Zero-power noise up to 100 kHz in the IPEN/MB-01 research reactor facility

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Abstract

Subcritical noise experiments at frequencies up to 100 kHz employing two distinct boron dilution in the reactor tank water (286.8 and 578.6 ppm of natural boron) were performed in the IPEN/MB-01 reactor. The Auto Power Spectral Densities (APSD) were inferred employing the IPEN/MB-01 Correlator and were best described by a four-mode decay model up to about 70 kHz. The analyses reveal that the first two modes were related to thermal neutron and the other two to the fast ones. The coupling between thermal and fast neutrons was weak, and they could be considered uncoupled. A two-region two-group kinetic model was built and for the first time important parameters, such as the subcritical reactivities, generation times, and the prompt neutron decay constants all in the core and reflector in the 286.8 ppm case were inferred. The experiments can be considered unique of its kind and the theory/experiment comparisons reached good agreements.

1. Introduction

Subcritical measurement techniques are considered an open area in the Reactor Physics field. The techniques used to determine subcritical reactivity can be classified into three categories: 1) Static or quasi-static methods, 2) Dynamic methods, and 3) Neutron noise methods. The static or quasi-static methods rely mostly on the detector signals placed strategically around or inside of the reactor core. The reactor system is considered at steady-state and the detector responses, mainly counts, are collected for further analyses. The main assumption is that detector count rates are linked to reactivity variation between two configurations. The macroscopic noise occurs when ensembles of neutrons are detected, as in ionization chamber current or the counts of a pulse mode detector. The count rate fluctuations are transformed into spectral densities in the frequency domain to build a Power Spectral Density distribution, where the Auto Power Spectral Density (APSD) and the Cross Power Spectral Density (CPSD) can be obtained. The microscopic noise occurs when neutrons are detected individually, only one neutron per bin, and they are analyzed in the time domain. The counts acquired are distributed directly into the time channels, generating a distribution as in the Rossi-α method. Other subcritical reactivity measurement techniques can be found in (Misawa and Unesaki, 2003; Shahbunder et al., 2010).
not applicable in all levels of subcriticality, experimental facilities, or explain all kinds of anomalies found in subcritical measurements (Spriggs et al., 1997). Most of the anomalies are related to the kinetic behavior of reflected reactors. Furthermore, the utilization of calculated factors (Gajda et al., 2013) imposes severe restrictions and limitations on these techniques. The ICSBEP (International Criticality Safety Benchmark Evaluation Project) handbook presents the state of art regarding criticality safety benchmarks. Out of around 5000 configurations, only two of them refer to subcritical states (Bess et al., 2017). The remainder of them are critical configurations. The lack of benchmarked subcritical experiments reinforces the necessity to perform more experiments of this class that attend the requirements of the ICSBEP.

Around the world, there are approximately 245 (IAEA, 2013) research reactors out of which a good fraction are thermal reactors. This class of reactors has one thing in common, they are all small size cores reflected by some light elements such as light water, heavy water, Graphite or Beryllium. The physics involved in these compact thermal cores considers the dynamic behavior of both core and reflector, which can exhibit large differences between them. Numerous theories and models have been developed for reflected system (Avery et al., 1958; Cohn, 1962; Kobayashi, 1990; Nishina and Yamane, 1985; Shinkawa et al., 1978; Spriggs et al., 1997; Van Dam, 1996; Wasserman, 1960; Yamane et al., 1980). Several studies performed at the IPEN/MB-01 research reactor facility indicate that its core is more appropriately described by a two-region model as proposed by Spriggs (Spriggs et al., 1997). Several parameters such as \( \mu_\text{eff} \) (Kuramoto et al., 2008) and subcritical reactivities (Gonnelli and dos Santos, 2016) were measured employing the two-region model. The neutron lifetimes and several other kinetic constants in the core and reflector play an important role, since they will rule out the time response after some perturbation in these systems. Particularly, the neutron lifetimes are distinct in the core and reflector, usually, they are of the order of tenths of microseconds in the core and fractions of milliseconds in the reflector.

The magnitude of these constants changes a lot depending on the physics characteristics of the core and reflector as well as they exhibit neutron energy dependency. The necessity of new physics and mathematical models for the dynamic description of these thermal systems and the experimental results to serve as benchmarks for the validation of these methods are of extreme importance. In this context, the IPEN/MB-01 research reactor can help because its core has been benchmarked in a lot of experiments (Diniz and dos Santos, 2006; dos Santos et al., 2014; 2012; 2006b; dos Santos and Diniz, 2014; Kuramoto et al., 2008, 2007).

The main concern of this paper is to present a new experiment based on the utilization of the macroscopic reactor noise technique and a new kinetic model based on a two-group and two-region kinetic model to interpret the experiment and from that to infer some important physical quantities related to the subcritical reflected reactor systems. To accomplish the work, the specific purposes are threefold: a) to present new experimental results for the measurement of the APSD to frequencies up to 100 kHz, b) to describe the main steps of a developed two-region two-group kinetic model considering the interpretation of the measured data, and c) to analyze the kinetics of the thermal neutrons in the reactor system poisoned with 286.8 ppm of natural boron. Two subcritical cases with distinct boric acid concentrations in the moderator water tank were considered in this work. The experimental data can be useful to infer important quantities of the reactor physics field, such as subcritical reactivity, neutron lifetimes, generation times, and prompt neutron decay constants in the core and reflector regions. The focus is devoted to the zero-power research reactor and the utilization of the macroscopic neutron noise technique.

The applications of the reactor noise in a zero-power reactor are extensive (dos Santos et al., 2006a; Kitamura et al., 1999; Mihalcz et al., 1990; Soule et al., 1990; Suzuki, 1966). Some recent applications of the reactor noise in research reactors were performed in Japan (Sakon et al., 2014, 2013), Sweden (Szieberth et al., 2015), and India (Kumar et al., 2016).

The experiments of high frequency performed at the IPEN/MB-01 are unique of its kind and several important aspects of the dynamic behavior of such compact reflected-core systems will become evident from the analyses of the experiments (dos Santos, 2020). Particularly, the dynamics of thermal neutrons will come out explicitly from the new model developed in this work. Kinetic parameters such as prompt neutron generation time, prompt neutron decay constants, partial reactivities all in the core and reflector are reported for the first time. A two-group and two-region kinetic model and a special numerical approach were specially developed to infer these kinetic parameters. The experimental approach developed in this work does not require any sort of correction factors. The procedure developed here is purely experimental.

2. Experimental approach

The core configuration considered a short version of the IPEN/MB-01 core in a 26 × 24 rectangular array of fuel rods. For this experiment, the outer row of fuel rods was removed in each face from the standard IPEN/MB-01 configuration (28 × 26), i.e., 104 fuel rods. Thus, almost all the reactivity excess was removed from the core. The measured reactivity was equal to (10 ± 3) pcm, with the control and the safety banks completely withdrawn. A complete description of the IPEN/MB-01 core can be found elsewhere (dos Santos et al., 2014).

The subcritical states were reached by diluting boric acid (H\(_3\)BO\(_3\)) solutions in the moderator water. The boron concentrations were (286.8 ± 10) and (578.6 ± 10) ppm (parts per million or µg/g) of natural boron for the two experiments considered here. The uncertainty of 10 ppm corresponds to the accuracy of the chemical analysis equipment. The core was driven by the 2\(^{24}\)Am–Be start-up source (~1.0 Ci) of the facility placed in the reflector. In these experiments, each 1 ppm of natural boron in the water was equivalent to about 23.6 pcm of negative reactivity (dos Santos et al., 2019). This high value of specific reactivity is due to the neutronic characteristics of the IPEN/MB-01 core which has its pitch very close to the optimum pitch (maximum \( k_c \)).

Two \(^{3}\)He Centronic detectors were placed 167.6 mm away from the outermost fuel rods of the reactor and inside the detection channels. As shown in Fig. 1, the detection channel was at the same level as the upper surface of the bottom grid plate. Due to the high level of subcriticality, the two most sensitive detectors available in the IPEN/MB-01 facility were selected. The detectors had the thermal neutron sensitivity of 202 cps/σw (s/\( 8739 \)) and 297 cps/σw (s/\( 8740 \)). They had 100 cm of sensitive height and covered the entire active height given by the fuel rods, exceeding it by 42.5 cm approximately.

The detectors were symmetrically located in the reflector region to get the neutron counts for the IPEN/MB–01 Correlator. The counts were then summed to get better statistics in the measurements. The data acquisition and processing system employ the following procedure shown in Fig. 2.

The temperature in the fuel region was monitored by means of a set of 12 thermocouples distributed uniformly in the active region of the reactor core. The average temperature for the experiment with 286.8 ppm of natural boron was (19.82 ± 0.37) °C and that for the 578.6 ppm was (19.89 ± 0.09) °C.
2.1. Description of the IPEN/MB-01 Correlator

According to Fig. 2 neutron pulses from the detectors were formatted and amplified by preamplifiers and amplifiers with the shaping time set to 2 μs. Subsequently, they were discriminated from the γ-radiation through the Lower Level Discriminator of the Single-Channel Analyzer (SCA). Negative logical pulses were generated in the output of the single-channel (standard NIM fast negative) with 25 ns width and −5 V of amplitude on 50 Ω impedance. Since the subcritical level was high for the two boron diluted cases considered here, the negative logic pulses were summed (merged) in a logic OR (X = A + B) employing an Input Logic Unit. A Multichannel Scaler (MCS) board registered the time intervals between a trigger signal and the subsequent logical pulses. The dwell time chosen for the MCS board provided the maximum frequency to be analyzed, and the number of channels gave the corresponding frequency resolution. The whole system of acquisition and signal processing is called IPEN/MB-01 Correlator, and it is a program written in LabView 5.1 (Laboratory Virtual Instrument Engineering Workbench).

The data processing is written in C/C++. Fig. 3 illustrates the data processing. The MCS transforms the pulses from the detectors into a time spectrum which represents the count rates in the detector. Once the detector count rates are obtained, the level DC (average count rate) is removed from these count rates. This is like an electronic filter, however, performed by a software. Finally, the APSD is constructed consistently to that employed in current mode by the DSA (Agilent 35670A Dynamical Signal Analyzer) (Keysight, 2017) as:

\[
\text{APSD} = \sum_{i=1}^{N} \left( \text{FFT}(\text{CR}(t)) \right) \cdot \left( \text{FFT}(\text{CR}(t))^{*} \right),
\]

where \( \text{FFT} \) is the Fast Fourier Transform of \( \text{CR}(t) \) (counts), \( \text{CR}(t) \) is the count rate minus DC at time \( t \) (in units of cps), \( N \) is the number of averages, \( B \) is the bandwidth (Hz), and the symbol \( (*) \) represents the complex conjugate of \( \text{FFT}(\text{CR}(t)) \).

In this work the dwell time was set at 5 μs, the bandwidth was 4.8 Hz, the number of channels, in the time domain, was set to be 62,500 (double-sided), which resulted in a resolution of 3.2 Hz in...
the frequency domain and a frequency domain from 9.6 Hz up to
100 kHz (single-sided spectrum).

The APSD unity arising from Eq. (1) is $\text{count}^2/\text{Hz}$. Experimentally, the IPEN/MB-01 Correlator is very similar to the one developed by Kitamura although there are some differences in the CPSD magnitude and its unity. The APSD derived from the IPEN/MB-01 Correlator preserves the correct units and magnitude. It has been successfully applied in the determination of the relative power in other subcritical experiments of the IPEN/MB-01 (dos Santos et al., 2013).

The APSD measurements performed by the IPEN/MB-01 Correlator considered 3,000 averages in each acquisition. Each acquisition took about 35 min to be completed. The average count rates (cps) were, respectively, $(923 \pm 11)$ cps and $(198.2 \pm 1.1)$ cps for the 286.8 and 578.6 ppm of natural boron cases. The number of acquisitions was, respectively, 12 and 163 for the 286.8 ppm and 578.6 ppm cases. The 578.6 ppm case has more acquisitions due to its high degree of subcriticality and lower detector counts. Each natural boron case has an average APSD curve calculated by the uncertainty propagation of the mean of the acquired individual APSDs. The uncertainty of the natural boron case has an average APSD curve calculated by the uncertainties inside 3 times the coefficient $A_j$ (from lower frequencies and two roots $(\omega_iC_0)$ at higher frequencies. It will be shown forward that the first two modes of the APSD given by Eq. (2) represent the effects of thermal neutrons and the last two models those of fast neutrons. Furthermore, the similarity of the coefficients $\omega_iC_0$ and $\omega_iC_0$ for both boron diluted cases is very noticeable. The presence of the boric acid does not change appreciably the neutron spectra in the fast neutron energy range. Finally, the absolute values of the coefficients $A_j$ and $A_j$ are anticorrelated ($A_j \approx -A_j$) considering their uncertainty range inside 3$\sigma$. This condition is best satisfied for the 286.8 ppm case. This feature of the fitted data will be an important finding to identify the first two modes as the kinetic behavior of the thermal group. Particularly, the ratio of the $A_j$ coefficients of the two boron dilutions are very close to the ratio of the square of the corresponding detector counts as it was expected to be.

Figs. 4 and 5 also show the calculated reactivities in units of dollar. Here, the calculated values are from MCNP6 (Goorley et al., 2013) in conjunction with the ENDF/B-VII.0 library. The reactivity was considered relative to the case without boron in the water tank. The effective delayed neutron fraction $(\rho_{eff} = 750 \pm 5\text{pcm})$ was taken from (dos Santos et al., 2006b).

The rise shown by the APSD curves toward higher frequencies in Figs. 4 and 5 were similar to that observed in other experiments performed in the IPEN/MB-01 reactor. These experiments were performed without boron with the core configuration of the paper or the rectangular standard configuration of 28 x 26 fuel rod array. The subcritical reactivity was reached by inserting the control banks. The experiments employed many variations, as a variety of detectors, some different detector positions, and different associated instrumentation. These experiments were done to choose and test the detectors and the instrumentation to certify that everything was suitable for the boron poisoned reactor measurements. However, during the boron experiments, the dwell time and the number of the time channels for the two-side spectrum were altered and that behavior at higher frequencies was observed.

The boron experiments were followed afterward employing the best achievable electronic equipment setup. Particularly, the shap-

### Table 1

<table>
<thead>
<tr>
<th>Number of Modes</th>
<th>Maximum Frequency (Hz)</th>
<th>Reduced $\chi^2$</th>
<th>Prob.($%$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,175</td>
<td>0.9956</td>
<td>51.36</td>
</tr>
<tr>
<td>2</td>
<td>9,650</td>
<td>0.9982</td>
<td>52.39</td>
</tr>
<tr>
<td>3</td>
<td>27,000</td>
<td>0.9999</td>
<td>50.02</td>
</tr>
<tr>
<td>4</td>
<td>75,800</td>
<td>0.9998</td>
<td>50.58</td>
</tr>
</tbody>
</table>

**Fig. 4.** APSD for the case of 286.8 ppm of natural boron.
ing time (pulse processing time) was selected to be 2 μs after several tests. This was the shortest possible shaping time that formed a spectrum of neutrons according to what was expected for the $^3$He detectors (Knoll, 2000) used in the experiments. The ideal would be an almost zero shaping time. A longer shaping time has the complexity and its lengthy development, only the main steps of the development are shown here. A complete description of this kinetic model can be found in (dos Santos, 2020).

Consider that a reactor in a critical state is suddenly perturbed and the physical constants of the transport equation are changed by a specific amount, e.g., $\sigma = \sigma_0 + \Delta \sigma$, and a neutron source is suddenly inserted into the system. The temperature feedback is neglected for this transient. Consider further the traditional way to get the time behavior of the transient, as in ref. (Bell and Glasstone, 1970), and a two-region problem consisted of a core and a reflector region, as shown in Fig. 7.

Here the notation is the same as that in (Bell and Glasstone, 1970). The fundamental equations governing this transient is the neutron transport equation given by:

$$
\frac{1}{p(E)} \frac{\partial \Phi(r, \Omega, E, t)}{\partial t} + \Omega \cdot \nabla \Phi(r, \Omega, E, t) + \sigma(r, E, t) \Phi(r, \Omega, E, t) = \int \int \sigma_i \left( r, E, t \right) f_i \left( r, \Omega, E, t \right) \Phi \left( r, \Omega, E, t \right) d\Omega dE' + \int \int \chi_i(E) \left[ 1 - \beta(E) \right] \sigma_i \left( r, E, t \right) \Phi \left( r, \Omega, E, t \right) d\Omega dE' + \sum_j \int C_j(r, t) \chi_j(E) + Q(r, \Omega, E, t),
$$

and the delayed neutron precursor equation given by:

$$
10 \text{ kHz} \text{ are approximately constants. On the other hand, for frequencies higher than around 7 kHz the contributions of modes 1 and 2 go to zero. Consequently, modes 1 and 2 dominates the measured APSD at frequencies lower than 10 kHz, and conversely modes 3 and 4 dominates at higher frequencies. The frequency spectrum is, somehow, proportional to the neutron energy and the time elapsed between neutron birth and death is also implicitly linked to that. If the fission neutron does not collide with any particles on the way to the detector, then it will arrive in a very short time, resulting in a high frequency neutron. If the neutron collides near the detector and is detected, then it will also be in the high frequency spectrum. These two types of situations correspond to the fast neutrons, but of course it depends on the cross-section of $^3$He to occur an interaction that generates electrons to transform in counts. Now as the neutron diffuses through the medium, the time from birth to detection increases, forming the low frequency spectrum. These findings will give rise to the development of the two-group two-region kinetic model for the interpretation of the measured APSD.

4. The Two-Group Two-Region neutron kinetic Model

The available theoretical models (Avery et al., 1958; Cohn, 1962; Kobayashi, 1990; Nishina and Yamane, 1985; Shinkawa et al., 1978; Spriggs et al., 1997; Van Dam, 1996; Wasserman, 1960; Yamane et al., 1980) do not explain all the findings the experiment revealed. The developed model shown in this section is a tentative to explain the experiment and to infer some important properties of the physics of the IPEN/MB-01 reactor.

The experiments clearly show that there are two distinct frequency ranges. It suggests that the appropriate model to describe the high-frequency experiment performed in the IPEN/MB-01 reactor is a two-group and two-region kinetic model. Due to its complexity and its lengthy development, only the main steps of the development are shown here. A complete description of this kinetic model can be found in (dos Santos, 2020).

Consider that a reactor in a critical state is suddenly perturbed and the physical constants of the transport equation are changed by a specific amount, e.g., $\sigma = \sigma_0 + \Delta \sigma$, and a neutron source is suddenly inserted into the system. The temperature feedback is neglected for this transient. Consider further the traditional way to get the time behavior of the transient, as in ref. (Bell and Glasstone, 1970), and a two-region problem consisted of a core and a reflector region, as shown in Fig. 7.

Here the notation is the same as that in (Bell and Glasstone, 1970). The fundamental equations governing this transient is the neutron transport equation given by:

$$
\frac{1}{p(E)} \frac{\partial \Phi(r, \Omega, E, t)}{\partial t} + \Omega \cdot \nabla \Phi(r, \Omega, E, t) + \sigma(r, E, t) \Phi(r, \Omega, E, t) = \int \int \sigma_i \left( r, E, t \right) f_i \left( r, \Omega, E, t \right) \Phi \left( r, \Omega, E, t \right) d\Omega dE' + \int \int \chi_i(E) \left[ 1 - \beta(E) \right] \sigma_i \left( r, E, t \right) \Phi \left( r, \Omega, E, t \right) d\Omega dE' + \sum_j \int C_j(r, t) \chi_j(E) + Q(r, \Omega, E, t),
$$

and the delayed neutron precursor equation given by:

$$
\frac{1}{p(E)} \frac{\partial \Phi(r, \Omega, E, t)}{\partial t} + \Omega \cdot \nabla \Phi(r, \Omega, E, t) + \sigma(r, E, t) \Phi(r, \Omega, E, t) = \int \int \sigma_i \left( r, E, t \right) f_i \left( r, \Omega, E, t \right) \Phi \left( r, \Omega, E, t \right) d\Omega dE' + \int \int \chi_i(E) \left[ 1 - \beta(E) \right] \sigma_i \left( r, E, t \right) \Phi \left( r, \Omega, E, t \right) d\Omega dE' + \sum_j \int C_j(r, t) \chi_j(E) + Q(r, \Omega, E, t),
$$

and the delayed neutron precursor equation given by:

$$
\frac{1}{p(E)} \frac{\partial \Phi(r, \Omega, E, t)}{\partial t} + \Omega \cdot \nabla \Phi(r, \Omega, E, t) + \sigma(r, E, t) \Phi(r, \Omega, E, t) = \int \int \sigma_i \left( r, E, t \right) f_i \left( r, \Omega, E, t \right) \Phi \left( r, \Omega, E, t \right) d\Omega dE' + \int \int \chi_i(E) \left[ 1 - \beta(E) \right] \sigma_i \left( r, E, t \right) \Phi \left( r, \Omega, E, t \right) d\Omega dE' + \sum_j \int C_j(r, t) \chi_j(E) + Q(r, \Omega, E, t),
$$

and the delayed neutron precursor equation given by:
To derive the two-group two-regions kinetic model consider the neutron transport adjoint equation at the critical state as:

\[
-\Omega \cdot \nabla \Phi_0(r, \Omega, E) + \sigma_0(r, E) \Phi_0(r, \Omega, E) = \frac{\sum_{\nu} \sigma_{\alpha\nu}(r, \Omega) \int_{\Omega' \rightarrow \Omega} \Phi_0(r, \Omega', E) d\Omega' d\Omega}{dE} 
+ \psi(r, E) \sigma_0(r, E) \int \tilde{\chi}(E, E') \Phi_0(r, E') d\Omega' dE,
\]

(5)

where:

\[
\tilde{\chi}(E, E') = [1 - \beta(E) \tilde{\chi}_p(E')] + \sum_{j=1}^{6} \beta_j(E) \tilde{\chi}_j(E')
\]

(6)

The kinetic equations will be derived in a two-group model following the traditional way as:

a) Initially, the direct and the adjoint transport equations are written in the core and reflector regions.

b) In each one of these regions, the adjoint transport equation is multiplied by the corresponding direct neutron flux and the direct transport equation by the adjoint flux.

c) The resultant equations for each region are then subtracted.

d) The neutron flux is factored for group \( g \) and region \( x \) in an amplitude and a shape factor as:

\[
\Phi_x^g(r, \Omega, E, t) = N_x^g(t) \psi_x^g(r, \Omega, E, t),
\]

(7)

where \( \Phi_x^g(r, \Omega, E, t) \) and \( \psi_x^g(r, \Omega, E, t) \) are respectively the neutron flux, the amplitude and the shape factors for group \( g \) and region \( x \). The shape factor depends on the energy group (thermal and fast) and the reactor region (core or reflector). The IPEN/MB-01 is a zero-power reactor, thus the shape factor can be considered constant over time for the subcritical experimental considered here.

e) The set of equations is integrated over the phase space \( (r, \Omega, E) \), where \( r \) represents the neutron flux position, \( \Omega \) the neutron direction, and \( E \) the neutron energy. Here the space integral is performed in the core and reflector regions individually and the integral in the neutron energy is performed in the thermal and fast groups, also individually.

The interval of the thermal group is between 0 and \( E_T \), where \( E_T \) is the maximum thermal energy, and that of the fast group between \( E_T \) and infinite.

f) The final set of equations derived consider specifically the case of poisoning the moderator water, both in the core and reflector, due to the insertion of boric acid in the reactor tank water. This specific case of perturbation possesses the special characteristic of inducing reactivity predominantly in the thermal neutron energy range, where the boron absorption cross-section is more effective. The reactivity effect in the fast neutron energy range will be considered negligible and it will not be in the coming development.

The final set of linear coupled differential equations for the amplitude factor in each region (core and reflector) and each neutron energy group (thermal and fast) is:

\[
\frac{dN_T^j}{dt} = -\frac{\beta_T^{jF}}{\lambda_T^c} N_T^j - \frac{j_T^c}{\lambda_T^c} N_T^j + \frac{j_T^{jT}}{\lambda_T^c} N_T^j + \frac{F_T^{jF}}{\lambda_T^c} N_T^j + \frac{6}{j_T^{jT}} \lambda_T^c + S_T^j;
\]

(8)

\[
\frac{dN_F^j}{dt} = -\frac{\beta_F^{jF}}{\lambda_F^c} N_F^j - \frac{j_F^c}{\lambda_F^c} N_F^j + \frac{j_F^{jT}}{\lambda_F^c} N_F^j + \frac{F_F^{jF}}{\lambda_F^c} N_F^j + \frac{6}{j_T^{jT}} \lambda_F^c + S_F^j;
\]

(9)

\[
\frac{d\psi_T^j}{dt} = \frac{\beta_T^{jF}}{\lambda_T^c} \psi_T^j - \frac{j_T^c}{\lambda_T^c} \psi_T^j + \frac{j_T^{jT}}{\lambda_T^c} \psi_T^j + \frac{F_T^{jF}}{\lambda_T^c} \psi_T^j;
\]

(10)

\[
\frac{d\psi_F^j}{dt} = \frac{\beta_F^{jF}}{\lambda_F^c} \psi_F^j - \frac{j_F^c}{\lambda_F^c} \psi_F^j + \frac{j_F^{jT}}{\lambda_F^c} \psi_F^j + \frac{F_F^{jF}}{\lambda_F^c} \psi_F^j;
\]

(11)

\[
\frac{dN_T^j}{dt} = \frac{\beta_T^{jF}}{\lambda_T^c} N_T^j + \frac{j_T^c}{\lambda_T^c} N_T^j - \frac{j_T^{jT}}{\lambda_T^c} N_T^j + \frac{F_T^{jF}}{\lambda_T^c} N_T^j + \frac{6}{j_T^{jT}} \lambda_T^c + X_T^{jT} N_T^j - F_T^{jF} N_T^j;
\]

(12)

where the superscripts \( T \) and \( F \) refer respectively to the thermal and fast neutron energy group, the subscripts \( c \) and \( r \) refer respectively to the core and reflector region, \( c_T^j \) is the adjoint weighted precursor density for family \( j \), \( \lambda_T^c \) is the precursor decay constant for family \( j \), \( \beta_T \) is the effective delayed neutron fraction either total or in the family \( j \), \( \rho \) is the radioactivity, \( j_T^{jT} \) is the neutron current from the core to the reflector and \( j_T^{jF} \) is the neutron current in the opposite direction. The superscripts \( FF, FT, TF, \) and \( TT \) represents that a double integral is being performed inside the triple integral. \( X_T^{jT} \) represents the transfer of fast neutrons to the thermal group, \( F_T^{jF} \) represents the production of prompt fast neutrons due to thermal fissions. \( A \) is the prompt neutron generation time. The quantities involved in Eqs. (8) through (12) are defined as follow:

\[
\Lambda_T^c = \int_E \frac{1}{\nu} \int_{\Omega_T} \int_{\Omega_T' \rightarrow \Omega_T} \frac{1}{\nu} \frac{\Phi_0^c}{\psi} dV d\Omega dE;
\]

(13)

\[
c_T^j = \frac{1}{\lambda_T^c} \int_E \int_{\Omega_T} \int_{\Omega_T' \rightarrow \Omega_T} \frac{\Phi_0^c \lambda_T C_j}{\psi} dV d\Omega dE;
\]

(14)

\[
S_T^c = \frac{1}{\lambda_T^c} \int_E \int_{\Omega_T} \int_{\Omega_T' \rightarrow \Omega_T} \Phi_0^c Q dV d\Omega dE;
\]

(15)

\[
j_T^{c-r} = \frac{1}{\Omega_T} \int_E \int_{\Omega_T' \rightarrow \Omega_T} \int_{\Omega_T} \Phi_0^c \psi d\Omega dV dE;
\]

(16)
\[ \int_{r_c-r_c}^{r_c-r_c} \int_{0}^{\infty} \int_{\Omega=0}^{\Omega=0} \int_{\Omega=0}^{\Omega=0} \hat{n} \cdot \Omega \Phi_{\Omega(\psi)} d\Omega d\Omega d\Omega; \quad (17) \]

\[ X_{T}^{TT} = \frac{1}{G_{c}} \int_{0}^{r_{c}} \int_{0}^{r_{c}} \Phi_{0} \left( \int_{0}^{r_{c}} \int_{0}^{r_{c}} \sum_{s \neq f} \sigma_{s \rightarrow f} \Phi_{s \rightarrow f} d\Omega d\Omega d\Omega; \quad (18) \]

\[ F_{T}^{TT} = \frac{1}{G_{c}} \int_{0}^{r_{c}} \int_{0}^{r_{c}} \Phi_{0} \left( \int_{0}^{r_{c}} \int_{0}^{r_{c}} \sum_{s \neq f} \sigma_{s \rightarrow f} \Phi_{s \rightarrow f} d\Omega d\Omega d\Omega; \quad (19) \]

\[ \rho_{\text{diff}}^{TT} = \frac{1}{G_{c}} \int_{0}^{r_{c}} \int_{0}^{r_{c}} \Phi_{0} \left( \int_{0}^{r_{c}} \int_{0}^{r_{c}} \sum_{s \neq f} \sigma_{s \rightarrow f} \Phi_{s \rightarrow f} d\Omega d\Omega d\Omega; \quad (20) \]

\[ X_{T}^{TT} = \frac{1}{G_{c}} \int_{0}^{r_{c}} \int_{0}^{r_{c}} \Phi_{0} \left( \int_{0}^{r_{c}} \int_{0}^{r_{c}} \sum_{s \neq f} \sigma_{s \rightarrow f} \Phi_{s \rightarrow f} d\Omega d\Omega d\Omega; \quad (21) \]

\[ \Lambda_{T} = \frac{1}{G_{c}} \int_{0}^{r_{c}} \int_{0}^{r_{c}} \frac{1}{p} \Phi_{0} \psi d\Omega d\Omega d\Omega; \quad (22) \]

\[ S_{T} = \frac{1}{\Lambda_{T} G_{c}} \int_{0}^{r_{c}} \int_{0}^{r_{c}} \Phi_{0} Q d\Omega d\Omega d\Omega; \quad (23) \]

\[ \rho_{\text{diff}}^{TT} = \frac{1}{G_{c}} \int_{0}^{r_{c}} \int_{0}^{r_{c}} \Phi_{0} \left( \int_{0}^{r_{c}} \int_{0}^{r_{c}} \sum_{s \neq f} \sigma_{s \rightarrow f} \Phi_{s \rightarrow f} d\Omega d\Omega d\Omega; \quad (24) \]

\[ \rho_{\text{diff}}^{TT} = -\frac{1}{G_{c}} \int_{0}^{r_{c}} \int_{0}^{r_{c}} \int_{0}^{r_{c}} \Phi_{0} \Lambda_{T} d\Omega d\Omega d\Omega; \quad (25) \]

\[ j_{c-r}^{TT} = \frac{1}{G_{c}} \int_{0}^{r_{c}} \int_{0}^{r_{c}} \int_{0}^{r_{c}} \hat{n} \cdot \Omega \Phi_{\Omega(\psi)} d\Omega d\Omega d\Omega; \quad (26) \]

\[ j_{c-r}^{TT} = -\frac{1}{G_{c}} \int_{0}^{r_{c}} \int_{0}^{r_{c}} \int_{0}^{r_{c}} \hat{n} \cdot \Omega \Phi_{\Omega(\psi)} d\Omega d\Omega d\Omega; \quad (27) \]

\[ \Lambda_{T} = \frac{1}{G_{c}} \int_{0}^{r_{c}} \int_{0}^{r_{c}} \int_{0}^{r_{c}} \frac{1}{p} \Phi_{0} \psi d\Omega d\Omega d\Omega; \quad (28) \]

\[ \rho_{\text{diff}}^{TT} = -\frac{1}{G_{c}} \int_{0}^{r_{c}} \int_{0}^{r_{c}} \int_{0}^{r_{c}} \Phi_{0} \Lambda_{T} d\Omega d\Omega d\Omega; \quad (29) \]

\[ \Lambda_{T}^{c} = \frac{1}{G_{c}} \int_{0}^{r_{c}} \int_{0}^{r_{c}} \int_{0}^{r_{c}} \frac{1}{p} \Phi_{0} \psi d\Omega d\Omega d\Omega; \quad (30) \]

\[ G_{c}^{c} = \int_{0}^{r_{c}} \int_{0}^{r_{c}} \Phi_{0} \left( \int_{0}^{r_{c}} \int_{0}^{r_{c}} \sum_{s \neq f} \sigma_{s \rightarrow f} \Phi_{s \rightarrow f} d\Omega d\Omega d\Omega; \quad (31) \]

The single quotes (‘) in the quantities of Eqs. (13) through (31) indicates the incident neutron direction \( \Omega \) and energy \( E \) and those without single quotes indicate the quantities are relative to the outgoing neutron direction \( \Omega \) and energy \( E \).

An implicit equation that is not explicitly written in the text is:

\[ \rho_{\text{diff}}^{TT} = \rho_{\text{diff}}^{TT} + \rho_{\text{diff}}^{TT} \]

The total reactivity induced by the addition of the boric acid in the moderating water is a sum of these two contributions. The effective delayed neutron fraction was also split into a fast \((\rho_{eff}^{TT})\) and a thermal \((\rho_{e}^{TT})\) parts. The total effective delayed neutron fraction is a sum of these two parts.

4.1. Reflected core equations for the thermal group

The experiment results showed that the coupling between fast and thermal modes is weak and it is a good approximation to consider the set of Eqs. (8) through (12) uncoupled. This can be accomplished neglecting the coupling parameters \( X_{T}^{TT}, X_{T}^{TT}, \) and \( P_{T}^{TT} \) in Eqs. (8), (9), (11), and (12). Due to this extensive work to be performed both analytically and numerically only the thermal group will be analyzed and applied in the experiment of 286.6 ppm of natural boron in this work.

Considering these approximations, the set of thermal neutron amplitude equations in the core and reflector regions is given by:

\[ \frac{dN_{T}^{c}}{dt} = \frac{\rho_{T}^{c} - \rho_{T}^{R} \frac{N_{T}^{c}}{A_{c}}}{\Lambda_{c}} - \frac{j_{c-r}^{c} N_{T}^{c} - j_{c-r}^{c} N_{T}^{c} + j_{c-r}^{c} N_{T}^{c}}{A_{c}} \quad (32) \]

and

\[ \frac{dN_{T}^{R}}{dt} = \frac{\rho_{T}^{R} N_{T}^{R}}{A_{c}} + j_{c-r}^{c} N_{T}^{R} - j_{c-r}^{c} N_{T}^{R} \quad (33) \]

Following the traditional way of solving the kinetic equations, first, it is considered a Laplace transform of Eqs. (32) and (33). The result for the core region is:

\[ L_{T}^{c}(s) = \frac{N_{T}^{c}(s)}{s A_{c} + (\rho_{T}^{c} - \rho_{T}^{R} \frac{N_{T}^{c}}{A_{c}})} \quad (34) \]

where \( L_{T}^{c}(s) \) is the Laplace transform of \( N_{T}^{c}(t) \), \( N_{T}^{R} \) and \( N_{T}^{R} \) are the initial conditions for the core and reflector neutron amplitude, respectively.

The core time-dependent equation associated to the thermal amplitude factor is the inverse Laplace transform of Eq. (34) and it is given by:

\[ N_{T}^{c}(t) = \sum_{j} N_{T}^{c} \omega_{j}^{c} \exp(\omega_{j}^{c} t) \quad (35) \]

where \( j \) is either 7 or 8 to be compatible to the experimental results, \( s = \omega_{j} \) are the poles of Eq. (34), and
\[
N_i^T = \frac{N_{i0}^T (\alpha_i \Lambda^T - \frac{\beta_i}{\omega})}{\Lambda^T + \frac{\beta_i}{\omega(\omega_i - \alpha_i)}},
\]

(36)

where
\[
\alpha_i = \rho_{iT}^{\text{ref}} - \rho_{eff}^{\text{ref}} - \frac{1}{\Lambda^T},
\]

(37)
\[
\alpha_i = \rho_{iT}^{\text{ref}} - \frac{1}{\Lambda^T},
\]

(38)
\[
\beta_i = \frac{\beta_{iT}^{\text{ref}}}{\Lambda^T}.
\]

(39)

The poles of Eq. (34) can be found by setting its denominator equals to zero. This gives rise to the inhour equation for the two-group two-region thermal kinetic model as:
\[
\rho_i^{\text{ref}} = \omega_i \Lambda^T + \left( \frac{\alpha_i \Lambda^T - \rho_{eff}^{\text{ref}}}{\omega_i (\omega_i - \alpha_i)} \right) + \rho_{eff}^{\text{ref}}.
\]

(40)

Eq. (40) for a given thermal reactivity \(\rho_i^{\text{ref}}\) and a set of parameters \(\Lambda^T, \alpha_i\) and \(\beta_i\), is a quadratic equation and possesses two roots, which is designated as \(\omega_i\) and \(\omega_a\) for the consistency to the experimental thermal roots (Fig. 4).

In the experiments, the detectors were placed in the reflector and the APSD measurements were acquired in that region with the consideration of the events caused in the reactor core. Thus, the major concern here is to obtain the time-dependent equation associated with the thermal amplitude factor in the reflector region. This is obtained by substituting Eq. (35) into Eq. (33) and solving the resulting equation. The final solution is written as:
\[
N_i(t) = \left[ N_{i0}^T - \frac{j_i}{\Lambda_i^T} \sum_j \frac{N_j^T}{\omega_j (\omega_j - \alpha_i)} \right] e^{\omega_i t} + \frac{\beta_i}{\Lambda_i^T} \sum_j \frac{N_j^T}{\omega_j (\omega_j - \alpha_i)} e^{\omega_a t},
\]

(41)

The APSD in the reflector region is the Fourier transform of Eq. (41) and it is found as:
\[
N_i^T(\omega) = \frac{-2 \pi \left[ N_{i0}^T - \frac{j_i}{\Lambda_i^T} \sum_j \frac{N_j^T}{\omega_j (\omega_j - \alpha_i)} \right] \right]}{\omega_i^2 + \omega^2} + \sum_j \frac{A_j}{\omega_j^2 + \omega^2},
\]

(42)

where
\[
A_j = -2 \pi \frac{\beta_i}{\Lambda_i^T} \frac{N_j^T}{\omega_j (\omega_j - \alpha_i)}.
\]

(43)

It can be demonstrated that \(N_i^T(\omega)\) possesses the following property:
\[
-2 \pi \left[ N_{i0}^T - \frac{j_i}{\Lambda_i^T} \sum_j \frac{N_j^T}{\omega_j (\omega_j - \alpha_i)} \right] = 0
\]

(44)

and consequently,
\[
N_i^T(\omega) = \sum_{j=1}^{n} \frac{A_j}{\omega_j^2 + \omega^2}.
\]

(45)

### 4.2. Additional equations arising from the thermal neutron kinetic model

The purpose of this subsection is to show a new form of some equations from the thermal neutron kinetic model. Whenever is possible, the equations will be written as a function of the basic parameters \(\alpha_i, \omega_i, \beta_i, \alpha_i^T, \rho_{iT}^T, \rho_{ref}^T, j_{ref}^T, j_{ref}^T\), and \(j_{ref}^T\), as arising from the two-group two-region kinetic model derived previously. These equations are properties of the two-region two-group model and they will be used in the numerical approach developed in this work to infer the basic thermal parameters which are of interest for the reactor physics area.

The Inhour equation given by Eq. (40) can be written in a more convenient form employing Eqs. (37) to (39) as:
\[
\rho_i^{\text{ref}} = \omega_i \Lambda_i^T + \frac{j_i}{\Lambda_i^T} \left( \frac{\beta_i}{\omega_i - \alpha_i} \right).
\]

(46)

A similar equation for the thermal reflector reactivity \(\rho_i^{\text{ref}}\) can be found employing Eq. (40) as:
\[
\rho_i^{\text{ref}} = \omega_i \Lambda_i^T + j_i \left( \frac{\alpha_i}{\Lambda_i^T} \right) \left( \frac{\beta_i}{\omega_i - \alpha_i} \right).
\]

(47)

Eq. (40) gives raise to another equation for thermal reflector reactivity given by
\[
\rho_i^{\text{ref}} = \omega_i \Lambda_i^T + \frac{j_i}{\Lambda_i^T} + \frac{1}{\Lambda_i^T} \left( \frac{\beta_i}{\omega_i - \alpha_i} \right).
\]

(48)

The addition of boric acid into the reactor tank water inserted a negligible reactivity in the fast group, so the total reactivity is basically the thermal one; \(\rho \approx \rho_i^{\text{ref}}\). Accordingly, the total reactivity \(\rho\) is defined as:
\[
\rho = \rho_i^{\text{ref}} + \rho_i^{\text{ref}}.
\]

(49)

Consequently, \(\rho\) can be found from the sum of Eqs. (46) and (47) as
\[
\rho = \omega_i \Lambda_i^T + \frac{j_i}{\Lambda_i^T} + \frac{1}{\Lambda_i^T} \left( \frac{\beta_i}{\omega_i - \alpha_i} \right).
\]

(50)

where \(\Lambda_i^T\) is the prompt neutron generation time in the thermal group and it is given by:
\[
\Lambda_i^T = \Lambda_i^T + \Lambda_i^T.
\]

(51)

Very useful equation for \(\Lambda_i^T\) can be obtained by making \(\rho_i^{\text{ref}}(\omega) = \rho_i^{\text{ref}}(\omega_0)\) in Eq. (46):
\[
\Lambda_i^T = \frac{1}{(\omega_i - \alpha_i)}(\omega_i - \alpha_i).
\]

(52)

Eq. (46) can be rewritten in a more convenient form by employing Eq. (37) as:
\[
\omega_i^2 - (\alpha_i + \omega_0) \omega_i + \alpha_i \omega_i - \frac{\beta_i}{\Lambda_i^T} = 0,
\]

(53)

which possesses the following properties:
\[
\omega_0 + \omega_0 = \alpha_i + \alpha_i
\]

(54)

and
\[
\omega_0 \omega_0 = \alpha_i + \alpha_i.
\]

(55)

The roots of Eq. (53) can be found employing the Bhaskara (Stillwell, 2004) formula as:
\[
\omega_i = \frac{\alpha_i + \alpha_i}{2} \pm \sqrt{\left(\frac{\alpha_i + \alpha_i}{2}\right)^2 - \alpha_i \alpha_i + \frac{\beta_i}{\Lambda_i^T}}.
\]

(56)

where the positive signal corresponds to \(\omega_0\) and the negative to \(\omega_0\).

Another useful equation can be found by combining Eqs. (37), (38) and (40). In this case the final equation is:
where the first term in the right-hand side is the prompt neutron generation time ($\tau = \frac{1}{\lambda e f f}$).

In a thermal reactor such as IPEN/MB-01, the fast neutrons are promptly thermalized; therefore the neutron lifetime is essentially the thermal neutron lifetime ($\tau = \tau^t$) (Lamarsh, 1966) and the thermal neutrons lifetime can be written as:

$$\tau^t = \lambda^t e f f,$$

where the effective multiplication factor $e f f$ is given by:

$$e f f = \frac{1}{1 - \rho},$$

and $\rho$ is given by Eq. (49). Eq. (67) shows that for a subcritical system the thermal lifetime must be lower than the thermal neutron generation time.

Another way to get the lifetime is considering the amplitude of the neutron flux as a function of time which is given by:

$$N^t = N^t_0 A^t e^{(\kappa^t - 1)\tau^t},$$

where $N^t_0$ represents the initial condition for the thermal neutron flux amplitude (Lamarsh, 1966). Because the root $\omega_k$ is much lower than the root $\omega_f$, as the time increases, the exponential function for $\omega_k$ decay more rapidly than that for $\omega_f$. Consequently, the amplitude of the thermal neutron flux can be written approximately as:

$$N^t \approx N^t_0 A^t e^{\kappa^t \tau^t}.$$ (Equation 69)

The amplitude of the thermal neutron flux is also given by (X-5 Monte Carlo Team, 2008):

$$N^t = N^t_0 A^t e^{(\kappa^t - 1)\tau^t}.$$ (Equation 70)

Making Eq. (70) equal to Eq. (71), the thermal neutron lifetime can be written as:

$$\tau^t = \frac{k^t - 1}{c o f f}.$$ (Equation 72)

5. Inference of the basic parameters of the Two-group Two-Region kinetic model

The quantities measured in the experimental approach are the four roots $\omega_f$ and the four coefficients $A_i$. The thermal neutron energy region model developed in Section 4 considers only part of these measured data. Namely, they are $\omega_f$, $\omega_k$, $A_f$, and $A_k$. The unknowns in the thermal neutron energy region are $c o f f$, $\beta$, $\gamma$, $\mu$, $\mu'$, $\lambda^t$, $\rho$, $\rho_f$, $\rho_k$, and $f_c$. The number of unknowns is higher than the number of equations which makes impossible an analytic solution. The least-squares technique also showed a severe degeneracy among the variables making this technique useless for solving the thermal constant unknowns. Consequently, this work developed a special approach to cope with this task.

The approach of determining the quantities of the experiment with 286.8 ppm of natural boron was divided into two parts using numerical calculations with iterative algorithms. Initially, the determination of $c o f f$, $\beta$, $\gamma$, $\mu$, $\mu'$, $\lambda^t$, $\rho$, $\rho_f$, $\rho_k$, and $f_c$ was carried out. The only hypothesis made was to assume that the thermal effective delayed neutron fraction ($f_{c o f f}$) was independent of the subcritical reactivity level. This parameter was estimated to be 90% of the total $f_c$, i.e. the sum of the thermal and fast groups. The value of $\rho_{c o f f}$, (750 ± 5) pcm, utilized in this work was taken from a previous benchmark available at the NEA IRPhE handbook (dos Santos et al., 2006b). This benchmark describes the evaluation of...
of the IPEN/MB-01 reactor in the $28 \times 26$ fuel rod array configuration without boron in the moderator water.

5.1. The inference of $x_c$, $x_r$, $\beta_r$, and $\lambda_c^T$

Fig. 8 shows the procedure developed in this work for the determination of $x_c$, $x_r$, $\beta_r$, and $\lambda_c^T$. Basically, the procedure is a numerical approach and starts by attributing tentative ranges for the variables $x_c$, $x_r$, $\beta_r$, and $\lambda_c^T$. As shown in Fig. 8, inside of each specific range the values of the variables $x_c$, $x_r$, $\beta_r$, and $\lambda_c^T$ were varied by loops in a stepwise way from the inferior to superior limits of the ranges. $x_c$ was the outermost loop while $\beta_r$ was the innermost one. There are two sorts of unknowns in the proposed procedure: the specific range for each one of the variables and the variables themselves. The values attributed to $x_c$, $x_r$, $\beta_r$, and $\lambda_c^T$ in every step of this procedure were tested if they simultaneously satisfy several criteria inside of a specific precision. If so, the set of $x_c$, $x_r$, $\beta_r$, and $\lambda_c^T$ values were considered acceptable values and were written in the output file, otherwise, it was neglected. This procedure was iterated until reach the superior limit of the outermost loop given by $x_c$. At this point, the set of acceptable values for $x_c$, $x_r$, $\beta_r$, and $\lambda_c^T$ was tested if they obey a normal distribution. If not the range for the variables $x_c$, $x_r$, $\beta_r$, and $\lambda_c^T$ were narrower and the criteria precision were increased. This procedure was repeated several times until the final acceptable criterion precision attained a desirable solution in an almost normal distribution shape.

The criteria and the precision utilized for the variables in the last phase of the algorithm are shown as follow:

Criterion (1). Consider Eq. (46) and make $\rho_c^T(\omega_r) = \rho_c^T(\omega_b)$. This criterion yielded

$$\omega_r \lambda_c^T - \frac{\beta_r}{\omega_r - x_c} = \omega_b \lambda_c^T - \frac{\beta_r}{\omega_b - x_c},$$

(73)

The acceptability criterion was

$$\frac{|\rho_c^T(\omega_r) - \rho_c^T(\omega_b)|}{\rho_c^T(\omega_b)} < 0.02,$$

(74)

where 0.02 is the relative error,

$$\rho_c^T(\omega_r) = \omega_r \lambda_c^T - \frac{\beta_r}{\omega_r - x_c},$$

(75)

and

$$\rho_c^T(\omega_b) = \omega_b \lambda_c^T - \frac{\beta_r}{\omega_b - x_c}.$$  

(76)

Criterion (2) The calculated anticorrelated condition $A_{\text{calc}}$ as a function the variables $x_c$, $x_r$, $\beta_r$, and $\lambda_c^T$ was given by Eq. (62). The experimental anticorrelated condition was

![Flowchart](image.png)  

Fig. 8. Flowchart to calculate the distributions for $x_c$, $x_r$, $\beta_r$, and $\lambda_c^T$.  

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The goodness of fit and the final values of $x_c, x_r, \beta_r, \text{ and } \Lambda^T_r$ are shown in Table 2. The goodness of fit was calculated using the reduced chi-squared statistic, $\chi^2_{\text{red}}$, which is defined as

$$
\chi^2_{\text{red}} = \frac{\chi^2}{\nu - 1}
$$

where $\chi^2$ is the chi-squared statistic and $\nu$ is the number of degrees of freedom. The goodness of fit is considered good if $\chi^2_{\text{red}}$ is close to 1.

The proposed procedure to obtain $x_c, x_r, \beta_r, \text{ and } \Lambda^T_r$ was successfully implemented. Except for the $\beta_r$ histogram, all other thermal constants follow a normal distribution. Fig. 9 shows an example of the final histogram for the prompt neutron decay constant in the core ($x_c$). The other thermal quantities ($x_r, \beta_r, \text{ and } \Lambda^T_r$) show similar behavior. This finding makes the whole procedure valid because the histograms do not show any specific tendency or bias. Also, it must be noted that the average value of the distributions corresponds to the maximum of the normal distributions.

The average values of $x_c, x_r, \beta_r, \text{ and } \Lambda^T_r$ arising from the proposed procedure were used as initial guesses in a least-squares approach employing in the ROOT program. The fitting employed the APFSD experimental data of the 286.8 ppm of natural boron. The most probable values (averages) were used as initial guesses because several true values could be calculated by the fit program. Therefore, the averages of the distributions were the best initial guesses for the ROOT program to calculate the standard deviations and correlation matrices of the fitted parameters. The constants $\Lambda$, $\Omega_0$, and $\omega_0$ of Eq. (2) were fixed, but the roots $\omega_T$ and $\omega_0$ were replaced by Eq. (58). The least-squares procedure with the most probable values of $x_c, x_r, \beta_r, \text{ and } \Lambda^T_r$ converged in a single iteration, thus demonstrating that the adopted procedure to infer these parameters was successful and accurate. The final values along with their uncertainties are given in Table 2.

The correlation matrix is shown in Table 3. According to (Devore, 2014), the correlation will be classified as strong if the absolute value of the correlation coefficients are greater than 0.8, moderate if it lies between 0.5 and 0.8, and weak if it is lower than 0.5. It shows that $x_r$ and $x_c$ are antecorrelated with a relatively strong correlation coefficient. Also, $\Lambda^T_r$ and $\beta_r$ have a moderate correlation coefficient. The correlation among the other variables are classified as weak.

5.2. The inference of $\rho^T_{\text{TT}}, \rho^T_{\text{TE}}, \rho^T_{\text{EC}}, \rho^T_{\text{EE}}, \rho^T_{\text{EC}}$, and $\Lambda^T_r$

The quantities $\rho^T_{\text{TT}}, \rho^T_{\text{TE}}, \rho^T_{\text{EC}}, \rho^T_{\text{EE}}, \rho^T_{\text{EC}}$, and $\Lambda^T_r$ were determined by using a procedure similar to that described in the previous section. The algorithm is shown in Fig. 10. The fitted values of $\omega_T, \omega_0, x_c, x_r, \beta_r, \text{ and } \Lambda^T_r$ were used as constraints in the calculation procedure of this work. An additional condition employed in this section was to get the total reactivity ($\rho$) from another experiment (dos Santos et al., 2019). After several attempts to calculate $\rho$, it was found out that this quantity could not be obtained from the set of equations derived in Section 4. The value employed was

$$
\rho = -0.06772 \pm 0.00146
$$

The criteria utilized for the variables in this last phase of the algorithm are shown below:

- **Criterion (1)**. Root $\omega_T$ could be calculated by employing Eq. (48) as:

$$
\omega^\text{calc}_T = \frac{\rho^T_T}{\Lambda^T_r} - \left[\frac{(\omega_T - x_c)\Lambda^T_r}{J_{\text{EC}}}\right] \left(\omega_T - x_c\right)
$$

where $\omega^\text{calc}_T$ is the calculated value of $\omega_T$. **Criterion (1)** was
where the experimental value \( \omega_{b}^{exp} = -2490 \text{ s}^{-1} \) and its uncertainty \( \sigma_{\omega_{b}^{exp}} = 11 \text{ s}^{-1} \), as given in Fig. 4.

Criterion (2) Similarly as the previous criterion, the root \( \omega_{b} \) could be calculated as:

\[
\omega_{b}^{calc} = \rho_{c}^{TT} - \frac{\left((\omega_{b} - \alpha_{c})\Lambda_{c}^{T} - \frac{J_{c}}{j_{c}}\right)(\omega_{b} - \alpha_{c})}{J_{c}^{c}}
\]

where \( \omega_{b}^{calc} \) is the calculated value of \( \omega_{b} \). Criterion (2) was

\[
|\omega_{b}^{exp} - \omega_{b}^{calc}| < 0.1 \sigma_{\omega_{b}^{exp}}
\]

(86)

being the experimental value \( \omega_{b}^{exp} = -6438 \text{ s}^{-1} \) and its uncertainty \( \sigma_{\omega_{b}^{exp}} = 132 \text{ s}^{-1} \), as given in Fig. 4.

Criterion (3) \( \alpha_{c} \) could be calculated with Eq. (60) and employing \( \omega_{b}^{exp} \) given previously. The criterion (3) was

\[
|\alpha_{c}^{exp} - \alpha_{c}^{calc}| < \sigma_{\alpha_{c}^{exp}}
\]

(87)

where the experimental value is \( \alpha_{c}^{exp} = -3581 \text{ s}^{-1} \) and its uncertainty is \( \sigma_{\alpha_{c}^{exp}} = 40 \text{ s}^{-1} \), as in Table 2.

Criterion (4) \( \alpha_{c} \) could be calculated employing Eq. (60) with \( \omega_{b}^{exp} \). Criterion (4) was

\[
|\alpha_{c}^{exp} - \alpha_{c}^{calc}| < \sigma_{\alpha_{c}^{exp}}
\]

(88)

where the experimental value is \( \alpha_{c}^{exp} = -3581 \text{ s}^{-1} \) and its uncertainty is \( \sigma_{\alpha_{c}^{exp}} = 40 \text{ s}^{-1} \), as in Table 2.

Criterion (5) \( \alpha_{c} \) could be calculated employing Eq. (38), obtaining \( \alpha_{c}^{calc} \). Criterion (5) was

\[
|\alpha_{c}^{exp} - \alpha_{c}^{calc}| < 2 \sigma_{\alpha_{c}^{exp}}
\]

(89)

where \( \alpha_{c}^{exp} \) and its corresponding uncertainty is given above.

Criterion (6) \( \beta_{c} \) could be calculated employing Eq. (39). Criterion (6) was

\[
|\beta_{c}^{exp} - \beta_{c}^{calc}| < 2 \sigma_{\beta_{c}^{exp}}
\]

(90)

where the experimental value is \( \beta_{c}^{exp} = -63.4 \text{ s}^{-1} \) and its uncertainty is \( \sigma_{\beta_{c}^{exp}} = 1.9 \text{ s}^{-1} \), as in Table 2.

Criterion (7) The total thermal reactivity is given by Eq. (49), resulting in the criterion (7) which was given by

\[
|\rho_{exp} - \rho_{calc}| < 2 \sigma_{\rho_{exp}}
\]

(91)

where \( \rho_{calc} \) is the calculated value, the experimental value is \( \rho_{exp} = -0.06772 \) and its uncertainty is \( \sigma_{\rho_{exp}} = 0.00146 \).

Criterion (8) This criterion was

\[
|F_{exp} - F_{calc}| < 2 \sigma_{F_{exp}}
\]

(92)

where the calculated quantity was obtained from Eq. (37) and it was given by:

\[
F_{calc} = \rho_{c}^{TT} - J_{c}^{T}
\]

(93)

\( F_{exp} \) was determined as:

\[
F_{exp} = \chi_{c} \Lambda_{c}^{T} + \beta_{c}^{TT}
\]

(94)

where, from the data of Table 2, \( F_{exp} = -0.1021 \) and its uncertainty is \( \sigma_{F_{exp}} = 0.0035 \).

Criterion (9) This criterion was obtained from Eq. (40). After some manipulations:

\[
G_{calc} = \rho_{c}^{TT} - \frac{(\omega_{b} \Lambda_{c}^{T} - \rho_{c}^{TT})J_{c}^{T}}{\omega_{b} \Lambda_{c}^{T} - \rho_{c}^{TT} + J_{c}^{T}}
\]

(95)

\( G_{exp} \) was given by:

\[
G_{exp} = \omega_{b} \Lambda_{c}^{T} + \rho_{c}^{TT}
\]

(96)

Criterion (9) could be written as:

\[
|G_{exp} - G_{calc}| < 2 \sigma_{G_{exp}}
\]

(97)

where \( G_{exp} = -0.0439 \) and its uncertainty is \( \sigma_{G_{exp}} = 0.0015 \).

Criterion (10) Similar to the previous criterion, the calculated unknown given by

\[
H_{calc} = \rho_{c}^{TT} - \frac{(\omega_{b} \Lambda_{c}^{T} - \rho_{c}^{TT})J_{c}^{T}}{\omega_{b} \Lambda_{c}^{T} - \rho_{c}^{TT} + J_{c}^{T}}
\]

(98)

and the experimental one by

\[
H_{exp} = \omega_{b} \Lambda_{c}^{T} + \rho_{c}^{TT}
\]

(99)

The criterion (10) is

\[
|H_{exp} - H_{calc}| < 2 \sigma_{H_{exp}}
\]

(100)

where \( H_{exp} = -0.1243 \) and its uncertainty is \( \sigma_{H_{exp}} = 0.0047 \).

All histograms for the thermal constants \( \rho_{c}^{TT}, \rho_{c}^{TT}, J_{c}, J_{c}, J_{c}, \) and \( \Lambda_{c}^{T} \), which simultaneously satisfied the criteria shown previously, follow a normal distribution. Similarly to the cases of \( \alpha_{c}, \chi_{c}, \beta_{c}, \) and \( \Lambda_{c}^{T} \), these averages values were used as initial guesses in a least-squares approach employing the ROOT program. The constants \( A_{i}, \omega_{a}, \) and \( \omega_{b} \) of Eq. (2) were fixed again, but the roots \( o_{r} \) and \( o_{b} \) were replaced by Eq. (59). The least-squares procedure with the most probable values of \( \rho_{c}^{TT}, \rho_{c}^{TT}, J_{c}, J_{c}, J_{c}, \) and \( \Lambda_{c}^{T} \) converged in a single iteration, thus demonstrating that the adopted procedure was successful and accurate. The final values along with their uncertainties are given in Table 4.

The correlation matrix is shown in Table 5. Here, it shows that \( \Lambda_{c}^{T} \) and \( \rho_{c}^{TT} \) are anticorrelated and their correlation coefficient is very close to being classified as strong. The other variables show a weak correlation (Devore, 2014).

5.3. Determination of the lifetimes

Section 4.3 shows two ways to get the thermal neutron lifetime. Here, the numerical values of this quantity are presented along with their uncertainty. Table 6 shows these values.
Fig. 10. Flowchart to calculate the distributions of $\rho_{T c}^{TT}$, $\rho_{T r}^{TT}$, $\Lambda_{T r}^T$, $j_{T c \rightarrow r}^T$, $j_{T r \rightarrow c}^T$, and $\Lambda_T$. 
Table 4
The goodness of fit and the final values of $\rho^{TT}_{\text{c}}, \rho^{TT}_{\text{r}}, j^{\text{T}}_{\text{c},-}, j^{\text{T}}_{\text{r},-}$, and $\Lambda^\text{T}/\text{C}$.  

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reduced $\chi^2$</td>
<td>0.99967</td>
</tr>
<tr>
<td>Prob (%)</td>
<td>51.31</td>
</tr>
<tr>
<td>$\rho^{TT}_{\text{c}}$</td>
<td>-0.05336 ± 0.00050</td>
</tr>
<tr>
<td>$\rho^{TT}_{\text{r}}$</td>
<td>-0.01556 ± 0.00055</td>
</tr>
<tr>
<td>$j^{\text{T}}_{\text{c},-}$</td>
<td>0.04966 ± 0.00080</td>
</tr>
<tr>
<td>$j^{\text{T}}_{\text{r},-}$</td>
<td>0.093936 ± 0.00023</td>
</tr>
<tr>
<td>$\Lambda^\text{T}/\text{C}$</td>
<td>6.96 ± 0.14</td>
</tr>
</tbody>
</table>

6. Theoretical results and comparison to experiments

Many of the inferred quantities reported previously are not determined by the several reactor codes available today. The theory/experiment comparisons shown are only for the quantities $\rho^{TT}_{\text{c}}, \rho^{TT}_{\text{r}}, \rho, \Lambda^\text{T}$ and $\tau^\text{T}$, and they will be done in an approximated way. Particularly, $\rho^{TT}_{\text{c}}$ and $\rho^{TT}_{\text{r}}$ reactivity variations are assumed to be independent of each other. The correct way of making these comparisons is to employ equations (25) and (29) and to perform the integrals together with the direct and adjoint fluxes. The theory/experiments comparisons of the thermal prompt neutron generation time ($\Lambda^\text{T}$) and the thermal lifetime ($\tau^\text{T}$) will also be performed in an approximated way since the thermal energy cutoff is not well defined in the experiments.

The theoretical analyses of the experiment were performed employing MCNP6 (Goorley et al., 2013) in conjunction with ENDF/B-VII.0 (Chadwick et al., 2006) and JENDL 3.3 (Shibata et al., 2002) nuclear data libraries. The nuclear data of $^{235}$U and $^{238}$U were taken from JENDL 3.3 and the remainders from ENDF/B-VII.0. The main reason for the utilization of JENDL 3.3 nu clear data was its best nuclear data for the delayed neutrons, as described in (dos Santos and Diniz, 2014).

The radial and axial geometric models follow closely the several benchmark models of the IPEN/MB-01 reactor (dos Santos et al., 2014, 2006b) available in the IRPhE handbook. The radial model is based in a rectangular array of 26 × 24 fuel rods immersed in a light water tank. The control and safety banks were considered totally withdrawn. The major distinction here is the utilization of a light water tank. The control and safety banks were considered is based in a rectangular array of 26 fuel rods immersed in a light water tank. The notation “Water” means that the moderator water was not poisoned with boric acid. The notation “286.8 ppm” means that the moderator water was poisoned with 286.8 ppm of natural boron. The boron poisoning was done in both core and reflector regions or in just one of these regions, as shown in Table 7.

The core is the region that contains the 26 × 24 fuel rods array. The pitch of the facility is 1.50 cm and the radial size of the core is 39 × 36 cm. Its height is the height of the fuel region, which is 54.84 cm. The reflector region is all other regions that complement the reactor model.

The reference simulation (case 1) in Table 7 is the configuration with water in the two regions. Its measured reactivity excess was $\Delta \rho = (10 \pm 3) \text{ pcm}$ and the calculated reactivity between case 1 (reference) and $K_{\text{eff}} = 1$ was $\Delta \rho = (-83 \pm 5) \text{ pcm}$. JENDL-3.3 library underestimated the value of $K_{\text{eff}}$ in approximately 100 pcm. Even so, the calculated reactivities showed excellent estimates compared to the experimental ones.

All reactivity variations $\Delta \rho/\text{pcm}$ shown in Table 7 are calculated relative to case 1 (the reference case). The hypotheses of making $\rho^{TT}_{\text{c}}$ and $\rho^{TT}_{\text{r}}$ independent of each other are made assuming that these two quantities are, respectively, given by the reactivity variations of cases 2 and 3. The total reactivity variation, given by case 4, is more reliable for a comparison between theory and experiment.

Table 6
Experimental thermal neutron lifetime.

<table>
<thead>
<tr>
<th>Lifetime</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^\text{T} = (25.55 \pm 0.57)\mu s$</td>
<td>(67)</td>
</tr>
<tr>
<td>$\tau^\text{T} = (25.89 \pm 0.24)\mu s$</td>
<td>(72)</td>
</tr>
</tbody>
</table>

Table 5
Correlation matrix for the parameters $\rho^{TT}_{\text{c}}, \rho^{TT}_{\text{r}}, j^{\text{T}}_{\text{c},-}, j^{\text{T}}_{\text{r},-}$, and $\Lambda^\text{T}/\text{C}$.  

<table>
<thead>
<tr>
<th>$\rho^{TT}_{\text{c}}$</th>
<th>$\rho^{TT}_{\text{r}}$</th>
<th>$j^{\text{T}}_{\text{c},-}$</th>
<th>$j^{\text{T}}_{\text{r},-}$</th>
<th>$\Lambda^\text{T}/\text{C}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho^{TT}_{\text{c}}$</td>
<td>1</td>
<td>-0.35012</td>
<td>0.46867</td>
<td>-0.35654</td>
</tr>
<tr>
<td>$\rho^{TT}_{\text{r}}$</td>
<td>-0.35012</td>
<td>1</td>
<td>0.34492</td>
<td>0.19493</td>
</tr>
<tr>
<td>$j^{\text{T}}_{\text{c},-}$</td>
<td>0.46867</td>
<td>0.34492</td>
<td>1</td>
<td>0.35816</td>
</tr>
<tr>
<td>$j^{\text{T}}_{\text{r},-}$</td>
<td>-0.35654</td>
<td>0.19493</td>
<td>0.35816</td>
<td>1</td>
</tr>
<tr>
<td>$\Lambda^\text{T}/\text{C}$</td>
<td>0.25923</td>
<td>-0.79027</td>
<td>-0.2526</td>
<td>0.25832</td>
</tr>
</tbody>
</table>
7. Conclusions

The subcritical experiment for the APSD measurements at frequencies up to 100 kHz has been successfully performed at the IPEN/MB-01 reactor. The experiment was very challenging because it required a long acquisition time and it was very sensitive to the electronic setup. The analyses revealed that the APSD modes occur uncoupled in the theoretical model. These pairs of modes was weak, therefore, they could be considered uncoupled in the theoretical model.

The recent neutron kinetic models (Dulla et al., 2006; Gandini, 2004; 2001; Sugawara et al., 2012) and even the two-region kinetic model of Spriggs (Spriggs et al., 1997) could not explain the experimental data. For this reason, a theoretical two-group two-region kinetic model was developed starting from the neutron transport equations and considering the interpretation of the measurements. In this model, the thermal group was completely solved. The coupling of these pairs of modes was weak, therefore, they could be considered uncoupled in the theoretical model.

The theory-to-experiment comparison was performed only for the case of 286.8 ppm of natural boron and employed MCNP6 in conjunction with ENDF/B-VII.0 and JENDL 3.3 nuclear data libraries. The comparison reveals that the values of \( \rho^T \), \( \rho \), \( \Lambda^T \), and \( \tau^T \) agreed within 3\( \sigma \), but \( \rho^T \) shows some discrepancy.

In general, the experimental data are of excellent quality and suitable for an international benchmark, since the entire experimental procedure was carefully performed, as described in the reference (dos Santos, 2020). Several important thermal constants for the kinetic behavior of the IPEN/MB-01 reactor such as the prompt neutron decay constant \( (\chi_2) \), the partial reactivity \( (\rho^T \) and \( \rho^T \)), and the prompt neutron generation time \( (\Lambda^T \) and \( \Lambda^T \)) all in the core and reflector were experimentally determined for the first time. The data can be useful for the validation of neutron cross-section libraries employed in the simulation programs.

Declarations of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References


