Development of a Three-dimensional Cell Suitable for Electromagnetic Field Calculation

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***Abstract*—This paper describes a method suitable for electromagnetic (EM) field calculation in the space and time domain; based on the development of a three-dimensional cell, which is formed by orthogonal transmission lines. This method employs the transmission line theory and wave propagation model to calculate the current distribution, and the infinitesimal time-varying dipole and image method to calculate the resulting electromagnetic field of each cell. The total field is then calculated by assuming the array of conductors as a chain of cells. Some results and conclusions are presented, where the advantages of the considered method are pointed out. This method provides a useful tool for problems considered too large to be modeled by other methods. For example, in the evaluation of EM fields regarding transient phenomena in substations. Now, authors are implementing the electric field calculation. This paper presents only the magnetic field calculation methodology.**

I. Introduction

The evaluation of electromagnetic (EM) fields assumes a special concern in the modern society. The desire to predict and control the radiated EM fields at locations and equipment have been increasing since high susceptibility equipment has been used in almost all branches of activities.

Aside these, the safety aspects and the recent concern about the possible environmental and biological effects of radiated EM fields should be considered .

Although Electromagnetic Compatibility (EMC) regulations specify maximum permissible radiated fields at various distances from a equipment, and some international regulations specify EMC zones inside a building; the EM fields should be evaluated for further studies of their effects on locations and on electric and electronic systems.

An example of such a concern is the evaluation of EM radiated fields to define the best lay-out of a control room, for which safety aspects are relevant to the performance of a complex industrial plant or substation. In this example, the best lay-out can be defined by comparing the calculated EM fields with some EMC limits, like the susceptibility of electric and electronic systems.

Manuscript received July 10, 1995.

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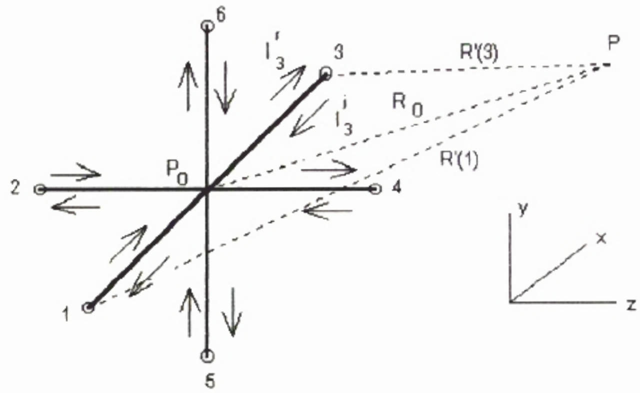

Fig. 1. The considered three-dimensional element or cell.

This paper describes a method suitable for EM field calculation in the space and time domains, in particular the magnetic field calculation.

Although this method can be used in applications regarding steady-state or transient phenomena, it provides a useful tool for problems considered too large to be modeled by other methods. For example, the evaluation of EM radiated fields regarding transient phenomena in big structures, like buildings and substations.

II. The Method

A. *Description of the Method*

This method is based on the development of a three-dimensional element or cell, where the central node corresponds to a junction of transmission lines; forming a impedance discontinuity in each line. Fig. 1 shows this cell.

The length of each transmission line is chosen to be electrically short enough when compared with the impulse wavelength.

The response of the cell to incident-voltage pulses and the current distribution are determined by the transmission line and wave propagation theories [1]. Once the current distribution is determined, the contribution of each reflected and incident current of the cell to the magnetic field is calculated using the dipole and images theory [2].

After subdividing the array of conductors into a finite number of elementary units, the calculation of the total magnetic field as a function of the time in any point of the space is then determined by superposition.

Conference on the Computation of Electromagnetic Fields, Berlin, Alemanha, julho, 1995

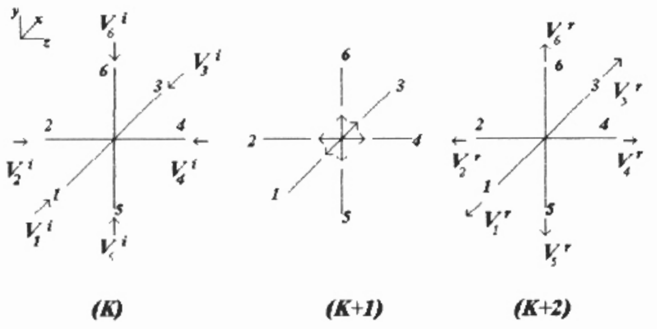

Fig. 2. Impulse response of the cell at instants k, $k+1$ and $k+2$

This method differs from others presented so far in the scientific literature. Thanks to its intrinsic aspects, it is not necessary to solve any equation system, and there are not problems regarding the convergence of the solution. Thus this method is unconditionally stable.

Another characteristic to be mentioned about this method is concerned with the determination of magnetic field through the incident and reflected current; which is not from the current resulted from the sum of them.

It is a very important characteristic of the method: first, because the field calculation is directly related to the current path; second, because the consideration of the incident and reflected current allow us to use bigger cells without introducing significant errors, specially in the near-field calculation.

The field contribution from each dipole is delayed by the retarded or validation time, a factor that depends on the current path. If the center of each dipole is considered when the retarded factor is calculated, like in other methods, some restriction will need to be included in the EM field calculation.

This is particularly significant when the dimension of the element and the distance from which the field is evaluated are similar.

B. *Impulse Response of The Cell*

To implement this method on a digital computer, the equations are formulated in the discretized form, where both space and time are represented in terms of finite, elementary units Δl and Δt, which are related by the speed of light c.

If at time $t=(k-2)\Delta l/2c$, voltage impulses ${}_{k-2}V_n^i$ on lines 1 to n are incident on any series node of the cell, the reflected voltages on these lines at time $t=k\,\Delta l/2c$ will be given by:

$$_K\left[V_n^r(z,x,y)\right]=\left[\sigma(I,J)\right]_{K-2}\left[V_n^i(z,x,y)\right] \tag{1}$$

The variable Δl is the length between two series nodes of the cell; $\Delta l/2c$ is the propagation time along each segment of transmission line, which is taken as the time step of the calculation.

$$\left[\sigma(I,J)\right]=\begin{bmatrix}(\sigma_h-1) & \sigma_h & \sigma_h & \sigma_h & \sigma_v & \sigma_v\\ \sigma_h & (\sigma_h-1) & \sigma_h & \sigma_h & \sigma_v & \sigma_v\\ \sigma_h & \sigma_h & (\sigma_h-1) & \sigma_h & \sigma_v & \sigma_v\\ \sigma_h & \sigma_h & \sigma_h & (\sigma_h-1) & \sigma_v & \sigma_v\\ \sigma_h & \sigma_h & \sigma_h & \sigma_h & (\sigma_v-1) & \sigma_v\\ \sigma_h & \sigma_h & \sigma_h & \sigma_h & \sigma_v & (\sigma_v-1)\end{bmatrix} \tag{2}$$

Equation (2) represents $\left[\sigma(I,J)\right]$, the transmission-coefficient matrix, where the coefficients σ_h and σ_v, are the transmission coefficients of lines 1-4 and 5-6, respectively.

When the central node *(z,x,y)* of the cell is the excited node, the pulse voltage at instant $(k+1)$ must be added in (1).

The impulse response of the cell at instants *k, k+1,* and *k+2* is illustrated by Fig. 2.

Equation (3) shows us how reflected impulses from neighboring cells become incident impulses on the cell, which coordinates of the central node are *(z,x,y)*:

$$\begin{bmatrix}{}_KV_1^i(z,x,y)\\ {}_KV_2^i(z,x,y)\\ {}_KV_3^i(z,x,y)\\ {}_KV_4^i(z,x,y)\\ {}_KV_5^i(z,x,y)\\ {}_KV_6^i(z,x,y)\end{bmatrix}=\begin{bmatrix}0&0&1&0&0&0\\0&0&0&1&0&0\\1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&0&0&0&1\\0&0&0&0&1&0\end{bmatrix}\begin{bmatrix}{}_KV_1^r(z,x+\Delta l,y)\\ {}_KV_2^r(z+\Delta l,x,y)\\ {}_KV_3^r(z,x-\Delta l,y)\\ {}_KV_4^r(z-\Delta l,x,y)\\ {}_KV_5^r(z,x,y+\Delta l)\\ {}_KV_6^r(z,x,y-\Delta l)\end{bmatrix} \tag{3}$$

The authors have named the matrix that relates incident and reflected impulses by *Connection-Matrix, C(I,J)*.

C. *The Boundary Conditions*

Equation (3) represents the general boundary condition for series nodes of cells.

Other conditions can be imposed, depending on how the cells are connected in the boundary. For example, a boundary condition can be imposed through a factor that gives us the relation between the incident and reflected impulse in the boundary, like the reflection coefficient.

D. *Current Distribution*

The reflected and incident current along line "n" at instant "k" are expressed by (4) and (5), where Zn is the characteristic impedance of line "n", and SR^D is a scattering factor regarding the position and direction of the reflected-wave propagation:

$$_K\left[I_n^r(z,x,y)\right]=\left[\frac{1}{Z_n}\right]SR^D\,{}_K\left[V_n^r(z,x,y)\right] \tag{4}$$

$$ {}_K\left[I_n^i(z,x,y)\right] = \left[\frac{1}{Z_n}\right]\left[C(I,J)\right]SR^D\,{}_{K-1}\left[V_n^r(z,x,y,\Delta l)\right] \quad (5) $$

Table I shows the values of SR^D, where $z,x,$ and y are positive (0) or negative (1) coordinates of the cell, and n is the series-node number.

TABLE I
VALUE OF SR^D AS A FUNCTION OF THE COORDINATES OF CELLS

Z	X	Y	l/n	1	2	3	4	5	6
0	0	0	1	-1	-1	1	1	-1	1
0	0	1	2	-1	-1	1	1	1	-1
0	1	0	3	1	-1	-1	1	-1	1
0	1	1	4	1	-1	-1	1	1	-1
1	0	0	5	-1	1	1	-1	-1	1
1	0	1	6	-1	1	1	-1	1	-1
1	1	0	7	1	1	-1	-1	-1	1
1	1	1	8	1	1	-1	-1	1	-1

If (z,x,y) is the excited node, then SR^D assumes the value 1, i. e., the direction of the wave propagation of the phenomenon is the same of the reflected impulses on the lines of this cell.

E. *Magnetic Field Calculation*

Rubinstein and Uman [02] have derived the equation for the magnetic field generated by a elemental dipole of current propagating along y axis with speed v, where the geometrical factors are shown in fig. 3:

$$ dB_\phi(r,\phi,y,t) = \frac{\mu_0 dy'}{4\pi}\left(\frac{r}{R^3}i(y',t-\frac{R}{c}) + \frac{r}{cR^2}\frac{\partial i(y',t-\frac{R}{c})}{\partial t}\right) \quad (6) $$

In this work, the spatial-temporal distribution of the current in each radiating dipole is considered as a step function to obtain the magnetic field:

$$ i(y',t) = I_0 u(t-\frac{y'}{v}), \text{ where } u(\xi) = \begin{matrix} 0, \xi \langle 0 \\ 1, \xi \geq 0 \end{matrix} \quad (7) $$

Therefore, from the knowledge of the reflected and incident currents along lines 5 and 6 of the cell; the magnetic field as a function of the time can be found by using the method of images after integrating properly (8) plus (9).

$$ \frac{\mu_0 I_0 r}{4\pi R^3}dy', \; t \rangle R/c + y'/v \quad (8) $$

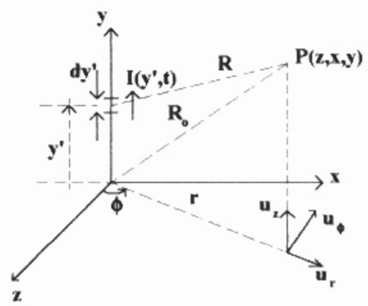

Fig. 3. Definition of geometrical factors used in the field computation.

$$ \frac{\mu_0 I_0 r}{4\pi c[(y-y')^2+r^2]}\frac{1}{\left[\frac{1}{v}-\frac{(y-y')}{c\sqrt{(y-y')^2+r^2}}\right]}dy', \; t \rangle R/c + y'/v \quad (9) $$

The time $(R/c + y'/v)$ is the retarded or validation time.

The same procedure can be used to obtain the magnetic field owing to the current distribution along lines 1 and 3; and 2 and 4 of the cell. The total field at time t in any point P of the space is then obtained by superposition.

III. APPLICATION

To evaluate the physical and mathematical consistencies of this method, some numerical results were obtained, like shown in the following example.

Although this method can be used in applications regarding steady-state or transient phenomena, an application regarding lightning was chosen by authors: a steel-structure building struck by lightning. Fig. 4 represents the building struck by lightning; and Fig. 5, the struck point and the region where the magnetic field is analyzed.

A. *General Criteria*

In this example, each column and beam of the structure is considered as part of the lightning protection system: the down-conductors and the roof-grid conductors.

Then, based on the length of beams and columns; and on the dimensions of the building, the structure was divided in identical cells, which length between two series nodes of the cell is 4m.

It was assumed in this example some simplifications without damaging the general philosophy of the method.

The average characteristic impedance of vertical (500.93Ω) and horizontal lines (483.23Ω) were calculated through traditional formulas, without considering the non-linear ionization phenomena. Therefore, the impulse-propagation speed was considered constant.

The lightning stroke was simulated by a unidirectional current source given by:

$I = 8.3\,t$ *(kA) for* $t \leq 1.2\ \mu s$, and
$I = 10\,[\,0.83\,t - 0.84\,(\,t - 1.2\,)\,]$ *(kA)* for $t \rangle 1.2\mu s$ (10)

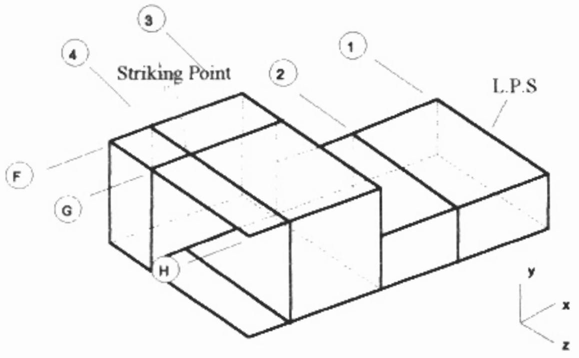

Fig. 4. Sketch of the building and the lightning protection system (L.P.S), which average dimensions are (28 x 16 x 10) m.

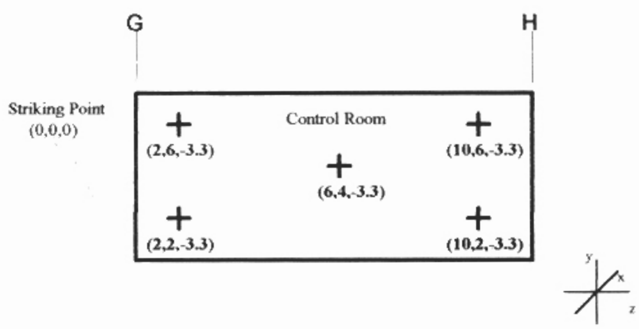

Fig. 5. Sketch of the considered region and the lightning struck point.

The incident voltage-impulses at node (0,0,0) is given by (10) times the resulting impedance "seeing" by the lightning stroke.

The earth resistance of the grounding system was assumed constant, equal to 5Ω.

B. *Results*

The magnetic field owing to the current distribution in the lightning protection system as a function of time in the points of the region represented by fig. 5 were calculated and its profile is represented in fig. 6.

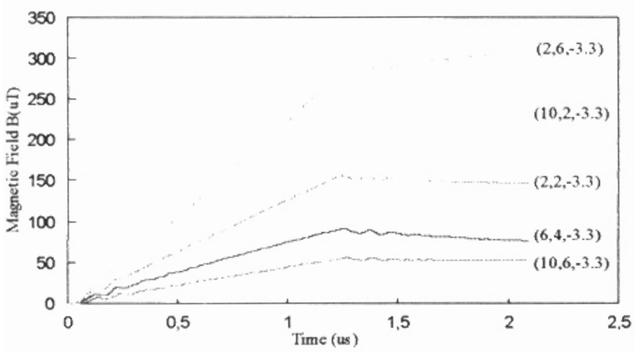

Fig. 6. Magnetic field B (μT) as a function of time (μs) at points shown in fig. 3; due the current distribution.

This is an interesting example because the point regarding the maximum magnetic field is not the one near the striking point. The most intense fields are around the points which coordinates are (2,6,-3.3) and (10,2,-3.3).

This fact is owing to the current distributions along the two down-conductors placed at the junction of axis 3 and G, and 4 and H.

IV. Conclusions

A method suitable for EM field calculation in the space and time domain has been described; in particular the magnetic field calculation.

This method is simple; but, fast and direct when computational aspects are considered. This is a useful tool for problems considered too large to be modeled by other methods. For example, in the evaluation of EM field regarding transient phenomena in buildings and substations.

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