t) \,]$. In the case of a periodic signal it is more useful to calculate the average power over one period.

$$P(t) = \frac{1}{T} \int_0^T [\, x(t) \,]^2 \, dt$$

A related concept is that of correlation. The auto correlation function $R_{xx}(\tau)$ is defined as the square of the value of x(t), separated by a time τ More precisely we have

$$R_{xx}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T x(t)\, x(t-\tau)\, dt = E[\, x(t) \,.\, x(t-\tau) \,]$$

Since an infinite period is not pratical only an estimate of $E[x]$ can be obtained.

In the frequency domain the power, $P(\omega, \omega + \Delta\omega)$, of a signal in the range ω to $\omega + \Delta\omega$ is similarly defined as the average squared value of the signal within this range, or

$$P(\omega, \omega + \Delta\omega) = \lim_{T \to \infty} \frac{1}{T} \int_0^T [\, x(t, \omega, \omega + \Delta\omega) \,]^2 \, dt$$

The power spectral density function (PSD), $G_{xx}(\omega)$, can be defined at all such frequencies through the relation

$$G_{xx}(\omega) = \lim_{\Delta\omega \to 0} \frac{1}{\Delta\omega} P(\omega, \omega + \Delta\omega)$$

This is generally known as the one sided PSD and is the function calculated. A function symmetrical about the origin, $S_{xx}(\omega)$ is defined such that

$$G_{xx}(\omega) = 2S_{xx}(\omega) \qquad 0 \leqslant \omega \leqslant \infty$$

If $X(\omega)$ is the Fourier transform of $x(t)$, it can be shown that

$$G_{xx}(\omega) = X(\omega) \, . \, X^*(\omega) \qquad 0 \leqslant \omega \leqslant \infty$$

It is noted that the PSD function, $S_{xx}(\omega)$, and the auto-correlation function constitute a Fourier Pair, that is

$$S_{xx}(\omega) = \int_{-\infty}^{\infty} R_{xx}(\tau) \, e^{-j\omega\tau} \, d\tau$$

and

$$R_{xx}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{xx}(\omega) e^{j\omega\tau} \, d\omega$$

For the case of a sinusoidal signal of frequency ω_1, $G_{xx}(\omega)$ reduces to the delta function.

$G_{xx}(\omega) = \frac{a^2}{2} \, \delta(\omega - \omega_1)$, where a is the peak amplitude of the sinewave.

1.4 — Cross Correlation and Cross Power Spectral Density

A number of joint properties of two different signals may be inferred by combining them in correlation and power spectral density functions. The concepts are identical to those concerning Auto Correlation and Auto-PSD. The definitions of the Cross Correlation, and the Cross Power Spectral Density, (CPSD) functions, $R_{xy}(\tau)$ and $S_{xy}(\omega)$, are given by

$$R_{xy}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T x(t) \, y(t - \tau) dt.$$

$$G_{xy} = X(\omega) \, Y^*(\omega) \qquad 0 \leqslant \omega \leqslant \infty$$

Cross-Correlation and the CPSD constitute a Fourier Transform pair, exactly analogous to the auto PSD case, *i.e,*

$$S_{xy}(\omega) = \int_{-\infty}^{\infty} R_{xy}(\tau) e^{-j\omega\tau} \, d\tau$$

and

$$R_{xy}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{xy}(\omega) e^{j\omega\tau} \, d\omega$$

It is seen that the Fourier transform transforms information from the time domain to the frequency domain and vice-versa, as schematically illustrated below.

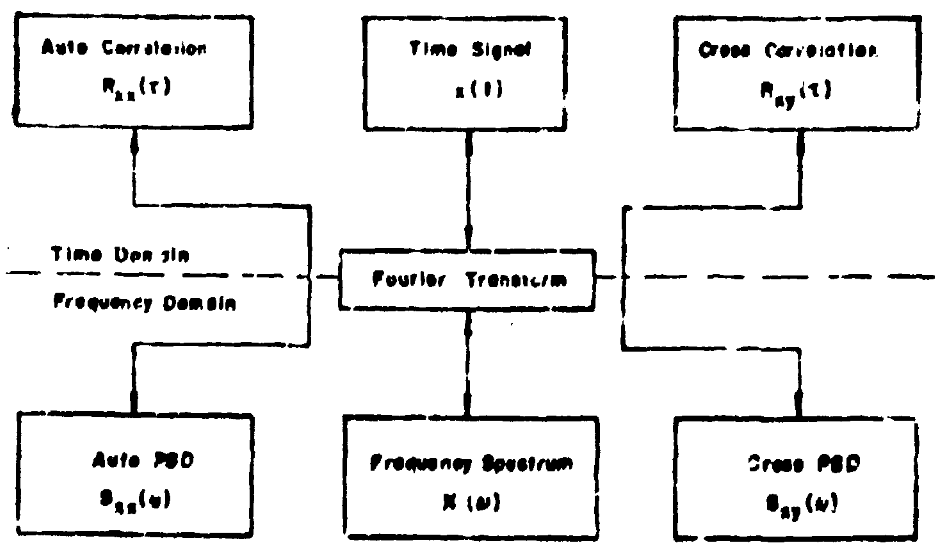


Figure 1 – Data Representation in the Time and Frequency Domains

1.5 – Convolution

If $y(t) = \int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau$, (frequently written as $y(t) = x_1(t) * x_2(t)$) then the Fourier transform of y(t) is

$$Y(\omega) = X_1(\omega) \,.\, X_2(\omega)$$

where $Y(\omega)$, $X_1(\omega)$ and $X_2(\omega)$ are the corresponding FT's of y(t), $x_1(t)$ and $x_2(t)$ Similarly the convolution of two functions in the frequency domain is equivalent to the algebric product in the time domain.

Thus if $y = x_1(t) - x_2(t)$

Then $Y(\omega) = X_1(\omega) - X_2(\omega)$

and if $Y(\omega) = X_1(\omega) - X_2(\omega)$

then $y(t) = x_1(t) \,.\, x_2(t)$

The significance of these concepts is perhaps clarified by considering a linear system, input x(t) and output y(t).

x(t) → | h(t) | → y(t)

For such a system the output is given by the convolution of the input signal with the system impulse response or weighting function, h(t). i.e

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau) dt$$

In most practical cases $x(t) = 0$ for $t < 0$ and therefore the lower limit of the integral may be equated to zero rather than minus infinity. Since the system is defined to be line: the Fourier transform of the convolution integral, y(t), is given by

$$Y(\omega) = H(\omega) \,.\, X(\omega)$$

where $Y(\omega)$, $H(\omega)$, and $X(\omega)$ are respectively the Fourier transforms of y(t), h(t) and x(t) $H(\omega)$ is termed the transfer function or frequency response of the system.

Physically the equation $Y(\omega) = H(\omega) \,.\, X(\omega)$ defines the characteristics of the system in terms of frequency transmission; the output spectrum is the input spectrum modified by the frequency response

1.6 – Windows

When calculating the FT of a time varying signal the signal is sampled for a finite time only This is equivalent to multiplying the signal by a window function which is non zero only during the measuring time. The simplest time window, the unit amplitude or box window, is defined as

$$x(t) = 1 \text{ for } 0 \leqslant t \leqslant T$$

$$= 0 \text{ elsewhere}$$

It has a Fourier transform

$$X(\omega) = T[\,\mathrm{Sin}\,(\frac{\omega t}{2})\,/\,(\frac{\omega t}{2})]$$

The calculated Fourier transform of a signal will thus be the FT of the signal convoluted with the window function, that is all frequencies in the signal are convoluted with the FT of the window function. This results in the power of each discrete frequency in the signal being distributed over a range of frequencies. This is termed frequency "leakage".

Leakage may be reduced using a time window[6] with attenuated side lobes: a common such function is the Hanning function $\frac{1}{2}(1 + \cos 2\pi t/T)$. As this is not generally zero for $|T| > T/2$, in order to obtain a window of finite sampling time the Hanning window is multiplied with the unit amplitude or box window. The figure below shows the box window, the Hanning function and the Hanning window and their respective Fourier transforms:

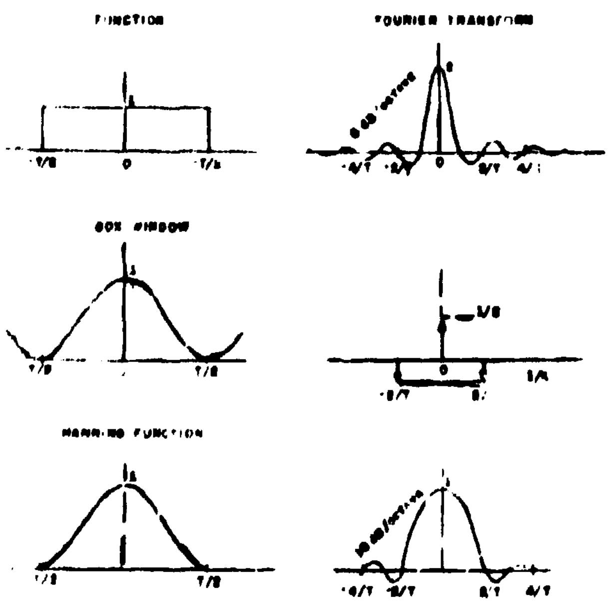


Figure 2 -- Box and Hanning windows and their Fourier transforms

The Hanning window spectrum compared to the box window spectrum is seen to have a wider flatter peak and reduced side lobes. This form reduces the uncertainty in the calculated amplititude of a given frequency but increases the frequency uncertainty, i.e. it reduces the frequency resolution.

Amplitude resolution is further improved by 'Hanning' the data two or more times. However, when Hanning or some other window function is applied, the amplitude of the power or amplitude spectra must be appropriately corrected. Thus for single Hanning the power spectrum attenuation is 6.02 dB (Multiplication factor of 2), for double Hanning the attenuation is 9.52 dB (multiplication factor of 2.67) etc.

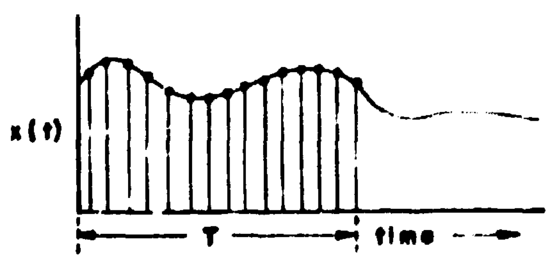
2 – DIGITAL DATA PROCESSING

2.1 – Sampling

The HP Fourier analyser, in common with all digital Fourier analysers, processes discrete blocks of digital data. Each "block of analog data is digitised by an analog to digital converter (ADC) into N discrete samples, each separated by Δt. The collecting time T (the time window) is given by

$$T = N\Delta t$$

which is equivalent to a sampling frequency $F_s = 1/\Delta t = N/T$. Schematically we have



Since data is discrete rather than continuous and is collected for a finite time, frequencies may only be calculated at discrete values between zero and a maximum value (the Nyquist frequency) given by $F_{max} = [\ N/2\ ]\Delta f$.

Now by Shannon's sampling theorem, the maximum frequency uniquely defined for a sampling time Δt is

$F_{max} = 1/2\Delta t$ (More precisely the theorem states that F_{max} is slightly less than $1/2\Delta t$)

$\therefore\ \Delta f = 1/T$

In digital Fourier analysis frequencies are calculated at discrete intervals using the Discrete Fourier Transform (DFT)

$$S_x(m\Delta f) = \Delta t \sum_{n=1}^{N-1} x(n\Delta t)e^{-2\pi i m\Delta f n\Delta t}$$

where

$$m = 0,1,2,\ldots \ldots N-1$$

As this is a series rather than continuous, it is implicit that the input function is periodic with period T. Since each frequency point is defined by a real and an imaginary (or amplitude and phase) quantity, N time samples define N/2 frequency samples.

i.e.

$$F_{max} = N/2$$

and so information is conserved.

When calculating the FT of a time varying signal in a finite window it is assumed that both the sample is periodic in the window and that the window is itself periodic. A consequence of the constant or unit amplitude of the box window is that this does not in itself introduce frequency leakage. However, for data samples which are not periodic in this window, frequency leakage results from the discontinuities in the repeated samples at the window boundary. The envelope of the leakage is the FT of the box window (or whatever window function is used).

Figure 3 below illustrates why leakage occurs for the box window; for each case the right hand diagram shows the FT of the function on the left. In (a) the analog sine signal is sampled for one complete period in the time window. Its periodic extension is consequently still a sine wave and no leakage occurs, (b). In (c) because the sine wave is not periodic in the sampling window and the extended signal is no longer a sine wave, leakage occurs, as shows in (e).

To reduce the leakage a different window is often used to make the signal more periodic in the window and reduce discontinuities. As before, the Hanning window has good characteristics. The ends of the windows are zero, and the side lobes of the envelope are considerably reduced compared with those of the box window.

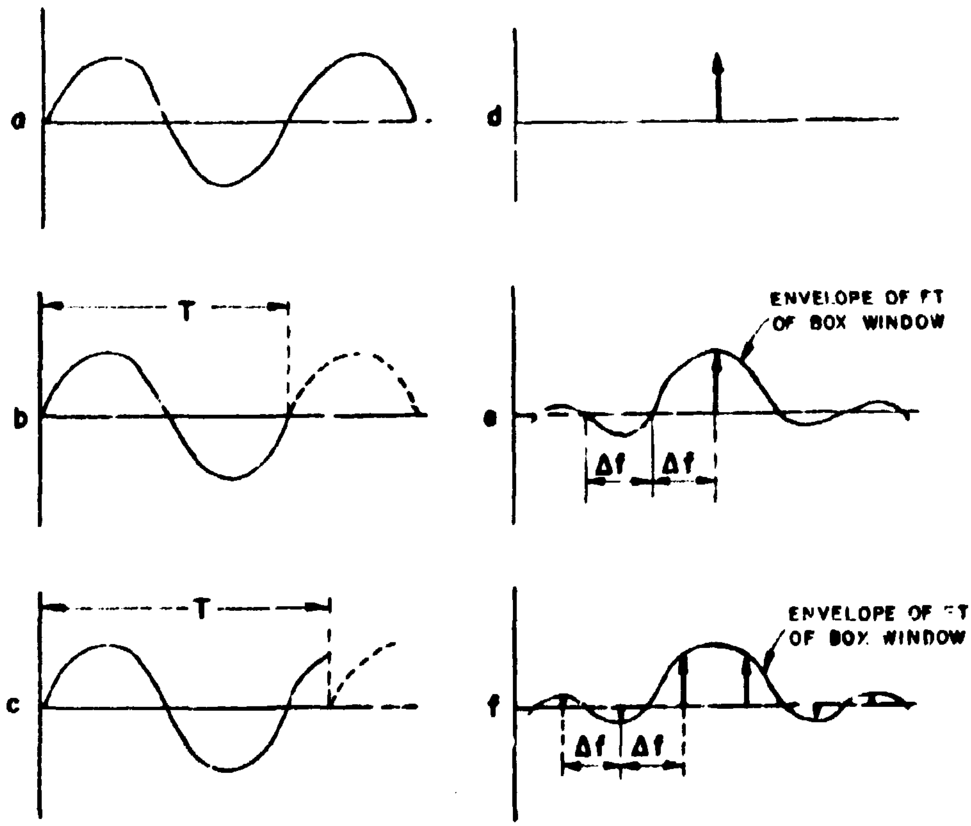
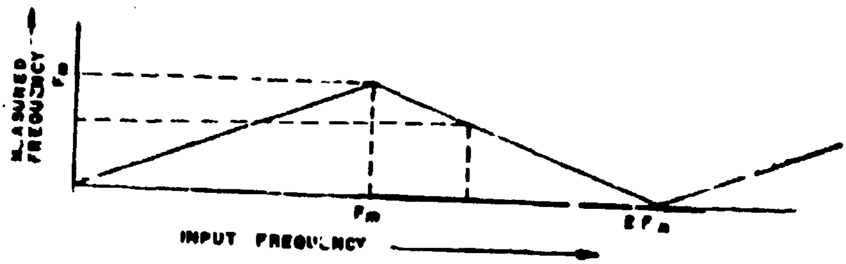


Figure 3 – Illustration of why leakage occurs for the discrete Fourier transform of a sinusoidal signal. For the FT of b) no leakage occurs, whilst for that of c) it does.

2.2 – Aliasing

If the input signal contains frequencies higher than the maximum or Nyquist frequency, F_m, then in numerical analysis these higher frequencies "fold back" and appear as lower frequencies. This aliasing is illustrated in the figure below and is a direct result of the sampling theorem; it is common to all frequency digital analysis processes.

The problem is avoided by ensuring that the signal does not contain any frequencies higher than F_m. When such frequencies are present in the "raw data", they must be removed using a low pass analog filter before digital processing. Typically the attenuation at frequencies above the cut off frequency, i.e. "the roll off", of such filters is about 40 dB/octave.



2.3 – Wrap Around Error

When convoluting two functions x(t) and y(t), we may consider that y(t) is progressively displaced with respect to x(t). For each displacement the convolution with x(t) is calculated and a succession of values of the convolution function is built up. Errors are introduced in digital processing because convolution values are computed only between the limits 0 and T rather than $-\infty$ and $+\infty$. Since the window length for the two functions is equal (i.e. equals the period T) any relative shift of the two results in y(t) overlapping the repeated values of x(t), and the convolution function is incorrectly calculated, when x(t) is not exactly cyclic in T. If x(t) were exactly cyclic in T it could be "wrapped around" upon itself to give a continuous trace. When this is not the case a "wrap around error" results.

This error is inherent in numerical calculations of functions which are non-periodic in the window T and is elliminated as follows. After equating the signal to zero over T/4 at each end of the sample window the convolution function is calculated to give good data in the two outer quarters. The meaningless data from T/4 to 3T/4, is rejected.

Although correlation and convolution are physically and mathematically different, in numerical computation the two processes may be treated as similar. Wrap around error is elliminated in the above described manner for the two processes.

3 – THE H. P. FOURIER ANALYSER

3.1 – *Introduction*

This description is intended to outline in general terms how data is entered into and analysed by the HP 5451B Fourier analyser. Figure 4 below is a schematic of the analyser and peripherals currently used at the Instituto de Energia Atômica.

The central control unit of the HP 5451B Fourier Analyser is the 2100S microprogramable systems computer. This may be used either in the "computer" or "Fourier" modes. Certain operational details of the latter are here given. Information is entered either through the appropriate external peripheral under direct computer control or from the magnetic tape store using the mass storage control system. Data may be displayed on the oscilloscope and an output obtained on punched tape, printout, magnetic tape or x-y graphical plots. It is only possible to plot information displayed on the oscilloscope. A Fourier Analyser System program controls access to the various peripherals and contains those software routines most frequently used. Additional software routines may be overlayed into the FA system tape and called by specifying the appropriate user program. Data is either directly analysed using the keyboard unit or is analysed using a keyboard program. This may be written on line, pre-punched and entered through a paper tape reader or entered from the mass storage system.

The foolowing brief description outkines data organisation in and input to the Fourier Analyser

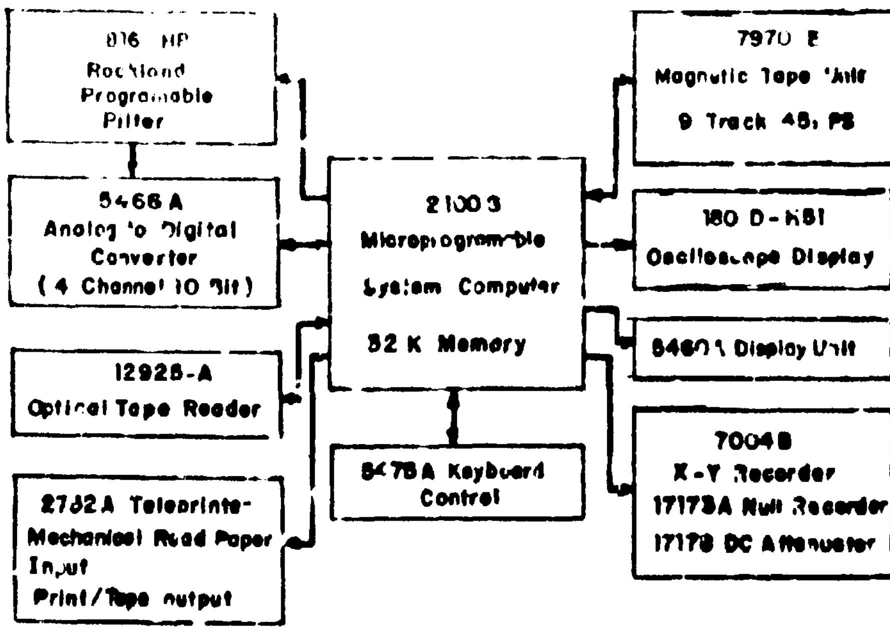


Figure 4 – Schematic of HP 5451 B Fourier Analyser System

3.2 – Data Organisation

Data words are stored as 16 bit binary numbers and represent integrals between -32768 and +32767; they cover a dynamic range of greater than 96 dB. Data is stored in data blocks in power of 2 from 2^6 to 2^{12} (64 to 4096). In the time domain data is stored in consecutive channels; in the frequency domain the real and imaginary (or amplitude and phase) values are interlaced. Thus in a given data block there are N channels in the time domain and N/2 channels in the frequency domain. When double precision is used in the frequency domain, interlacing is not used and two blocks are required to define a given data set.

Data in a specified block may represent one element in an ensemble average or it may be the result of a complex operation, such a Fourier Transform. Each data block is specified by an amplitude scale factor (a power of 10) and a block calibration, A(0,1 < A ≤ 1). Automatic adjustament of these block definers and the "data word size" ensures that amplitude overflow does not occur during calculations. A further block parameter, the frequency code, defines the sampling parameters set on the ADC.

The maximum possible block size will depend on the number of data blocks processed and the particular processing operation. Thus calculation of coherence functions requires 6 blocks, cross correlation requires 2 blocks etc. A maximum of 4 x 4096 data words, i e 4 x maximum block capacity, is available for data storage. During the execution of a given operation all blocks must be of equal size, but this may be changed between operations.

3.3 – Mass Storage System

When the FA is used in the mass store mode, data and programs are stored in and retrieved from the 7970 digital tape recorder (other mass store devices may be used). The mass store system is accessed through the keyboard control unit; execution and "book keeping" is performed by the softwave drivers. Information handling is simplified by using distinct files, in each of which a particular type of information is stored. These are as follows.

Data Block File: Store whole Fourier data blocks for later analysis. Gain core efficiency by using this file for storage of intermediate results while making a measurement (e g. running sums, reference levels) For example, a transfer and coherence function measurement requires only two core data blocks instead of the usual five. At the same time, maintain real-time processing rates.

ADC Throughput File: Input data directly from the ADC into this file. Later bring the data back into core for processing, or transcribe it directly to the data block file for later analysis The software is optimized for throughput of single or multiple inputs with little or no data loss between records.

System Coreload File: Swap complete core images instantly between computer memory and this file. Eliminate paper tape loading to change from one system program to another. Store multiple Fourier coreloads, or non-Fourier programs such as BASIC or FORTRAN systems to make the computer an effective general-purpose processor.

Program Stack File: Hold additional keyboard programs on this file, each equal in size to the entire core program stack. Link programs together in subroutine fashion, if desired, to execute a sequence of keyboard programs automatically.

Ascii Text File: Store multiple lines of alphanumeric information at 70 characters per line. Annotate experiments with labels dates, or comments. Then see the information printed on the system terminal anytime the experiment is repeated

Index Block File: Restore all the files to the positions held at some previous event in time. When files contain associated information, for example ASCII text with keyboard programs and blocks of data, re-position the files with one command instead of separate positioning commands

3.4 – Keyboard Control Unit

Computer operation in the FA mode is usually controlled through the keyboard control unit, shown in figure 5, with which single operations or complex programs involving many operations are grouped into six general categories, namely: editing, data input/output, programming, data manipulation, measurements and arithmetic. A detailed description of these would be inappropriate but suffice it to say that each key activates the corresponding command or operation. Thus F (Fourier) causes data in block 0 to be Fourier Transformed (or inverse transformed), **Polar** causes the oscilloscope display to be give in polar form etc Data may be entered into the analyser using keyboard statements.

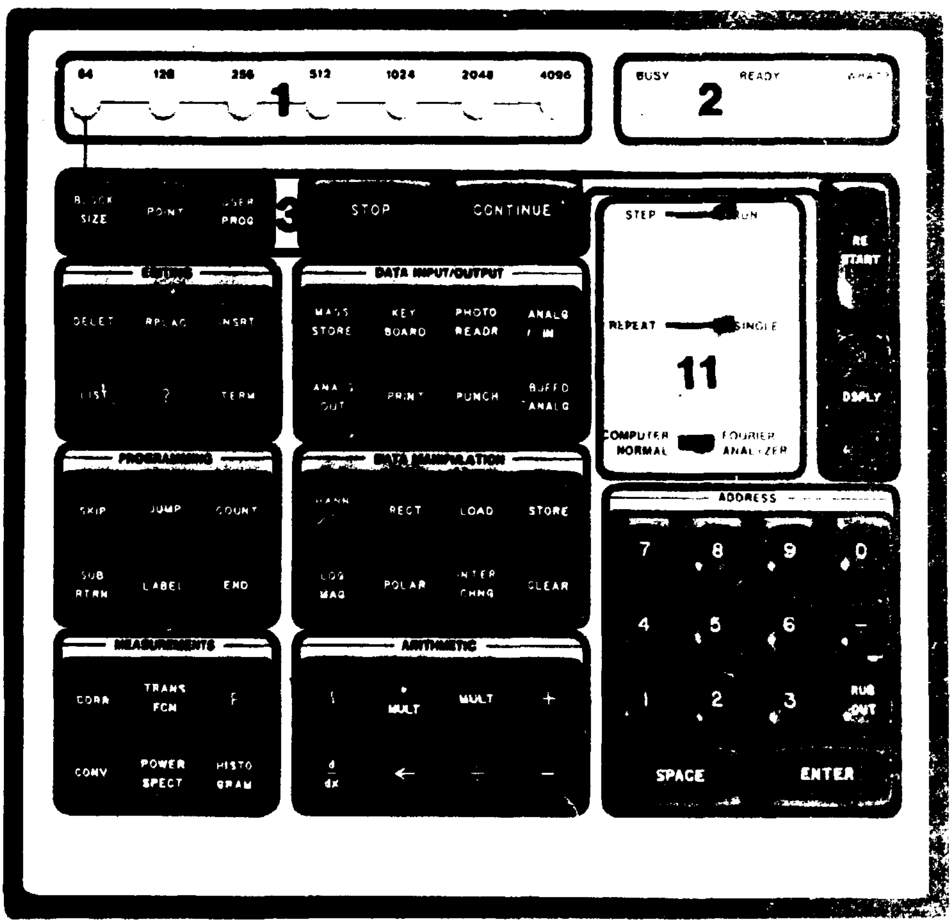


Figure 5 – HP 5451B Keyboard Unit

1 **BLOCKSIZE**—All data is stored on a block basis with an identifying number for each block. To operate on a block, simply enter the operation and the block number. One of seven indicators shows the size of the data block currently being used.

2 **STATUS LIGHTS**—One of three indicators shows system busy, ready, or illegal command given.

3 **FUNCTION KEYS**—Re-initialize the system executive routine with the restart button. Display a data block or portion of a data block with the display button. Stop or continue a keyboard program, select a data block size; set the keyboard program pointer; or execute a user program.

4 **EDITING KEYS**—Do on-line editing of a keyboard program such as delete, replace, or insert lines; list a keyboard program on the system terminal; interrogate the pointer position; terminate editing to return to the system ready mode.

5 **PROGRAMMING KEYS**—Put together on-the-spot keyboard programs; tag with identifying labels; form subroutines, tag these with identifying labels; exit via the subreturn command; form count loops, jump unconditionally or skip conditionally to various points in a program; end a physical program.

6 **MEASUREMENT KEYS**—Press one key to perform any of six complex measurement functions such as auto or cross correlation, transfer and coherence function, Fourier transform, convolution, power spectrum ensemble average (either stable or exponentially weighted), and histogram.

7 **DATA INPUT/OUTPUT KEYS**—Press a key to enter data from a mass storage device, the keyboard itself, or a photoreader. Activate the ADC for analog or buffered analog inputs. Output the data to a system terminal, punched paper tape device, or mass storage device. Output a digitally synthesized analog waveform for stimulus/response measurements.

8 **DATA MANIPULATION KEYS**—Perform a Hanning (Windowing) function; take the log magnitude of a data block; convert a data block to rectangular or polar coordinates and back again; load, store, and interchange data blocks; clear data blocks or portions of data blocks.

9 **ARITHMETIC KEYS**—Do complex arithmetic operations on a data block, or portions of a data block in some cases, such as integration, differentiation, conjugate multiplication, simple and complex multiplication, division, addition, subtraction, and block rotation.

10 **ADDRESS KEYS**—Enter number combinations as quantitative data or to identify data blocks or portions of data blocks. End commands with ENTER key or correct wrong keystrokes with RUB OUT key.

11 **FUNCTION SWITCHES**—Single step a program or let run automatically; continuously repeat the analog display selected by the ADC display switch or take a single sample; operate the computer as a stand-alone minicomputer or as a Fourier Analyzer.

3.5 – Analog to Digital Converter ADC

Analog data is entered into the FA via the ADC unit (usually preceeded by the Rockland filter to avoid aliasing). The overload level is appropriately set ($\pm 2^0 \times 0{,}125$ to $\pm 2^6 \times 0{,}125$ volts) depending on the amplitude of the input signal. Any one of the channels into which data is entered (A, AB, ABC, or ABCD) may be displayed. Check pulses of length 12[illegible]0 μS and amplitude 51 mV may be repetitively or singly entered from an internal source.

The appropriate triggering level is set for the triggering mode chosen. On line, triggering is at the line frequency. On internal (A) triggering is from a signal of minimum level ± 1 division peak to peak on the oscilloscope display. In the free run mode data is collected as fast as the Fourier analyser can accept it on receipt of an encode command from the computer. Triggering in the external trigger mode operates from any external signal of peak to peak voltage greater than 100 mV. In the AC mode the average DC levels is blocked and in the DC mode signals enter directly.

The maximum frequency which may be analysed in each ADC input channel (one to four channels being entered simultaneously) is 100 kHz. This may be increased to 150 kHz when and external clock is used. Settings of sample interval Δt, collection time T, frequency resolution Δf and maximum frequency F_{max} are interdependent and chosen according as table I.

Table I

Choise of Data Sampling Parameters of ADC Unit

Choose convenient round number for parameter shown (x)	Dependent parameter (fixed by x)	Either of remaining two parameters fixed by defining N (block size = 2^n)
Δt	$F_{max} = \frac{1}{2\Delta t}$	$T = N\Delta t$ $\Delta f = 1/N\Delta t$
F_{max}	$\Delta t = \frac{1}{2 F_{max}}$	$T = N\Delta t$ $\Delta f = 1/N\Delta t$
Δf	$T = 1/\Delta f$	$\Delta t = T/N$ $F_{max} = \Delta f\, N/a$
T	$\Delta f = \frac{1}{T}$	$\Delta t = T/N$ $F_{max} = \Delta f\, N/2$

4 – EXPERIMENTS

4.1 – Introduction

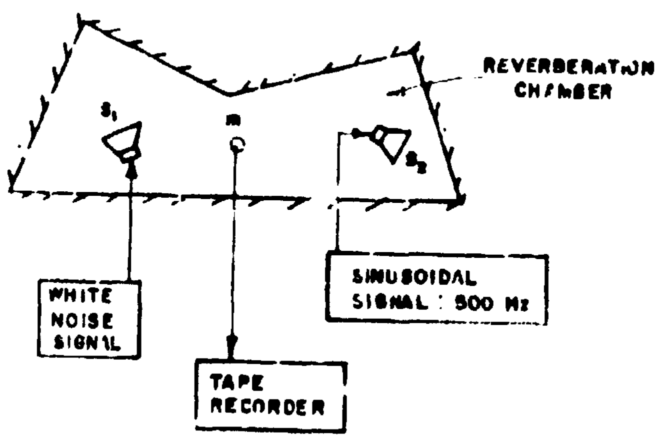
A series of experiments were undertaken to illustrate the use of the HP 5451 Fourier analyser in the analysis of time varying signals. Results of calculations of power spectral densities and correlations, the two functions most usually evaluated in such analyses, are given. Characteristic features of the shapes of these functions are discussed with reference to presented results. The same data has been analysed for various combinations of frequency and time analysis parameters (table I, pg 2) in order to illustrate how these may clarify and influence data interpretation.

Modification of signal spectra by intermediate devices is illustrated in graphs showing results of 'analysing' pure white noise 'and' white noise with superimposed resonances' In practical situations, changes in the characteristic signature spectra of a device are indicative of structure or operational changes of the device. Interpretation of such changes is an important branch of noise analysis.

4.2 – General Details of Experiments and Equipment

Electrical signals from a beat frequency oscillator (Bruel & Kjaer type 1410) producing a sine signal and a white noise generator (BK type 1402) were analysed directly. Acoustic signal were detected with microphones (BK type 4134) and recorded on a portable tape recorder (BK type 7004) for later analysis.

Acoustic signals were produced by feeding speakers with amplified signals taken from the above generators. Speakers were located in a 200 m^3 reverberation chamber so as to simulate the more difficult situations under which acoustic and other signals are usually measured. A schematic of the equipment is shown below.



4.3 – Presentation of Data

Simple programs were written to calculate power spectra and correlation functions. Wrap around error, due to the periodicity implicit in digital analysis has been eliminated in the manner described on page 11. Power spectra were obtained after Hanning the time data once.

Although graphical representation of result simplifies data visualization, precise data is obtained from numerical printouts. The vertical scale of the presented graphs has been arbitrarily chosen as it is unimportant for the present work. It should be noted, however, that when exact data is required the calculated PSD must be suitably corrected. Thus a correction must be made for window used (Hanning window: + 6.02 dB for example), Δf and the transfer functions (frequency characteristics) of transducers. Data is presented using a linear frequency scale. For many cases it is conventional to use log frequency scales as used for frequency analysis using analog methods.

Results of analysing sine and white noise signals in different ways are presented graphically and a discussion for each is presented. A summary of the calculation relevent to each graph is given in table II below Under the heading 'signal type', 'acoustical' indicates analyses are of microphone signals produced as in the schematic above. "Electrical" indicates analyses are of signals taken directly from the signal generators.

Table II

Summary of Graphical Data

Figure	Display	Display signal	Analysis parameters	Signal type
	Power Spectral Density (PSD) Auto-correlation (AC) Cross Correlation (CC) Cross Power Spectral Density (CPSD)	Sine S White W Sine + white SW	H = Δf (Hz) S = Number of Samples averaged Δt = ADC setting	Acoustical A Electrical E
6	Input signal	S, W, SW	Δt = 100 μS, 50 μS	E
7	PSD, AC	S, W, SW	H2, S50	E
8	PSD, AC	S, W, SW	H2, S50	A
9	PSD	SW	H20; 5; 2; 0,5 S50	E
10	PSD	SW	H20; 5; 2; 0,5; S50	A
11	PSD	SW	H2 S1, 5, 50, 500	E
12	PSD	SW	H5 S1, 5; 50; 500	A
13	AC	W	H S1; 5; 50; 500	E
14	CPSD, PSD	SW	H5; S50	A
15	AC, CC	SW	S50; Δt = 100 μS	A
16	PSD Using Band Selective Frequency Analysis	S	S50	E

4 — Comments on Graphs of Analysed Data

Figure 6: Digitized time signals taken directly from the generator.

a) Sinewave (500 Hz)

b) White Noise

c) Sinewave (500 Hz) plus white noise

These were the signals analysed in graphs 2, 4, 6, 8 below.

The different resolutions times $\Delta t = 50\ \mu S$ (a, b, c) and $\Delta t = 100\ \mu S$ (d) do not show different information. Results are for a single data set, rather than the average of several sets as in most of the following figures.

It would be difficult to detect the presence of the sinewave in c), by inspection of the noise signal itself.

Figure 7: Illustrations of the general shapes of Power Spectral densities and correlations.

a) PSD of white noise: Having equal energy per unit bandwidth of all frequencies the PSD is a constant. In the graph the noise signal has been bandlimited by low pass filters to avoid aliasing. The high frequency roll-off is due to the anti-aliasing filter.

b) PSD of sinewave (500 Hz): This would be a vertical line if $\Delta f \rightarrow 0$ and for a perfectly stable oscillator. The figure is more representative of pratical situations for which neither of these conditions are satisfied.

c) PSD of white noise plus a sinewave (500 Hz): This is the simple linear (voltage) sum of the two signals. It should be noted that decibels (dB) are not added linearly since they are logarithmic ratios.

Auto-correlations of the same signals are presented in d, e and f.

d) Auto-correlation of white noise: A perfectly random signal is only correlated with itself at a zero time lag; for $\tau \neq 0$ the correlation is zero. Because the analysed signal is not perfectly white and has been analysed for a finite time only, the auto-correlation function is of finite width about $\tau = 0$.

e) Auto-correlation of sine wave (500 Hz): This is periodic with the frequency of the sine wave. Since the correlation function is a measure of the dependence of data values at one time on the values at another, this periodicity of the auto-correlation of the sine signal is to be expected.

f) Auto-correlation of white noise plus sine wave (500 Hz): The clearly visible periodicity demostrates that the correlation function provides a sensitive test of the presence of periodic signals in a noisy background.

Figure 8: The same effects demonstrated in figure 2 are illustrated for acoustic signals recorded in a reverberation chamber.

PSD of a) white noise b) sine wave (500 Hz) c) sine wave (500 Hz) + white noise.

Auto-correlation of d) white noise e) sine wave (500 Hz) and f) sine wave (500 Hz) + white noise).

It will be noticed that the "white noise" is no longer white. The reverberation chamber has preferred frequency modes or resonances which selectively reinforce or cancel; these are clearly indicated in a) This is an illustration of the fact that the analysis of acoustic signals which in general are periodic may be complicated by periodic components of associated "background noise"

Figure 9: The effect of varying the frequency resolution on the calculated power spectra: Power spectra of a sinusoidal signal mixed with white noise were calculated for frequency resolutions of a) 10 Hz, b) 5 Hz, c) 2 Hz and d) 0,5 Hz.

It will be noticed that the "separation" of the sine wave from the background noise is progressively improved as the resolution is improved; i e. as Δf is decreased.

In each case 50 samples were averaged.

Figure 10: The effect of varying frequency resolution on the calculated power spectra of acoustic signals recorded in a reverberation chamber. Calculations were made for frequency resolutions of a) 20 Hz, b) 5 Hz, c) 2 Hz and d) 0,5 Hz. For each case 50 samples were averaged. High frequency roll off is due to the anti-aliasing filters

Figure 11: Effect of averaging many samples on the calculated PSD of a mixture of sine and white noise signals:

The power spectra of a sine wave (500 Hz) mixed with white noise was calculated for the average of a) 1 sample, b) 5 samples, c) 50 samples and d) 500 samples The frequency resolution, Δf, was 2 Hz in each case.

As earlier emphasised, digital analysis is carried out on data blocks of finite length. When a single frequency only is present the finite data block has no effect on information retrieval. There is no gain in information by averaging more than 1 data block For random or white noise, however, smoothness of the PSD improves as the number of blocks over which the PSD is average, is increased. For perfectly white signals the PSD amplitude is of constant amplitude and independent of frequency. This is clearly demonstrated in the figure It is also clearly shown that averaging improves the recovery of repetitive signals in a noisy background. For one sample the sine wave is not distinguishable from the background

Figure 12: Showing the effect of averaging many samples on the calculated power spectra obtained for acoustic signals recorded in the reverberation chamber. A sine wave (500 Hz) and white noise were used the mean power spectra were calculated for a) 1 sample, b) 5 samples, c) 50 samples and d) 500 samples. The frequency resolution, Δf, was 5 Hz in each case

Figure 13: This figure shows the influence of sample averaging on the calculated auto-correlation of white noise. The white noise signal was taken directly from the generator, and analysed for a) 1 sample, b) 5 samples, c) 50 samples and d) 500 samples

It will be noticed that the amplitude of the side lobes reduces as the number of samples is increased. In this case, because the noise signal was of necessity "band limited" to prevent aliasing, complete elimination of the side lobes is impossible.

For a random signal the auto-correlation function obtained for an infinitely long sample, equivalent to the average of an infinitive number of finite samples, will be a single line at the origin.

Figure 14: Cross power spectral densities (CPSD): Signal isolation (or identification) in background noise is greatly improved by taking the CPSD of the signal with a second signal which contains the

same signal information. This is demonstrated for "a noisy" sine signal detected in a reverberation chamber (microphone m1). A correlated signal was obtained from the output of a second detector microphone m2) placed immediately in front of the sine-wave emitting speaker as show in the schematic diagram on the figure.

The following were calculated.

a) CPSD of signals from microphones 1 and 2

b) PSD of signal from microphone 1

c) PSD of signal from microphone 2

For each case a frequency resolution of 5 Hz was used and 50 samples were averaged.

It is noted that the sine-wave "suggested" in the "noisy" PSD of graph a), is clearly isolated in the CPSD of graph a). This technique of increasing signal definition is much used. For more complex spectra the coherence function of two signals is used to quantify the degree of causality between them.

Figure 15: Cross and Auto-correlations: As before the effect of background noise in the test signal has been reduced by correlation with a second signal containing the same information but a lower "noise" content. It is not inherently necessary that the background signal level be reduced. Providing the background is uncorrelated in the two signals then the "signal information" will be enhanced.

As before the second signal was taken from a microphone placed close to the speaker emitting the sine-wave. The following were calculated.:

a) Cross-correlation of signals from microphones 1 and 2

b) Auto-correlation of signal from microphone 1

c) Auto-correlation of signal from microphone 2

The auto-correlation functions have a peak at the origin, as expected, but the cross correlation shows a displacement from the origin. This displacement, or lag, corresponds to the time lag between the arrival of the source signal at the two microphones. Measured time lags may be used to locate noise sources by the method of triangulation. For two dimensional location, three microphone in the same plane as the source are used. For three dimensional location four microphones must be used. When a periodic signal or signals are used, a family of source locations, rather than a particular location, are identified.

In the simple example given the separation of the two microphones m1 and m2 is evaluated from the measured time lags of the signal detected by them. The source is a 30 cm diameter speaker emitting a 500 Hz sinusoidal signal. M2 was positioned immediately adjacent to the speaker cone and is considered coincidant with it. From the printout of the graphically presented data the time lag, τ and the period T are respectively $(31.5 \pm 1) \times 10^{-4}$ s and $(41 \pm 1) \times 10^{-4}$ s. The source distance is $d = C\ (\tau \pm nT)$, where C = velocity of sound ($340\ ms^{-1}$) and n is an integer. For n = 1, d = 2.46 m. This is compared with the measured d = 2,40 m. The difference between the measured and calculated separations lies within the precision with which the source position is defined.

Figure 16: Improvement in frequency resolution is obtained using Band Selective Frequency Analysis. a) Shows the PSD of the sinusodal signal from the generator calculated over the frequency

range 0 to 500 Hz. A double peak at about the position of the first harmonic is clearly visible. In b) the frequency scale in the range 100 to 200 Hz has been expanded and from the corresponding printout the oscillater frequency is more precisely defined to be f = 143.6 Hz. In the expanded scale of 250 to 350 Hz shown in c) the double peak is clearly resolved. The second peak has a frequency of 287.3 Hz and is clearly the first harmonic of the fundamental frequency.

It is observed that the frequency band, chosen as 100 Hz in this case, may be increased or decreased as desired.

CONCLUSIONS

The aim of this report has been to illustrate the way in which the Fourier Analyser may aid in the analysis, and hence understanding, of problems involving frequency analysis of electrical signals. Although the experiments were highly simplified, it is hoped that the presented results have indicated to non-specialist the type of information which may be obtained using the HP 5451 Fourier Analyser and also some of the precautions necessary to avoid erronious analysis and interpretation of signal data.

RESUMO

Este relatório está diretamente relacionado com a análise de freqüência por HP5451B, analisador Fourier de freqüência discreta. É dada uma teoria básica importante com ênfase nos problemas associados com análise da freqüência discreta. A vantagem do uso da correlação cruzada na identificação das freqüências discretas dentro de um ruído de fundo é discutida em conjunto com a eliminação de 'aliasing' e erro de 'wrap-around'. As janelas de freqüência são ilustradas com referência à janela da unidade de amplitude e da janela tipo "Hanning".

As análises e a organização de dados pelo HP5451B são tratadas com ênfase no sistema de armazenamento de massa e na unidade de conversão analógica digital. O uso de uma unidade especial de controle com teclado facilita bastante as análises de freqüências para problemas de diversas complexidades que podem ser tratados sem a necessidade de especialistas em computação.

A identificação de freqüências discretas é ilustrada por uma série de gráficos que fornecem resultados de análises elétrica e acústica, ruído branco e sinais senoidais.

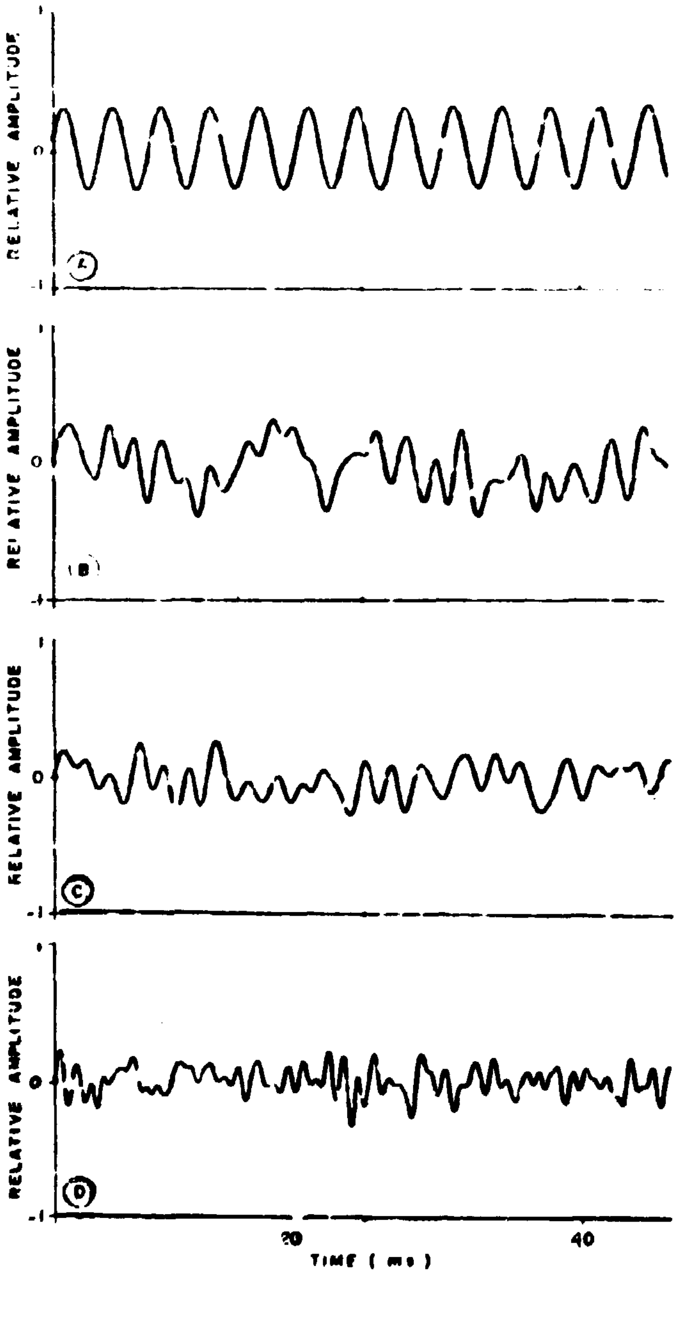


Figure 6 — Digitised time domain plots of analysed signals shown in graphs figures 7, 9 and 11. a) sine wave (500 Hz) b) White noise c) , d) sine wave (500 Hz) + white noise. Time resolution; $\Delta t = 50\ \mu s$ for a, b and c and $100\ \mu s$ for d.

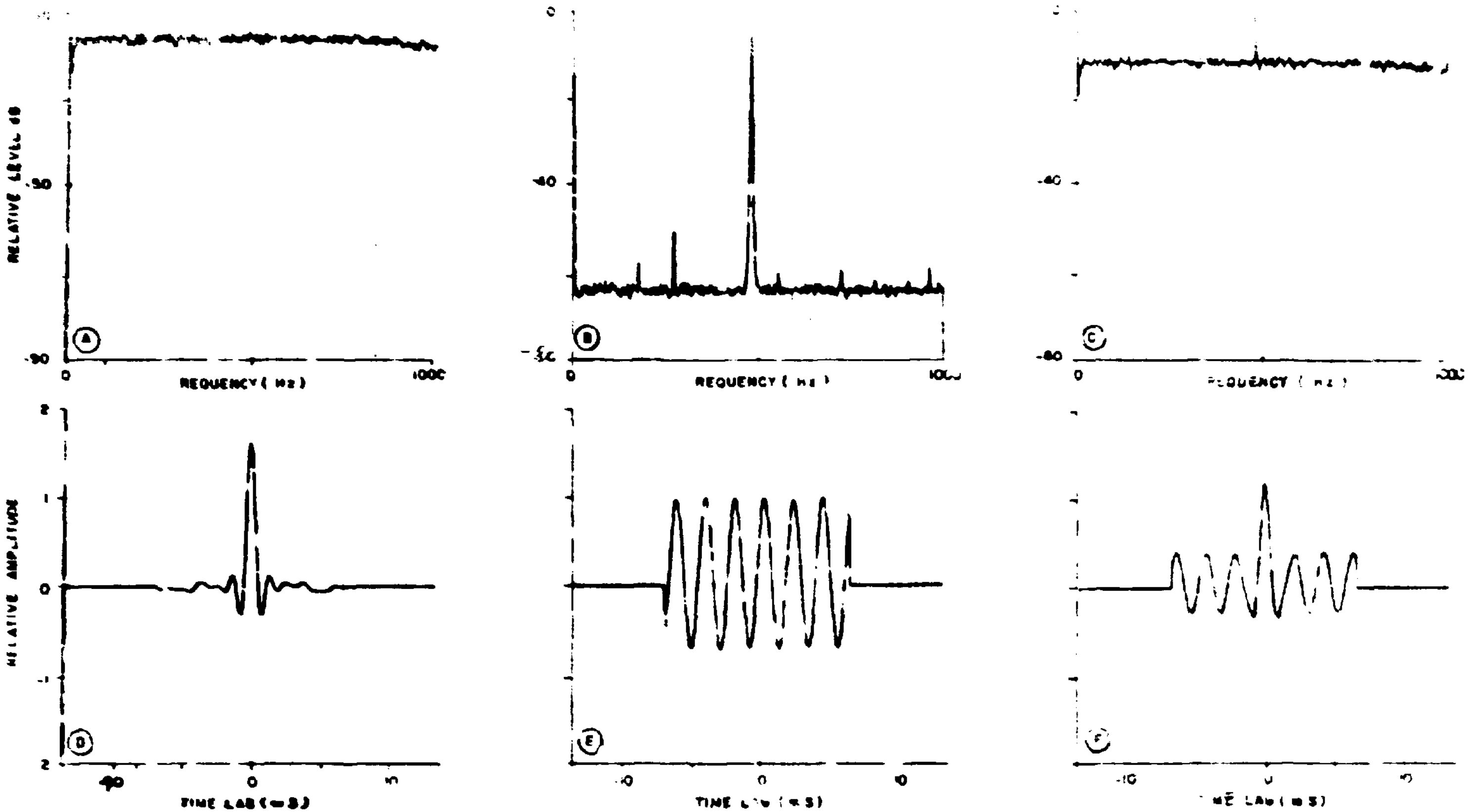


Figure 7 – Illustrative graphs showing power spectral densities (above) and auto-correlation (below) for signals from sine (500 Hz) and white noise generators. a) white noise b) sine wave c) sine wave + white noise. $\Delta f = 2$ Hz, Average of 50 samples

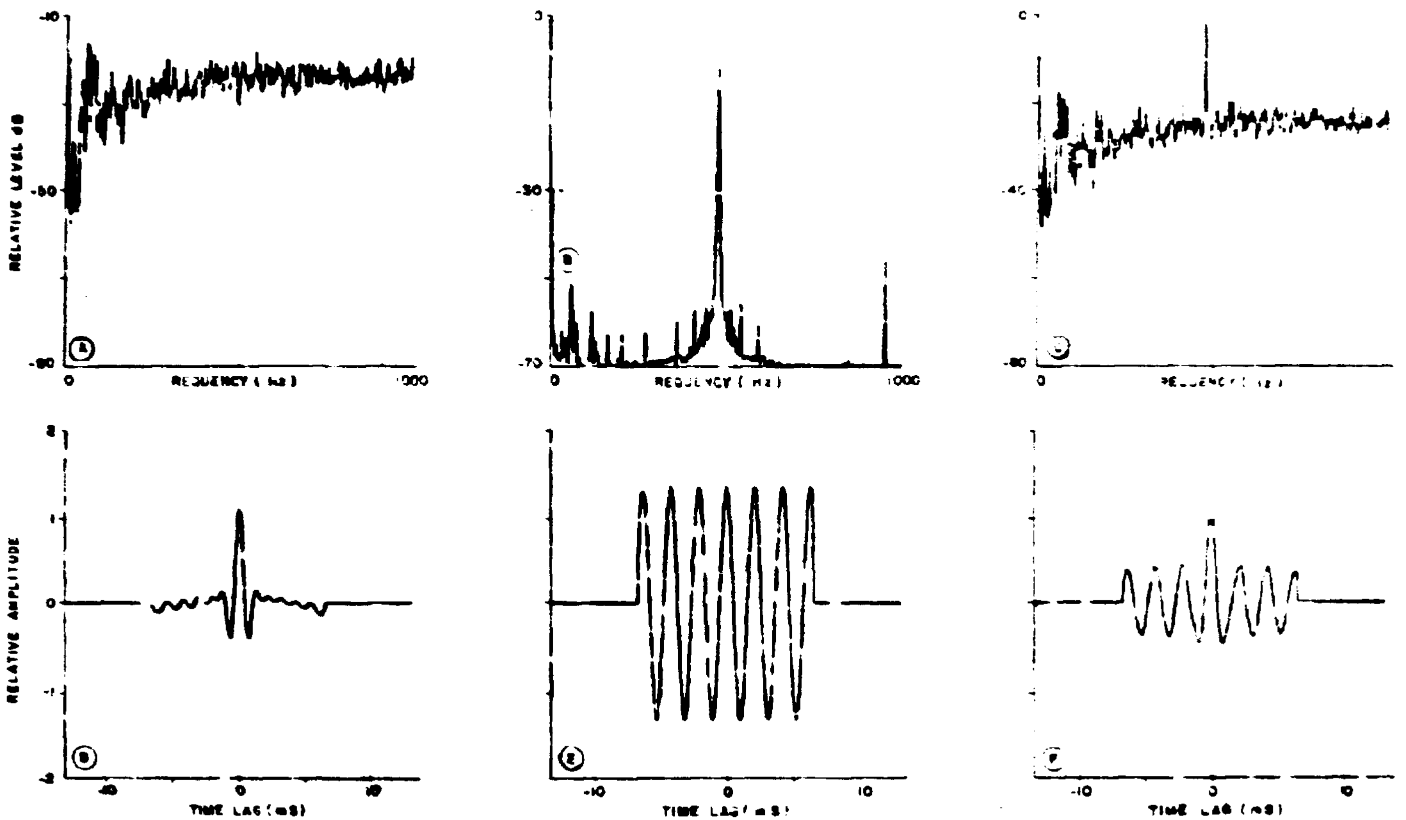


Figure 3 – Illustrative graphs showing power spectral densities (above) and auto correlations (below) for acoustic signals in a reverberation chamber for a) white noise b) sine wave (500 Hz) c) sine wave + white noise

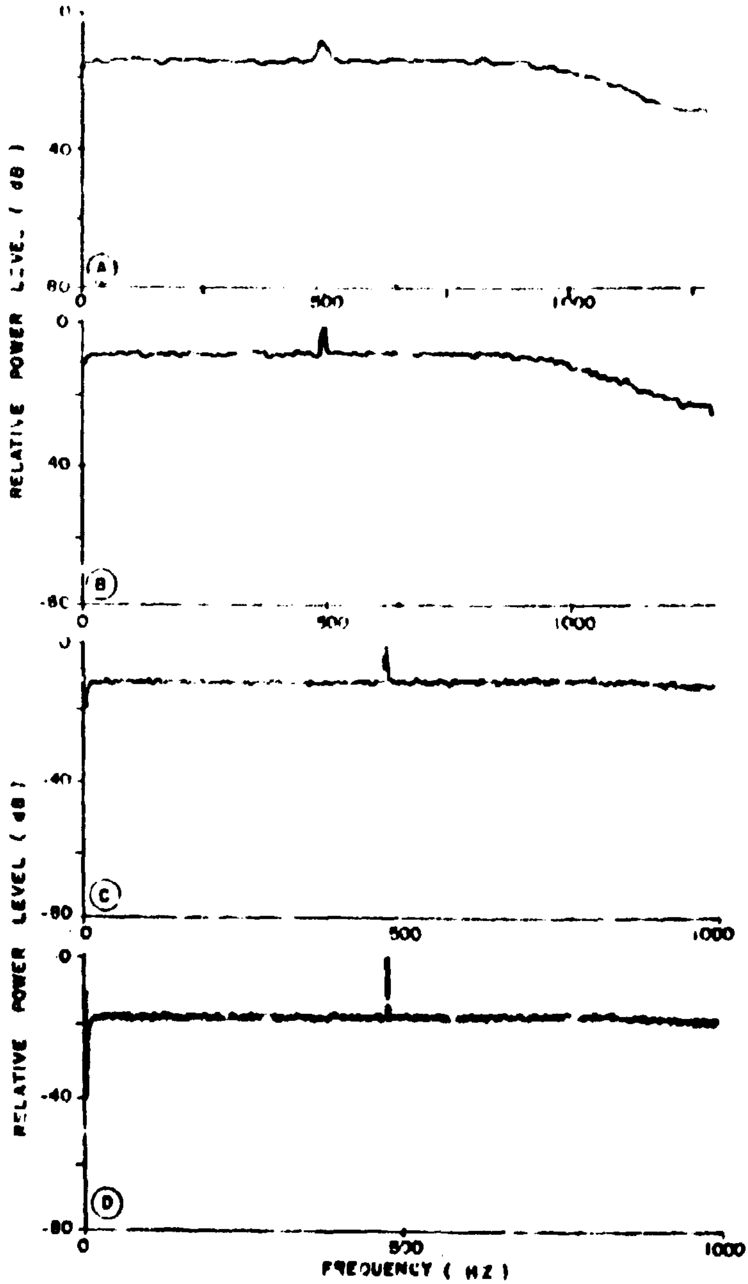


Figure 9 – Effect of varying the frequency resolution on the calculated power spectra obtained for signals from sine and white noise generators. 500 Hz sine and white noise signals of constant amplitude were analysed for a Δf of a) 10 Hz b) 5 Hz c) 2 Hz d) 0.5 Hz. For each case the average power spectrum of 50 samples was calculated.

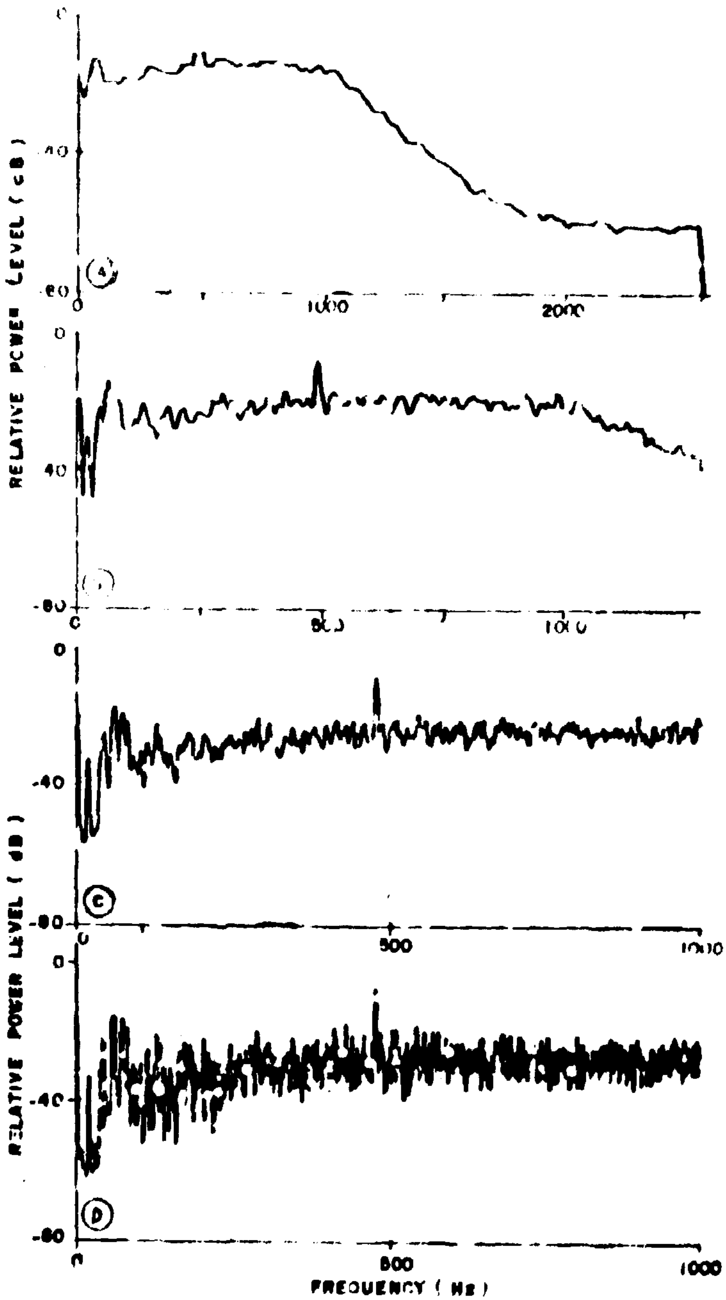


Figure 10 – Effect of varying the frequency resolution on the calculated power spectra of acoustic signals *recorded in a reverberation chamber. Sine wave (500 Hz) and white noise generators were* used. The recorded signal was analysed for a frequency resolution, Δf, of a) 20 Hz, b) 5 Hz, c) 2 Hz, d) 0.5 Hz. In each case 50 samples were averaged.

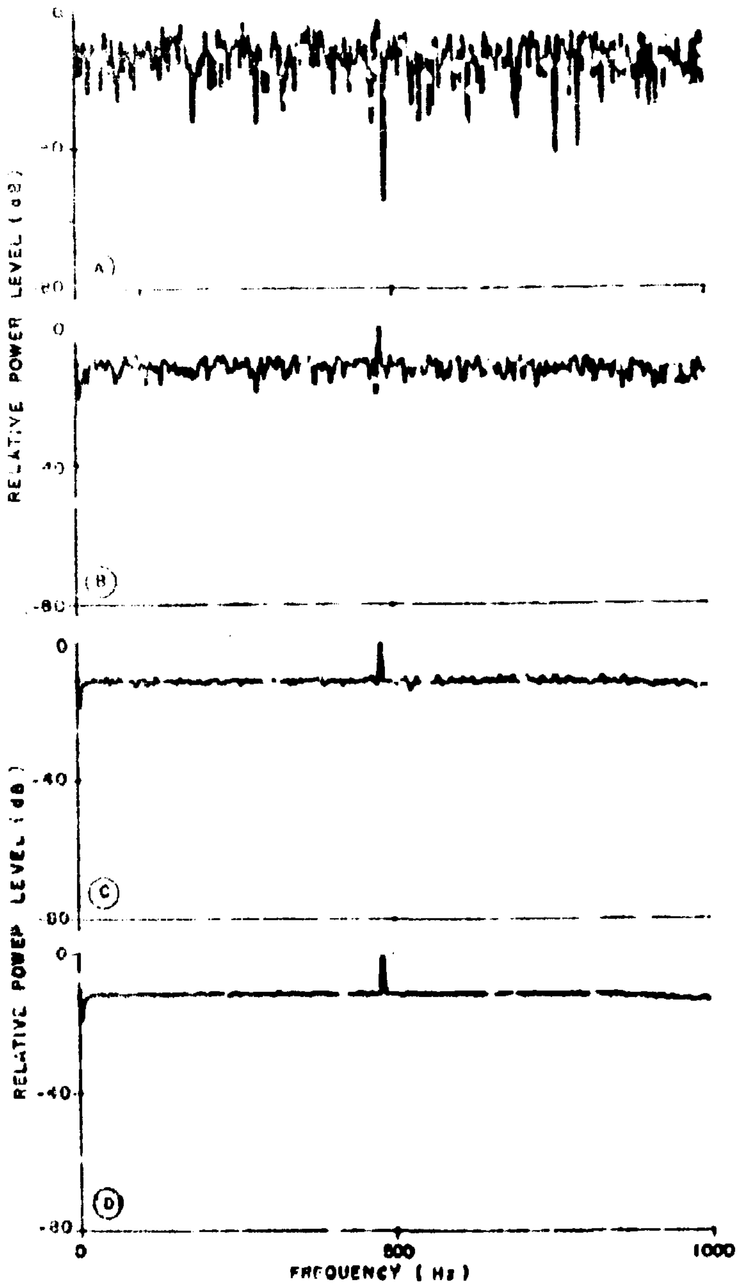


Figure 11 – Effect of averaging many samples on the calculated power spectra of signals from sine and white noise generators. 500 Hz sine and white noise signals of constant amplitude were analysed for the cases of a) 1, b) 5, c) 50 and d) 500 samples. A frequency resolution (Δf) of 2 Hz was used in each case.

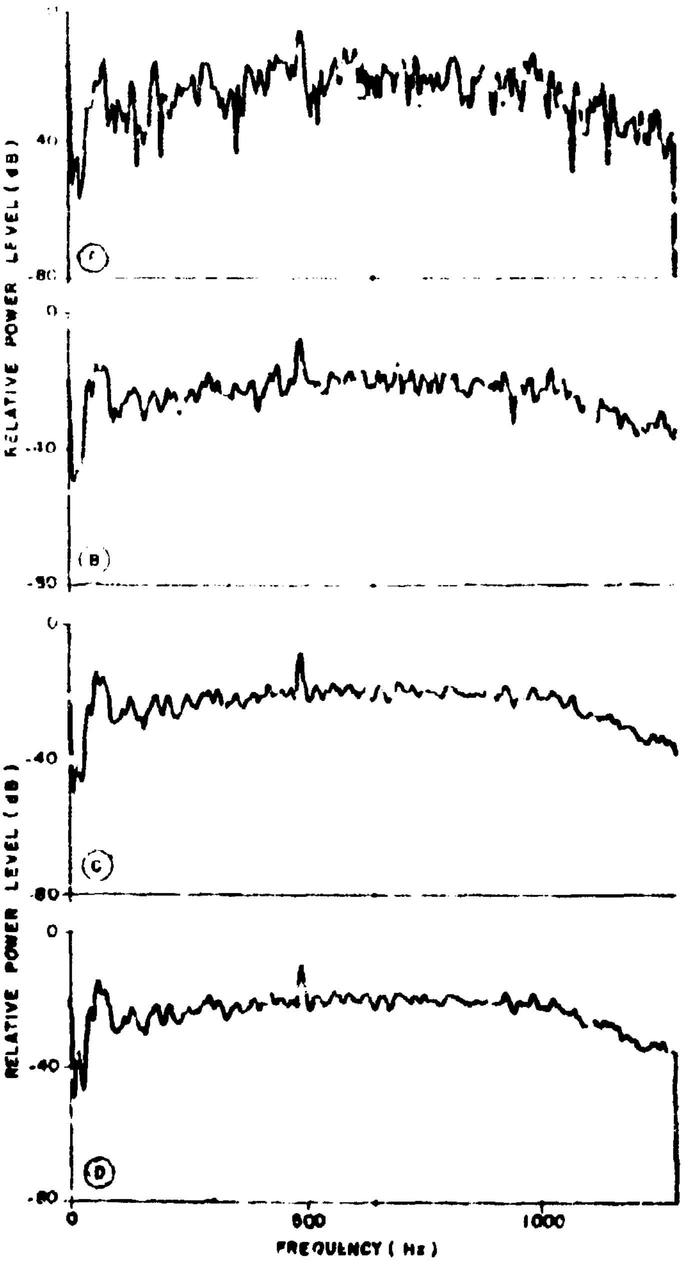


Figure 12 – Effect of averaging many samples on the calculated power spectra obtained for acoustic signals recorder in a reverberation chamber. Sine wave (500 Hz) and white noise generators were used. The recorded signal was analysed for the case of a) 1 sample b) 5 samples c) 50 samples d) 500 samples. For each case Δf = 5 Hz.

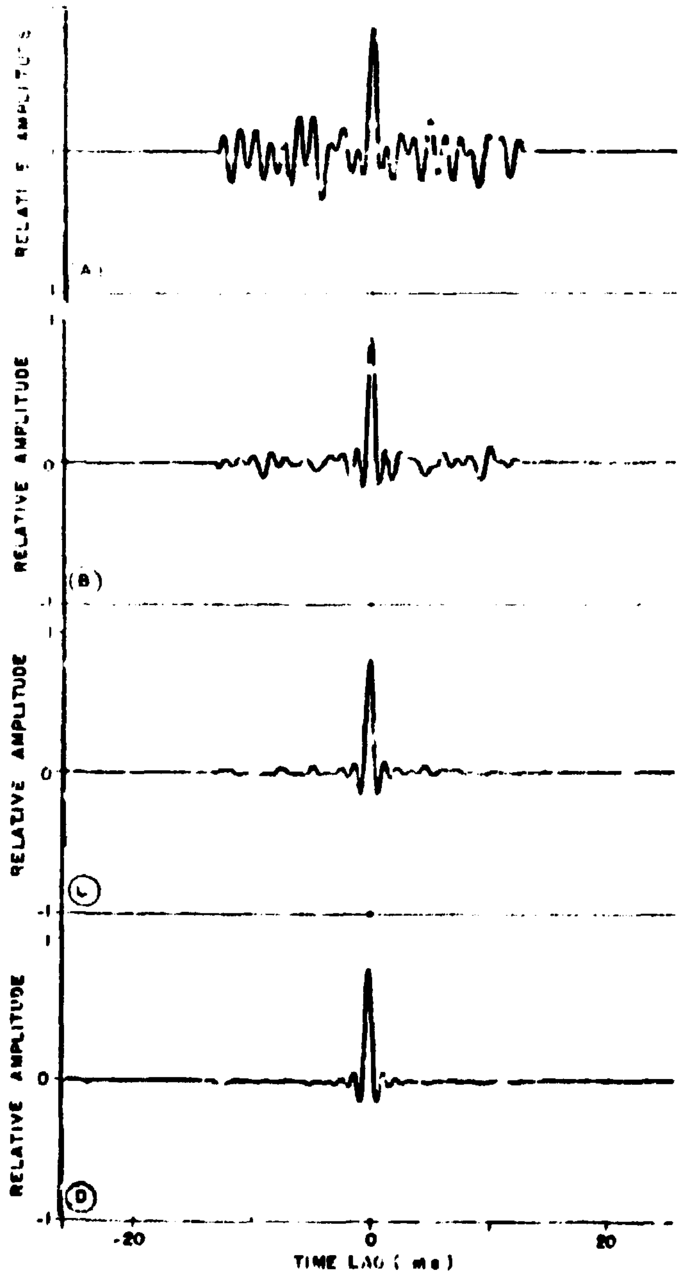


Figure 13 – Auto correlation of band limited "white noise" for one or more data samples obtained directly from a white noise generator a) 1 sample b) 5 samples c) 50 samples d) 500 samples. $\Delta t = 100\ \mu s$.

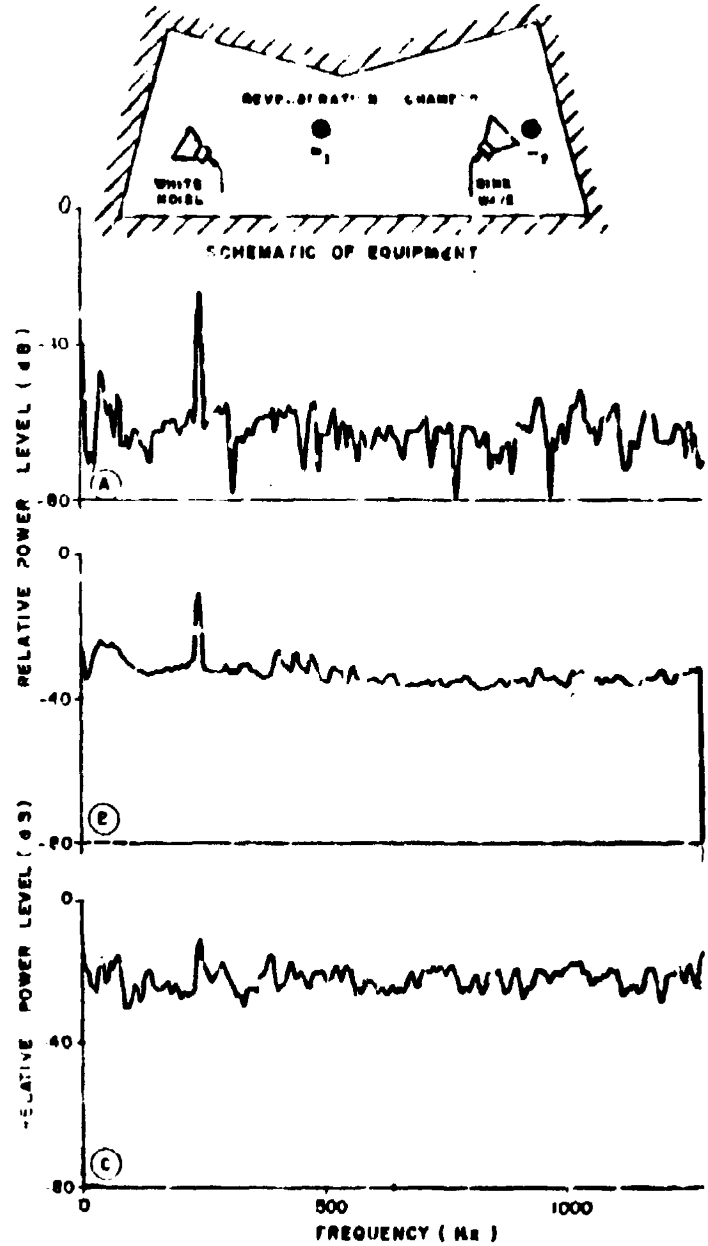


Figure 14 – The use of cross and auto power spectral densities to identify discrete frequencies in random noise signals. The equipment was arranged as shown above.

a) CPSD of the signals from microphones 1 and 2

b) PSD of the signal from microphone 1

c) PSD of the signal from microphone 2

In each case 50 samples were averaged for $\Delta f = 5$ Hz

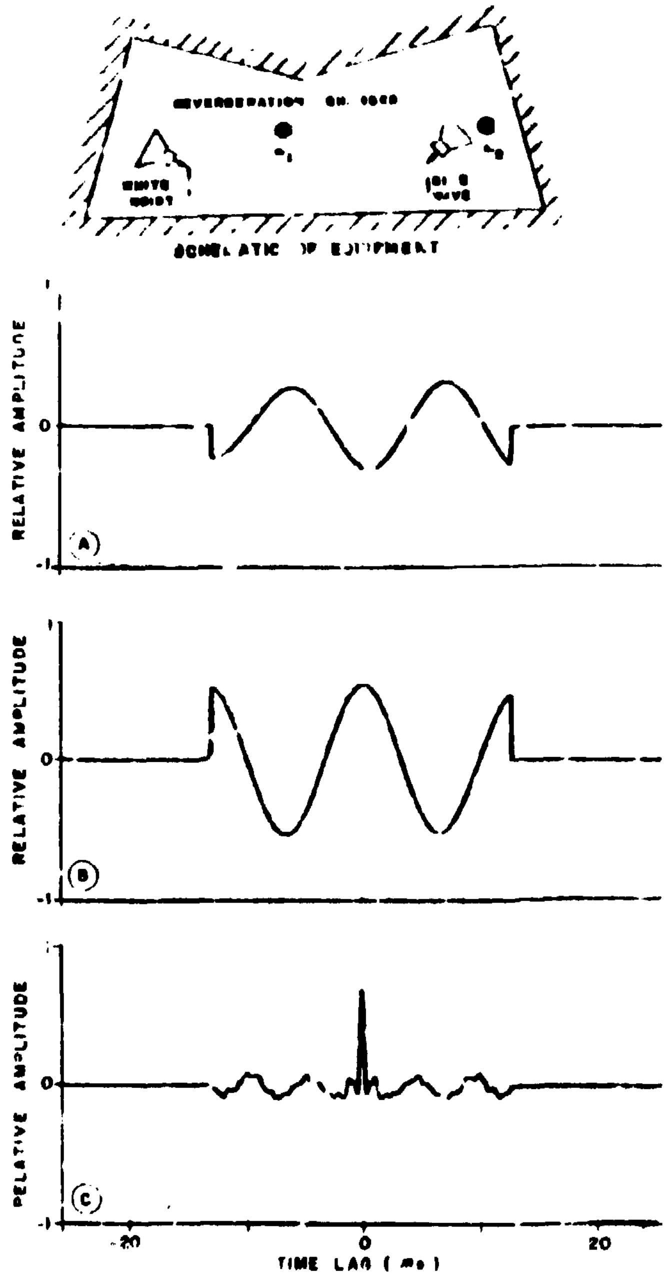


Figure 15 – Cross and Auto Correlations of "white" noise and sine wave (243.9 Hz) signals. The equipment was arranged as shown above.

a) cross correlation of the signals from microphones 1 and 2

b) auto correlation of the signal from microphone 1

c) auto correlation of the signal from microphone 2

In each case 50 samples were averaged for $\Delta t = 100 \mu s$

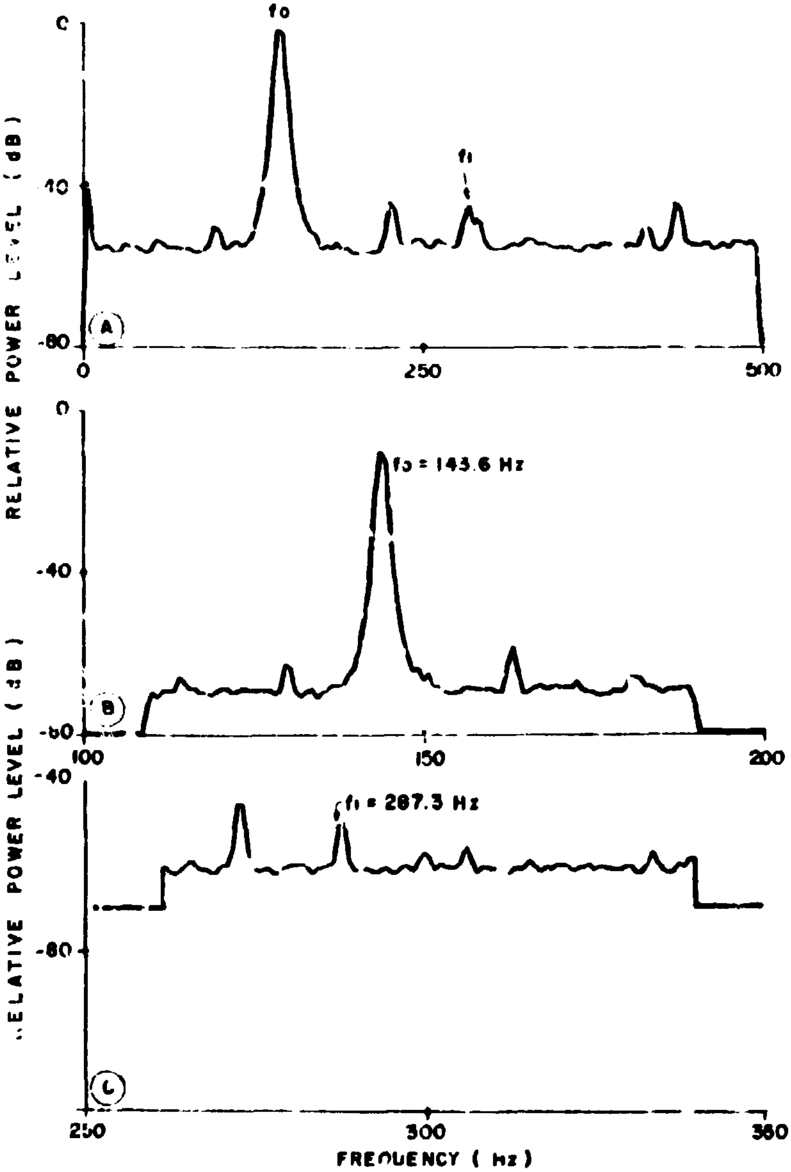


Figure 16 — Improvement of frequency res 'ution using Band Selective Frequency Analysis. In the power spectral densities shown above the first harmonic of the sine wave in a) is clearly resolved in the expanded frequency scale of c) Exact frequencies were obtained from numerical printouts. In each case 50 samples were averaged.

REFERENCES

1. BENDANT, J. S. & PERSOL, A. G. *Random data: analysis and measurement procedures.* New York, Wiley-Interscience, 1971.

2. HEWLETT PACKARD, Santa Clara, Calif. Application notes 140-1 to 140-6 [Progress reports on applications of Fourier analysis]. Santa Clara, Calif. [s.d.]

3. KERLIN, T. W. *Frequency response testing in nuclear reactors.* New York, Academic Press, 1974.

4. *ON-LOAD surveillance of nuclear power plant components by noise and vibration analysis.* Luxembourg, Commission of the European Communities, May 1974. (EUR 5036).

5. PAPOULIS, A. *Probability, Random variables and stochastic processes.* New York, McGraw-Hill, 1965.

6. POTTER, R. W. *Compilation of time windows and line shapes for Fourier analysis.* Santa Clara, Calif., Hewlett Packard [n.d.]

7. UHRIG, R. E. *Random noise techniques in nuclear reactors.* New York, Ronald Press, 1970.

8. WILLIAMS, M. M. R. *Random processes in nuclear reactors.* Oxford, Pergamon, 1974.