Non-Linear Thermal Stresses in an Aluminum Composite Reinforced with SiC Particles

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Abstract. Boari [1] developed an analytical methodology, based on statistics mechanics and modified Maxwell-Boltzmann distribution, predicts the most probable thermal stress level in an MMC due to its fabrication process which induces a temperature range of -580ºC. The beneath hypothesis in that analytical work was the elastic behavior of the materials and particles with round geometry. The analytical results were verified against several numerical analyses using properties of an Al-SiC composite and considering random distribution of particles and linear material properties. The numerical results in terms of isostress curves when ΔT=-580 ºC were analyzed with an image analyzer. The analytical and numerical results compared very well. The stresses locally reached values over the Aluminum yielding limit.

This work adopted a non-linear behavior for the Aluminum matrix once the plastic behavior can alter significantly the stress field in the material. Two set of analyses were done with 30 and 40 models. Firstly, using rounded particles, to allow a direct comparison with the previous [1] numerical work, and secondly, using quadrilateral particles, to allow a less simplified modeling hypothesis. Also, an algorithm was developed to analyze the new results/isostress curves. Some of the numerical results, in terms of the non-linear stress distribution, as well as the most probable stress values in the aluminum matrix, using both sets, were presented along with a discussion about the obtained results. In short, the result analysis shows: (a) the strong influence of the aluminum elastic-plastic behavior on the composite thermal stress distribution due to its manufacturing process and (b) a less strong influence of the particle geometry which is influenced by misfit strain which is relaxed by the punching of dislocation loops. The plastic deformation in aluminum matrix indicates high work-hardening around the particles, increasing the dislocation density through the relaxation of thermal stress.

1 Introduction

The typical dimension for SiC particles, in a Al-SiC composite, is about L = 0.20 10^{-6} m. This material can be obtained from the components mixing and extrusion at about 600 ºC and it was used at the room temperature (20 ºC). The difference in the thermal expansion coefficients and the temperature range induces thermal stresses in the material. The particles distribution, the particles geometry and size are factors that alter the thermal stress field and contribute to the development of its strength mechanisms. Due to the random distribution of particles in the composite matrix there isn't a unique stress value in the material but, instead, a stress distribution with different values from one point to another which depends, among others, on the volumetric ratio. This work uses the most probable stress value obtained by a statistical treatment from the results of several analyses, each one with a random distribution of particles, to have a statistically consistent data base.

Usually, in an MMC composite the particles distribution is not uniform nor the particles have a perfectly round or rectangular geometry. The analytical work [1] deals with round particles and material linear behavior. To stick with these hypotheses, the numerical models to check the analytical development adopted perfectly round particles and the linear behavior. In these models the volumetric ratio values, *f*, ranges from about 15% to 40% to allow the verification over a wide range of *f* values. The numerical and analytical results, in terms of the most probable thermal stress value in the material compared very well.

However, the results were not directly applicable to an actual MMC composite, in particular the Al-SiC one, once the stress maximums reached values over the Aluminum yielding limit. This fact implies actual stress fields being different from the ones predicted in [1] which implies, also, a different average and most probable thermal stress values in the material.

In this work two set of analyses were done with 30 and 40 models, considering non-linear behavior for the Aluminum matrix and linear behavior for the SiC particles. In the first set, round particles were used to allow a direct comparison with the previous (linear) numerical work. In the second set of analyses, quadrilateral particles were used to allow a less simplified particle geometry modeling and a tight volumetric ratio – from 20% to 25% to simulate a more realistic material (nominal *f* value of 22.5% with a variation of ± 2.5%, to simulate the clusters formation as observed in an actual composite). Also, a specific algorithm based on the Matlab [2] image processing toolbox was developed to analyze the new results/isostress curves.

2 Background for the Round Particles and Linear Behavior

The analytical model developed by Boari [1] to get the thermal stress distribution in an MMC composite is based on the statistical mechanics principle adapted to composites by the Eshelby's method. It is a modification of the Maxwell- Boltzmann statistics as formulated by Beiser [3]. As general idea, let's suppose the Aluminum matrix divided in K cells each one with area a_1, a_2, a_3,..., a_k. By throwing N particles in a random fashion one can know how many particles fall in each cell. Repeating this procedure a great number of times one can verify that there is a most frequent distribution or, the most probable one which is directly associated with: (a) the cell size (area) and (b) the number of ways the particles can be distributed with no change in their number in each cell (for this the particles can be identical but distinguishable). These conditions above are associated with (a) the "a priori" probability and (b) the thermodynamic probability, respectively.

2.1 The Maxwell – Boltzmann Statistics Model

The Maxwell-Boltzmann distribution law is given by eq. (1), [3], where n_i is the most probable number of particles in a given cell *i*, g_i is the "a priori" probability, the probability of one particle falls in the cell *i*, α and β are Lagrange multipliers (used to find out the maximum or the minimum of a function with restrictions).

$$n_i = g_i\, e^{-\alpha}\, e^{-\beta u} \tag{1}$$

The term $e^{-\alpha}$ is associated with the particle quantity while the term $e^{-\beta u}$ is associated with the particles energy (u). The above described equation is associated with a discrete set of energy. To have an expression associated with a continuum range of energy the eq. (1) is re-written as eq. (2) where the left hand side, *n(u)du*, represents the quantity of particles with energy between *u* and *u+du*.

$$n(u)du = g e^{-\alpha}\, e^{-\beta u} du \tag{2}$$

2.2 Boari's Analytical Development

Originally, *u* represents the kinetic energy while for his work Boari assumed it represents the elastic potential energy of a particle within the metal matrix. So, eq. (2) can be rewritten as eq. (3) where E is the Young's modulus and σ is the stress.

$$n(\sigma)\, d\sigma = g e^{-\alpha}\, e^{-\beta \frac{\sigma^2}{2E}}\, d\sigma \tag{3}$$

From this eq. (3), considering the modified Eshelby's method presented elsewhere, [4] and [5], and based on the balance of the internal (thermal) stress, and after some manipulation Boari [1] get the modified Maxwell-Boltzmann distribution as given by eq. (4) where the term *K* is given by eq. (5). In these expressions the subscripts M and I stand for matrix and particles, respectively, N is the number of particles, f is the composite volumetric ratio, S is the Eshelby's tensor, I is the identity matrix, and α_M and α_I are tensors with the thermal expansion coefficients of the matrix and the particles, respectively, and ΔT is the temperature range, C_M and C_I are the elastic tensors. The most probable stress value σ_p in the composite material is associated with the maximum of eq. (4) and is given by eq. (7).

$$n(\sigma) = 4\pi N \left(\frac{3}{2 C_M K \pi} \right)^{3/2} \sigma^2\, e^{-\frac{3\sigma^2}{K 2 C_M}} \tag{4}$$

$$K = f\, C_M \{(S - I)\{(\mathrm{C_M} - \mathrm{C_I})\,[\mathrm{S} - \mathrm{f}(\mathrm{S} - \mathrm{I})] - \mathrm{C_M}\}^{-1} \mathrm{C_I}\,(\alpha_I - \alpha_M)\Delta T\}^2 \tag{5}$$

$$\sigma_p^2 = \frac{2 f C_M^2 \{(S - I)\{(\mathrm{C_M} - \mathrm{C_I})[\mathrm{S} - \mathrm{f}\,(\mathrm{S} - \mathrm{I})] - \mathrm{C_M}\}^{-1} \mathrm{C_I}\ (\alpha_I - \alpha_M)\Delta T\}^2}{3} \tag{7}$$

2.3 Analytical Model Verification

To verify the analytical development partially presented above, Boari [1] performed a set of numerical analyses using the ANSYS program. Using the ANSYS uniformly distributed random number generator and its programming resources to define the particles dimensions (radius) and position and to eliminate some particle superposition, in each analysis several round particles were generate, defining a certain volumetric ratio, *f*. The loading consisted of uniform temperature field imposed to the model to simulate the hot condition, immediately after the extrusion at 600 ºC, and the slowly cooling down to the room temperature, 20 ºC. Appropriated boundary conditions were defined to simulate the central portion of a composite bar made with an Aluminum matrix reinforced with SiC particles (Al-SiC). To be coherent with the analytical development, each analysis was a linear one although locally the thermal stresses in the Aluminum matrix reached values over the Aluminum yielding limit.

To obtain the most probable stress from each numerical analysis, the figures with the isostress curves associated with ΔT = -580 ºC were processed in an image processor to obtain the percentage of each stress value (represented by a given color) in the material. As shown in figure 1, the numerical results matched very well with the analytical ones despite the large range of values in the volumetric ratio.

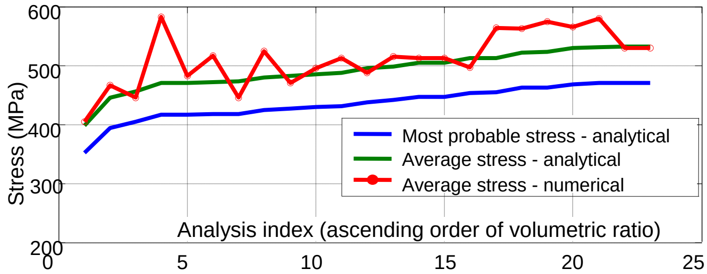

Figure1: Comparison of linear results [1]: analytical x numerical (round particles)

3 Present Work – Numerical Analyses

In this work, the basic Boari's numerical procedure was followed using two sets of analyses with rounded and quadrilateral particles.

As the stresses in the Aluminum matrix reached values over the yielding limit each analysis now was a non-linear one with a defined Aluminum stress x strain curve, table 1, once the plastic behavior can alter significantly the stress field in the material.

To allow a direct comparison between the linear *versus* the non-linear behavior, the same models (particle form, size and distribution) used in [1] were adopted in these new analyses using the ANSYS program [6]. Also, to verify the influence of the particles geometry a new set of numerical simulations with quadrilateral particles were performed using a very tight volumetric ratio range (20%-25%).

Table 1: Adopted Aluminum stress-strain curve values (MPa *x* %)

σ - stress	146	175	195	205	215	220	225	228	229	230
ε - strain	0.2	0.45	0.8	1.0	1.5	2.0	3.0	4.0	6.0	8.0

3.1 Material Properties and Boundary Conditions

The adopted physical properties for the Aluminum matrix are: Young's modulus, 73 GPa, specific mass, 2800 Kg/m^3, thermal dilatation coefficient, 23.6 10^{-6} $^{o}C^{-1}$, Poisson's ratio, 0.33, transversal elastic modulus, 27.4 GPa.

For the SiC particles the adopted properties are: Young's modulus, 450 GPa, specific mass, 3200 Kg/m^3, thermal dilatation coefficient, 4.0 10^{-6} $^{o}C^{-1}$, Poisson's ratio, 0.17, transversal elastic modulus, 192 GPa.

As boundary conditions, the nodes on the X axis were restrained in the Y direction while the nodes on the Y axis had their X direction restrained.

The loading consisted of uniform temperature field imposed to the model in several load steps to simulate its slowly cooling down from 600 °C to 20 ºC. So, the last load step is associated with the ΔT=-580 °C.

3.2 FE Results – Round Particles

Each isostress curve has its own color defined by the R(ed), G(reen) and B(lue) values defined previously in the ANSYS pos-processor. This color map is passed to the Matlab program developed to analyze the images/figures. A typical result obtained from the numerical analyses with round particles is shown in Figure 2 where one can see the isostress lines within the composite, their scale and, also, the particle distribution for that specific model which is one out of thirty.

3.3 FE Results – Quadrilateral Particles

Initially the same number (9 curves) used in the round particle analyses was used for these quadrilateral particles post-processing analyses. However, each isostress was associated with a large range of values – about four times the range value associated with each isostress in the round particles analyses. So, the results were pos-processed again twice: defining 20 isostress curves in each figure at ΔT = -580 °C, and with 40 isostress curves. For the image processing step the 40 isostress curves figures were used.

A typical result obtained from the numerical analyses with quadrilateral particles is shown in Figure 3 (for better visualization the one presented was among those generated with 20 isostress curves), which is one out of forty.

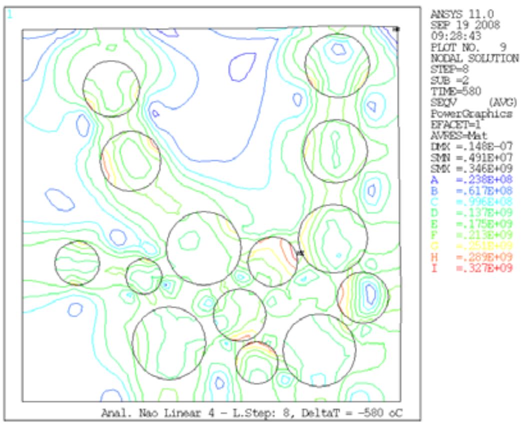

Figure2: Typical results at ΔT = -580 ºC - using round particles

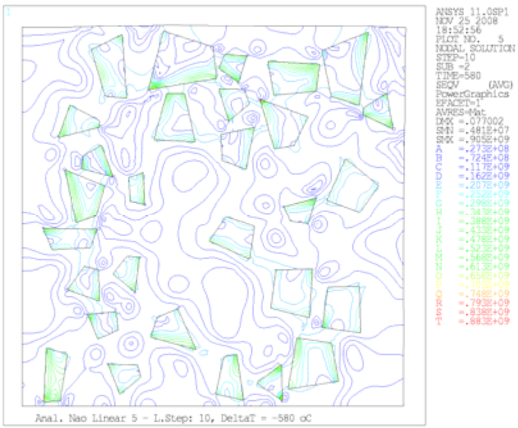

Figure3: Typical results at ΔT = -580 ºC - using quadrilateral particles

As expected, the calculated average composite thermal stress changed as the number of isostress curves changed toward a ‘converged’ value as the number of isostress increased. This is shown in the next section.

4 Analysis of the Numerical Results

For each analysis with round and with quadrilateral particles, the corresponding figure with the isostress curves at ΔT=-580 °C was saved in TIFF format to be analyzed by an image analyzer algorithm specifically developed in Matlab language [2]. This algorithm was already described elsewhere, including the performed tests [7]. The result from the image analysis is the percentage associated with each color or isostress curve/value in the figure. These percentages are used to obtain the average stress value for each analysis as well as the mode stress value (the most frequent value).

4.1 FE Results – Round Particles

Figure 4 shows the averaged and the mode stress values for each one of the analysis with round particles. The overall calculated average thermal stress, from all analyses, is 171.9 MPa with 12.8 MPa of standard deviation.

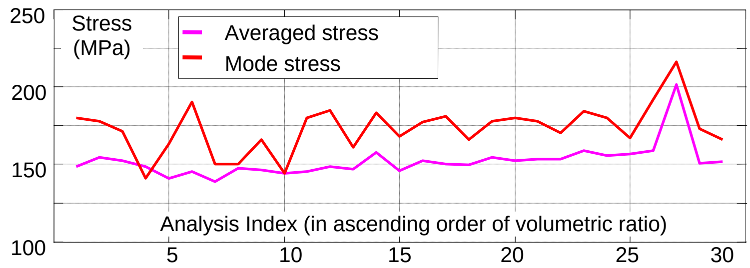

Figure4: Mode and averaged stress values for each analysis - round particles

4.2 FE Results – Quadrilateral Particles

Figure 5 shows the mode stress for each one of the analyses with quadrilateral particles and their averaged values (dotted line) for 9, 20 and 40 isostress curves. The calculated overall averaged thermal stress 'converged' to 180.8 MPa with 6.5 MPa of standard deviation, as clearly shown in figure 6. Both figures show that, for practical use the results obtained with 20 isostress curves were already good to evaluate the average thermal stress in the composite. Figure 7 shows, for each one of the analyses with quadrilateral particles, the averaged stress and the mode stress values.

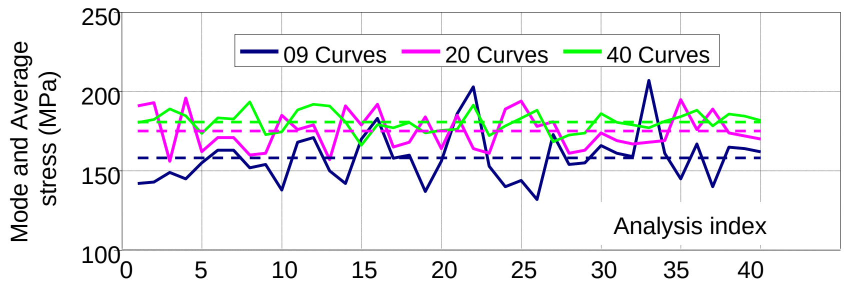

Figure5: Mode stress and averaged values at ΔT = -580 ºC - quadrilateral particles

5 Results Discussion and Conclusions

The figure 7 shows the averaged stress greater then the mode stress in the quadrilateral particles. This is a different behavior respect to the results found with the round particles and presented in figure 4. This is supposed to be due to the particles geometry. Each one of the performed analyses can be seen as the stresses in a small region of the composite with its locally different volumetric ratio value. The thermal stress level in the analyzed Al-SiC composite was evaluated as about 172 MPa with ~13 MPa of standard deviation and about 181 MPa with ~6.5 MPa of standard deviation (STD) considering round particles and quadrilateral particles respectively.

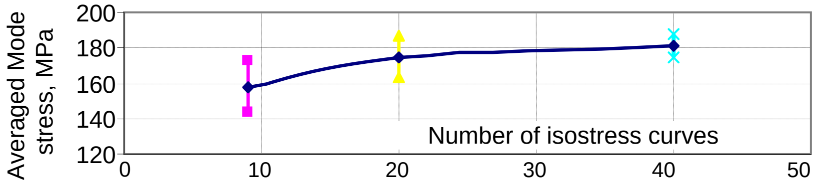

Figure6: Convergence of the averaged mode stress values - quadrilateral particles

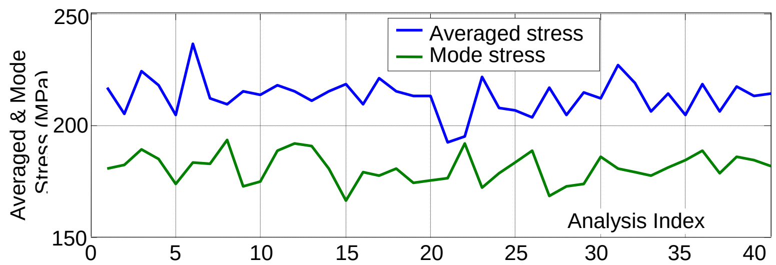

Figure7: Mode and Averaged stress values for each analysis - quadrilateral particles

It is known that, from the statistics and for a normal distribution, a given value has a ~95% accumulated probability to be within the range of ±2.0*STD around the average value. So, interpreting this as a measure of confidence in a given value, with the above hypothesis, one can say, with a 95% of certainty, the thermal stress level is in the range 146 – 198 MPa or 168 – 194 MPa considering, respectively, round or quadrilateral particles. The differences can be attributed to the number of analyses in each set, the particles geometry and the range of the volumetric ratio in each set (about 17% to 35% in the 30 round particles analyses and about 20 to 25% in the 40 quadrilateral particles). However, these thermal stress ranges are essentially the same as one encompasses the other.

By other hand, from the comparison between the linear results [1], figure 1, and the non-linear ones, figure 4 and 7, the main factor to define the thermal stress level is the Aluminum non-linear behavior. The particles geometry takes a less important role as can be seen by comparing the figures 4 and 7 despite the differences in the volumetric ratio range. This can be explained by the material behavior which is influenced by misfit strain which is relaxed by the punching of dislocation loops. The plastic deformation in the aluminum matrix indicates high work-hardening around the particles, increasing the dislocation density through the relaxation of thermal stress.

Figures 8a and 8b show the obtained results when the percentages were averaged at each isostress index curve (30 values for the round particles and 40 values for the quadrilateral ones) where the ‘error’ bar represents the respective standard deviation.

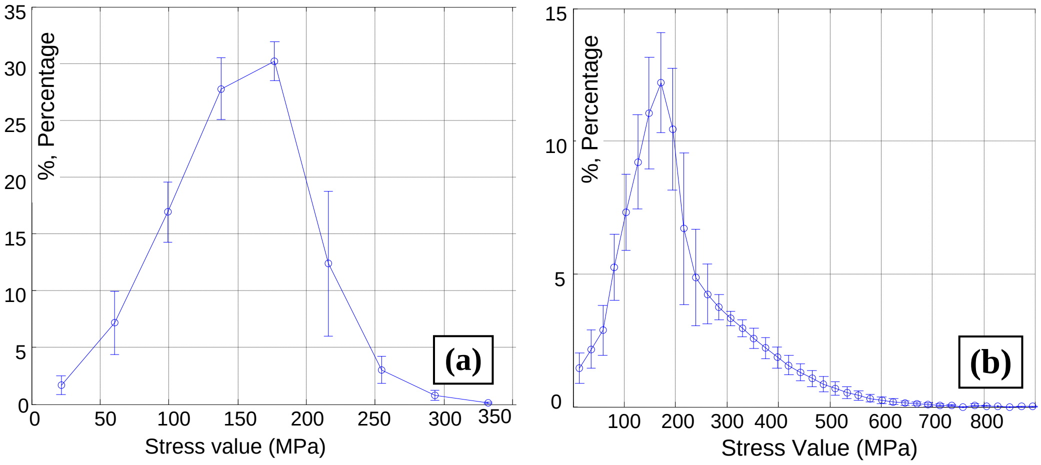

Figure8: Averaged percentages - using (a) 30 isostress curves with circular particles and (b) 40 isostress curves with quadrilateral particles

The curves depicted in figure 8 shows how the actual thermal stress distribution in the composite material should look like when plasticity is taken into account. These curves confirm that the most probable thermal stress value in the Al-SiC studied composite is around 175 MPa.

As final conclusion, is possible to take the average range values above to say, with 95% of confidence, the thermal stress values in this Al-SiC composite is in the range 155 MPa to 195 MPa with an average value of ~175 MPa and a standard deviation of ~10 MPa.

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