SOME EFFECTS OF A POSSIBLE

T.R.I. VIOLATION IN NUCLEAR PHYSICS

by

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PREFACE

The research described in this thesis was carried out at the University of Sussex from the autumn term of 1969 to the spring term of 1972 under the supervision of Professor R. J. Blin-Stoyle.

I should like to express my sincere thanks to Professor R. J. Blin-Stoyle for suggesting the problem and for his guidance. It is a pleasure to acknowledge also the help of Professor J. P. Elliott in various phases of this work.

A. M. Kahn and Drs. G. Barton, J. A. Evans and D. Hamilton have always been prepared to give me their time and attention.

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Finally I would like to thank Sue Newman for typing this thesis rapidly and efficiently.

SUMMARY

This thesis deals with several aspects of a possible violation of Time Reversal Invariance in the electromagnetic interaction.

In Chapter 2 it is shown that such a Time Reversal Invariance (T. R. I.) violation contributes to low energy nuclear physics in the form of a T. R. I. violating electromagnetic transition operator and also as a T. R. I. violating two and three body potential. The T. R. I. violating transition operator is likely to dominate in light nuclei whereas the three body T. R. I. violating potential becomes more important for heavy nuclei. Two possible forms for a T. R. I. violating $NN^*\gamma$ vertex are considered and the above mentioned operators calculated.

In Chapter 3 T. R. I. violation is assumed to occur through the $N^*N\gamma$ vertex and again T. R. I. violating electromagnetic transition operators and potential operators are calculated.

In Chapter 4 the effect of the T. R. I. violating electromagnetic transition operators obtained in Chapters 2 and 3 are estimated for a light nucleus and the operators stemming from the T. R. I. violating $N^*N\gamma$ vertex are found to be more important. The calculated effect is represented by an imaginary part for the "mixing ratio" δ (the ratio between the reduced matrix elements of two competing multipoles) and is found to be of the order of 10^{-3}.

Finally in Chapter 5 an experiment performed at the University of Sussex on T-violation is analysed. It is found that if the accuracy of the experiment can be increased by an order of magnitude, information of the nature on the T. R. I. violation will be forthcoming.

CONTENTS

LIST OF APPENDIX

CHAPTER 1

INTRODUCTION

1.1 Introduction

In 1964 Christenson et. al. observed the decay of a long lived neutral K-meson into two charged π-mesons. This has been interpreted as a violation of C. P. invariance or if the C. P. T. theorem is true as a violation of Time Reversal invariance. Since this first experiment many more experimental results have established that CP invariance is indeed violated. The present day experimental evidence is expressed by the ratio between the transition amplitudes for the decay of the long lived K-meson into two pions, $M(K_L^0 \rightarrow \pi^+\pi^-)$ and $M(K_L^0 \rightarrow \pi^0\pi^0)$ and the corresponding transition amplitudes for the decay of the short lived K-meson $M(K_S^0 \rightarrow \pi^+\pi^-)$ and $M(K_S^0 \rightarrow \pi^0\pi^0)$. Both ratios ought to be zero if CP is conserved but in fact one has (Cronin (1968)) Particle data Group (1970) and Chollet et. al. (1969)) for

$$\eta^{+} = \frac{M(K_L^0 \rightarrow \pi^+\pi^-)}{M(K_S^0 \rightarrow \pi^+\pi^-)} = |\eta^{+-}|\, e^{i\varphi^{+-}}.$$

and

$$\eta^{00} = \frac{M(K_L^0 \rightarrow \pi^0\pi^0)}{M(K_S^0 \rightarrow \pi^0\pi^0)} = |\eta^{00}|\, e^{i\varphi^{00}}$$

the values

$$|\eta^{+-}| = (1.90 \pm 0.05) \times 10^{-3} \qquad \varphi^{+-} = (43.5 \pm 5.1)^0$$

$$|\eta^{00}| = (2.5 \pm 0.3) \times 10^{-3} \qquad \varphi^{00} = (23 \pm 32)^0$$

Several theories have been put forward to explain this observed CP violation in the decay of the long lived neutral K-meson. Some of these will now be very briefly mentioned.

(i) Superweak interaction (Wolfenstein (1964)).

This theory assumes that CP violation is due to a first order effect of a CP violating term in the Hamiltonian with a selection rule $|\Delta Y| = 2$. The coupling constant F of such an interaction turns out to be of the order 10^{-8}G where G is the coupling constant for the weak interaction.

(ii) Milliweak Theory.

There are several theories which assume that the CP invariance violating part of the Hamiltonian has a coupling constant 10^{-3}G and therefore are called milliweak. Among these the theories of Glashow (1965), Oakes (1968) and Das (1968) are small modifications of the usual current-current theory of the weak interaction.

(iii) Semiweak Theory.

This theory proposed by Nishijima and Swank (1966 and 1967) is a radical modification of the usual current-current theory of non leptonic weak interaction. The coupling constant of the proposed Hamiltonian is of the order of magnitude of the usual weak interaction strength. However, C. P. violating processes occur only in third order in perturbation theory and therefore the theory accounts for the smallness of the observed C. P. violation.

(iv) Electromagnetic Theory.

This theory put forward by Bernstein et. al. (1965) and Lee (1965a,b) assumes that C. P. invariance is violated by the electromagnetic interaction. Since there is good experimental evidence that P is conserved by the electromagnetic interaction these authors assume that the total electromagnetic current is composed of a normal part j_μ (odd under C) and a C-even part K_μ which would produce CP violation.

Among the proposed theories described above the assumption that CP invariance violation occurs in the electromagnetic interaction is more likely to produce a bigger effect in nuclear physics than the other proposals (for example $|\Delta Y| = 2$ effects would be quite undetectable).

Among the possible tests in low energy nuclear physics this thesis concentrates on those which aim to discover a small imaginary part in the ratio between the reduced matrix elements of two competing multipoles in a given electromagnetic transition. In fact it was shown by Lloyd (1951) that if Time Reversal Invariance (TRI) holds then this ratio must be real to first order in the electromagnetic interaction. On the other hand if TRI (TRI and CP invariance are the same if the PCT theorem is assumed to be true) is violated in the electromagnetic interaction or in the nucleon-nucleon interaction a small imaginary part appears.

The observable effects of an imaginary part on the ratio between reduced matrix elements of competing multipoles in a given electromagnetic transition was first worked out by Henley and Jacobson (1958) and in more detailed form by Lobov (1969). These authors show that the effect appears in the form of T. R. invariance violating asymetries in appropriate angular correlation and polarisation sensitive experiments. The reader is refered to the work of Henley (1969) for a comprehensive review of these effects. In this thesis only one of the possible experiments is examined and is described in Chapter 4. This experiment involves the measurement of the angular correlation between two γ-rays from an oriented ensemble of nuclei. This particular experiment was chosen because it is likely to be the most sensitive (Hamilton (1971)) and also because it is under study experimentally in this university.

1.2 Content of the thesis

In this section the contents of the thesis will be described. Since this thesis is concerned with several different features of time reversal invariance (T.R.I.) violation this section is intended as a guide to the contents.

In Chapter 2 the electromagnetic current of nucleons is examined with the intention of constructing a part K_μ which is T.R.I. violating (or CP invariance violating by PCT). The matrix elements $< N | K_\mu | N' >$ of such a current between nucleon states have been examined before by several authors: Bincer (1960), Lipshutz (1968) and Huffman (1970). From the results of the last named author two alternative forms for the matrix elements of K_μ are taken. Both forms satisfy the properties usually required for an acceptable electromagnetic current. Parity conservation, hermiticity and Gauge Invariance.

Both forms of $< N | K_\mu | N' >$ (and indeed any possible form) vanish when both the initial and final nucleons are on the mass shell. This feature turns out to be very important since because of this it follows that K_μ can only contribute to low energy nuclear physics when other nucleons are involved, i.e. as an "exchange effect", in the form of a two body short range transition operator. Therefore K_μ is only likely to contribute a few per cent of the total transition probability.

Two different T.R.I. transition violating operators are derived corresponding to the two alternative forms for $< N | K_\mu | N' >$ and are subsequently expanded in multipoles. An estimate of the effect in nuclear physics is postponed to Chapter 4.

To end Chapter 2 an estimate is made of the magnitude of the T.R.I. violating force between nucleons deriving from K_μ. Both two and three body potential operators are derived. The two body operator is shown to be negligible compared to the transition operators mentioned before and the three body forces.

In Chapter 3 the violation of T.R.I. is assumed to occur in the vertex where the nucleon resonance N* (J = 3/2, T = 3/2, M = 1236) (that is isospin and spin equals 3/2 and has a mass M = 1236 MeV) goes into a nucleon with the emission of a γ-ray. Again two body transition operators corresponding to this model are derived and expanded in multipoles. Two and three body forces are also calculated. The calculation of the effect of transition operators is postponed to Chapter 4.

In Chapter 4 the effect of the transition operators is examined. The operators derived in Chapter 2 are considered separately from those derived in Chapter 3. A general estimate is given for the operators derived in Chapter 2 and the effect is found to be too small to be detected with present day experimental techniques. A more detailed calculation of the effect in a particular transition in ^{18}F is given for the operators derived in Chapter 3. A measurable effect is found.

As mentioned above, all the models of T.R.I. violation treated in this thesis give a T.R.I. violating contribution to the force between the nucleons in the form of a three body potential. No detailed calculation of this effect is presented in this thesis. It is however shown that their effect is smaller than the effect of the transition operators for light nuclei and therefore unlikely to have much effect on the conclusions of Chapter 4. They are however likely to be the dominating contribution in heavy nuclei and this will make the calculation of T.R.I. violating effects very difficult for these nuclei.

Finally in Chapter 5 an experiment performed by Holmes (1972) in Pt^{192} is analysed. Because of the fact that microscopic wave function for Pt^{192} are not available, the experiment is analysed in terms of a phenomenological two body T.R.I. violating force which is assumed to include three body effects.

First the most general form of a two body T.R.I. violating potential is derived. It is then noted that all terms in this potential are velocity dependent so that the overall Hamiltonian is not gauge invariant unless further terms are introduced. It is shown by an example that these terms can be very important. It is further shown that it is not possible, having only a two body phenomenological potential to write down unambiguously the complete gauge invariant Hamiltonian. The Siegert theorem is therefore used to calculate the contribution of these unknown parts to the electric multipoles and thus permits us to relate the observed effect to one matrix element of the chosen T.R.I. violating interaction.

By means of an averaging procedure this matrix element is evaluated and so an approximate value for the upper limit of the coupling constant of the T.R.I. violating interaction is deduced.

1.3 Content of the Appendix

A great deal of the work presented in this thesis has been separated from the main body and put in the appendix. About half of the appendix consists of detailed calculation of the results presented in the text. The other half is described below.

In Appendix 4 the Siegert theorem is derived following Sachs and Austern (1951) as a consequence of Gauge Invariance. This theorem is usually stated as forbidding the contribution of exchange effects to the electric multipoles in a given electromagnetic transition. This usage as a simplifying tool is examined critically and found to be somewhat misleading. The work by Michell (1965) on parity violating potentials is used as an example in this connection.

In recent years (see Green and Schucan (1971) for a review) the role of nucleon resonances like the $N^*(J = 3/2,\ T = 3/2,\ M = 1236)$ in nuclear physics has been seriously considered. Inevitably several different approaches have been proposed and one of these is used in Chapter 3 to calculate the effects of a possible failure of T.R.I. in the $N^*N\gamma$ vertex. In Appendix 5 the alternative approach of treating the N^* explicitly in the nuclear wave function is outlined. This approach is compared with the one used in Chapter 3.

Finally Appendix 8 presents some more complete formulae of T-violating angular correlations and Appendix 9 examines the problem of making an arbitrary potential gauge invariant.

CHAPTER 2

TWO SIMPLE FORMS OF A TRI VIOLATING CURRENT

2.1 Introduction

It is well known that because of the strong interaction the matrix element of the electromagnetic current between nucleons is modified by the introduction of form factors. In the first section of this chapter the possibility of introducing T.R.I . violation by use of appropriate form factors is examined.

The results of section 2.2 are fully relativistic and so cannot be used in nuclear physics. In section 2.3 a method for deriving non-relativistic operators from the covariant results of section 2.2 is given. In particular it is found that T.R.V. terms in the electromagnetic current, can only contribute as an "exchange effect" and this in turn implies that the effect of these terms is just a few percent of the normal part of the current.

In section 3, the methods described in section 2 is applied to two possible forms for the T.R.I. violating electromagnetic current introduced in section 1 and two body T.R.V. transition operators are calculated. Finally in section 4 two and three body operators, contributing to the interaction between the nucleons is derived.

Specific calculations of the effect of the operators derived in this chapter are postponed until Chapter 4.

2.1a General Considerations

The usual form for the matrix elements of the electromagnetic current between two nuclear states is (Drell and Zachariasen 1960)

$$<p_1 | J_\mu^{e.m.}(0) | p_2> = e\, (\bar{u}_{p_1} | i F_1(q^2)\gamma_\mu - i F_2(q^2)\sigma_{\mu\nu} q_\nu | u_{p_2}) \tag{2.1}$$

where $F_i(q^2) = F_i^s(q^2) + F_i^v(q^2)\,\tau^z$ $(i = 1,2)$ are the usual form factors, τ^z is the z component of the i-spin Pauli matrix and $q^2 = (p_1 - p_2)^2$.

This is in fact the most general form assuming besides T.R. invariance, Parity invariance, current conservation and that the initial and final nucleons are on the mass shell.

If one assumes that both T.R. and Parity invariance are violated but the initial and final nucleons are still on the mass shell, the matrix elements are complicated by the appearance of two further terms. The electric dipole moment and the anapole moment (Broadhurst 1971)

$$\begin{aligned} <p_1 | J_\mu^{e.m.}(0) | p_2> = e\, (\bar{u}_{p_1} | \; & i F_1(q^2)\gamma_\mu - i F_2(q^2)\sigma_{\mu\nu} q_\nu + F_4(q^2)\sigma_{\mu\nu} q_\nu \gamma_5 + \\ & + F_5(q^2)\,(q_\mu + \frac{q^2 \gamma_\mu}{2m})\,\gamma_5 | u(p_1)) \end{aligned} \tag{2.2}$$

where F_4 and F_5 are additional form factors in the notation of Broadhurst. If one assumes parity conservation (a well established fact for the electromagnetic interaction) then both the additional terms vanish. Therefore one can conclude that terms in the electromagnetic current matrix elements that violate T but conserve P can only appear if at least one of the nucleons is off the mass shell.

The most general expression for the matrix elements of the current satisfying only covariance and with both nucleons off the mass shell have been given by Bincer (1960) and Lipshutz (1967). It contains 24 terms but if Parity conservation is assumed then there are only 12 terms. These 12 terms further reduce to only six if just one of the nucleons is assumed off the mass shell. The twelve terms have to satisfy hermiticity and the Ward identity (Gauge Invariance). This last requirement gives a complicated relationship among most of the twelve terms. To simplify the Ward identity Huffmann (1970) took only three terms and constructed a current which has both TRI normal and violating terms. These terms will be taken as a starting point for this Chapter and the reader is refered to Huffmann (1970) for details. The matrix elements for the T.R.I. normal current J_μ is

$$<p'|J_\mu|p> = (\bar{u}_{p'}|F_1\, i\gamma_\mu - i\, F_2\, \sigma_{\mu\nu}\, q_\nu - i\,[1 - F_1(q^2)]\, \frac{\not{q}\, q_\mu}{q^2}\, |u_p) \tag{2.3}$$

The matrix elements for the T.R.I. violating current K_μ is

$$\begin{aligned} <p'|K_\mu|p> = {} & (\bar{u}_{p'}|F_1'(q^2)(\gamma_\mu P\cdot q - \not{q}\, P_\mu)|u_p) + \\ & + (\bar{u}_{p'}|(p'^2 - p^2)\, F_2'(q^2)\, \sigma_{\mu\nu}\, q_\nu\, |u_p) + \\ & + (\bar{u}_{p'}|-(p'^2 - p^2)\, F_3'(q^2)\, (q_\mu - \frac{q^2 P_\mu}{P\cdot q})\,|u_p) \end{aligned} \tag{2.4}$$

The notation is $P = (p' + p)$ $q = (p' - p)$ and the dash in the form factors of $<p'|J^{TRV}|p>$ indicates that these are different functions from the T-normal form factors. The form factor $F_i(q^2)$ is of course $F_i(q^2) = F_i^S(q^2) + F_i^V(q^2)\,\tau_z$.

2.2 Derivation of the two body electromagnetic transition operators

The two body transition operators will now be derived by using the S-matrix method. This method has been explained in detail (Chemtob (1968), Chemtob (1969), Chemtob and Rho (1971) and Tadic and Fischback (1971)) and therefore will not be repeated here. However, in order to illuminate the calculations which follow, an intuitive discussion of the method will be given following the exposition of Akhiezer and Berestetskii (1965).

The problem is how to derive an operator from field theory for use in non relativistic calculations. For simplicity a system of electrons is taken and the interaction between the electromagnetic field and the electron current is taken, of course, to be

$$\mathcal{H} = \int - ie \left[\bar{\psi}(x_1)\, \gamma_\mu\, \psi(x_1) \right] A_\mu\, (x_1)\, d^4\, x_1$$

The first approximation to the S-matrix is then

$$S_1 = - e \int d^4 x_1\, \bar{\psi}_{p'}\, (x_1)\, \gamma_\mu\, \psi_p\, (x_1)\, A_\mu\, (x_1)$$

Integration in time gives

$$S_1 = - e \int d^3\, r_1\, \psi_{p'}\, (r_1)\, \gamma_\mu\, \psi_p\, (r_1)\, A_\mu (r_1)\, 2\pi\, \delta\, (\epsilon_{p'} + \omega - \epsilon_p\,) =$$

$$= - 2\pi i\, \delta\, (\quad)\, \left[- ie \int d^3\, r_1\, \psi_{p'} (r_1)\, \gamma_\mu\, \psi_p\, (r_1)\, A_\mu\, (r_1) \right]$$

The "effective transition operator" U_{if} is the defined to be

$$U_{if} = - ie \int d^3 r_1\, \psi_{p'} (r_1)\, \gamma_\mu\, \psi_p (r_1)\, A_\mu\, (r_1)$$

Finally the non-relativistic limit is taken by using the Dirac representation for the γ matrices and separating the spinors into large and small components.

The result is of course (Spacial Component)

$$U_{if} = \frac{-e}{2m} \int d^3 r_1 \, \psi_L^* (r_1) \{ [\vec{A}(r_1) \cdot \vec{p} + \vec{p} \cdot \vec{A}(r_1)] + \sigma \cdot (\nabla \times \vec{A}(r_1)) \} \psi_L (r_1$$

from which the usual operator is extracted

$$V_{elet.} (\vec{A}) = -\frac{e}{2m} [\vec{A}(r_1) \cdot \vec{p}_1 + \vec{p}_1 \cdot \vec{A}(r_1)] \frac{e}{2m} (\nabla \times \vec{A}(r_1)) \qquad (2.5)$$

This method has been used frequently in the past. For example, Blin-Stoyle and Nair (1966) used this method to extract the effective β-operator from the most general on shell form of the weak current.

If instead of electrons, nucleons are considered, then from the first term in (2.1) it follows, bearing in mind that $F_1(q^2) = F_1{}^S(q^2) + F_1{}^V(q^2)\,\tau_z$, that

$$V_{nucl.} (\vec{A}) = \frac{+e}{2m} [\vec{A}(r_1) \cdot p_1 + p_1 \cdot \vec{A}(r_1)] (F_1{}^S(0) + F_1{}^V(0)\,\tau_z)$$

$$+ \frac{e}{2m} [F_1{}^S(0) + F_1{}^V(0)\tau_z] \, \sigma \cdot \vec{B}(r_1) \qquad (2.6)$$

Consider now the problem of using this operator to calculate say Bremstralung due to the collision of two nucleons.

Before solving this problem it is convenient first to solve the problem of elastic scattering of two nucleons. In nuclear physics this is done by finding the scattering solutions of the Hamiltonian

$$H = \sum_{i=1}^{2} \frac{p_i^2}{2m} + U(r_{12}) \tag{2.7}$$

The operator $U(r_{12})$ should be obtained from field theory by reducing non relativistically the matrix element U_{if} obtained from the diagram of fig. 1, by means of

$$S_{if} = -2\pi i \;\delta\;(\text{Energ.})\;U_{if}$$

The diagram is considered as a pole diagram and therefore U_{if} is the one pion exchange potential (see e. g. Berestetskii andAhkiezer 1965 p509)

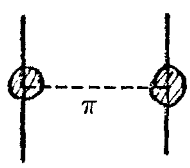


Fig. 1

The non relativistic potential $U(r_{if})$ is represented diagramatically by Fig. 2

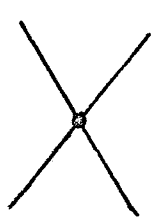


Fig. 2

Now calling $\psi_f^{(-)}$ and $\psi_i^{(+)}$ the scattering solution of (7) the relevant matrix element for Bremstralung is (Goldberger and Watson 1964 pp. 202-209)

$$M = < \psi_f^{(-)} | V_{nucl.}(\vec{A}) | \psi_i^{(+)} >$$

This is equivalent to calculating the graphs in old fashioned pertubation theory of Fig. 3 below

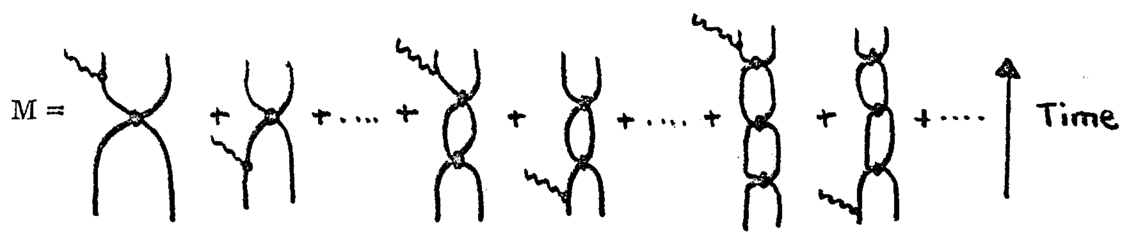


Fig. 3

Is this however a complete solution? If the Bremstralung calculation were to be carried out by old fashioned pertubation theory in a pure field theoretical framework there would be many more graphs not included in the procedure just described. For example the graphs of Fig. 4

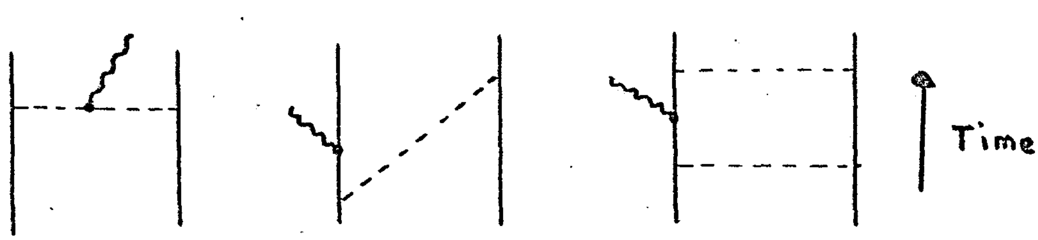


Fig. 4

These graphs constitute what are called exchange effects. The usual way to deal with them is to extract, in the non relativistic limit, an equivalent transition operator for each of them.

Now if the procedure leading to $V_{nucl.}(\vec{A})$ in equation (6) is applied to a vertex like the ones in equation (4) that vanish when both nucleons are on the mass shell, it is easy to see that one would get zero in the non relativistic limit. The conclusion is that there is no one body T.R.V. transition operator. It is therefore necessary to consider exchange graphs from which one can then extract two body T.R.V. transition operators. Because of the short range two body nature of these operators their effect is expected to be just a few percent of the usual one body operators. So it seems justified to expect only small effects from these vertices in nuclear physics as predicted by Bernstein et. al. (1965).

2.3 Calculation of the two body T.R.V. transition operators

2.3.1 The Lee Vertex

In this section the non relativistic transition operators corresponding to the first term of equation 2.4 will be calculated. A vertex of this type has the theoretical appeal that it arises naturally (Huffmann 1970) from a more fundamental theory of T.R.I. violation such as the theory by Lee (1965-b) of T.R.I. violating "a" particles. To stress this point and also for the sake of notational simplicity the notation used in equation 2.4 will be modified. The form factor $F_1'(q^2) = F_1'^{s}(q^2) + F_1'^{v}(q^2)\tau_z$ will be called $F'_{Lee}(q^2) = F'^{s}_{Lee}(q^2) + F'^{v}_{Lee}(q^2)\tau_z$ and all the operators arising from this vertex will have a sufix "Lee" so the first term of the equation 2.4 is written

$$\langle p' | (K_\mu(0))_{Lee} | p \rangle = \langle \bar{u}_{p'} | F'_{Lee}(q^2)\, (\gamma_\mu P \cdot q - \not{q} P_\mu) | u_p \rangle \tag{2.8}$$

The matrix elements corresponding to the graphs of Fig. 5 are calculated by applying the usual Feynman rules

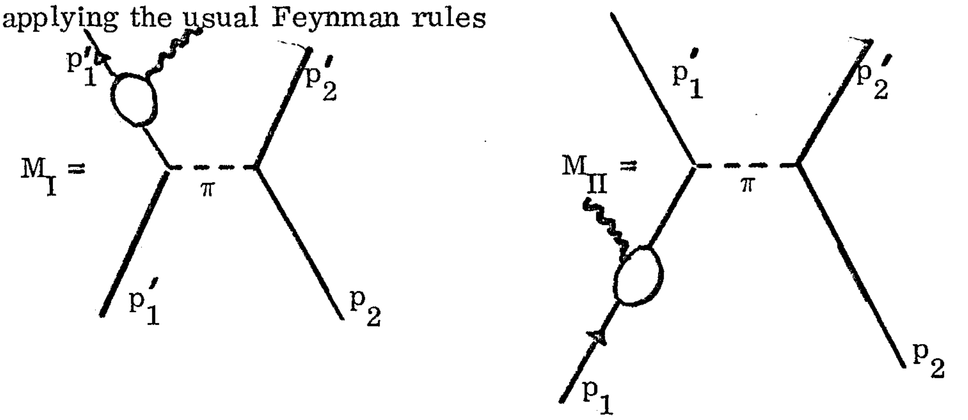


Fig. 5

Following Tadic and Fischbach (1972) the calculation is carried out from the outset in co-ordinate space. Alternatively one could (Chemtob and Rho - 1971) carry out all the calculations in momentum space and transform back to co-ordinate space at the end of the calculation.

So the matrix element given by equation 2.8 is written in co-ordinate space. By using

$$\bar{\psi}_{p_1'}(x_1)\overleftarrow{\partial}_\nu = -i p'_{1\nu} \bar{\psi}_{p_1'} \quad \overrightarrow{\partial}_\nu \psi_{p_1}(x_1) = i p_1 \psi_{p_1} \quad \text{and} \quad \partial_\nu A(x_1) = -i k_\nu A(x_1)$$

there results

$$<p'\,|\,J_\mu \cdot A_\mu\,|\,p> = -\,F'_{\mathrm{Lee}}(0)\;\bar{\psi}_{p_1'}(x_1)\,\gamma_\mu\left(-\frac{\overleftarrow{\partial}}{\partial x_{1\rho}}+\frac{\overrightarrow{\partial}}{\partial x_{1\rho}}\right)\psi_{p_1}(x_1)\left[\frac{\partial A_\rho}{\partial x_{1\mu}}-\frac{\partial A_\mu}{\partial x_{1\rho}}\right]$$

The form factor $F'_{\mathrm{Lee}}(q^2)$ is of course not known and was approximated by its value at $q^2 = 0$.

Using as interaction between the pions and the nucleons the form

$$\mathcal{L}_I = i\,f\;\bar{\psi}(x_1)\,\gamma_5\,\vec{\tau}\,\psi(x)\cdot\vec{\phi}(x)$$

the matrix element corresponding to the graphs of Fig. 5 can be written as follows

$$M_I = if^2\iiint d^4x_1\,d^4x_2\,d^4x_3\,\{[\bar{\psi}(x_1)\gamma_\mu F_1'(0)\left(-\frac{\overleftarrow{\partial}}{\partial x_{1\rho}}+\frac{\overrightarrow{\partial}}{\partial x_{1\rho}}\right)S_F(x_1-x_2)\gamma_5\,\psi(x_2)]$$
$$\left[\frac{\partial A_\rho}{\partial x_{1\mu}}-\frac{\partial A_\mu}{\partial x_{1\rho}}\right]\Delta_F(x_2-x_3)\,[\bar{\psi}(x_3)\gamma_5\,\psi(x_3)]\,[F'^{\,s}_{\mathrm{Lee}}+F'^{\,v}_{\mathrm{Lee}}\,\tau_z^{(1)}]\,\tau_{(1)}\cdot\tau_{(2)} \tag{2.9}$$

and

$$M_{II} = if^2\iiint d^4x_1\,d^4x_2\,d^4x_3\,\{[\bar{\psi}(x_3)\gamma_5 S_F(x_1-x_2)\left(-\frac{\overleftarrow{\partial}}{\partial x_{1\rho}}+\frac{\overrightarrow{\partial}}{\partial x_{1\rho}}\right)\gamma_\mu\,\psi(x_1)\,[$$
$$\left[\frac{\partial A_\rho}{\partial x_{1\mu}}-\frac{\partial A_\mu}{\partial x_{1\rho}}\right]\Delta_F(x_2-x_3)\,[\bar{\psi}(x_3)\gamma_5\,\psi(x_3)]\,\tau_{(1)}\cdot\tau_{(2)}\,[F'^{\,s}_{\mathrm{Lee}}+F'^{\,v}_{\mathrm{Lee}}\,\tau_z^{(1)}] \tag{2.10}$$

The isospin factors have been written outside the wave functions for the sake of notational simplicity. Of course they should be written inside the brackets in an obvious manner.

After the manipulation described in Appendix 2-A, the following T.R.I. violating two body transition operator is extracted

$$V^{TRV\vec{E}}_{Lee}(\vec{E}) = \frac{f^2 F'^{\,v}_{Lee}}{m} \sigma_1 \cdot \vec{E}(r_1)\, \sigma_2 \cdot (\vec{r}_1 - \vec{r}_2)\, (\tau_{(1)} \times \tau_{(2)})_z\, \mathcal{Y}(r_{12}) + (1 \leftrightarrow 2) \qquad (2.11)$$

where $\vec{E}(r_1)$ is the electric field and $\mathcal{Y}(r_{12}) = -\frac{1}{4\pi} \left[\frac{\mu}{|r_1 - r_2|^2} + \frac{1}{|r_1 - r_2|^3}\right] e^{-\mu|r_1 - r_2|}$

The transition operator of equation (2.11) can now be expanded in multipoles by the procedure described in Appendix III. The contribution to the electric multipoles is

$$(EL)^{TRV}_{Lee} = \frac{f^2 F'^{\,v}_{Lee}}{m} \left(\frac{4\pi}{3}\right)^{\frac{1}{2}} 2\left[L(2L+1)\right]^{\frac{1}{2}} \sum_{i<j} (\tau_{(i)} \times \tau_{(j)})_z \frac{d}{dr_{ij}} \left(\frac{\exp(-\mu r_{ij})}{r_{ij}}\right)$$

$$\left\{\frac{1}{3} \sigma_i \cdot \sigma_j R^{L-1}_{ij} \left[Y_{L-1}(R_{ij}) \otimes Y_1(r_{ij})\right]^{(L)}_M - \sum_{L'} \left[5(2L'+1)\right]^{\frac{1}{2}} \begin{Bmatrix} L' & 2 & L \\ 1 & L-1 & 1 \end{Bmatrix} R^{L-1}_{ij}\right.$$

$$\left.\left[\left[Y_{L-1}(R_{ij}) \otimes Y_1(r_{ij})\right]^{(L')} \otimes \left[\sigma_i \otimes \sigma_j\right]^{(2)}\right]^{(L)*}_M \right\} \qquad (2.12)$$

$$(ML)^{TRV}_{Lee} = -\frac{f^2 F'^{\,v}_{Lee}}{m}\, 2 \left(\frac{4\pi}{3}\right)^{\frac{1}{2}} K \left(\frac{L}{L+1}\right)^{\frac{1}{2}} \sum_{i<j} (\tau_{(i)} \times \tau_{(j)})_z \frac{d}{dr_{ij}} \left(\frac{\exp(-\mu r_{ij})}{r_{ij}}\right)$$

$$\left\{\frac{1}{3} R^L_{ij}\, \underline{\sigma}_i \cdot \underline{\sigma}_j \left[Y_L(R_{ij}) \otimes Y_1(r_{ij})\right]^{(L)*}_M + R^L_{ij} \sum_{L'} \left[5(2L'+1)\right]^{\frac{1}{2}} \begin{Bmatrix} L' & 2 & L \\ 1 & L & 1 \end{Bmatrix}\right.$$

$$\left[\left[Y_L(R_{ij}) \otimes Y_1(r_{ij})\right]^{(L')} \otimes \left[\sigma_i \otimes \sigma_j\right]^{(2)}\right]^{(L)*}_M + \left[\frac{(2L+1)^2 L}{16\pi}\right]^{\frac{1}{2}} \begin{Bmatrix} L-1 & 1 & L \\ 1 & L & 1 \end{Bmatrix}$$

$$\left. r_{ij} R^{L-1}_{ij} \left[Y_{L-1}(R_{ij}) \otimes \left[\sigma_i \otimes \sigma_j\right]^{(1)}\right]^{(L)*}_M \right\} \qquad (2.13)$$

It should be noted at this point that the transition operator of equation (2.11) is exactly the same (apart from constants) as the one obtained by Clement and Heller (1971), although they use a quite different approach. One can in fact at this stage take over almost all the conclusions of this work as far as the consequences of this particular transition operator are concerned. These conclusions together with an analysis of the implications of the transition operators to be derived in the next chapter will be given in Chapter 4.

To finish this section two further points will now be remarked upon. The first one is concerned with the Siegert theorem. One might think that the Siegert theorem, which prohibits exchange effects in the electric multipoles is violated by a contribution such as the operator in equation (2.12). The Siegert theorem is discussed in Appendix 4 where it is shown that although these exchange contributions are small they do not vanish identically. The second point is much more important and concerns the possible effects of two and three body T.R.V. potentials, due to the vertex we have been discussing. This matter will be discussed in the fourth section of this chapter.

2.3-2 The Lipshutz vertex

In this subsection the transition operators arising from the second term of equation 2.4 will be calculated. A vertex of this type has been discussed by Lipshutz (1967) in his analysis of possible T.R.I. violating effects in proton Compton scattering. The notation used in equation 2.4 will therefore be modified to stress this point. The form factor $F_2'(q^2) = F_1'^S(q^2) + F_2'^V(q^2)\tau_z$ will be called

$F'_{Lip}(q^2) = F'^{S}_{Lip}(q^2) + F'^{V}_{Lip}(q^2)\,\tau_z$ and all the operators arising from this vertex will have a sufix "Lip". So the second term of equation 2.4 is written

$$<p'|\,(K_\mu(0))_{Lip}\,|p> = <u_{p'}|\,F'_{Lip}(q^2)\,(p'^2 - p^2)\,\sigma_{\mu\nu}q_\nu\,|u_p) \qquad (2.14)$$

Using the same procedure described in 3.1 the matrix elements corresponding to the graphs of Fig. 5 can be written

$$M_I = f^2 \iiint d^4x_1 d^4x_2 d^4x_3 [\bar{\psi}(x_1)(-\overleftarrow{\partial}^2 + \overrightarrow{\partial})\,\sigma_{\mu\nu} S_F(x_1-x_2)\gamma_5\psi(x_2)]$$

$$\left(\frac{\partial A_\mu(x_1)}{\partial x_{1\nu}}\right) \Delta_F(x_2-x_3)\,[\bar{\psi}(x_3)\gamma_5\,\psi(x_3)]\,[F'^{S}_{Lip} + F'^{V}_{Lip}\tau_z^{(1)}]\,\tau_{(1)}\cdot\tau_{(2)} \qquad (2.15)$$

$$M_{II} = f^2 \iiint d^4x_1 d^4x_2 d^4x_3 [\bar{\psi}(x_1)\gamma_5 S_F(x_2-x_1)(-\overleftarrow{\partial}^2 + \overrightarrow{\partial}^2)\,\sigma_{\mu\nu}\,\psi(x_1)]$$

$$\left(\frac{\partial A_\mu(x_1)}{\partial x_{1\nu}}\right) \Delta_F(x_2-x_3)\,[\bar{\psi}(x_3)\gamma_5\psi(x_3)]\,\tau_{(1)}\cdot\tau_{(2)}\,[F'^{S}_{Lip} + F'^{V}_{Lip}\tau_z^{(1)}] \qquad (2.16)$$

After the manipulations outlined in Appendix 2-B, one can extract from the matrix elements (14) and (15) the following leading term.

$$V_{Lipshutz}(\vec{B}) = \frac{2f^2}{4m}\,[\vec{p}_1\cdot\vec{B}(\vec{r}_1)\,\sigma_2\cdot(\vec{r}_2-\vec{r}_1)\,\mathcal{G}(r_{21}) + \sigma_2\cdot(\vec{r}_2-\vec{r}_1)\,\mathcal{G}'(r_{21})\,B(\vec{r}_1)\cdot\vec{p}_1]$$

$$\{F'^{S}_{Lip}\,\tau_{(1)}\cdot\tau_{(2)} + F'^{V}_{Lip}\,\tau_z^{2}\} + (2 \longrightarrow 1) +$$

$$+\frac{2f^2}{4m}\,i\,[\sigma_1\cdot(\vec{p}_1 x\vec{B}(r_1))\,\sigma_2\cdot(\vec{r}_2-\vec{r}_1)\,\mathcal{G} + \sigma_2\cdot(\vec{r}_2-\vec{r}_1)\,\mathcal{G}'\sigma_1\cdot(B(\vec{r}_1)\;x\,\vec{p}_1)]$$

$$\{F'^{S}_{Lip}\,\tau_{(1)}\cdot\tau_{(2)} + F'^{V}_{Lip}\,\tau_z^{(2)}\} + (2 \longrightarrow 1) +$$

$$+\frac{f^2 F'^{v}_{Lip}}{4m}\left[\vec{p}_1\cdot\vec{B}(\vec{r}_1)\,\sigma_2\cdot(\vec{r}_2-\vec{r}_1)\mathcal{Y}(r_{12})-\sigma_2\cdot(\vec{r}_2-\vec{r}_1)\mathcal{Y}(r_{12})\vec{B}(\vec{r}_1)\cdot\vec{p}_1\right]\frac{(-i)}{2}\,(\tau_{(1)}\times\tau_{(2)})_z$$

$$+\frac{f^2 F'^{v}_{Lip}}{4m}\left[\sigma_1\cdot(\vec{p}_1\times\vec{B}(r_1))\,\sigma_2\cdot(\vec{r}_2-\vec{r}_1)\mathcal{Y}(r_{12})-\sigma_2\cdot(\vec{r}_2-\vec{r}_1)\mathcal{Y}(r_{12})\,\sigma_1\cdot(\vec{B}(\vec{r}_1)\times\vec{p}_1)\right]$$

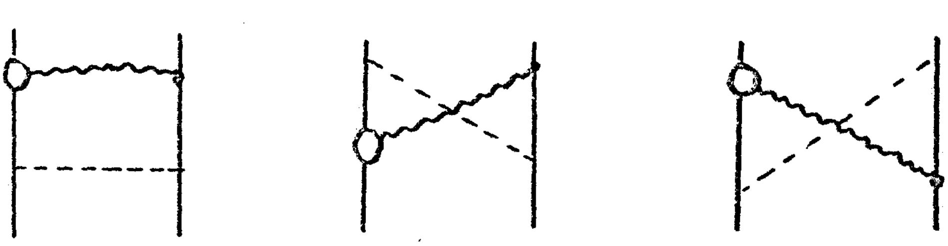
$$\tfrac{1}{2}(\tau_{(1)}\times\tau_{(2)})_z + (1\rightarrow 2) \qquad (2.17)$$

Again by using the techniques described in Appendix 3 it is easy to expand the operators of equation (2.17) in multipoles. However, no further use will be made of the operators in equation 2.17 in this thesis. This is for two reasons. Firstly because there is no detailed theory that predicts matrix elements of the form (2.14) and secondly because all the operators are seen to depend on p_1, that is, velocity dependent and therefore their effect is expected to be small compared with the operator given by equation 2.2. Since no use will be made of 2.17 the very lengthy expressions for the multipole operators will be omitted.

2.4 Two and three body operators

In this section the possibility of two and three body T.R.V. forces arising from T.R.I. violation in the electro-magnetic interaction is discussed. A calculation of two body forces is given in a paper by Huffmann (1970).

Two body forces arise from a large set of graphs a selection of which is given below in Fig. 6.



(a) (b) (c)

Fig. 6

In those graphs the bubble vertex represents the T.R.V. vertex and the point on the other end of the virtual photon is the normal electromagnetic vertex. Because of the extra e in this normal vertex, it is possible at least as a first approximation to assume their effect to be small compared with the effect of the transition operators derived earlier in this chapter.

The three body potential arises from the graphs of Fig. 7

(a) (b)

Fig. 7.

To derive the three body potential from the graphs the method of Clement and Heller (1971) will be followed. Using this method it is trivial to derive the three body operators from the transition operators already presented.

Firstly the case where the bubble in Fig. 7 represents the Lee vertex given by equation 2. 8 will be considered. The three body operator is obtained from the transition operator $V_{Lee}^{T.R.V.}(E)$ in equation 2.11 by replacing E (r_1) by

$$\vec{E}(\vec{r}_1) = \frac{e(\vec{r}_1-\vec{r}_3)}{|\vec{r}_1-\vec{r}_3|^3} \quad \tfrac{1}{2}(1+\tau^z_{(3)}) \qquad (2.18)$$

The right hand side of equation 2.18 is of course the electric field $\vec{E}$ produced by the third nucleon in the position of the first one. By doing this replacement there results

$$V_{Lee}^{T.R.V.}(r_1 r_2 r_3) = \frac{f^2 F'^{\,v}_{Lee}}{m} \; \frac{e}{2}\,(1+\tau^z_{(3)})\,\sigma_1 \cdot \frac{\vec{r}_1-\vec{r}_3}{|\vec{r}_1-\vec{r}_3|^3}\,\sigma_2 \cdot (\vec{r}_1-\vec{r}_2)\,(\tau_{(1)} \times \tau_{(2)})_3 \mathcal{Y}(r_{12})$$

$$+ (1 \longrightarrow 2) \tag{2.19}$$

In the case where the bubble in Fig. 7 represents the Lipshutz vertex given by equation 2.14, the three body operators are obtained by replacing $\vec{B}(r_1)$ in equation 2.17 by

$$\vec{B}(r_1) = \frac{e}{m}\;\left[3\,\frac{\mu\,\sigma_3 \cdot (\vec{r}_1-\vec{r}_3)}{|\vec{r}_1-\vec{r}_3|^5}\,(\vec{r}_1-\vec{r}_3) - \mu\,\frac{\vec{\sigma}_3}{|r_1-r_3|^3}\right] \tag{2.20}$$

where

$$\mu = \tfrac{1}{2}\,(\mu_\mu + \mu_p) - \tfrac{1}{2}\,(\mu_\mu - \mu_p)\;\tau^z_{(3)} \tag{2.21}$$

Equation 2.20 gives the magnetic field produced by the third nucleon in the position of the first.

By doing the replacement indicated above there results

$$V_{Lipshutz}^{T.R.V.}(r_1 r_2 r_3) = \frac{2f^2}{4m^2}\,e\,[\,\vec{p}_1 \cdot \left(3\,\frac{\mu\,\sigma_3 \cdot (\vec{r}_1-\vec{r}_3)}{|\vec{r}_1-\vec{r}_3|^5}\,(\vec{r}_1-\vec{r}_3) - \mu\,\frac{\vec{\sigma}_3}{|r_1-r_3|^3}\right)\sigma_2 \cdot (\vec{r}_2-\vec{r}_1)\mathcal{Y}(r_{12})$$

$$+\,\sigma_2 \cdot (\vec{r}_2-\vec{r}_1)\mathcal{Y}(r_{12})\,\left(3\,\frac{\mu\,\sigma_3 \cdot (\vec{r}_1-\vec{r}_3)}{|\vec{r}_1-\vec{r}_3|^5}\,(\vec{r}_1-\vec{r}_3) - \mu\,\frac{\vec{\sigma}_3}{|\vec{r}_1-\vec{r}_3|^3}\right)\cdot \vec{p}_1\,]\,\{F'^{\,s}_{Lip}\,\tau_{(1)} \cdot \tau_{(2)} + F'^{\,v}_{Lip}\,\tau^z_{(2)}\,\}+$$

$$+\,\frac{2f^2}{4m^2}\,e\,i\,[\sigma_1 \cdot \left(\vec{p}_1 \times \left(3\,\frac{\mu\,\vec{\sigma}_3 \cdot (\vec{r}_1-\vec{r}_3)}{|\vec{r}_1-\vec{r}_3|^5}\,(\vec{r}_1-\vec{r}_3) - \mu\,\frac{\vec{\sigma}_3}{|\vec{r}_1-\vec{r}_3|^3}\right)\right)\vec{\sigma}_2 \cdot (\vec{r}_2-\vec{r}_1)\mathcal{Y}(r_{21}) +$$

$$+\,\sigma_2 \cdot (\vec{r}_2-\vec{r}_1)\mathcal{Y}(r_{12})\,\sigma_1 \cdot \left(3\,\frac{\mu\,\vec{\sigma}_3 \cdot (\vec{r}_1-\vec{r}_3)}{|\vec{r}_1-\vec{r}_3|^5}\,(\vec{r}_1-\vec{r}_3) - \mu\,\frac{\vec{\sigma}_3}{|\vec{r}_1-\vec{r}_3|^3}\right) \times \vec{p}_1\;]$$

$$\{ F'^{S}_{Lip} \tau_{(1)} \cdot \tau_{(2)} + F'^{V}_{Lip} \tau^{z}_{(2)} \} +$$

$$+ \frac{f^2 F'^{V}_{Lip}}{4m^2} e \left[\vec{p}_1 \cdot \left(3 \frac{\mu \vec{\sigma}_3 \cdot (\vec{r}_1 - \vec{r}_3)}{|\vec{r}_1 - \vec{r}_3|^5} (\vec{r}_1 - \vec{r}_3) - \mu \frac{\vec{\sigma}_3}{|\vec{r}_1 - \vec{r}_3|^3} \right) \sigma_2 \cdot (\vec{r}_2 - \vec{r}_1) \mathcal{F}(r_{12}) - \right.$$

$$\left. - \sigma_2 \cdot (\vec{r}_2 - \vec{r}_1) \mathcal{F}(r_{12}) \left(3 \frac{\mu \vec{\sigma}_3 \cdot (\vec{r}_1 - \vec{r}_3)}{|\vec{r}_1 - \vec{r}_3|^5} (\vec{r}_1 - \vec{r}_3) - \mu \frac{\vec{\sigma}_3}{|\vec{r}_1 - \vec{r}_3|^3} \right) \cdot \vec{p}_1 \right] \frac{(-i)}{2} (\tau_{(1)} \times \tau_{(2)})_z$$

$$+ \frac{f^2 F'^{V}_{Lip}}{4m^2} e \left[\sigma_1 \cdot \left(\vec{p}_1 \times \left(3 \frac{\mu \vec{\sigma}_3 \cdot (\vec{r}_1 - \vec{r}_3)}{|\vec{r}_1 - \vec{r}_3|^5} (\vec{r}_1 - \vec{r}_3) - \mu \frac{\vec{\sigma}_3}{|\vec{r}_1 - \vec{r}_3|^3} \right) \right) \sigma_2 \cdot (\vec{r}_2 - \vec{r}_1) \mathcal{F}(r_{12}) - \right.$$

$$\left. - \sigma_2 \cdot (\vec{r}_2 - \vec{r}_1) \mathcal{F}(r_{12}) \sigma_1 \cdot \left(\left(3 \frac{\mu \vec{\sigma}_3 \cdot (\vec{r}_1 - \vec{r}_3)}{|\vec{r}_1 - \vec{r}_3|^5} (\vec{r}_1 - \vec{r}_3) - \mu \frac{\vec{\sigma}_3}{|\vec{r}_1 - \vec{r}_3|^3} \right) \times \vec{p}_1 \right) \right]$$

$$\tfrac{1}{2} (\tau_{(1)} \times \tau_{(2)})_z + (1 \longrightarrow 2) \qquad (2.22)$$

As is clear from equation (2.19) and (2.22) the part of the three body potential connecting the third nucleon is a long range one. (This is obvious from the fact that the exchanged photon in the left hand side of each graph in Fig. 7 is a massless particle). This implies that its matrix elements between states diagonal with respect to A-2 nucleon orbitals (Clement and Heller, 1971) will have a factor Ze instead of the factor e for the two body potential. Therefore for heavy nuclei (and therefore large Z) it is perhaps fair to expect that the three body potentials will be the dominating effect of the T.R.I. violating vertex.

This is unfortunate since calculations with three body potentials are complicated. However as explained in Clement (1971), it is possible to replace, in certain circumstances a three body potential by a two body equivalent potential although of course this involves some approximation. We shall return to this subject in chapter V where the case of a heavy nucleus (Pt^{192}) is treated.

In chapter IV attention will be focused on light nuclei where the effect of three body potential is small compared with the effect of the transition operators presented earlier in this chapter. Before this however, in the following chapter another possible mechanism of T.R.I. violation is examined along the same lines as the analyses presented in this chapter.

CHAPTER 3

THE N*Nγ TRI VIOLATING VERTEX

3.1 Introduction

In recent years much attention has been paid to the role of nucleon resonances in contributing to the magnetic moment of nuclei and to the β-decay Gamow Teller matrix element. It has also been found that the N* ($J=\frac{3}{2}$ $T=\frac{3}{2}$ $M=1236$) resonance is an important factor in possibly removing the need for a hypothetical σ-meson in the one-boson exchange potential (see Green and Schucan (1971) for a survey).

Much earlier Barshay (1966) had suggested that the NN*γ vertex might violate T.R.I. invariance. He calculated the consequences of such an assumption on detailed balance for the reactions $\gamma + d \rightarrow n + p$ and $n + p \rightarrow \gamma + d$.

In this chapter, the possible effects of T.R.I. violation in the NN*γ vertex on γ-transitions in nuclei will be considered. In the approach used in this chapter the N* resonance contributes to the effect only as an intermediate state in the Feynman Graphs. In Appendix 5 however a different method of calculation is outlined. In this alternative approach the N* is introduced explicitly in the nuclear wave function. Also in Appendix 5 the two approaches are compared although no detailed calculation is presented.

3.2 Effective Interaction

The $N^*(J = \frac{3}{2}\ T = \frac{3}{2}\ M = 1236)$ is a nucleon resonance with both spin and isospin $\frac{3}{2}$ and a mass of $M = 1236$ MeV. The Rarita Schwinger (1941) formalism will be used to describe the N^*. The resonance has four charged states with charge 2e, e, 0 and -e. To describe the i-spin, column vectors Ψ_λ are introduced for the isobar. The corresponding spinor for the nucleon is denoted ψ, thus

$$\Psi_\lambda = \begin{pmatrix} \psi_\lambda^{++} \\ \psi_\lambda^{+} \\ \psi_\lambda^{0} \\ \psi_\lambda^{-} \end{pmatrix} \qquad \psi = \begin{bmatrix} \psi_p \\ \psi_n \end{bmatrix} \tag{3.1}$$

The $N^*N\pi$ vertex is taken to have the form (Sugawara - 1953)

$$\mathcal{L}_{NN^*} = -\frac{G}{\mu}\,\bar{\Psi}_\lambda T_\alpha \psi \left(\frac{\partial \phi_\alpha}{\partial x_\lambda}\right) - \frac{G}{\mu}\,\bar{\psi} T_\alpha^* \Psi_\lambda \left(\frac{\partial \phi_\alpha}{\partial x_\lambda}\right) \tag{3.2}$$

where the T_α are the following matrices

$$T_1 = \begin{bmatrix} 1 & 0 \\ 0 & -\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} & 0 \\ 0 & 1 \end{bmatrix} \qquad T_2 = \begin{bmatrix} -i & 0 \\ 0 & \frac{i}{\sqrt{3}} \\ -\frac{i}{\sqrt{3}} & 0 \\ 0 & i \end{bmatrix} \qquad T_3 = \begin{bmatrix} 0 & 0 \\ \frac{2}{\sqrt{3}} & 0 \\ 0 & -\frac{2}{\sqrt{3}} \\ 0 & 0 \end{bmatrix} \tag{3.3}$$

and ϕ_α is the pion field.

The interaction (2) conserves charge and is the one usually adopted (e. g. Salin - 1963).

The $NN^*\gamma$ vertex is taken to be (Salin - 1963 and Gourdin - 1966)

$$\mathcal{L}_{NN^*\gamma} = \frac{-i}{m} \bar{\Psi}_\lambda \, \epsilon \, \gamma_\mu \gamma_5 \, \psi \, F_{\lambda\mu} - \frac{i}{m} \bar{\psi} \, \epsilon^* \gamma_\mu \gamma_5 \Psi_\lambda \, F_{\lambda\mu}$$

where $F_{\lambda\mu} = \partial_\lambda A_\mu - \partial_\mu A_\lambda$ is the electromagnetic tensor and ϵ is a matrix given below.

T. R. I. is violated if ϵ is complex and so the T. R. I. violating vertex is taken to be

$$\mathcal{L}^{T.R.V.}_{NN^*\gamma} = \frac{-i}{m} \left[\bar{\Psi}_\lambda \begin{pmatrix} 0 & 0 \\ \epsilon_p - \epsilon^*_p & 0 \\ 0 & \epsilon_N - \epsilon^*_N \\ 0 & 0 \end{pmatrix} \gamma_\mu \gamma_5 \psi \right] F_{\lambda\mu} - \frac{i}{m} \left[\bar{\psi} \begin{pmatrix} 0 & \epsilon^*_p - \epsilon_p & 0 & 0 \\ 0 & 0 & \epsilon^*_N - \epsilon_N & 0 \end{pmatrix} \gamma_\mu \gamma_5 \Psi_\lambda \right] F_{\gamma\mu} \tag{3. 4}$$

and the normal time reversal invariant part is taken to be

$$\mathcal{L}^{NORMAL}_{NN^*\gamma} = \frac{-i}{m} \left[\bar{\Psi}_\lambda \begin{pmatrix} 0 & 0 \\ \epsilon_p + \epsilon^*_p & 0 \\ 0 & \epsilon_N + \epsilon^*_N \\ 0 & 0 \end{pmatrix} \gamma_\mu \gamma_5 \psi \right] F_{\lambda\mu} - \frac{i}{m} \left[\bar{\psi} \begin{pmatrix} 0 & \epsilon^*_p + \epsilon_p & 0 & 0 \\ 0 & 0 & \epsilon^*_N + \epsilon_N & 0 \end{pmatrix} \gamma_\mu \gamma_5 \psi_\lambda \right] F_{\lambda\mu} \tag{3. 5}$$

With these interactions it would appear to be an easy matter to calculate the diagrams of Fig. 8 below. In this figure the line represents the isobar and the vertex is taken to be T. R. I. violating i. e. given by equation (3. 4)

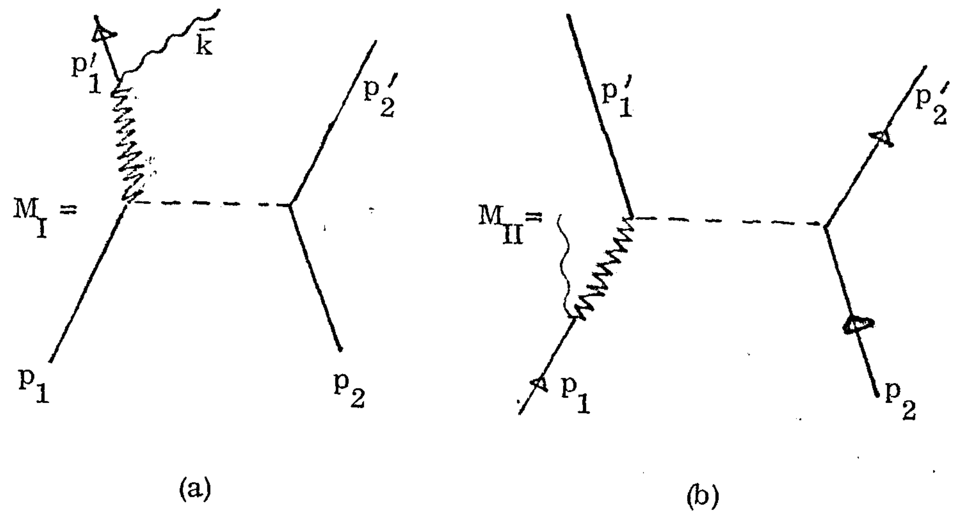


Fig. 8

The only difficulty is that normal dependent terms in the Hamiltonian do not cancel exactly with the terms coming from boson contractions (Lurie - 1968). This is a difficult matter the solution of which has been given by Takahashi-Umezawa (1953). Their result is that one should (wrongly) assume that $H = -\mathcal{L}$ and use the common Feynman rules with the following propagator for the N*

$$S_{F\,\mu\nu}(x_1-x_2) = \partial_{\mu\nu}\,\Delta_F(x_1-x_2) - \frac{2i}{3M^2}\left[(\gamma_\mu\partial_\nu-\gamma_\nu\partial_\mu)+(\gamma\cdot\partial-M)\gamma_\mu\gamma_\nu\right]\delta^4(x_1-x_2) \tag{3.6}$$

where

$$\partial_{\mu\nu} = -(\gamma\cdot\partial - M)\left[\delta_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu + \frac{1}{3M}(\gamma_\mu\partial_\nu-\gamma_\nu\partial_\mu) - \frac{2}{3M^2}\partial_\mu\partial_\nu\right] \tag{3.7}$$

and

$$\Delta_F(x) = \frac{-i}{(2\pi)^4}\int d^4k\,\frac{e^{ikx}}{k^2+M^2-i\epsilon}$$

Now using the usual Feynman rules one gets for the matrix element of Fig. 8-a

$$M_I = i\iiint d^4x_1\, d^4x_2\, d^4x_3 \frac{Gf}{m\mu} [\bar{\psi}(x_1)\gamma_\mu\gamma_5 S_{\lambda\rho}(x_1-x_2)\, \vec{T}'_{(1)}\psi(x_2)]\, F_{\lambda\mu}(x_2)$$

$$\left[\frac{\partial \Delta_F(x_2-x_3)}{\partial x_{2\rho}}\right] \cdot [\bar{\psi}(x_3)\vec{\tau}_{(2)}\gamma_5\,\psi(x_3)] \tag{3.8}$$

where

$$\vec{T}'_{(1)} = \epsilon^* T_{(1)} \tag{3.9}$$

The suffices (1) and (2) refer to particles (1) and (2) respectively. The matrix element of M_{II} can be written down but it is easily verified that the operators coming from M_{II} will be the Hermitian Conjugates of the ones coming from M_I. This was also the case for the operators derived in Chapter 2.

The i-spin dependence can be further simplified as shown in Appendix 6. The results for the normal and T.R.I. violating (T.R.V.) cases are given below.

$$(\vec{T}'_{(1)}\cdot\vec{\tau}_{(2)})_{TRV} = \frac{1}{\sqrt{2}} T_{20}(\tau_{(1)}\,\tau_{(2)})(\overset{*}{\epsilon}_p-\epsilon_p+\overset{*}{\epsilon}_N-\epsilon_N) + \frac{i}{2\sqrt{3}}(\overset{*}{\epsilon}_p-\epsilon_p-\overset{*}{\epsilon}_N+\epsilon_N)(\tau^{(1)}\times\tau^{(2)})_z +$$

$$+\frac{1}{\sqrt{3}}(\overset{*}{\epsilon}_N-\epsilon_N-\overset{*}{\epsilon}_p+\epsilon_p)\,\tau_z^{(2)} \tag{3.10a}$$

$$(\vec{T}'_{(1)}\cdot\vec{\tau}_{(2)})_{NORM} = \frac{1}{\sqrt{2}} T_{20}\,(\tau^{(1)}\tau^{(2)})(\overset{*}{\epsilon}_p+\epsilon_p+\overset{*}{\epsilon}_N+\epsilon_N) + \frac{i}{2\sqrt{3}}(\overset{*}{\epsilon}_p+\epsilon_p-\overset{*}{\epsilon}_N-\epsilon_N)(\tau^{(1)}\times\tau^{(2)})_z +$$

$$+\frac{1}{\sqrt{3}}(\overset{*}{\epsilon}_N+\epsilon_N-\overset{*}{\epsilon}_p-\epsilon_p)\,\tau_z^{(2)} \tag{3.10b}$$

Obviously if ϵ_p and ϵ_N are real then the T.R.I. violating term vanishes.

Substituting the $N^*(J = \frac{3}{2}\ T = \frac{3}{2}\ M = 1236)$ nucleon resonance propagator given by equation (3.6) into equation (3.7) the non-relativistic transition operator corresponding to the graphs of Fig. 8 are obtained. However, because the $N^*(J = \frac{3}{2}\ T = \frac{3}{2}\ M = 1236)$ propagator is so complicated the whole calculation is rather long and the transition operator includes very many terms. To present the results in an orderly fashion it is convenient to divide the propagator into six parts as follows.

$$S_{F_{\mu\nu}} = X + Y + Z + U + V + K$$

where

$$X = M\left[\delta_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu\right]\Delta_F(x) \tag{3.11a}$$

$$Y = -\gamma\cdot\partial\left[\delta_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu\right]\Delta_F(x) + \frac{1}{3}(\gamma_\mu\partial_\nu - \gamma_\nu\partial_\mu)\Delta_F(x) \tag{3.11b}$$

$$Z = -\gamma\cdot\partial\ \frac{1}{3M}(\gamma_\mu\partial_\nu - \gamma_\nu\partial_\mu)\Delta_F(x) \tag{3.11c}$$

$$U = -\frac{2i}{3M^2}\gamma_\mu\gamma_\nu\ \delta^4(x) \tag{3.11d}$$

$$V = +(\gamma\cdot\partial - M)\frac{2}{3M^2}\partial_\mu\partial_\nu\Delta_F(x) \tag{3.11e}$$

$$K = \frac{2i}{3M^2}\left[(\gamma_\mu\partial_\nu - \gamma_\nu\partial_\mu) + \gamma\cdot\partial\ \gamma_\mu\gamma_\nu\right]\delta^4(x) \tag{3.11f}$$

Among the transition operators listed in the Appendix 2-C only the leading ones in the static limit are taken, that is, only the operators in the lowest order in $(\frac{p}{m})$ are taken. On this basis the terms stemming from (3.11c) and (3.11e) are all neglected.

The transition operators from the part of the propagator given by (3.11a) are larger than the transition operators from the other parts (equations 3.11b, 3.11d and 3.11f). The transition operators from 3.11a have a factor $\frac{M}{M^2 - m^2}$. The corresponding factors in the operators deriving from 3.11b, 3.11c and 3.11f are respectively $\frac{m}{M^2 - m^2}$, $\frac{1}{M}$ and $\left(\frac{m}{M}\right)\frac{1}{M}$. Therefore the terms arising from 3.11c and 3.11f can be neglected since they are of the order of 10^{-2} smaller than the terms arising from 3.11a.

In the final result given below the operators stemming from 3.11a and 3.11b combine term by term so that the overall factor is

$$\frac{M}{M^2 - m^2} + \frac{m}{M^2 - m^2} = \frac{1}{M - m}$$

The resulting transition operators are given in the form

$$W = W_1 + W_2$$

They have been separated according to whether the isospin space part is itself odd under time reversal or not. Thus W_2 has $(\tau_{(1)} \times \tau_{(2)})$ which is T.R.I. violating.

$$W_1(\vec{B}) = \frac{1}{3}\,\frac{G f i}{\mu (M-m) 2m^2}\,\vec{B}(r_1) \cdot \vec{\sigma}_1 \times \vec{\sigma}_2\, \mathcal{Y}'(r_{12})\, \Big\{ \frac{2}{\sqrt{2}} T_{20}(\tau_{(1)}, \tau_{(2)}) (\epsilon_p^* - \epsilon_p + \epsilon_N^* - \epsilon_N)$$

$$+ \frac{2}{\sqrt{3}} (\epsilon_N^* - \epsilon_N - \epsilon_p^* + \epsilon_p) \tau_z^{(2)} \Big\} + \frac{1}{3}\,\frac{G f (-i)}{\mu (M-m) 2m^2}\,\vec{B}(r_1) \cdot \vec{\sigma}_1 \times (\vec{r}_1 - \vec{r}_2)\, \sigma_2 \cdot \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|}\, K(|r_1 - r_2|)$$

$$\Big\{ \frac{2}{\sqrt{2}} T_{20}(\tau_{(1)}, \tau_{(2)}) (\epsilon_p^* - \epsilon_p + \epsilon_N^* - \epsilon_N) + \frac{2}{\sqrt{3}} (\epsilon_N^* - \epsilon_N - \epsilon_p^* + \epsilon_p) \tau_z^{(2)} \Big\} + (1 \rightarrow 2) \qquad (3.12)$$

$$W_2(\vec{B}) = \frac{(-)2}{3} \frac{Gf}{\mu(M-m)2m^2} \vec{B}(r_1)\cdot\vec{\sigma}_2 \mathcal{Y}(r_{12}) \left\{\frac{i}{\sqrt{3}}(\epsilon_p^*-\epsilon_p-\epsilon_N^*+\epsilon_N)(\tau^{(1)}x\tau^{(2)})_z\right\}$$

$$+\frac{2}{3} \frac{Gf}{\mu(M-m)2m^2} \vec{B}(r_1)\cdot(\vec{r}_1-\vec{r}_2)\,\vec{\sigma}_2\cdot\frac{\vec{r}_1-\vec{r}_2}{|\vec{r}_1-\vec{r}_2|} K(|\vec{r}_1-\vec{r}_2|) \left\{\frac{i}{\sqrt{3}}(\epsilon_p^*-\epsilon_p-\epsilon_N^*+\epsilon_N)(\tau^{(1)}x\tau^{(2)})_z\right\}$$

(3.13)

By using the techniques described in Appendix 3, the transition potentials are expanded in multipoles. The results are listed below. The electric-multipole stemming from $W_1(B)$ is written $W_1(EL)$ and analogously the magnetic L multipole is written $W_1(ML)$.

$$W_1(EL) = \frac{1}{2} \frac{Gf}{\mu(M-m)2m} \frac{2}{2\sqrt{3}} i\,(\epsilon_N^*-\epsilon_N-\epsilon_p^*+\epsilon_p) \sum_{i<j} K\left(\frac{L}{L+1}\right)^{\frac{1}{2}} R_{ij}^{\ L} \left[Y_L(R_{ij})\otimes(\sigma_i x\sigma_j)\right]^{*L}_M$$

$$(\tau_z^{\ j}-\tau_z^{\ i})\mathcal{Y}(r_{ij}) + \frac{1}{3} \frac{Gf}{\mu(M-m)2m^2} \frac{2}{\sqrt{3}} i\,(\epsilon_N^*-\epsilon_N-\epsilon_p^*+\epsilon_p) \sum_{i<j} R_{ij}^{\ L} \frac{K(|r_i-r_j|)}{|r_i-r_j|}$$

$$\left[Y_L(R_{ij})\otimes\{[\sigma_i\, x\,(\vec{r}_i-\vec{r}_j)]\,\sigma_j\cdot(\vec{r}_i-\vec{r}_j)\tau_z^{\ j} + [\sigma_j\, x\,(\vec{r}_i-\vec{r}_j)]\,\sigma_i\cdot(\vec{r}_i-\vec{r}_j)\tau_z^{\ i}\}\right]^{*L}_M +$$

$$\frac{1}{3} \frac{Gf}{\mu(M-m)2m^2} \frac{2}{\sqrt{2}} i\,(\epsilon_p^*-\epsilon_p+\epsilon_N^*-\epsilon_N) K\left(\frac{L}{L+1}\right)^{\frac{1}{2}} \sum_{i<j} R_{ij}^{\ L} \frac{K(|\vec{r}_i-\vec{r}_j|)}{|r_i-r_j|} T_{20}(\tau_{(i)},\tau_{(j)})$$

$$\left[Y_L(R_{ij})\otimes\{[\sigma_i\, x\,(r_i-r_j)]\sigma_j\cdot(r_i-r_j) + [\sigma_j x(r_i-r_j)]\sigma_i\cdot(r_i-r_j)\}\right]^{*L}_M \qquad (3.14)$$

$$W_2(EL) = (-)\frac{2}{3} \frac{Gf}{(M-m)2m^2\mu} \frac{i}{\sqrt{3}} (\epsilon_p^*-\epsilon_p-\epsilon_N^*+\epsilon_N) K\left(\frac{L}{L+1}\right)^{\frac{1}{2}} \sum_{i<j}\mathcal{Y}(r_{ij})R_{ij} \left[Y_L(R_{ij})\otimes(\sigma_j-\sigma_i)\right]^{*L}_M$$

$$(\tau^{(i)}x\tau^{(j)})_z + \frac{2}{3} \frac{Gf}{(M-m)2m^2\mu} \frac{i}{\sqrt{3}} (\epsilon_p^*-\epsilon_p-\epsilon_N^*+\epsilon_N) K\left(\frac{L}{L+1}\right)^{\frac{1}{2}} \sum_{i<j} |r_i-r_j|\, K(|r_i-r_j|)$$

$$R_{ij}^{\ L} \left[Y_L(R_{ij})\otimes(\sigma_j-\sigma_i)\right]^{*L}_M (\tau^{(i)}x\tau^{(j)})_z + \frac{2}{3} \frac{Gf(-)}{\mu(M-m)2m} \frac{i}{2\sqrt{3}} (\epsilon_p^*-\epsilon_p-\epsilon_N^*+\epsilon_N)\ K$$

$$\left(\frac{L}{L+1}\right)^{\frac{1}{2}} \sum_{i<j} R_{ij} \left[Y_L(R_{ij})\otimes\{[(\vec{r}_i-\vec{r}_j)\,x\,(\sigma_j-\sigma_i)]x\,(\vec{r}_i-\vec{r}_j)\}\right]^{*L} \frac{K(|\vec{r}_i-\vec{r}_j|)}{|\vec{r}_i-\vec{r}_j|} (\tau^{(i)}x\tau^{(j)})_3$$

(3.15)

Analogously the magnetic multipoles are

$$W_1(ML) = \frac{1}{2}\frac{Gf}{\mu(M-m)2m^2}\frac{2}{\sqrt{3}}\, i\,(\epsilon_N^* - \epsilon_N - \epsilon_p^* + \epsilon_p)\,[(2L+1)L]^{\frac{1}{2}} \sum_{i>j} R_{ij}^{L-1}[Y_{L-1}(R_{ij}) \otimes (\sigma_i x \sigma_j)]_M^{*L}$$

$$(\tau_z^j - \tau_z^i)\mathcal{Y}(r_{ij}) + (-)\frac{1}{3}\frac{Gf}{\mu(M-m)2m^2}\frac{2}{\sqrt{3}}\, i\,(\epsilon_N^* - \epsilon_N - \epsilon_p^* + \epsilon_p)[(2L+1)L]^{\frac{1}{2}} \sum_{i<j} R_{ij}^{L-1}\,[Y_{L-1}(R_{ij}) \otimes$$

$$\{[\sigma_i \times (\vec{r}_i - \vec{r}_j)]\,\sigma_j\cdot(\vec{r}_i - \vec{r}_j)\tau_z^j + [\sigma_j x (r_i - r_j)]\,\sigma_i\cdot(r_i - r_j)\tau_z^i\}]_M^{*L}\frac{K(|\vec{r}_i - \vec{r}_j|)}{|\vec{r}_i - \vec{r}_j|} +$$

$$+\frac{1}{3}\frac{Gf}{\mu(M-m)2m^2}\frac{2}{\sqrt{3}}\, i(\epsilon_p^* - \epsilon_p + \epsilon_N^* - \epsilon_N)\,[L(2L+1)]^{\frac{1}{2}} \sum_{i<j} R_{ij}^{L-1}[Y_{L-1}(R_{ij}) \otimes \{[\sigma_i x(\vec{r}_i - \vec{r}_j)]\sigma_j\cdot(\vec{r}_i - \vec{r}_j)$$

$$+ [\sigma_j \times (\vec{r}_i - \vec{r}_j)]\,\sigma_i\cdot(\vec{r}_i - \vec{r}_j)\}]_M^{*L}\, T_{20}(\tau_{(i)}, \tau_{(j)})\frac{K(|\vec{r}_i - \vec{r}_j|)}{|\vec{r}_i - \vec{r}_j|} \tag{3.16}$$

$$W_2(ML) = (+)\frac{2}{3}\frac{Gf}{\mu(M-m)2m^2}\,[L(2L+1)]^{\frac{1}{2}}\frac{i}{\sqrt{3}}(\epsilon_p^* - \epsilon_p - \epsilon_N^* + \epsilon_N)\sum_{i<j} R_{ij}^{L-1}$$

$$[Y_{L-1}(R_{ij}) \otimes (\sigma_j - \sigma_i)]_M^{L*}\mathcal{Y}(r_{ij})\quad(\tau^{(i)} x \tau^{(j)})_z + (-)\frac{2}{3}\frac{Gf}{\mu(M-m)2m^2}\frac{i}{\sqrt{3}}(\epsilon_p^* - \epsilon_p - \epsilon_N^* + \epsilon_N)$$

$$[L(2L+1)]^{\frac{1}{2}} \sum_{i<j} |\vec{r}_i - \vec{r}_j|\; K(|\vec{r}_i - \vec{r}_j|)\, R_{ij}^{L-1}\;[Y_{L-1}(R_{ij}) \otimes (\sigma_j - \sigma_i)]_M^{L*}(\tau^{(i)} x \tau^{(j)})_z +$$

$$+\frac{2}{3}\frac{Gf}{(M-m)2m^2\mu}\;\frac{i}{\sqrt{3}}(\epsilon_p^* - \epsilon_p - \epsilon_N^* + \epsilon_N)\,[(2L+1)]^{\frac{1}{2}} \sum_{i<j} R_{ij}^{L-1}\,[Y_{L-1}(R_{ij}) \otimes \{(\vec{r}_i - \vec{r}_j) \times$$

$$[(r_i - r_j) \times (\sigma_j - \sigma_i)\}]_M^{*L}\frac{K(|\vec{r}_i - \vec{r}_j|)}{|\vec{r}_i - \vec{r}_j|}(\tau^{(i)} x \tau^{(j)})_z \tag{3.17}$$

The effect of the transition operators will be given in Chapter 4. In the next section of this chapter the three body T.R.I. violating potentials arising from the $NN^*\gamma$ vertex will be briefly examined.

3.3 Two and three body T.R.I. violating forces

In this section it will be shown that the T.R.I. violation in the $NN^*\gamma$ vertex also contributes to low energy nuclear physics in the form of two and three body T.R.I. violating potentials between the nucleons.

The contribution in the form of a two body T.R.I. violating potential stems from a set of graphs a few of which are shown in Fig. 9.

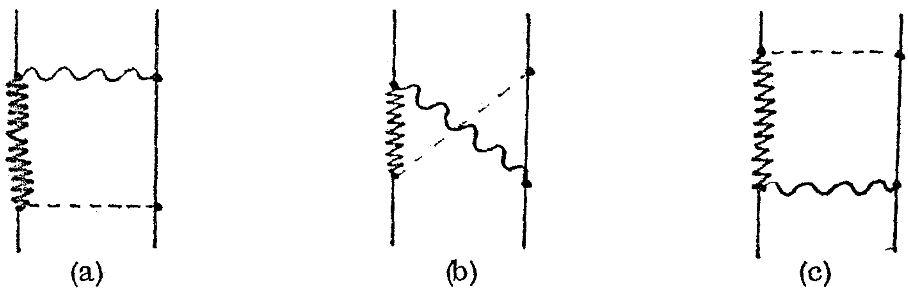


Fig. 9

The wavy line in each graph represents a virtual photon. The $NN\gamma$ vertex in the right hand side of each graph is taken to be the normal electromagnetic vertex. This extra photon vertex contributes an extra small factor e in addition to the small $N^*N\gamma$ T.R.I. violating vertex. Therefore the effect of the two body T.R.I. violating potential is expected to be small with respect to the T.R.I. violating transition operators already derived.

The three body T.R.I. violating potential arises from the graphs of Fig. 10, below

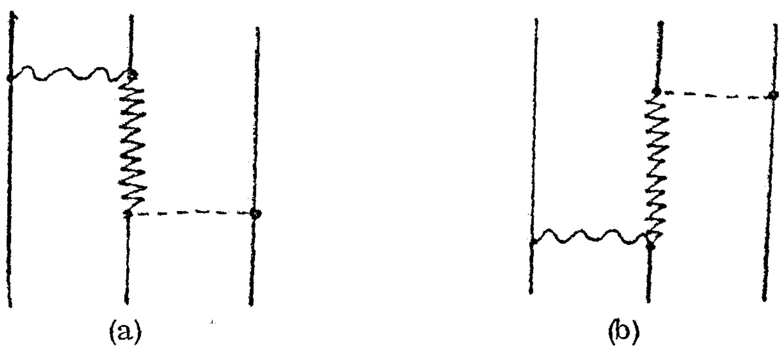


Fig. 10

As already explained in section 2.4 it is easy to calculate the three body potential from the transition operators given by equation 3.12 and 3.13, by using the method given by Clement (1971). This method consists of replacing $B(r_1)$ in $W_1(\vec{B})$ and $W_2(\vec{B})$ (equations 3.12 and 3.13) by the magnetic field produced by the third nucleon in the position of the first, viz

$$\vec{B}(r_1) = \frac{e}{m} \left[3 \frac{\mu \sigma_3 \cdot |\vec{r}_1 \cdot \vec{r}_3|}{|\vec{r}_1 - \vec{r}_3|^5} (\vec{r}_1 \cdot \vec{r}_3) - \mu \frac{\vec{\sigma}_3}{|\vec{r}_1 - \vec{r}_3|^3} \right] \tag{3.18}$$

where

$$\mu = \frac{1}{2} (\mu_\mu + \mu_p) - \frac{1}{2} (\mu_\mu - \mu_p) \tau^z_{(3)} \tag{3.19}$$

The resulting formulae however are very complicated and therefore will not be written down here. Note however that the part of the three body potential connecting the third nucleon is a long range one due to the virtual photon shown in the left hand side of the graphs of Fig. 10. As already explained in section 2.4 this implies that its matrix elements between states diagonal with respect to A-2 nucleons orbitals will have an extra factor Ze due to the normal electromagnetic vertex. Since for heavy nuclei Z can be very large the effect of the three body potential for these nuclei is expected to be larger than the effect of the transition operators derived in section 3.2.

In the next chapter the effect of the transition operators derived in Chapter 2 and in this chapter will be estimated. Because of the effect of the three body operators discussed above, the calculations are confined to the light nuclei.

CHAPTER 4

ESTIMATE OF THE TRV TRANSITION OPERATORS IN LIGHT NUCLEI

4.1 Introduction

In this chapter attention will be devoted to the experimental consequences of the T.R.I. violating transition operators derived in Chapters 2 and 3. In the next section the experimental method to detect a possible failure of T.R.I. which has been used in this University will be discussed. This method is based on a theorem due to Lloyd (1951) and presented below.

Consider a "mixed γ-transition" between two nuclear levels, that is, a transition such that both the electric multipole operators E(L+1) and the magnetic multipole operator M(L) contribute significantly. One can define the reduced matrix elements of the transition operators by the Wigner-Ekhart theorem (see Brink and Satchler (1968) pp 56). For example

$$<\psi_f|\,E(L+1)\,|\psi_i> = (-)^{2L} <I_i m_i LM|\,I_f m_f><\psi_f||\,E(L+1)\,||\,\psi_i>$$

where I and m refer to the spin and its third component of the state indicated.

The result given by Lloyd is that if T.R.I. holds then the ratio between the reduced matrix elements of the competing multipoles is real. Thus, if T.R.I. holds the imaginary part of the "mixing ratio" δ vanishes ($\mathrm{Im}\,\delta = 0$), where δ is defined as*

$$\delta = \frac{<\psi_f||\,E(L+1)\,||\,\psi_i>}{<\psi_f||\,M(L)\,||\,\psi_i>}$$

---

* This definition is in accordance with the convention of Lobov (1969) and Frauenfelder and Steffen (1965).

Conversely (Lobov - 1969) if T.R.I. is violated then Im $\delta \neq 0$. Both statements however ignore final state interaction effects. This point is taken up later in this chapter.

The observable effects of an imaginary part in δ were first worked out by Henley and Jacobson (1958). Their analysis is based on the result that the expectation value in the final state of an operator Q_{odd} with respect to T.R.I. (that is $TQT^{-1} = -Q$) vanishes if T.R.I. holds. This result is valid if the decay is weak (in the sense that first order pertubation theory is adequate) and if final state interactions can be neglected (see Sakurai (1964) for a discussion).

Table 1 below, taken from the work by Henley and Jacobson (1958) presents in the second column a list of T.R.I. violating quantities which can be measured in an electromagnetic transition from a nuclear level A (spin I_A) to a nuclear level B (spin I_B). The quantities $\vec{K}$, $\vec{\Sigma}$ and $\vec{\epsilon}$ are the momentum, the circular polarisation and the linear polarisation respectively of the emitted γ-ray.

γ-ray polarisation	Quantity measured	Degree of orientation Ω_A	Ω_B
None	$(\vec{K}\cdot I_B)\ (\vec{K}\cdot I_B \times I_A)$	1	2
	$(\vec{K}\cdot I_A)\ (K\cdot I_A \times I_B)$	2	1
Circular	$(\vec{K}\cdot \vec{\Sigma})\ (K\cdot I_A \times I_B)$	1	1
	$(\vec{K}\cdot \vec{\Sigma})\ (K\cdot I_A \times I_B)\ (I_A . I_B)$	2	2
Linear	$(\vec{K}\cdot I_A \times \vec{\epsilon})\ (\vec{K}\cdot I_A)\ (\vec{\epsilon}\cdot I_A)$	3	0

TABLE 1

The first column indicates whether γ-ray polarisation measurements are necessary and where they are which kind of measurements (circular or linear) is required.

In order to understand the meaning of the third column the concept of "degree of orientation" is introduced below. Other concepts to be used in the next section will also be discussed here.

Consider an assembly of identical nuclei subjected to a strong magnetic field in a certain direction and at a very low temperature. Due to the interaction of the magnetic moment of the nucleus with the external magnetic field the assembly becomes "oriented", that is, the probability a_m of finding a nucleus of the assembly in a state $|Im>$ (assume for example that the quantisation axis is in the direction of the magnetic field) varies with m.

Consider now the density matrix of this assembly $\rho = \sum_m |Im> a_m <Im|$ and its matrix elements $<Im|\rho|Im'>$. One can expand $<Im|\rho|Im'>$ in terms of the "statistical tensors" $R_q^{(k)}$ introduced by Fano (1951).

$$<Im|\rho|Im'> = \sum_{k,q} (-)^{I-m} <Im\; I - m'|\, kq> R_q^{(k)} \tag{4.1}$$

or

$$R_q^{(k)} = \sum (-)^{I-m} <Im\; I - m'|\, kq><Im|\rho|Im'> \tag{4.2}$$

In the representation chosen above $<Im|\rho|Im'>$ is diagonal ($<Im|\rho|Im'> = a_m \delta_{mm'}$) and therefore only tensors with q = 0 survive, viz

$$R_0^{(k)} = \sum_m (-)^{I-m} <Im\; I - m|\, k\, 0> a_m$$

The "degree of orientation" of an assembly of nuclei is Ω if in equation 4.1 the greatest value of k is Ω.

Returning now to Table 1, the degree of orientation shown in the third and fourth columns gives the minimum degree of orientation required for the states A and B respectively so that the expectation value, for the assembly, of the corresponding T.R.I. violating terms do not vanish.

In the next section the experimental method used in this University to search for a possible failure in T.R.I. will be explained in detail. It aims to detect the first T.R.I. violating term shown in Table 1, namely $(\vec{K}_1 \cdot I_B)\,(\vec{K}_1 \cdot I_B \times I_A)$. The degree of orientation required for the initial state A is $\Omega_A = 1$. This is obtained by subjecting the nuclei to a strong magnetic field in a certain direction (taken for convenience as the z-axis) at very low temperatures. The measurement of the orientation Ω_B of the state B is carried out by detecting a second γ-ray emitted when the nucleus decays from the state B to a third state C (see Fig. 11). This is effected via a measurements of the direction k_2 of the second γ-ray through an implicit correlation involving $(\vec{K}_2 \cdot I_B)$. The overall quantity measured is therefore obtained from $(\vec{K}_1 \cdot I_B)(\vec{K}_1 \cdot I_B \times I_A)$ by replacing j_B by $\vec{K}_2$ and is $(\vec{K}_1 \cdot \vec{K}_2)$ $(\vec{K}_1 \cdot \vec{K}_2 \times \vec{S})$ where $\vec{S}$ is a unit vector in the direction of the assembly expectation value of the angular momentum of the state A, that is, $\langle\overrightarrow{I_A}\rangle$. This will be seen in detail in the next section.

4.2 Angular correlation from oriented nuclei

In this section it is shown in detail that examination of the angular distribution of γ-rays emitted from an oriented assembly of nuclei can be used to detect a violation of T.R.I. Of course as mentioned earlier there is always the possible effect of final state interactions that simulates the effect of T.R.I. violation. This will be considered briefly in the last section of this chapter.

Consider two γ-rays emitted in succession and with no perturbation of the intermediate state as shown in Fig. 11 and consider the probability W(1,2) of detecting photon γ_1 in detector 1 in coincidence with photon γ_2 in detector 2 as in Fig. 12. The assembly of nuclei is considered to be oriented by a strong magnetic field B also shown in Fig. 12.

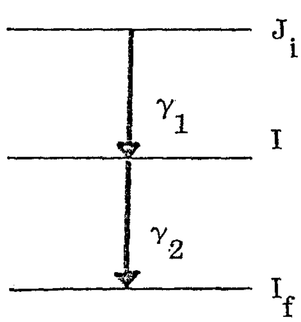


Fig. 11

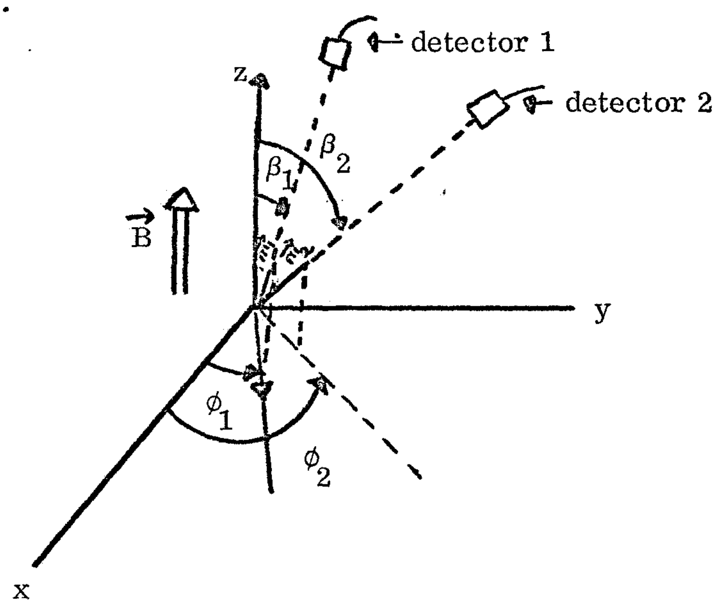


Fig. 12

The function W(1,2) is called the angular correlation function and can be decomposed as follows,

$$W(1,2) = W^{(0)}(1,2) + W^{(1)}(1,2) + \ldots\ldots + W^{(\ell)}(1,2) \tag{4.3}$$

$$0 \le \ell \le 2I_i$$

where each term $W^{(k)}(1,2)$ is proportional to the corresponding statistical tensor $R_0^{(k)}$.

Lobov (1969) has obtained expression for $W^{(0)}(1,2)$ and $W^{(1)}(1,2)$ in the case of γ_1 and γ_2 being mixed transitions of multipolarities $M(L_1)$, $E(L_1+1)$ and $M(L_2)$, $E(L_2+1)$ respectively, in terms of the reduced matrix elements already defined.

$$W^{(0)} = \sum_{k(\text{even})} \{F_k(L_1 L_1 I_i I) + 2\,\mathrm{Re}\,\delta(1)\, F_k(L_1 L_1{+}1\; I_i I) + |\delta(1)|^2 F_k(L_1{+}1\, L_1{+}1\, I_i I)\}$$

$$\{F_k(L_2 L_2 I_f I) - 2\,\mathrm{Re}\,\delta(2)\, F_k(L_2 L_2{+}1\; I_f I) + |\delta(2)|^2 F_k(L_2{+}1\, L_2{+}1\, I_f I)\}\; P_k(\cos\theta_{12})$$

(4.4)

$$W^{(1)} = \frac{-1}{(1+|\delta_1|^2)(1+|\delta_2|^2)} \sum_k \{F_k(L_2 L_2 I_f I) - 2\,\mathrm{Re}\,(\delta(2))\, F_k(L_2 L_2{+}1\; I_f I) + |\delta(2)|^2$$

$$F_k(L_2{+}1\, L_2+1\; I_f I)\}$$

$$\frac{GP}{[I_i(I_i+1)]^{\frac{1}{2}}}\; \vec{S}\cdot[\vec{m}_1 \times \vec{m}_2]\; \mathrm{Im}\,\delta(1)\; \frac{d\,P_k(\cos\theta_{12})}{d(\cos\theta_{12})} \left[\frac{2k+1}{k(k+1)}\right]^{\frac{1}{2}} F^{k}_{1k}(L_1 L_1{+}1\; I\, I_i)$$

(4.5)

In the two formulae above $\vec{S}$, $\vec{m}_1$ and $\vec{m}_2$ are the directions of the magnetic field and the directions of photons (1) and (2) respectively. The coefficients are

$$F_k(L L' I_f I) = (-)^{I_f+I-1}\, [(2L+1)(2L'+1)\,(2I+1)\,(2k+1)]^{\frac{1}{2}} \begin{pmatrix} L & L & k \\ 1 & -1 & 0 \end{pmatrix} \begin{Bmatrix} L & L' & k \\ I & I & I_f \end{Bmatrix}$$

$$F^{k}_{1k}(L_1 L_1{+}1\; I\, I_i) = (-)^{L-1}[(2L_1+1)(2(L_1+1)+1)(2I+1)(2I_i+1)]^{\frac{1}{2}} \langle L_1 1\, L_1{+}1\, {-1} \,|\, k\, 0\rangle$$

$$\begin{Bmatrix} I_i & I_i & 1 \\ L_1 & L_1{+}1 & k \\ I & I & K \end{Bmatrix}$$

In equations (4.4) and (4.5) the angle between the directions m_1 and m_2 is denoted by θ_{12} and $P_k(\cos\theta_{12})$ is a Legendre Polynomial.

The constant P in equation (4.5) is related to the statistical tensor $R_0^{(1)}$ by

$$P = \left(\frac{(2I_i+1)\, I_i\, (I_i+1)}{3}\right)^{\frac{1}{2}} R_0^{(1)}$$

The terms $W^{(\ell)}(1,2)$ for an arbitrary ℓ can also be easily obtained (see Appendix 8 and Coutinho and Ridley (1972)) but these more general terms are not needed here nor for the experimental analysis given in this thesis (Chapter 5). This is because the method of orienting the assembly of nuclei by applying a strong magnetic field at low temperatures produces negligible $R_0^{(\ell)}$ for ℓ greater than $\ell = 1$ (Siegbahm - 1965). From equation (4.5) we see that the effect of T violation manifests itself through the first transition of the cascade. (Therefore it is essential that the first transition be of mixed multipolarity). The effect is seen to be proportional to E defined below. The proportionality constant consists of some geometrical factor plus some nuclear factors refering to the second transition only.

$$E = \frac{1}{1 + |\delta(1)|^2}\ \mathrm{Im}\,\delta(1) \tag{4.6}$$

In the next two sections $\mathrm{Im}\,\delta(1)$ will be calculated for the transition operators derived in Chapters 2 and 3. It is therefore convenient to relate $\mathrm{Im}\,\delta(1)$ to the matrix elements of a general T.R.I. violating transition operator.

The majority of mixed transitions met with in nuclear γ decay are mixtures of a magnetic M(L) and electric E(L + 1) multipoles. In what follows $E^{TRV}(L+1)$ and $M^{TRV}(L)$ are the electric (L + 1) and magnetic (L) multipoles stemming from a T.R.I.

violating transition operator. The electric (L + 1) and magnetic (L) T.R.I. conserving operators are $E^{NOR}_{(L+1)}$ and $M^{NOR}_{(L)}$ respectively.

From the definition of the mixing ratio (first equation in this chapter) it follows that δ (1) is given by

$$\delta (1) = \frac{<I|| E^{NOR}_{(L+1)} || I_i> + <I|| E^{TRV}_{(L+1)} || I_i>}{<I|| M^{NOR}_{(L)} || I_i> + <I|| M^{TRV}_{(L)} || I_i>} \approx$$

$$\approx \frac{<I|| E^{NOR}_{(L+1)} || I_i>}{<I|| M^{NOR}_{(L)} || I>} + \frac{<I|| E^{NOR}_{(L+1)} || I_i>}{<I|| M^{NOR}_{(L)} || I_i>} \left\{ \frac{<I|| E^{TRV}_{(L+1)} || I_i>}{<I|| E^{NOR}_{(L+1)} || I_i>} - \frac{<I|| M^{TRV}_{(L)} || I_i>}{<I|| M^{NOR}_{(L)} || I_i>} \right\}$$

where $<I|| E^{TRV}_{(L+1)} || I_i>$ and $<I|| M^{TRV}_{(L)} || I_i>$ are imaginary relative to $<I|| E^{NOR}_{(L+1)} || I_i>$ or $<I|| M^{NOR}_{(L)} || I_i>$. Therefore the imaginary parts of δ (1) (I m δ (1)) is

$$\mathrm{Im}\delta (1) = \delta_N (1) (-i) \left\{ \frac{<I|| E^{TRV}_{(L+1)} || I_i>}{<I|| E^{NOR}_{(L+1)} || I_i>} - \frac{<I|| M^{TRV}_{(L)} || I_i>}{<I|| M^{NOR}_{(L)} || I_i>} \right\} \tag{4.7}$$

where $\delta_N(1) = \dfrac{<I|| E^{NOR}_{(L+1)} || I_i>}{<I|| M^{NOR}_{(L)} || I_i>}$ is the real part (Re δ (1)) of δ (1). Hence (4.6) becomes

$$E = \frac{(-i)\ \delta_N(1)}{1+|\delta(1)|^2} \left\{ \frac{<I|| E^{TRV}_{(L+1)} || I_i>}{<I|| E^{NOR}_{(L+1)} || I_i>} - \frac{<I|| M^{TRV}_{(L)} || I_i>}{<I|| M^{NOR}_{(L)} || I_i>} \right\} \tag{4.8}$$

From equation (4.8) it follows that the best experimental situation is one in which the transition under study has $|\delta (1)| \approx 1$. Also from equation (4.8) it is seen that the effect depends on the ratio between the T.R.I. violating multipoles and the corresponding normal ones. This is useful since it implies that it is possible to use any one of the many definitions of multipole operators found in the literature (see Brink and Rose (1967) for a review)to calculate these ratios without worring about phase and normalisation problems. One must however take note of these conventions when using values of δ from experimental data.

In the next section the value for E in equation (4. 8) will be calculated for the case in which the T. R. I. violating transition operator is derived from the Lee vertex discussed in Chapter 2.

4. 3 Consequences of the Lee Vertex

In this section the possible effects of the operators derived in Chapter 2 (Equations 2. 12 and 2. 13) will be considered.

Full advantage will be taken of the fact that these operators have the same form as the one derived by Clement and Heller (1971).

In equations (2. 11), (2. 12) and (2. 13) the value of $F'^{V}_{Lee}(0)$ is not known. For the purposes of this section we take

$$F'^{V}_{Lee}(0) = \frac{e}{m^2} \tag{4.9}$$

This value obtained on purely dimensional grounds is usually referred to in the literature as a "maximal value" for the form factor (see Clement (1971) and Huffman (1970) for example).

Now a close look at equations (2.12) and (2.13) will reveal that all the multipole operators are such that as with the usual multipole operatore an additional factor KR_0 (where R_0 is the nuclear radius) is introduced when there is a unit increase in multipolarity. Thus to compare to the normal transition operators, it is sufficient to consider the ratio of the lowest multipole operators.

This fact together with equation (4. 8) shows that to obtain an order of magnitude estimate of the effect it is sufficient to estimate the ratio between the matrix element of the T. R. I. violating electric and magnetic dipole operators and the corresponding matrix elements of the normal transition operators.

As was remarked before the transition operators V_{Lee}^{TRV} obtained in Chapter 2 have the same form as the operators obtained by Clement and Heller (1971) who make an equivalent comparison (see equation (4) in this paper and compare with equation (2. 11) in this thesis).

Allowing for the difference in proportionality constant, the following estimates are obtained for the different matrix elements.

$$< (E\cdot 1)_{Lee} > \approx \frac{2ef^2}{\mu} \left(\frac{\mu}{m}\right)^3 \frac{1}{(\mu R_0)^4}$$

$$< (M\cdot 1)_{Lee}^{TRV} > \approx \frac{2ef^2}{\mu} \left(\frac{\mu}{m}\right)^3 \frac{(KR_0)}{(\mu R_0)^4}$$

$$< (E\cdot 1)_{NORM} > \approx e\, R_0$$

$$< (M\cdot 1)_{NORM} > \approx \frac{e}{m}$$

where R_0 is the nuclear radius, K is the energy of the γ-ray, μ is the mass of the pion and m the mass of the nucleon.

With these estimates the ratios necessary to evaluate expression (4. 8) can be calculated. The results are

$$\frac{<(E\cdot 1)^{TRV}_{Lee}>}{<(E\cdot 1)_{NOR}>} = i\ 1.25 \times 10^{-5} \tag{4.10}$$

$$\frac{<(M\cdot 1)^{TRV}_{Lee}>}{<(M\cdot 1)_{NOR}>} = i\ 7.25 \times 10^{-6}\ (E_\gamma \text{ in MeV}) \tag{4.11}$$

With these figures it is possible to make an estimate of the effect given by equation (4. 7). We have taking $\delta \approx 1$ and $E_\gamma \approx 1$ MeV

$$E = \frac{\delta_N}{1 + |\delta|^2}\left\{\frac{<I||E^{TRV}_{Lee}(L+1)||I_i>}{<I||E^{NOR}_{(L+1)}||I_i>} - \frac{<I||M^{TRV}_{(L)}||I_i>}{<I||M^{NOR}_{(L)}||I_i>}\right\} \approx 10^{-5} \tag{4.12}$$

This value of E is much too small to be detected at present, even allowing an extra factor of 10 to allow for the crudity of the estimates.

No estimate for the transition operators stemming from the Lipshutz vertex will be given. This is because as remarked already in Chapter 2 all the transition operators resulting from the Lipshutz vertex have a factor $(\frac{p}{m})$ and therefore their effect is expected to be even smaller than the effect of the Lee vertex.

In the next section the effect of the transition operator stemming from a possible T.R.I. violation in the $N^*N\gamma$ vertex will be given.

4.4 Consequences of T.R.I. violation in the $N^*N\gamma$ vertex

4.4.1 Introduction

This section sets out to obtain a realistic estimate of the effect of the transition operator $V^{TRV}_{N^*N\gamma}$ (B) stemming from a T.R.I. violation in the $N^*N\gamma$ vertex. This transition operator (eq. 3.12 and 3.13) was obtained in Chapter 3 and expanded in electric and magnetic multipoles (eq. 3.16 and 3.17).

Before proceeding with the more detailed calculation it is worth noting that the effect of the electric multipoles stemming from $V^{TRV}_{N^*N\gamma}$ (B) are negligible compared with the effect of the magnetic multipoles so that the following relation holds.

$$\frac{< M^{TRV}_{N^*N\gamma} (L) >}{< M^{NOR} (L) >} >> \frac{< E^{TRV}_{N^*N\gamma} (L+1) >}{< E^{NOR} (L+1) >} \tag{4.13}$$

To see this an estimate using the method of Clement (1971) will be made for one term only of the transition operator $V^{TRV}_{N^*N\gamma}$ (B) noting that all terms have the same order of magnitude. Firstly, inspection of equation s (3.16) and (3.17) reveals that all the multipole operators are such that an additional factor $K R_0$ (where R_0 is the nuclear radius) is introduced when there is a unit increase in multipolarity. Thus to compare to the normal transition operators, it is sufficient to consider the lowest multipoles.

The first term of the transition operator in equation (2.13) is

$$W_2^{(a)} = (-)\frac{2}{3}\ \frac{Gf}{\mu m\ (M-m)2m\sqrt{3}}\ \frac{i}{}\ (\epsilon_p^* - \epsilon_p - \epsilon_N^* + \epsilon_N)\ [\vec{B}(r_1)\cdot\vec{\sigma}_2 - \vec{\sigma}_1\cdot\vec{B}(r_2)]\ \mathcal{Y}(r_{12})(\tau^{(1)} x \tau^{(2)})$$

This, expanded in multipoles gives

$$W_2^{(a)}\ (EL) = (-)\frac{2}{3}\ \frac{Gf}{(M-m)2m^2\mu}\ K\left(\frac{L}{L+1}\right)^{\frac{1}{2}} \sum_{i<j} \mathcal{Y}(r_{12})\ R_{ij}^{L}\left[Y_L(R_{ij}) \otimes (\sigma_j - \sigma_i)\right]_M^{*(L)} \frac{i}{\sqrt{3}}$$

$$(\epsilon_p^* - \epsilon_p - \epsilon_N^* + \epsilon_N)\ (\tau^{(i)} x \tau^{(j)})_z$$

$$W_2^{(a)}\ (ML) = \frac{2}{3}\ \frac{Gf}{(M-m)2m^2\mu}\ \frac{i}{\sqrt{3}}(\epsilon_p^* - \epsilon_p - \epsilon_N^* + \epsilon_N)\ [\,L(2L+1)]^{\frac{1}{2}} \sum_{i<j} \mathcal{Y}(r_{12})\ R_{ij}^{L-1}$$

$$\left[Y_{L-1}(R_{ij}) \otimes (\sigma_j - \sigma_i)\right]_M^{(L)*} (\tau^{(i)} x \tau^{(j)})_z$$

On comparing the two operators ($W_2^{(a)}$ (EL) and $W_2^{(b)}$ (ML)) with the corresponding ones resulting from the special scalar meson vertex of Clement (1971) it will be noted that they have the same form. Therefore, by using the estimates given by Clement (1971) we have

$$\frac{<(E\cdot 1)_{NN^*\gamma}^{TRV}>}{<(E\cdot 1)^{NOR}>}\ \alpha\ \frac{(KR_0)}{(\mu R_0)^3} \tag{4.14}$$

and

$$\frac{<(M\cdot 1)_{N^*N\gamma}^{TRV}>}{<(E\cdot 1)^{NOR}>}\ \alpha\ \frac{(mR_0)}{(\mu R_0)^3} \tag{4.15}$$

Since m >> K the inequality (4.13) is justified and therefore the equation (4.8) reduces to

$$E = \frac{i\,\delta_N}{1+|\delta|^2}\left\{\frac{<I\,||\,M^{TRV}_{(L)}\,||\,I_i>}{<I\,||\,M^{NOR}_{(L)}\,||\,I_i>}\right\} \tag{4.16}$$

In the remainder of this section a more realistic estimate of the effect of the transition operator $V_{N^*N\ \gamma}(B)$ will be given. A particular transition in ^{18}F has been selected for the following reasons. Firstly because this is a transition in a light nucleus and so the effect of the three body potential is minimised. Secondly because this is a "simple" nucleus from the shell model point of view (the wave functions of certain levels are essentially a closed core with neutrons and protons filling completely the $n = 0$ s and p shells and the two remaining particles in the n = 1 s = 0 or n = 0d shells). The chosen transition is from the (J = 2 T = 1 E≃3.06) to the (J = 3 T = 0 E ≐ 0.94) levels. This choice considerably simplifies the problem because the change $\Delta T = 1$ in isospin and the two particle nature of the levels implies that only the operators which are antisymmetric in isospin coordinates contributes.

The nucleus ^{18}F has been studied both experimentally (see for example Warburton et. al. (1967) and references therein) and theoretically (Kuo and Brown (1966)). An earlier theoretical study of this nucleus was made by Elliott and Flowers (1955) and the wave functions given in this work will be used for the present calculation.

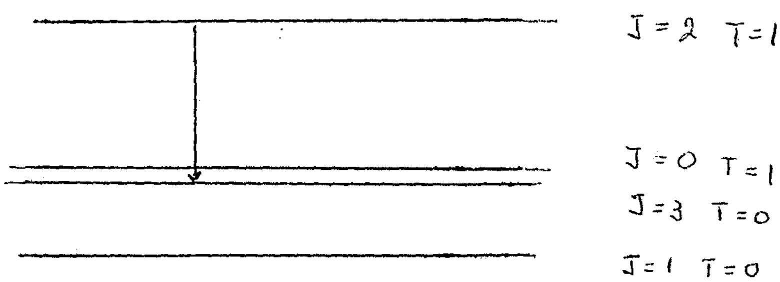


Fig. 13

The level scheme of ^{18}F is given in Fig. 13 above and the wave functions for the levels indicated are given in the L-S coupling representation in an obvious notation as follows

$$\psi(T = 0, J = 1) = 0.82\ (d^2)\ {}^{13}S - 0.07\ (d^2)\ {}^{13}D + 0.16\ (d^2)\ {}^{11}P + 0.02\ (ds)\ {}^{13}D$$

$$\psi(T = 0, J = 3) = -0.59\ (d^2)\ {}^{13}D + 0.03\ (d^2)\ {}^{13}G - 0.12\ (d^2)\ {}^{11}F + 0.79\ (sd)\ {}^{13}D$$

$$\psi(T = 1, J = 0) = 0.84\ (d^2)\ {}^{31}S - 0.38\ (d^2)\ {}^{33}P + 0.39\ (s^2)\ {}^{31}S$$

$$\psi(T = 1, J = 2) = 0.65\ (d^2)\ {}^{31}D + 0.33\ (d^2)\ {}^{33}P - 0.20\ (d^2)\ {}^{33}F - 0.61\ (sd)\ {}^{31}D + 0.22\ (sd)\ {}^{33}D$$

4.4.2 Operators

Given the details of the last subsection it is easy to extract from those operators in equation 3.16 and 3.17 the ones that contribute by using simple techniques of angular momentum algebra, as explained in Appendix 7. The relevant operators are all antisymmetric in isospin space. The operator W_1 (ML) given by equation (3.16) contributes with

$$W_1(M\cdot 1) = W_1^{(a)}(M\cdot 1) + W_1^{(b)}(M\cdot 1) + W_1^{(c)}(M\cdot 1)$$

where

$$W_1^{(a)}(M\cdot 1) = \frac{Gf}{\mu(M-m)2m^2}\frac{1}{\sqrt{3}}\, i\,(\epsilon_N^*-\epsilon_N-\epsilon_p^*+\epsilon_p)\left(\frac{3}{4\pi}\right)^{\frac{1}{2}}\sum_{i<j}(\sigma_i \times \sigma_j)_M^{*1}(\tau_z^j-\tau_z^i)\,\mathcal{J}(r_{ij})$$

$$W_1^{(b)}(M\cdot 1) = \frac{1}{3}\,\frac{Gf}{\mu(M-m)2m^2}\frac{2}{2\sqrt{3}}\, i\,(\epsilon_N^*-\epsilon_N-\epsilon_p^*+\epsilon_p)\left(\frac{1}{4\pi}\right)^{\frac{1}{2}}\begin{pmatrix}1&1&0\\0&0&0\end{pmatrix}\sum_{i<j}(\sigma_i \times \sigma_j)^{*1}$$

$$(\tau_z^j-\tau_z^i)\,|r_i-r_j|\,K(|r_i-r_j|)$$

$$W_1^{(c)}(M\cdot 1) = \frac{(-)}{3}\,\frac{Gf}{\mu(M-m)2m^2}\frac{2}{\sqrt{3}}\, i\,(\epsilon_N^*-\epsilon_N-\epsilon_p^*+\epsilon_p)\,[6\times5\times3]^{\frac{1}{2}}\begin{Bmatrix}1&1&1\\2&1&1\end{Bmatrix}\begin{pmatrix}1&1&2\\0&0&0\end{pmatrix}$$

$$\sum_{i<j}(-i)\,\{[\sigma_i\otimes\sigma_j]^{(1)}\otimes Y_2(r_{ij})\}_M^{*(1)}\,(\tau_z^{(i)}-\tau_z^{(j)})\,|r_i-r_j|\,K(|r_i-r_j|)$$

Analogously the operator W_2(ML) given by equation (3.17), can be written

$$W_2(M\cdot 1) = W_2^{(a)}(M\cdot 1) + W_2^{(b)}(M\cdot 1) + W_2^{(c)}(M.1) + W_2^{(d)}(M\cdot 1)$$

where

$$W_2^{(a)}(M\cdot 1) = (-)\frac{2}{3}\,\frac{Gf}{(M-m)2m^2\mu}\,\frac{i}{\sqrt{3}}\,(\epsilon_p^* - \epsilon_p - \epsilon_N^* + \epsilon_N)\,\left(\frac{3}{4\pi}\right)^{\frac{1}{2}} \sum_{i<j} \left[\sigma_i - \sigma_j\right]_M^{*1} (\tau^{(i)} \times \tau^{(j)})_z\, \mathfrak{N}($$

$$W_2^{(b)}(M\cdot 1) = (+)\frac{2}{3}\,\frac{Gf}{(M-m)2m^2\mu}\,\frac{i}{\sqrt{3}}\,(\epsilon_p^* - \epsilon_p - \epsilon_N^* + \epsilon_N)\,\left(\frac{3}{4\pi}\right)^{\frac{1}{2}} \sum_{i<j} |\vec{r}_i - \vec{r}_j|\, K(|\vec{r}_i - \vec{r}_j|)$$

$$\left[\sigma_i - \sigma_j\right]_M^{*1} (\tau^{(i)} \times \tau^{(j)})_z$$

$$W_2^{(c)}(M\cdot 1) = (-)\frac{2}{3}\,\frac{Gf}{\mu(M-m)2m^2}\; 3\,i\,(\epsilon_p^* - \epsilon_p - \epsilon_N^* + \epsilon_N)\; W\,(1111;01)\begin{pmatrix}1 & 1 & 0\\ 0 & 0 & 0\end{pmatrix}\left(\frac{1}{4\pi}\right)^{\frac{1}{2}} \sum_{i<j}$$

$$(\sigma_i - \sigma_j)_M^{*1}\, (\tau_i \times \tau_j)_z\; |\vec{r}_i - \vec{r}_j|\, K(|\vec{r}_i - \vec{r}_j|)$$

$$W_2^{(d)}(M\cdot 1) = (-)\frac{2}{3}\,\frac{Gf}{\mu(M-m)2m^2}\sqrt{3} \times 5\; i\,(\epsilon_p^* - \epsilon_p - \epsilon_N^* + \epsilon_N)\; W\,(1111;21)\begin{pmatrix}1 & 1 & 2\\ 0 & 0 & 0\end{pmatrix} \sum_{i<j}$$

$$\left[Y_2(r_{ij}) \otimes (\sigma_i - \sigma_j)\right]_M^{*1}\; |\vec{r}_i - \vec{r}_j|\, K(|\vec{r}_i - \vec{r}_j|)\; (\tau_i \times \tau_j)_z$$

At this stage two points should be noted. Firstly, that the operators listed above are only the leading ones (see Chapter 3 and Appendix 2. c). Secondly, that due to the radial dependence of the operators their matrix elements would diverge if it were not for short range correlations. These can be taken into account by introducing a hard core r_c and by performing the radial integral from this value.

This introduces an uncertainty into the calculations and it was therefore decided to simplify the whole calculation by using an approximate procedure due to (Maqueda and Blin-Stoyle - 1967). This procedure will now be explained for $W_1^{(a)}(M\cdot 1)$. The results of similar calculations for the remaining operators $W_1^{(b)}(M\cdot 1)$, $W^{(c)}(M\cdot 1)$ and $W_1^{(a)}(M\cdot 1)$ to $W_1^{(d)}(M\cdot 1)$ will be quoted. First, write $W_1^{(a)}(M\cdot 1)$

$$W_1^{(a)}(M\cdot 1) = \frac{G f \mu^3}{\mu(M-m)m} \quad \frac{i(\epsilon_N^* - \epsilon_N - \epsilon_p^* + \epsilon_p)}{2m} \left(\frac{3}{4\pi}\right)^{\frac{1}{2}} \sum_{i<j} (\sigma_i \times \sigma_j)_M^{*1} (\tau_z^j - \tau_z^i) \mathcal{Y}(\mu r_{ij}) \tag{4.17}$$

where

$$\mathcal{Y}(\mu r_{ij}) = \frac{-1}{4\pi}\left(\frac{1}{\mu r} + \frac{1}{\mu^2 r^2}\right)\frac{e^{-\mu r}}{\mu r}$$

The method consists of using instead of the operator (14) the long range one

$$U_1^{(a)}(M\cdot 1) = \frac{G f \mu^3}{\mu(M-m)m} \quad \frac{i(\epsilon_N^* - \epsilon_N - \epsilon_p^* + \epsilon_p)}{2m} \left(\frac{3}{4\pi}\right)^{\frac{1}{2}} F \sum_{i<j} (\sigma_i \times \sigma_j)_M^{*1} (\tau_z^j \times \tau_z^i) \tag{4.18}$$

where F is defined as

$$F = \frac{\int_{r_c}^{\infty} \mathcal{Y}(\mu r_{ij})\, 4\pi\, (\mu r^2)\, d(\mu r)}{\frac{4}{3}\pi (\mu R_0)^3} \tag{4.19}$$

The operator $U_1^{(a)}$ (M.1) can be considered as the first term of a "multipole" expansion of the operatore $W_1^{(a)}(M\cdot 1)$. By doing this we get the following operators

$$U(M\cdot 1) = U^{(A)}(M\cdot 1) + W^{(B)}(M\cdot 1) + W^{(C)}(M\cdot 1) + W^{(D)}(M\cdot 1)$$

$$U^{(A)}(M\cdot 1) = G_A \left(\frac{3}{4\pi}\right)^{\frac{1}{2}} \sum_{i<j} (\sigma_i \times \sigma_j)^{*1}_M (\tau^j_z - \tau^i_z)$$

$$U^{(B)}(M\cdot 1) = G_B \left(\frac{3}{4\pi}\right)^{\frac{1}{2}} \sum_{i<j} (\tau_i \times \tau_j)_z \left[\sigma_i - \sigma_j\right]^{*1}_M$$

$$U^{(C)}(M\cdot 1) = G_C \left(\frac{3}{4\pi}\right)^{\frac{1}{2}} \sum_{i<j} i \{[\sigma_i \otimes \sigma_j]^{(1)} \otimes Y_2 (\hat{r}_{ij})\}^{*1}_M (\tau^j_z - \tau^i_z)$$

$$U^{(D)}(M\cdot 1) = G_D \left(\frac{3}{4\pi}\right)^{\frac{1}{2}} \sum_{i<j} (\tau_i \times \tau_j)_z \left[Y_2 (\hat{r}_{ij}) \otimes (\sigma_i - \sigma_j)\right]^{*1}_M$$

The calculation of the reduced matrix elements of the operators above was performed by using the techniques of angular momentum algebra. We have for example that the matrix elements of $U^{(A)}(M\cdot 1)$ is, using standard notation,

$$\langle n_1 l_1 n_2 l_2 \underline{\lambda}\, s_1 s_2 \underline{S}\, \underline{\underline{J}}\, \tau_1 \tau_1\, \underline{T}\, M_T \| U^{(A)}(M\cdot 1) \| n'_1 l'_1 n'_2 l'_2\, \underline{\lambda'}\, s_1 s_2 \underline{S'}\, \underline{\underline{J'}}\, \tau_1 \tau_2\, \underline{T'} M'_T \rangle =$$

$$= i \left(\frac{3}{4\pi}\right)^{\frac{1}{2}} G_A \langle \tau_1 \tau_2\, T M_T | (\tau^{(1)}_z - \tau^{(2)}_z) | \tau_1 \tau_2 T' M_T \rangle \; \{3(2J'+1)\}^{\frac{1}{2}} \begin{Bmatrix} J & J' & 1 \\ \lambda & \lambda & 0 \\ S & S' & 1 \end{Bmatrix}$$

$$(2\lambda+1)^{\frac{1}{2}} (2S+1)^{\frac{1}{2}} \left(\frac{2\,(2S'+1)}{(2S+1)}\right)^{\frac{1}{2}} (-)^{S-S'} \langle s_1 s_2 S' \| [\sigma_1 \otimes \sigma_2]^{(1)} \| s_1 s_2 S \rangle \delta_{n_1 n'_1} \delta_{n_2 n'_2} \delta_{l_1 l'_1}$$

$$\delta_{l_2 l'_2}\; \delta_{\lambda\lambda'}$$

It was found that the last two operators have matrix elements which are one order of magnitude smaller than the first two, because the small values of the reduced matrix elements $\langle n_1 l_1 n_2 l_2 \underline{\lambda} \| Y_2 (r_{ij}) \| n'_1 l'_1 n'_2 l'_2 \lambda' \rangle$ they contain. So they were neglected.

The values of G_A and G_B are

$$G_A = 0.013 \, \frac{i\,(\epsilon_N^* - \epsilon_N - \epsilon_p^* + \epsilon_p)}{2m}$$

and

$$G_B = -\,0.03 \, \frac{i\,(\epsilon_p^* - \epsilon_p - \epsilon_N^* + \epsilon_N)}{2m}$$

In calculating this number the following values have been taken from Gourdin (1966).

$$G = 2.07 \quad f = 13 \quad M = 1236 \text{ MeV} \quad m = 938 \text{ MeV} \quad \mu = 138 \text{ MeV}$$

4.4.3 Results

The following results are obtained:

$$\langle J = 3\ T = 0 \| W^{(A)}(M\cdot 1) \| J = 2\ T = 1 \rangle = (-)\, i\, 3.38 \times 0.013 \, \frac{i\,(\epsilon_N^* - \epsilon_N - \epsilon_p^* + \epsilon_p)}{2m} \left(\frac{3}{4\pi}\right)^{\frac{1}{2}}$$

$$\langle J = 3\ T = 0 \| W^{(B)}(M\cdot 1) \| J = 2\ T = 1 \rangle = (-)\, i\, 2.29 \times 0.03 \, \frac{i\,(\epsilon_p^* - \epsilon_p - \epsilon_N^* + \epsilon_N)}{2m} \left(\frac{3}{4\pi}\right)^{\frac{1}{2}}$$

The normal operator to be used must have the same conventions as far as phases and normalisation is concerned as the TRI violating ones. As explained in Appendix 3 the M·1 normal operator is

$$(M\cdot 1)^{NOR} = \left(\frac{3}{4\pi}\right)^{\frac{1}{2}} \frac{1}{2m} \sum_i \left\{ \frac{e}{2}\left(1 + \tau_z^{(i)}\right) \vec{l}_i + \left[\frac{1}{2}(\mu_n + \mu_p) - \frac{1}{2}(\mu_n - \mu_p)\, \tau_z^{(i)} \right] \vec{\sigma}_i \right\}^{*1}_{M}$$

and the relevant matrix element is found to be

$$<J = 3\ T = 0 || M^{NOR}(1) || J = 2\ T = 1> = \frac{e}{2m} (\frac{3}{4\pi})^{\frac{1}{2}} 5.26$$

Therefore

$$(-i) \frac{< I || (M\cdot 1)^{TRV}_{N^*N\gamma} || I_i>}{< I || (M\cdot 1)^{NOR} || I_i >} = \frac{- i(\epsilon^*_p - \epsilon_p - \epsilon^*_N + \epsilon_N)}{e} 4.5 \times 10^{-3}$$

A "maximal" estimate is of course

$$i(\epsilon^*_p - \epsilon_p - \epsilon^*_N + \epsilon_N) = e \quad \text{and therefore}$$

$$(-i) \frac{< || (M\cdot 1)^{TRV}_{N^*N\gamma} || >}{< || (M\cdot 1)^{NOR} || >} = (-)\ 4.5 \times 10^{-3}$$

This is a relatively large number and the result is therefore very encouraging. However, for this particular transition $\delta_N \approx 0.06$ and therefore

$$E = (-) \frac{\delta_N}{1+|\delta|^2} \left| \frac{< || M(1)^{TRV} || >}{< || M(1)^{NOR} || >} \right| = 2.7 \times 10^{-4}$$

This number is a little outside experimental possibility at present, but not far enough to preclude the experimental investigation in a few years time.

4.5 Final State Interaction

As was emphasised in the first section of this chapter Lloyd's theorem is valid only as far as the electromagnetic interaction can be treated in first order perturbation theory. If high order radiative corrections and final state interactions are taken into account a phase η appears in the "mixing ratio" $\delta = |\delta| e^{i\eta}$ which has nothing to do with T.R.I. violation.

The magnitude of this "spoiling" sphase has been studied by several authors (see Henley and Jacobson (1966) and Hannon and Trammell (1968) for example). This effect makes it possible for a T.R.I. odd operator Q to have non zero expectation value even if no T.R.I. violation occurs (see Sakurai (1964) for a discussion).

The important fact about this "spoiling" phase is that it can be calculated at least in principle and therefore its effects can be subtracted out from possible TRI-violating effects.

The phases introduced by radiative corrections were first calculated by Henley and Jacobson (1966). The contributing physical processes are shown in the graphs of Fig. 14 below, and the "spoiling" phase introduced were found to be roughly 10^{-6} and

(a) (b)

Fig. 14

therefore negligible.

The effect of the interaction with atomic electrons was calculated by Hannon and Trammel (1968) and found to be important. The most important process is shown in Fig. 15 where in the intermediate state one atomic electron is in an excited state leaving a hole as represented by the dashed loop

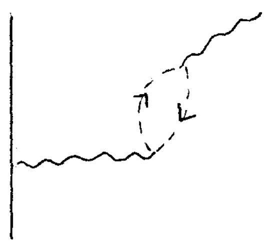


Fig. 15

The contribution to η due to the graph of Fig. 15 where the electron-hole pair occurs in a particular electron shell was found to be proportional to the internal conversion coefficient (Rose - 1958) of this shell. Since the internal conversion coefficient decreases with energy the effect is expected to be smaller for high energy transition.

The magnitude of η was calculated by Hannon and Trammel (1968) for two low energy Mossbauer transitions (the 90 keV transition in ^{99}Ru and the 73 keV transition in ^{193}Ir). The calculated values are

$$\eta(\mathrm{Ru}) = -6.5 \times 10^{-3}$$

$$\eta(\mathrm{Ir}) = 0.9 \times 10^{-3}$$

These values are seen to be of the same order of magnitude as the expected values from a possible T. R. I. violation. We must however remember that those values are for very low energy transitions. For transitions with energy around 1 MeV the values of η should be at least one order of magnitude smaller.

4.6 Conclusions

In this chapter the effects of the transition operators obtained in Chapter 2 and 3 have been calculated for a light nucleus where the effects of three body operators are minimised. Three models of TRI violation have been considered but only a "maximal" violation in the $N^*N\gamma$ vertex was found to contribute significantly to an imaginary part of δ, namely.

$$\delta = (\mathrm{Re}\ \delta) \times 10^{-3}$$

The effects of final state interactions were also considered qualitatively and also found to contribute an imaginary part to δ, we shall call δ_{FSI}. For transitions with an energy of about 1 MeV the value of δ_{FSI} is approximately

$$\delta_{FSI} \approx |\delta| \times 10^{-4}$$

In the next chapter an experiment performed at the University of Sussex in ^{192}Pt will be analysed.

CHAPTER 5

ANALYSIS OF AN EXPERIMENT IN ^{192}Pt

5.1 Introduction

A recent experiment at the University of Sussex (Holmes et. al. (1972)) has used the angular correlation techniques described in section 5.2 and the object of this chapter is to attempt a theoretical interpretation of the experimental results. For experimental reasons described in the paper by Holmes et. al. (1972) the nucleus chosen was ^{192}Pt the level scheme of which is given in Fig. 16 below.

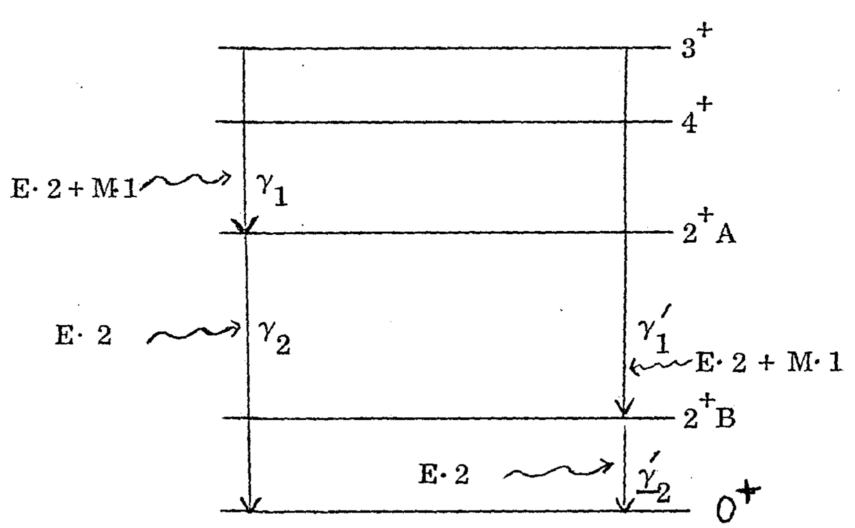


Fig. 16

Because the energy differences between γ_1 and γ_2' and between γ_1' and γ_2 are small the two cascades shown in Fig. 13 are detected simultaneously. From now on we shall denote the quantities referring to the cascade $\psi_{3^+} \xrightarrow{\gamma_1'} \psi_{2^+B} \xrightarrow{\gamma_2'} \psi_{0^+}$ by a prime. For example we write

$$\delta' = \frac{<\psi_{2^+B}||E\cdot 2||\psi_{3^+}>}{<\psi_{2^+B}||M\cdot 1||\psi_{3^+}>} \quad \text{and} \quad \delta = \frac{<\psi_{2^+A}||E\cdot 2||\psi_{3^+}>}{<\psi_{2^+A}||M\cdot 1||\psi_{3^+}>}$$

The experimental result (see formula 4 in Chapter 4) is

$$|\,\text{sen}\,\eta' + 0{,}19\ \text{sen}\,\eta\,| < 4 \times 10^{-3} \qquad (5.1)$$

where $\text{sen}\,\eta' = \frac{\text{Im}\,\delta'}{|\delta'|}$ and analogously for $\text{sen}\,\eta$.

It is the specific aim of this chapter to interpret this experiment in terms of a Phenomenological two body T. R. V. potential. If the violation does derive from the electromagnetic interaction, we have seen in Chapters2 and 3 that in heavy nuclei the effect is more likely to appear in the form of a three body T. R. I. violating potential. However, as stated there, such a three body potential can be simulated by an effective two-body potential.

The most general two body T. R. I. violating but parity conserving potential has been given by (Herczeg - 1965) and it should be noted that all the terms in this potential are momentum dependent. The simplest terms arising and to which the remainder of this discussion is restricted are given below.

$$V_{t.v.} = (\vec{r}_1 - \vec{r}_2)\cdot(\vec{p}_1 - \vec{p}_2)\ \{h_1(r_{12}) + h_2(r_{12})\,\tau_{(1)}\cdot\tau_{(2)} + h_3(r_{12})\,\tau^z_{(1)}\,\tau^z_{(2)} +$$

$$+ h_4(r_{12})\,[\tau^z_{(1)} + \tau^z_{(2)}]\}\cdot[a + b\,\sigma_{(1)}\cdot\sigma_{(2)}] + \text{Hermitian Conjugate} \qquad (5.2)$$

In (2) the $h_1(r_{12})$ to $h_4(r_{12})$ are arbitrary functions of $r_{12} = |\vec{r}_1 - \vec{r}_2|$. The fact that the potential is momentum dependent implies that as it stands it is not gauge invariant and the consequences of this will be discussed in the next section.

5.2 Gauge Invariance Requirements

For well known reasons, in nuclear physics one uses the Coulomb Gauge, where the electromagnetic vector potential $\vec{A}$ satisfies $\nabla \cdot \vec{A} = 0$. By gauge invariance we mean that everything should be unchanged if one makes the transformation $\vec{A} \rightarrow \vec{A} + \nabla G$ where G is an arbitrary function. More precisely we require that given an arbitrary G there exists g such that

$$H + \mathcal{H}(\vec{A} + \nabla G) = e^{+ig}\{H + \mathcal{H}(\vec{A})\}\, e^{-ig} \tag{5.3}$$

In Appendix 4 it is demonstrated that for the case of a system of point particles, g is required to have the form

$$g = \sum_i G(r_i)\, \frac{e}{2}\, (1 + \tau_z^{(i)}) \tag{5.4}$$

This gauge invariance requirement has powerful consequences for nuclear physics. The so called Siegert Theorem (Siegert (1937)) is one such and it is examined in detail in Appendix 4.

Here we note that gauge invariance requires that the total Hamiltonian of the system has the form

$$H_T = H_0 + V_{t.v.} + \mathcal{H}_0(\vec{A}) + \mathcal{V}_{t.v.}(\vec{A}) \qquad (5.5)$$

In this expression H_0 is the usual strong Hamiltonian, $V_{t.v.}$ is the T.R.V. two body operator, $\mathcal{H}_0(\vec{A})$ is the usual electromagnetic interaction and $\mathcal{V}_{t.v.}(\vec{A})$ is a two body T.R.V. interaction between the electromagnetic field and the nucleons. $\mathcal{V}_{t.v.}(\vec{A})$ has to be introduced so that the total Hamiltonian H_T given by (5.5) is gauge invariant.

The importance of $\mathcal{V}_{t.v.}(\vec{A})$ will now be illustrated with an example. Assume that

$$V_{t.v.} = \sum_{(i\neq j)} (\vec{r}_i - \vec{r}_j)\cdot(\vec{p}_i - \vec{p}_j)\, h_1(|\vec{r}_i - \vec{r}_j|) + h.c.$$

~~In this particular case~~ $\mathcal{V}_{t.v.}$ ~~$(\vec{A})$ can be obtained by means of the usual substitution~~ $\vec{p}_i \rightarrow \vec{p}_i - \frac{e}{2}(1 + \tau_z^{(i)})\vec{A}(r_i)$ so we have

$$V_{tv}(\vec{A}) = -2\,e\,G \sum_{i\neq j} \left(\vec{A}(r_i)\frac{1}{2}(1+\tau_z^{(i)}) - \vec{A}(r_j)\frac{1}{2}(1+\tau_z^{(j)})\right)\cdot(\vec{r}_i - \vec{r}_j)\, h_1(|\vec{r}_i - \vec{r}_j|)$$

Now it is easy to verify the following two remarkable equations

$$V_{tv} = i\,[S, H_0] \qquad (5.6)$$

$$\mathcal{V}_{tv}(\vec{A}) = i\,[S, \mathcal{H}_0(\vec{A})] \qquad (5.7)$$

where

$$H_0 = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} V(r_{ij})$$

and

$$\mathcal{H}_0(\vec{A}) = \frac{-e}{2m} \sum_i (\vec{p}_i \cdot \vec{A}(r_i) + \vec{A}(r_i)\cdot \vec{p}_i) \frac{1}{2} (1+\tau_z^{(i)}) + \sum_i \frac{\mu_i}{2m} \vec{\sigma}_i \times \vec{B}(r_i)$$

with

$$\mu_i = (\mu_n + \mu_p) - (\mu_n - \mu_p)\tau_z^{(i)} \quad \text{and } \vec{B} = \nabla \times \vec{A}$$

and finally

$$S = G \sum_{i \neq j} 2m\, H\,(|\vec{r}_i - \vec{r}_j|) \quad \text{where} \quad \frac{d\,H(r)}{dr} = -r\, h_I\,(r)$$

Because of equs. (5.6) and (5.7) the T-violating effect stemming from $V_{t.v.}$ cancels in first order with the effect stemming from $\mathcal{V}_{t.v.}(\vec{A})$ as shown below.

Let ψ_k denote eigenstates of $H = H_0 + V_{t.v.}$ and ϕ_k the corresponding eigenstates of H_0. Then the matrix element between two states ψ_i and ψ_f of the total electromagnetic operator can be written

$$M_{fi} = \langle \psi_f | \mathcal{H}_0(\vec{A}) + \mathcal{V}_{t.v.}(\vec{A}) | \psi_i \rangle = \langle \psi_f | \mathcal{H}_0(\vec{A}) | \psi_i \rangle + \langle \psi_f | \mathcal{V}_{t.v.}(\vec{A}) | \psi_i \rangle$$

Writing $\psi_k = e^{iS} \phi_k$, the first term in the preceding equation becomes

$$\langle \psi_f | \mathcal{H}_0(\vec{A}) | \psi_i \rangle = \langle \phi_f | e^{-iS} \mathcal{H}_0(\vec{A})\, e^{iS} | \phi_i \rangle \cong \langle \phi_f | \mathcal{H}_0(\vec{A}) | \phi_i \rangle - \langle \phi_f | i\, [S, \mathcal{H}_0(\vec{A})]\, | \phi_i \rangle$$

$\approx \langle \phi_f | \mathcal{H}_0(\vec{A}) | \phi_i \rangle - \langle \phi_f | \mathcal{V}_{t.v.}(\vec{A}) | \phi_i \rangle$ because of (5.7) and the second term to first order in (G) is

$$\langle \psi_f | \mathcal{V}_{t.v.}(\vec{A}) | \psi_i \rangle = \langle \phi_f | e^{-iS} \mathcal{V}_{t.v.}(\vec{A})\, e^{iS} | \phi_i \rangle \cong \langle \phi_f | \mathcal{V}_{t.v.}(\vec{A}) | \phi_i \rangle$$

so that M_{fi} becomes

$$M_{fi} = \langle \psi_f | \mathcal{H}_0(\vec{A}) + \mathcal{V}_{t.v.}(\vec{A}) | \psi_i \rangle \cong \langle \phi_f | \mathcal{H}_0(\vec{A}) | \phi_i \rangle \quad \text{to first order in G}$$

and therefore there is no T-violating effect in the matrix element.

This example shows how important $V_{t.v.}(\vec{A})$ can be and therefore we next examine the possibility of determining $V_{t.v.}(\vec{A})$ given solely a phenomenological $V_{t.v.}$. The cancellation we have seen only happens if equations (5.6) and (5.7) are both satisfied. This only occurs for a T.R.I. violating potential of the type given in equation (5.2) if <u>either</u> $h_2 = h_3 = h_4 = b = 0$ <u>or</u> the strong potential V in H_0 has no spin or isospin exchange terms.

The general problem of constructing a gauge invariant combination $V_{t.v.} + \mathcal{V}_{t.v.}(\vec{A})$ from a given $V_{t.v.}$ is trivial when $V_{t.v.}$ does not contain an isospin exchange term. In this case the gauge invariant replacement $\vec{p} \longrightarrow \vec{p} - e\,\vec{A}\,\frac{1}{2}(1 + \tau^z)$ leads directly to $\mathcal{V}_{t.v.}(\vec{A})$. The general case was dealt with by Sachs (1948) who first expressed the charge exchange interaction in terms of the space exchange operator p^x in the Wheeler (1936) representation and then made the gauge invariant replacement $\vec{p} \longrightarrow \vec{p} - e\,\vec{A}\,\frac{1}{2}(1 + \tau^z)$. A simpler and slighly more general procedure is explained in Appendix IX. However as shown there the terms $\mathcal{V}_{t.v.}(\vec{A})$ are not uniquely determined. Parts of it are found to depend on functions $\mathbb{F}(r_i r_j; \vec{A})$ of $\vec{A}$ such that

$$\mathbb{F}\,(r_i\, r_j \backslash \nabla\, G) = G\,(r_i) - G(r_j)$$

This is also true for the procedure given by Sachs (1948) mentioned above (see Bohr and Motelson (1969) page 392 for example). In fact the only way of obtaining

$\mathcal{V}_{t.v.}(\vec{A})$ uniquely is by going back to a field theoretical basis using Feynman Graphs from which $V_{t.v.}$ itself has been extracted (see Appendix 9). This clearly cannot be done for a phenomenological potential $V_{t.v.}$. Progress can however still be made as shown in the next section.

5.3 Use of Siegert Theorem

As was remarked in the last section (and in detail in Appendix 9) the terms $V_{t.v.}(\vec{A})$ cannot be uniquely determined from $V_{t.v.}$ and there are generally terms which depend upon arbitrary functions $\mathcal{F}(r_i, r_j, \vec{A})$.

One could however argue, that in a phenomenological treatment a simple form for $\mathcal{F}(r_i \cdot r_j\ \vec{A})$ could be chosen and the calculations carried out. This would indeed be the case, if it were not for the fact that for the nucleus Pt^{192} being considered particle wave functions for the levels are not available. (There are however good collective model wave functions).

However some progress can be made. Thus use can be made of the Siegert Theorem (see Appendix 4) which states that the electric multipoles resulting from the expansion in multipoles of $\mathcal{H}_0(A) + \mathcal{V}_{t.v.}(A)$ can be written as

$$E(L) + E_{t.v.}(L) = i\left[H_0 + V_{t.v.},\ D_L(K)\right] \tag{5.8}$$

where

$$D_L(K) = \sum_{(i)} \frac{e}{2}\left(1 + \tau_z^{(i)}\right) \frac{x_L^*}{K} \sqrt{\frac{L+1}{2L}}\ r_{(i)}^L\ C_{LM}^*(r_i)$$

where $x_L = \dfrac{(iK)^L}{(2L-1)!!}$ and K is the energy difference between the two levels.

Using the Siegert Theorem one obtains the following expression for the electric multipole matrix element.

$$<\psi_f|\,E(L) + E_{t.v.}(L)|\psi_i> \;=\; (\mathcal{E}_f - \mathcal{E}_i)\; <\psi_f|\,D_L(K)\,|\psi_i> \tag{5.11}$$

where $\mathcal{E}_f$ and $\mathcal{E}_i$ are the eigenvalues of the total Hamiltonian $H_T = H_0 + V_{t.v.}$ corresponding to ψ_f and ψ_i. If we now expand the ψ's in terms of the eigenstates ϕ of the unperturbed Hamiltonian H_0 one has

$$<\psi_f|\,E(L) + E_{t.v.}(L)|\,\psi_i> = (\mathcal{E}_f - \mathcal{E}_i)\,\Big[<\psi_f|D_L(K)|\phi_i> + \sum_{\mu \neq i} \frac{<\psi_f|D_L(K)|\phi_\mu><\phi_\mu|V_{t.v.}|\phi_i>}{E_i - E_\mu}$$

$$+ \sum_{\mu \neq f} \frac{<\phi_f|\,V_{t.v.}\,|\phi_\mu> <\phi_\mu|D_L(K)|\phi_i>}{E_f - E_\mu}\;\Big] \tag{5.12}$$

Note that it is not claimed that $E_{t.v.}(L)$ has no effect. This claim, usually found in the literature is based on the fact that one can calculate the effect without knowing the form of $E_{t.v.}(L)$ and is clearly misleading (see Appendix 4).

Since there is no Siegert Theorem for magnetic multipoles we have

$$<\psi_f|\,M(L) + M_{t.v.}(L)|\,\psi_i> \;=\; <\phi_f|M(L)\,|\phi_i> + \sum_{\mu \neq i} \frac{<\phi_f|\,M(L)\,|\phi_\mu><\phi_\mu|\,V_{t.v.}\,|\phi_i>}{E_i - E_\mu}$$

$$+ \sum_{\mu \neq f} \frac{<\phi_f|\,V_{t.v.}\,|\phi_\mu> <\phi_\mu|M(L)|\phi_i>}{E_f - E_i} + <\phi_f|\,M_{t.v.}(L)\,|\phi_i> \tag{5.13}$$

The "mixing ratio" δ can then be written

$$\delta = \frac{<\psi_f || E(L+1) + E_{t.v.}(L+1) || \psi_i>}{<\psi_f || M(L) + M_{t.v.}(L) \| \psi_i>} = \delta_0 + \delta_0 \, i \, \epsilon_{L+1}^{(e)} - \delta_o \, i \, \epsilon_L^{(m)} \simeq \delta_0 e^{i(\epsilon_{L+1}^{(e)} + \epsilon_L^{(m)})}$$

$$= |\delta| \; e^{i\eta} \tag{5.14}$$

where

$$\delta_0 = \frac{<\psi_f || E(L+1) || \phi_i>}{<\psi_f || M(L) || \phi_i>} \quad \text{and so} \quad |\delta_0| \approx |\delta|$$

and

$$i \, \epsilon_{L+1}^{(e)} = [<\phi_f | | D_{L+1}(k) | | \phi_i>]^{-1} \left\{ \sum_{\mu \neq i} \frac{<\phi_f || D_{L+1}(k) || \phi_\mu>< \phi_\mu || V_{t.v.} || \phi_i>}{E_i - E_\mu} \right.$$

$$\left. + \sum_{\mu \neq f} \frac{<\phi_f | V_{t.v.} | \phi_\mu> <\phi_\mu || D_{L+1}(k) | | \phi_i>}{E_f - E_\mu} \right\} \tag{5.15}$$

$$i \, \epsilon_L^{(m)} = [<\phi_f | |M(L) | | \phi_i>]^{-1} \left\{ \sum_{\mu \neq i} \frac{<\phi_f | | M(L) | | \phi_\mu> <\phi_\mu | V_{t.v.} | \phi_i>}{E_i - E_\mu} + \right.$$

$$\left. + \sum_{\mu \neq f} \frac{<\phi_f | V_{t.v.} | \phi_\mu><\phi_\mu || M(L) | | \phi_i>}{E_f - E_\mu} + <\phi_f | | M_{t.v.}(L) | | \phi_i> \right\} \tag{5.16}$$

5.4 Calculation for Pt^{192}

Using (15) and (16) derived in the previous section it is now possible to calculate sen η and sen η' needed for use in the experimental result (1). First of all the sums in (15) and (16) will be replaced by just one term resulting from the initial admixture of levels ϕ_{2^+A} and ϕ_{2^+B}. This is probably a good approximation because of the small energy denominator ($\Delta E \cong 300$ kev) and the fact that there seem to be no other nearby states having the same spins and parities as the states involved in the two transitions. The result is

$$i\,\epsilon_2^{(e)}(\gamma_1) = \frac{\langle \phi_{2^+B} || D_2(K_1) || \phi_{3^+} \rangle}{\langle \phi_{2^+A} || D_2(K_1) || \phi_{3^+} \rangle} \; \frac{\langle \phi_{2^+A} | V_{t.v.} | \phi_{2^+B} \rangle}{\Delta}$$

$$i\,\epsilon_1^{(m)}(\gamma_1) = \frac{\langle \phi_{2^+B} || M(1) || \phi_{3^+} \rangle}{\langle \phi_{2^+A} || M(1) || \phi_{3^+} \rangle} \; \frac{\langle \phi_{2^+A} | V_{t.v.} | \phi_{2^+B} \rangle}{\Delta} + \frac{\langle \phi_{2^+A} || M_{t.v.}(1) || \phi_{3^+} \rangle}{\langle \phi_{2^+A} || M(1) || \phi_{3^+} \rangle}$$

and analogously a similar expression for γ_1' transition. Using now the transition probability (Brink and Rose (1967))

$$P(\gamma_1) = 4\,K_1 \left[\frac{|\langle \phi_f || E\cdot 2 || \phi_i \rangle|^2}{5} + \frac{|\langle \phi_f || M(1) || \phi_i \rangle|^2}{3} \right] =$$

$$= 4\,K_1 \, |\langle \phi_f || E\cdot 2 || \phi_i \rangle|^2 \, \left[\frac{1}{5} + \frac{1}{3} |\delta(\gamma_1)|^{-2} \right]$$

together with the above expression for ϵ the following expression is obtained for $\sin\eta$

$$\sin\eta = \mp \frac{i \langle \phi_{2^+B} | V_{t.v.} | \phi_{2^+A} \rangle}{\Delta} \; \frac{K_1}{K_1'} \left[\frac{P(\gamma_1')}{P(\gamma_1)} \; \frac{K_1}{K_1'} \; \frac{(1 + \frac{5}{3} |\delta(\gamma_1)|^{-2})}{(1 + \frac{5}{3} |\delta(\gamma_1')|^{-2})} \right]^{\frac{1}{2}}$$

$$\left[\frac{K_1}{K_1'} - \frac{\delta(\gamma_1)}{\delta(\gamma_1')} \right] - \frac{\langle \phi_{2^+A} || M_{t.v.}(1) || \phi_{3^+} \rangle}{\langle \phi_{2^+A} || M(1) || \phi_{3^+} \rangle} \qquad (5.17)$$

and

$$\sin \eta' = \mp \frac{i < \phi_{2^+B} | V_{t.v.} | \phi_{2^+A} >}{\Delta} \frac{K_1'}{K_1} \left[\frac{P(\gamma_1')}{P(\gamma_1)} \frac{K_1'}{K_1} \frac{(1 + \frac{5}{3} |\delta(\gamma_1')|^{-2})}{(1 + \frac{5}{3} |\delta(\gamma_1)|^{-2})} \right]^{\frac{1}{2}}$$

$$\left[\frac{K_1'}{K_1} - \frac{\delta(\gamma_1')}{\delta(\gamma_1)} \right] - \frac{< \phi_{2^+B} || M_{t.v.}(1) || \phi_{3^+} >}{< \phi_{2^+B} || M(1) || \phi_{3^+} >} \tag{5.18}$$

In (17) and (18) all the values expect $< \phi_{2^+B} | V_{t.v.} | \phi_{2^+A} >$ and

$$\xi = \frac{< \phi_{2^+A} || M_{t.v.}(1) || \phi_{3^+} >}{< \phi_{2^+A} || M(1) || \phi_{3^+} >} \quad \text{and} \quad \xi' = \frac{< \phi_{2^+B} || M_{t.v.}(1) || \phi_{3^+} >}{< \phi_{2^+B} || M(1) || \phi_{3^+} >} \quad \text{can}$$

be taken from experiment. The values are

$K_1 = 0{,}3085$ MeV; $\quad K_1' = 0{,}6044$; $\quad \frac{P(\gamma_1)}{P(\gamma_1')} = \text{Branching Ratio} = \frac{77}{22}$

$\delta(\gamma_1) = \delta = 7{,}3 \quad \delta(\gamma_1') = \delta' = -2{,}1$

This then gives

$$\sin \eta = \mp 0{,}76 \frac{i < \phi_{2^+B} | V_{t.v.} | \phi_{2^+A} >}{\Delta} - \xi_1$$

$$\sin \eta' = \mp 11{,}8 \frac{i < \phi_{2^+B} | V_{t.v.} | \phi_{2^+A} >}{\Delta} - \xi_1'$$

Taken with the experimental result (1) by Holmes et. al. this gives

$$\left| \pm 11.8 \frac{i < \phi_{2^+B} | V_{t.v.} | \phi_{2^+A} >}{\Delta} - (\xi_1' + 0.19 \quad \xi_1) \right| \leq 4 \times 10^{-3}$$

Further, unless there is an accidental virtually complete cancellation, an upper limit for the matrix element $V_{t.v.}$ can be given, namely,

$$\left| \frac{i < \phi_{2^+B} | V_{t.v.} | \phi_{2^+A} >}{\Delta} \right| < (3.4) \times 10^{-4} \qquad (5.19)$$

Using the value of $\Delta = 0.2959$ MeV one has

$$| i < \phi_{2^+B} | V_{t.v.} | \phi_{2^+A} > | \sim 100 \text{ ev.}$$

In the next section the matrix element $< \phi_{2^+B} | V_{t.v.} | \phi_{2^+A} >$ is evaluated.

5.5 Evaluation of the Matrix Element

Up to this point it has been possible to avoid the consequences of our ignorance of the wave functions of the levels of Pt^{192}. In this section this is no longer possible and very crude simplifications have to be made.

The method that will be used consists in replacing the two body T.R.I. violating potential by an equivalent one body potential (see similar estimative in Bohr and Mottelson (1969), pages 259 and 393). The specific way of doing this is explained below.

First we expand ϕ_{2^+A} and ϕ_{2^+B} in a sum of Slater determinants constructed from single particle orbitals $\mu^i \equiv (m\ L\ M_L\ S\ M_S\ \tau\ M_\tau)$ where m is the principal quantum number and, M_L, M_S and M_τ are the z projections of the orbital, spin and i-spin quantum numbers.

$$\phi_{2^+B} = \sum_i b_i \, \Delta^i_{2^+B}$$

$$\phi_{2^+A} = \sum_i a_i \, \Delta^i_{2^+A}$$

where $\Delta^i_{2^+B}$ and $\Delta^i_{2^+A}$ are the Slater determinants building up ϕ_{2^+B} and ϕ_{2^+A} respectively and b_i and a_i are the expansion coefficients.

We divide the matrix elements $<\phi_{2^+A}| V_{t.v.} |\phi_{2^+B}>$ into three groups of terms

$$<\phi_{2^+A}| V_{t.v.} |\phi_{2^+B}> = \sum_{k,l} a_k b_l \, (\Delta^{(k)}_{2^+A} | V_{t.v.} | \Delta^{(l)}_{2^+B}) +$$

$$+ \sum_{m,n} a_m b_n \, (\Delta_{2^+A}{}^{(n)} | V_{t.v.} | \Delta_{2^+B}{}^{(m)}) +$$

$$+ \sum_{\sigma,p} a_\sigma b_p \, (\Delta_{2^+A}{}^{(\sigma)} | V_{t.v.} | \Delta_{2^+B}{}^{(p)})$$

In the first sum over the pairs (k, l), the Slater determinants $\Delta_{2^+A}{}^{(k)}$ and $\Delta_{2^+B}{}^{(l)}$ are identical. In the second sum, the pairs m and n differ by just one orbital and in the (σ, p) pair they differ by two orbitals.

Being T.R.I. violating, the potential $V_{t.v.}$ has no diagonal matrix elements (see e.g. - Bohr - Motelson (1969)) and therefore

$$\sum_{k,l} a_k b_l \, (\Delta_{2^+A}{}^{(k)} | V_{t.v.} | \Delta_{2^+B}{}^{(l)}) = 0$$

The third term involving a sum over (σ, p) where $V_{t.v.}$ connects Slater determinants differing by two orbitals cannot be taken into account in moving to a one-body approximation for $V_{t.v.}$ since a one-body operator cannot connect states differing by two orbitals. We shall therefore neglect this term.

$$<\phi_{2^+A} | V_{t.v.} | \phi_{2^+B}> \cong \sum_{m,n} a_m b_n (\Delta_{2^+A}{}^n | V_{t.v.} | \Delta_{2^+B}{}^m)$$

It is now possible to define a single particle potential which has the same matrix elements as $V_{t.v.}$ when calculated between Slater determinants differing by just one orbital. Indeed the matrix element of $V_{t.v.}$ between two Slater determinants differing in that the orbital u(i) in $\Delta_{2^+A}{}^n$ is ν (i) in $\Delta_{2^+B}{}^m$ is

$$< \Delta_{2^+A}{}^n | V_{t.v.} | \Delta_{2^+B}{}^m > = V \text{ (direct)} + V \text{ (exchange)} \tag{5.20}$$

$$V \text{ (direct)} = \sum_k \iint d(i)\, d(j)\; u^*(i)\, w_k{}^*(j)\, V_{t.v.}(i,j)\; \nu(i)\, w_k(j) \tag{5.21}$$

$$V \text{ (exchange)} = - \sum_k \iint d(i)\, d(j)\, u^*(i)\; w_k{}^*(j)\, V_{t.v.}(i,j)\, \nu(j)\, w_k(i) \tag{5.22}$$

where the sum over k runs over the common orbitals. On the other hand the matrix element of a single particle operator $F = \sum_i f(i)$ between the same states is

$$< \Delta_{2^+A}{}^{(n)} | F | \Delta_{2^+B}{}^{(m)} > = \int d(i)\, u^*(i)\, f(i)\; \nu(i) \tag{5.23}$$

Comparing (21) and (22) with (23) permits the definition of an equivalent one particle potential $V^{eq.}$

$$V^{eq.} = V^{eq.}\,(\text{direct}) + V^{eq.}\,(\text{exchange}) \tag{5.24}$$

where

$$V^{eq.}\ (\text{direct}) = \sum_k \int d(j)\, w_k^*\,(j)\ V_{t.\,v.}\,(i,j)\, w_k(j) \tag{5.25}$$

$$V^{eq.}\ (\text{exchange}) = -\tfrac{1}{2}\sum_k \int d(j)\, w_k^*\,(j)\left[V_{t.\,v.}\ P + P^*\, V_{t.\,v.}\right] w_k(j) \tag{5.26}$$

In (26) P is an operator which exchanges i and j, that is

$$P(\mu(i)\,\nu\,(j)) = \mu(j)\,\nu(i) \tag{5.27}$$

In order to define an equivalent operator that is the same for all (n) and (m) determinants one has to make a further simplication. The sum over k in (5.25) and (5.26) has to be restricted to be over the orbitals which are common to <u>all</u> the n, m determinants. We shall assume also that the common core is spherically symmetrical in orbital angular momentum and spin. This last assumption greatly simplifies the problem because now one can say that $V^{eq.}$ has the following simple form, purely on symmetry grounds.

$$V^{eq.} = \sum h_I(r_i)\,\vec{r}_i\cdot\vec{p}_i + \vec{p}_i\cdot\vec{r}_i\, h_I(r_i) + \sum \left[h_{II}(r_i)\,\vec{r}_i\cdot\vec{p}_i + \vec{p}_i\cdot\vec{r}_i\, h_{II}(r_i)\right]\boldsymbol{\gamma}_z^{(i)} \tag{5.28}$$

As an example we take the first part (and the only one that contributes in this approximation) of a 2-body T-violating potential derived by Huffman (1970).

$$V_{t.v.} \cong G \sum_{\substack{i,j \\ (i\neq j)}} \frac{(\vec{r}_i-\vec{r}_j)}{|r_i-r_j|} (p_i-p_j)\, Q\,(\mu|r_i-r_j|)\,(\sigma_i\cdot\sigma_j)\,(\tau_{(i)}\cdot\tau_{(j)}-\tau_{(i)}^{z}\,\tau_{(j)}^{z}) + h.c. \tag{5.29}$$

In Appendix 10 it is shown that

$$V^{eq.} = - GK \sum_i \{\vec{p}_i\cdot\vec{r}_i \frac{1}{r_i} \frac{d\,\bar{\rho}(r_i)}{dr_i} + h.c.\}\, \tfrac{1}{2}\,(1 - \frac{N-Z}{A}\tau_{(i)}^{z}) \tag{5.30a}$$

where

$$K = -2\pi \int Q(\mu s)\, s^3\, ds \tag{5.30b}$$

and $\rho(r_i)$ is the nucleon density.

We neglect the part proportional to $\frac{N-Z}{A}\tau_{(i)}^{z}$ and write the first part as

$$V^{eq.} = \frac{-GK}{2}\{\sum_i \vec{p}_i\cdot\vec{r}_i \frac{1}{r_i} \frac{d\,\rho\,(r_i)}{dr_i} + h.c.\} = mGKi\,[\sum_i \rho\,(r_i), H_0\,] \tag{5.31}$$

where

$$H_0 = \sum_i \frac{p_i^2}{2m} + U(|r_i-r_j|)$$

Substituting (5.31) in (5.19) we have

$$|G\,Km\,\rho_0| < 3 \times 10^{-4} \tag{5.32}$$

It is interesting to compare this with a theoretical estimate obtained by Huffmann (1970), namely

$$G = \frac{\mu^2}{m} \frac{g^2}{(2\pi)^3} F_1^{\,v} F_3^{\,v} \tag{5.33}$$

where

$$F_1^{\,v} = (4\pi\alpha)^{\frac{1}{2}}, \quad F_3^{\,v} = \frac{(4\pi\alpha)^{\frac{1}{2}}}{m} \quad \text{and } \alpha = \frac{1}{137}$$

We can take Q(x) in equation (5.29) as $Q(x) = A \frac{e^{-x}}{x^2}$ where $x = \mu|r_1 - r_2|$. This form for Q(x) agrees well with the curves presented by Huffmann(1970) for $|r_1 - r_2| > 1$ F (Fermi) with $A = 24.4 \times 10^{-4}$. The value of K given by equation (5.30b) is

$$K = -\frac{1.53}{\mu^4} \times 10^{-2} \tag{5.34}$$

With the values of G (equation 5.32) and K (equation 5.33) we obtain

$$|G\,K\,m\,\rho_0| \simeq 5 \times 10^{-5} \tag{5.35}$$

Therefore by comparing equation (5.32) and (5.35) we see that the experiment by Holmes does not rule out a two body T.R.I. violating potential having a strength of this order of magnitude. More accurate experimental work is clearly needed.

To conclude this section it is interesting to compare the strength of the Huffmann potential with the strength $G_{p.v.}$ of the parity violating potential derived by Michell (1965). We have

$$|G\,m\,R_0| \sim 10^{-3} \tag{5.36}$$

and

$$|G_{p.v.}\ m R_0| \sim 10^{-7} \tag{5.37}$$

As can be seen from equations (5. 36) and (5. 37) the Huffmann potential is relatively very strong. This is due to the fact that it derives from an assumed "maximal violation" in the electromagnetic interaction.

Conclusions

This thesis has studied the effects in low energy physics of a possible T.R.I. violation in the electromagnetic interaction. As shown in Chapters 2 and 3 T.R.I. violation effects appear as a two body short range transition operator or as a two and three body potential. It was shown by means of semi quantitative arguments that the three body potential operator probably dominates in heavy nuclei but has smaller effects in light nuclei than the T.R.I. violating transition operators.

In Chapter 4, therefore, attention was focussed on a light nucleus and an estimate of the effect of the T.R.I. violating transition operators derived in Chapters 2 and 3 was made. The effect of an assumed T.R.I. "maximal" violation in the $N^*N\gamma$ vertex was found to contribute significantly to the imaginary part of the "mixing ratio" δ in a 1 MeV M1-E2 γ transition, namely

$$\mathrm{I.m}\ \delta \approx \mathrm{Re}\ \delta \times 10^{-3}$$

Such a value is in principle within the region of possible experimental measurement.

Finally in Chapter 5 an experiment carried out at this University was used to set an approximate upper limit on the strength of a phenomenological T. R. I. violating two body potential. It was found that the experimental limit (Holmes et. al. - 1972) does not rule out a possible T. R. I. violating two body operator derived by Huffmann (1970). One should however keep in mind that due to the fact that the wave function of ^{192}Pt are not known in any detail only a crude estimate was possible.

In general it can be concluded that if the accuracy of experimental data can be improved by about an order of magnitude significant information about the origin of T. R. I. violation will be forthcoming.

APPENDIX 1

NOTATION

(a) The Pauli Metric is used throughout this thesis. So a four vector $f_\mu \equiv (\vec{f},\ if_0)$ has norm $f^2 = f_\mu f_\mu = \vec{f}\cdot\vec{f} - f_0^{\,2}$. The Dirac γ matrices satisfy

$$\{\gamma_\mu, \gamma_\nu\} = 2\,\delta_{\mu\nu}$$

and $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$, $\sigma_{\mu\nu} = \frac{1}{2i}\,[\gamma_\mu, \gamma_\nu]_-$

(b) Natural Units are used throughout so that $\hbar = c = 1$.

(c) The cross product $\vec{a} \times \vec{b}$ signifies the usual vector product of $\vec{a}$ and $\vec{b}$. The simbol $[\vec{a} \otimes \vec{b}]^{(l)}_{m}$ is the tensor product of $\vec{a}$ and $\vec{b}$, thus

$$\vec{a}\cdot\vec{b} = -\sqrt{3}\ [a \otimes b]^{(0)}$$

$$\vec{a} \times \vec{b} = -i\sqrt{2}\,[\vec{a} \otimes \vec{b}]^{(1)}$$

(d) The isospin operators T^+ and T^- are defined as follows

$$T^{\pm} = \tau_{(1)}^{(+)}\ \tau_{(2)}^{(-)} \pm \tau_{1}^{(-)}\ \tau_{(2)}^{(+)} \text{ where}$$

$$\tau^{\pm} = \frac{1}{2}\,(\tau_x \pm i\,\tau_y)$$

The identities below are also used

$$\tau_{(1)}^{z}\,(\vec{\tau}_{(1)}\cdot\vec{\tau}_{(2)}) = 2T^{-} + \tau_{(2)}^{z}$$

$$(\vec{\tau}_{(1)}\cdot\vec{\tau}_{(2)})\,\tau_{(1)}^{z} = -\,2T^{-} + \tau_{2}^{z}$$

$$\tau_{1}^{z}(\vec{\tau}_{(1)}\cdot\tau_{(2)})\,\tau_{(2)}^{z} = 2T^{+} + 1$$

APPENDIX 2

MANIPULATIONS TO EXTRACT OPERATORS

A. The Lee Vertex

We start from equations (9) and (10) of Chapter 2, viz

$$H_I = i f^2 \iiint d^4x_1 d^4x_2 d^4x_3 \left[\bar{\psi}(x_1) \gamma_\mu \left(-\frac{\overleftarrow{\partial}}{\partial x_{1\rho}} + \frac{\overrightarrow{\partial}}{\partial x_{1\rho}} \right) S_F(x_1 - x_2) \gamma_5 \psi(x_2) \right] \left[\frac{\partial A_\rho(x_1)}{\partial x_{1\mu}} - \frac{\partial A_\mu(x_1)}{\partial x_{1\rho}} \right]$$

$$\Delta_F(x_2 - x_3) \left[\bar{\psi}(x_3) \gamma_5 \psi(x_3) \right] \left[F^{1S}_{\ell\ell} + F^{1V}_{\ell\ell} \tau_z^{(1)} \right] \tau_{(1)} \cdot \tau_{(2)} \qquad (A2-1)$$

$$H_B = i f^2 \iiint d^4x_1 d^4x_2 d^4x_3 \left[\bar{\psi}(x_2) \gamma_5 S_F(x_1 - x_2) \left(-\frac{\overleftarrow{\partial}}{\partial x_{1\rho}} + \frac{\overrightarrow{\partial}}{\partial x_{1\rho}} \right) \gamma_\mu \psi(x_1) \right] \left[\frac{\partial A_\rho(x_1)}{\partial x_{1\mu}} - \frac{\partial A_\mu(x_1)}{\partial x_{1\rho}} \right]$$

$$\Delta_F(x_2 - x_3) \left[\bar{\psi}(x_3) \gamma_5 \psi(x_3) \right] \tau_{(1)} \cdot \tau_{(2)} \left[F^{1S}_{\ell\ell} + F^{1V}_{\ell\ell} \tau_z^{(1)} \right] \qquad (A2-1a)$$

by substituting in (A2-1) and (A2-1a) the propagators

$$S_F(x) = -\frac{1}{(2\pi)^4} \int d^4Q \, \frac{\gamma \cdot Q + im}{Q^2 + m^2 - i\varepsilon} \, e^{iQ\cdot x} \qquad (A2-2)$$

$$\Delta_F(x) = \frac{-i}{(2\pi)^4} \int d^4q \, \frac{e^{iq\cdot x}}{q^2 + \mu^2 - i\varepsilon} \qquad (A2-3)$$

and integrating first over $d^4 x_1$ and secondly over d^4Q there results

$$M_I = i f^2 \iint d^4x_1 d^4x_2 \left\{ \bar{\psi}(x_2) \gamma_\mu (-) \frac{i(p_1'+p_1+k)_\rho \, [\gamma\,(p_1'+k)+im]}{(p_1'+k)^2+m^2} \gamma_5 \psi(x_2) \right. \left[\frac{\partial A_\rho(x_2)}{\partial x_{2\mu}} - \frac{\partial A_\mu(x_2)}{\partial x_{2\rho}} \right]$$

$$\left[\frac{-i}{(2\pi)^4} \int d^4q \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon} \right] [\bar{\psi}(x_3)\gamma_5\psi(x_3)] \left[F^{'S}_{fee} + F^{'V}_{fee} \tau_z^{(2)} \right] \tau_{(2)} \tau_{(3)} \quad \cdots (A2\text{-}4)$$

$$M_{II} = i f^2 \iint d^4x_2 d^4x_2 \left\{ \bar{\psi}(x_2) \gamma_5 (-) \frac{[\gamma\cdot(p_1-k)+im]\, i\,[p_1+p_1-k]_\rho}{(p_1-k)^2+m^2} \gamma_\mu \psi(x_1) \right\} \left[\frac{\partial A_\rho(x_2)}{\partial x_{2\mu}} - \frac{\partial A_\mu(x_2)}{\partial x_{2\rho}} \right]$$

$$\left[\frac{-i}{(2\pi)^4} \int d^4q \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon} \right] [\bar{\psi}(x_3)\gamma_5\psi(x_3)] \tau_{(2)} \tau_{(3)} \left[F^{'S}_{fee} + F^{'V}_{fee} \tau_z^{(2)} \right] \quad \cdots (A2\text{-}4a)$$

Now commiting $[\gamma\cdot(p_1'+k)+im]$ in equation (A4-4) with γ_μ so that it acts on $\bar{\psi}(x_2)$ and similarly commiting $[\gamma\cdot(p_1-k)+im]$ in equation (A2-4a) with γ_μ so that it acts on $\psi(x_1)$, we have

$$M_I = i f^2 \iint d^4x_1 d^4x_2 \left\{ \bar{\psi}(x_2) \left[-\gamma\cdot(p_1'+k)+im\right] \frac{(-i)\,(p_1'+p_1'+k)_\rho}{(p_1'+k)^2+m^2} \gamma_\mu \gamma_5 \psi(x_2) \right\} \left[\frac{\partial A_\rho(x_2)}{\partial x_{2\mu}} - \frac{\partial A_\mu(x_2)}{\partial x_{2\rho}} \right]$$

$$\left[\frac{-i}{(2\pi)^4} \int d^4q \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon} \right] [\bar{\psi}(x_3)\gamma_5\psi(x_3)]. \left[F^{'S}_{fee} + F^{'V}_{fee} \tau_z^{(2)} \right] \tau_{(2)} \tau_{(3)} +$$

$$+ 2 i f^2 \iint d^4x_1 d^4x_2 \left[\bar{\psi}(x_2) (p_1'+k)_\mu \frac{\{-i(p_1+p_1'+k)_\rho\}}{(p_1'+k)^2+m^2} \gamma_5 \psi(x_2) \right] \left[\frac{\partial A_\rho(x_2)}{\partial x_{2\mu}} - \frac{\partial A_\mu(x_2)}{\partial x_{2\rho}} \right]$$

$$\left[\frac{-i}{(2\pi)^4} \int d^4q \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon} \right] [\bar{\psi}(x_3)\gamma_5\psi(x_3)] \left[F^{'S}_{fee} + F^{'V}_{fee} \tau_z^{(2)} \right] \tau_{(2)} \cdot \tau_{(3)} \quad \cdots (A2\text{-}5)$$

$$H_{II} = i f^2 \iint d^4x_1\, d^4x_3 \left\{ \bar{\psi}(x_2)\, \gamma_5 (-)\, \gamma_\mu \frac{i (P_1 + P_1 - K)_\rho \{ -\gamma \cdot (P_1 - K) + i m \}}{(P_1 - K)^2 + m^2} \psi(x_1) \right] \left[\frac{\partial A_\rho(x_2)}{\partial x_{2\mu}} - \frac{\partial A_\mu(x_2)}{\partial x_{2\rho}} \right]$$

$$\left\{ \frac{-i}{(2\pi)^4} \int d^4q \frac{e^{iq(x_2 - x_3)}}{q^2 + \mu^2 - i\varepsilon} \right\} \left[\bar{\psi}(x_3)\, \gamma_5 \psi(x_3) \right] \tau_{(2)} \tau_{(3)} \left[F'^S_{fee} + F'^V_{fee} \tau_t^{(2)} \right] +$$

$$+ i f^2 \iint d^4x_2\, d^4x_3 \left\{ \bar{\psi}(x_2)\, \gamma_5 (-)\, i \frac{(P_1 + P_1 - K)_\rho\, 2 (P_1 - K)_\mu}{(P_1 - K)^2 + m^2} \psi(x_2) \right] \left[\frac{\partial A_\rho(x_2)}{\partial x_{2\mu}} - \frac{\partial A_\mu(x_2)}{\partial x_{2\rho}} \right]$$

$$\left\{ \frac{-i}{(2\pi)^4} \int d^4q \frac{e^{iq(x_2 - x_3)}}{q^2 + \mu^2 - i\varepsilon} \right\} \left[\bar{\psi}(x_3)\, \gamma_5 \psi(x_3) \right] \tau_{(2)} \tau_{(3)} \left[F'^S_{fee} + F'^V_{fee} \tau_t^{(2)} \right] \quad \ldots \quad (A2\text{-}5a)$$

Using now the Dirac equation these simplify to

$$H_I = i f^2 \iint d^4x_2\, d^4x_3 \left\{ \bar{\psi}(x_2) \left[\gamma \cdot K\, \gamma_\mu \frac{i (P_1' + P_1' + K)_\rho}{(P_1' + K)^2 + m^2} \gamma_5 \psi(x_2) \right\} \left[\frac{\partial A_\rho(x_2)}{\partial x_{2\mu}} - \frac{\partial A_\mu(x_2)}{\partial x_{2\rho}} \right]$$

$$\left\{ \frac{-i}{(2\pi)^4} \int d^4q \frac{e^{iq(x_2 - x_3)}}{q^2 + \mu^2 - i\varepsilon} \right\} \left[\bar{\psi}(x_3)\, \gamma_5 \psi(x_3) \right] \left[F'^S_{fee} + F'^V_{fee} \tau_t^{(2)} \right] \tau_{(2)} \cdot \tau_{(3)} +$$

$$+ 2 i f^2 \iint d^4x_2\, d^4x_3 \left[\bar{\psi}(x_2)\, (P_1' + K)_\mu \left\{ \frac{-i (P_1' + P_1' + K)_\rho}{(P_1' + K)^2 + m^2} \right\} \gamma_5 \psi(x_2) \right\} \left[\frac{\partial A_\rho(x_2)}{\partial x_{2\mu}} - \frac{\partial A_\mu(x_2)}{\partial x_{2\rho}} \right]$$

$$\left\{ \frac{-i}{(2\pi)^4} \int d^4q \frac{e^{iq(x_2 - x_3)}}{q^2 + \mu^2 - i\varepsilon} \right\} \left[\bar{\psi}(x_3)\, \gamma_5 \psi(x_3) \right] \left[F'^S_{fee} + F'^V_{fee} \tau_t^{(2)} \right] \tau_{(2)} \cdot \tau_{(3)} \quad \ldots \quad (A2\text{-}6)$$

$$M_{II} = i f^2 \iint d^4x_2\, d^4x_3 \left\{ \bar\psi(x_2)\, \gamma_5 \gamma_\mu \frac{(+i)\,(P_1+P_1-\kappa)_\rho}{(P_1-\kappa)^2+m^2} \left[-\gamma\cdot\kappa\right] \psi(x_1) \right] \left[\frac{\partial A_\rho(x_2)}{\partial x_{2\mu}} - \frac{\partial A_\mu(x_2)}{\partial x_{2\rho}} \right]$$

$$\left[\frac{-i}{(2\pi)^4} \int d^4q \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon} \right] \left[\bar\psi(x_3)\, \gamma_5\, \psi(x_3)\right] \tau_{(2)}\, \tau_{(3)} \left[F^{S}_{fee} + F^{V}_{fee}\, \tau_t^{(2)} \right] +$$

$$+ i f^2 \iint d^4x_2\, d^4x_3 \left\{ \bar\psi(x_2)\, \gamma_5 \frac{(+i)\,(P_1+P_1-\kappa)_\rho}{(P_1-\kappa)^2+m^2}\, 2\,(P_1-\kappa)_\mu\, \psi(x_2) \right] \left[\frac{\partial A_\rho(x_2)}{\partial x_{2\mu}} - \frac{\partial A_\mu(x_2)}{\partial x_{2\rho}} \right]$$

$$\left[\frac{-i}{(2\pi)^4} \int d^4q \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon} \right] \left[\bar\psi(x_3)\, \gamma_5\, \psi(x_3)\right] \tau_{(2)}\, \tau_{(3)} \left[F'^{S}_{fee} + F'^{V}_{fee}\, \tau_t^{(2)} \right] \quad \cdots \cdot (A2-6a)$$

For convenience M_I and M_{II} are now decomposed into terms with $\mu \neq \rho$ $\rho = 4$, terms with $\mu = 4$ $\rho \neq 4$ and finally terms with $\mu \neq 4$ $\rho \neq 4$. They are written as follows

$$M_I = M_I^{(a)} + M_I^{(b)} + \cdots\cdots + M_I^{(f)}$$

$$M_{II} = M_{II}^{(a)} + M_{II}^{(b)} + \cdots\cdots + M_{II}^{(f)}$$

$$M_I^{(a)} = i\, f^2 \iint d^4x_2\, d^4x_3 \left\{ \bar\psi(x_2)\, \gamma\cdot\kappa\; \gamma_\mu \frac{i\,(P_1'+P_1'+\kappa)_4}{(P_1+\kappa)^2+m^2}\, \gamma_5\, \psi(x_2) \right\} \left[\frac{\partial A_4(x_2)}{\partial x_{2\mu}} - \frac{\partial A_\mu(x_2)}{\partial x_{24}} \right]$$

$$\left[\frac{-i}{(2\pi)^4} \int d^4q \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon} \right] \left[\bar\psi(x_3)\, \gamma_5\, \psi(x_3)\right] \left[F'^{S}_{fee} + F'^{V}_{fee}\, \tau_t^{(2)} \right] \tau_{(2)}\, \tau_{(3)}$$

$$M_I^{(b)} = i f^2 \iint d^4x_2 d^4x_3 \frac{[\bar{\psi}(x_2)\, \gamma \cdot K\, \gamma_4\, i\, (p_1' + p_1' + K)_\rho\, \gamma_5 \psi(x_2)]}{(p_1+K)^2 + m^2} \left[\frac{\partial A_\rho(x_2)}{\partial x_{24}} - \frac{\partial A_4(x_2)}{\partial x_{2\rho}} \right]$$

$$\left[\frac{-i}{(2\pi)^4} \int d^4q \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon} \right] [\bar{\psi}(x_3)\, \gamma_5 \psi(x_3)] \left[F_{\rho\ell\ell}^{\prime S} + F_{\rho\ell\ell}^{\prime V}\, \tau_\ell^{(2)} \right] \tau_{(2)} \cdot \tau_{(3)}$$

$$M_I^{(c)} = i f^2 \iint d^4x_2 d^4x_3 \frac{[\bar{\psi}(x_2)\, \gamma \cdot K\, \gamma_{\mu\neq 4}^{(i)} [p_1' + p_1' + K]_{\rho\neq 4}\, \gamma_5 \psi(x_2)]}{(p_1'+K)^2 + m^2} \left[\frac{\partial A_{\rho\neq 4}(x_2)}{\partial x_{2\mu\neq 4}} - \frac{\partial A_{\mu\neq 4}(x_2)}{\partial x_{2\rho\neq 4}} \right]$$

$$\left[\frac{-i}{(2\pi)^4} \int d^4q \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon} \right] [\bar{\psi}(x_3)\, \gamma_5 \psi(x_3)] \left[F_{\rho\ell\ell}^{\prime S} + F_{\rho\ell\ell}^{\prime V}\, \tau_\ell^{(2)} \right] \tau_{(2)} \cdot \tau_{(3)}$$

$$M_I^{(d)} = 2 i f^2 \iint d^4x_2 d^4x_3 \left[\bar{\psi}(x_2)\, (p_1'+K)_{\mu\neq 4} \left\{ \frac{-i\,(p_1' + p_1' + K)_{\rho\neq 4}}{(p_1'+K)^2 + m^2} \right\} \gamma_5 \psi(x_2) \right] \left[\frac{\partial A_{\rho\neq 4}(x_2)}{\partial x_{24}} - \frac{\partial A_4(x_2)}{\partial x_{2\rho}} \right]$$

$$\left[\frac{-i}{(2\pi)^4} \int d^4q \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon} \right] [\bar{\psi}(x_3)\, \gamma_5 \psi(x_3)] \left[F_{\rho\ell\ell}^{\prime S} + F_{\rho\ell\ell}^{\prime V}\, \tau_\ell^{(2)} \right] \tau_{(2)} \cdot \tau_{(3)}$$

$$M_I^{(e)} = 2 i f^2 \iint d^4x_2 d^4x_3 \left[\bar{\psi}(x_2)\, {}^{(i)}\, (p_1' + K)_{\mu\neq 4} \left\{ \frac{-i\,(p_1'+p_1'+K)_{\rho=4}}{(p_1'+K)^2+m^2} \right\} \gamma_5 \psi(x_2) \right] \left[\frac{\partial A_{\rho}(x_2)}{\partial x_{2\mu}} - \frac{\partial A_\mu(x_2)}{\partial x_{24}} \right]$$

$$\left[\frac{-i}{(2\pi)^4} \int d^4q \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon} \right] [\bar{\psi}(x_3)\, \gamma_5 \psi(x_3)] \left[F_{\rho\ell\ell}^{\prime S} + F_{\rho\ell\ell}^{\prime V}\, \tau_\ell^{(2)} \right] \tau_{(2)} \cdot \tau_{(3)}$$

$$M_I^{(f)} = 2 i f^2 \iint d^4x_2 d^4x_3 \left[\bar{\psi}(x_2)\, (p_1'+K)_{\mu\neq 4} \frac{\{-i\,(p_1'+p_1'+K)_{\rho\neq 4}\}}{(p_1'+K)^2+m^2} \gamma_5 \psi(x_2) \right] \left[\frac{\partial A_\rho(x_2)}{\partial x_{2\mu}} - \frac{\partial A_\mu(x_2)}{\partial x_{2\rho}} \right]$$

$$\left[\frac{-i}{(2\pi)^4} \int d^4q \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon} \right] [\bar{\psi}(x_3)\, \gamma_5 \psi(x_3)] \left[F_{\rho\ell\ell}^{\prime S} + F_{\rho\ell 0}^{\prime V}\, \tau_\ell^{(2)} \right] \tau_{(2)} \cdot \tau_{(3)}$$

Analogously

$$M_{II}^{(a)} = i f^2 \iint d^4x_2 d^4x_3 \left[\bar\psi(x_2)\, \gamma_5 \gamma_\mu \frac{(i)\,(P_1+P_1-K)_4}{[(P_1-K)^2+m^2]} (-\gamma\cdot K)\, \psi(x_2)\right] \left[\frac{\partial A_4(x_2)}{\partial x_{2\mu}} - \frac{\partial A_\mu(x_2)}{\partial x_{24}}\right]$$

$$\left\{\frac{-i}{(2\pi)^4}\int d^4q \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon}\right\} [\bar\psi(x_3)\,\gamma_5\,\psi(x_3)]\; \tau_{(2)}\cdot\tau_{(3)} \left[F_{fee}^{'S} + F_{fee}^{'V}\,\tau_z^{(2)}\right]$$

$$M_{II}^{(b)} = i f^2 \iint d^4x_2 d^4x_3 \left[\bar\psi(x_2)\, \gamma_5 \gamma_4 \frac{(i)\,(P_1+P_1-K)_{\rho\neq 4}}{[(P_1-K)^2+m^2]} (\gamma\cdot K)\, \psi(x_2)\right] \left[\frac{\partial A_\rho(x_2)}{\partial x_{24}} - \frac{\partial A_4(x_2)}{\partial x_{2\rho}}\right]$$

$$\left\{\frac{-i}{(2\pi)^4}\int d^4q \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon}\right\} [\bar\psi(x_3)\,\gamma_5\,\psi(x_3)]\; \tau_{(2)}\cdot\tau_{(3)} \left[F_{fee}^{'S} + F_{fee}^{'V}\,\tau_z^{(2)}\right]$$

$$M_{II}^{(c)} = i f^2 \iint d^4x_2 d^4x_3 \left[\bar\psi(x_2)\, \gamma_5 \gamma_{\mu\neq 4} \frac{(i)\,(P_1+P_1-K)_{\rho\neq 4}}{[(P_1-K)^2+m^2]} [-\gamma\cdot K]\, \psi(x_2)\right] \left[\frac{\partial A_\rho(x_2)}{\partial x_{2\mu\neq 4}} - \frac{\partial A_{\mu\neq 4}(x_2)}{\partial x_{2\rho}}\right]$$

$$\left\{\frac{-i}{(2\pi)^4}\int d^4q \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon}\right\} [\bar\psi(x_3)\,\gamma_5\,\psi(x_3)]\; \tau_{(2)}\cdot\tau_{(3)} \left[F_{fee}^{'S} + F_{fee}^{'V}\,\tau_z^{(2)}\right]$$

$$M_{II}^{(d)} = 2 i f^2 \iint d^4x_2 d^4x_3 \left\{\bar\psi(x_2)\, \gamma_5 \frac{(+i)\,(P_1+P_1-K)_{\rho\neq 4}}{(P_1-K)^2+m^2} (P_1-K)_4\, \psi(x_2)\right\} \left[\frac{\partial A_\rho(x_2)}{\partial x_{24}} - \frac{\partial A_4(x_2)}{\partial x_{2\rho}}\right]$$

$$\left\{\frac{(-i)}{(2\pi)^4}\int d^4q \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon}\right\} [\bar\psi(x_3)\,\gamma_5\,\psi(x_3)]\; \tau_{(2)}\cdot\tau_{(3)} \left[F_{fee}^{'S} + F_{fee}^{'V}\,\tau_z^{(1)}\right].$$

$$M_{II}^{(e)} = 2 i f^2 \iint d^4x_2\, d^4x_3 \left\{\bar\psi(x_2)\, \gamma_5 \frac{(i)\,(P_1+P_1-K)_4}{(P_1-K)^2+m^2} (P_1-K)_\mu\, \psi(x_2)\right\} \left[\frac{\partial A_4(x_2)}{\partial x_{2\mu}} - \frac{\partial A_\mu(x_2)}{\partial x_{24}}\right]$$

$$\left\{\frac{(-i)}{(2\pi)^4}\int d^4q \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon}\right\} [\bar\psi(x_3)\,\gamma_5\,\psi(x_3)]\; \tau_{(1)}\cdot\tau_{(2)} \left[F_{fee}^{'S} + F_{fee}^{'V}\,\tau_z^{(1)}\right]$$

$$M_{II}^{(f)} = 2i f^2 \iint d^4x_2\, d^4x_3 \left\{ \bar{\psi}(x_2)\, \gamma_5 \frac{(+i)\,(P_1+P_1-K)_{\rho\neq 4}}{(P_1-K)^2+m^2} (P_1-K)_{\mu\neq 4}\, \psi(x_2) \right\} \left[\frac{\partial A_\rho(x_2)}{\partial x_{2\mu}} - \frac{\partial A_\mu(x_2)}{\partial x_{2\rho}} \right]$$

$$\left\{ \frac{(-i)}{(2\pi)^4} \int d^4q\, \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon} \right\} \left[\bar{\psi}(x_3)\, \gamma_5 \psi(x_3) \right] \tau_{(1)}\, \tau_{(3)} \left[F^S_{fee} + F^V_{fee}\, \tau_t^{(2)} \right]$$

From each of these matrix elements a non-relativistic transition operator can be extracted. The specific procedure to do this is illustrated below for the matrix elements $M_I^{(a)}$ and $M_{II}^{(a)}$ since detailed calculation shows that the operators resulting from these two matrix elements are the leading ones in the non relativistic limit, i.e., the operators which are of the lowest order in $(\frac{P}{m})$.

Using

$$\gamma \cdot K = \vec{\gamma} \cdot \vec{K} + i\, \gamma_4 K_0$$

$$(P_1+K)^2 + m^2 \approx 2m K_0$$

$$(P_1-K)^2 + m^2 \approx -2m K_0$$

one gets

$$M = M_I^{(a)} + M_{II}^{(a)} = (-) f^2 \iint d^4x_2\, d^4x_3 \left\{ \bar{\psi}(x_2)\, \gamma_4 \gamma_\mu \gamma_5 \psi(x_2) \right\} \vec{E}_{\mu\neq 4}(x_2) \left[\frac{1}{(2\pi)^4} \int d^4q\, \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon} \right]$$

$$\left[\bar{\psi}(x_3)\, \gamma_5 \psi(x_3) \right] F^{IV}_{fee} \left[\tau_t^{(2)},\ \tau_{(1)} \cdot \tau_{(3)} \right]$$

Integrating over dt_2, dt_3 and dq_0 and noting that $q_0 = p_{20} - p'_{20} \approx 0$ gives

$$M = -2\pi i\, \delta(E\text{nergies}) \Bigg\{ (-) i f^2 \iint d^3 r_2\, d^3 r_3 \left[\bar{\Psi}(r_2)\, \gamma_4 \gamma_\mu \gamma_5 \Psi(r_2) \right] \vec{E}_{\mu \neq 4}(r_2)$$

$$\left[\frac{1}{(2\pi)^3} \int d^3 q\, \frac{e^{i\vec{q}\cdot(\vec{r}_2 - \vec{r}_3)}}{\vec{q}^{\,2} + \mu^2 - i\varepsilon} \right] \left[\bar{\Psi}(r_3)\, \gamma_5 \Psi(r_3) \right] F^{1V}_{Lee} \left[\tau_z^{(2)}, \tau_{(2)} \cdot \tau_{(3)} \right]_- \Bigg\}$$

Therefore

$$\langle f |\, V^{TRV}_{Lee}(E)\, | i \rangle = (-) i f^2 F^{1V}_{Lee} \iint d^3 r_2\, d^3 r_3 \left[\bar{\Psi}(r_2)\, \gamma_4 \gamma_\mu \gamma_5 \Psi(r_2) \right] E_\mu(r_2) \left[\frac{1}{(2\pi)^3} \int d^3 q\, \frac{e^{i\vec{q}\cdot(\vec{r}_2 - \vec{r}_3)}}{\vec{q}^{\,2} + \mu^2 - i\varepsilon} \right]$$

$$\left[\bar{\Psi}(r_3)\, \gamma_5 \Psi(r_3) \right] \left[\tau_z^{(2)}, \tau_{(1)} \cdot \tau_{(2)} \right]_-$$

where V^{TRV}_{Lee} (E) was introduced in Chapter 2 to simbolize the transition operator resulting from the Lee vertex. A non-relativistic reduction of this equation results in equation (11) of Chapter 2.

B. The Lipshutz vertex

In this case we start from equations (15) and (16) of Chapter 2

$$M_I = f^2 \iiint d^4x_1 d^4x_2 d^4x_3 \left[\bar{\psi}(x_1)\left(-\overleftarrow{\partial}^2 + \overrightarrow{\partial}^2\right)\sigma_{\mu\nu} S_F(x_1-x_2)\gamma_5 \psi(x_2)\right] \frac{\partial A_\mu(x_1)}{\partial x_{1\nu}} \Delta_F(x_2-x_3)$$

$$\left[\bar{\psi}(x_3)\gamma_5\psi(x_3)\right]\left[F'^S_{\mu p} + F'^V_{\mu p}\tau_z^{(1)}\right]\tau_{(1)}\cdot\tau_{(2)} \qquad \ldots \quad (A2\text{-}7)$$

$$M_{II} = f^2 \iiint d^4x_1 d^4x_2 d^4x_3 \left[\bar{\psi}(x_2)\gamma_5 S_F(x_2-x_1)\left(-\overleftarrow{\partial}^2 + \overrightarrow{\partial}^2\right)\sigma_{\mu\nu}\psi(x_1)\right] \frac{\partial A_\mu(x_1)}{\partial x_{1\nu}} \Delta_F(x_2-x_3)$$

$$\left[\bar{\psi}(x_3)\gamma_5\psi(x_3)\right]\tau_{(1)}\cdot\tau_{(2)}\left[F'^S_{\mu p} + F'^V_{\mu p}\tau_z^{(1)}\right] \qquad \ldots\ldots \quad (A2\text{-}7a)$$

Now we have to manipulate equations (A2.7) and (A2.7a) as in subsection A of this Appendix. For simplicity only the equation relating to M_I will be written down.

First the propagators S_F and Δ_F are substituted in equations (A2.7) and (A2.7a) giving

$$M_I = -f^2 \iiint d^4x_1 d^4x_2 d^4x_3 \left[\bar{\psi}(x_1)\sigma_{\mu\nu}\left[\frac{1}{(2\pi)^4}\int d^4Q\left(p_1'^2 - Q^2\right)\frac{\gamma\cdot Q + im}{Q^2+m^2} e^{iQ\cdot(x_1-x_2)}\right]\gamma_5\psi(x_2)\right]$$

$$\frac{\partial A_\mu(x_1)}{\partial x_{1\nu}}\left[\frac{-i}{(2\pi)^4}\int d^4q \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon}\right]\left[\bar{\psi}(x_3)\gamma_5\psi(x_3)\right]\left[F'^S_{\mu p} + F'^V_{\mu p}\tau_z^{(1)}\right]\tau_{(1)}\cdot\tau_{(2)} \cdots (A2\text{-}8)$$

Now since $p_1'^2 = -m^2$ and $p_1^2 = m^2$,

$$M_I = f^2 \iiint d^4x_1 d^4x_2 d^4x_3 \left[\bar{\psi}(x_1)\, \sigma_{\mu\nu} \left[\frac{1}{(2\pi)^4} \int d^4Q \, (\gamma Q + i m)\, e^{iQ(x_1-x_2)}\right] \gamma_5 \psi(x_2)\right] \frac{\partial A_\mu(x_1)}{\partial x_{1\nu}}$$

$$\left[\frac{-i}{(2\pi)^4} \int d^4q \, \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon}\right] \left[\bar{\psi}(x_3)\gamma_5 \psi(x_3)\right] \left[F_2^S + F_2^V \tau_z^{(1)}\right] \tau_{(1)}\cdot\tau_{(2)} \quad \ldots \quad (A2\text{-}9)$$

Integrating first over d^4x_1 and then over d^4Q gives

$$M_I = f^2 \iint d^4x_2 d^4x_3 \left\{\bar{\psi}(x_2)\, \sigma_{\mu\nu} \left(\gamma\cdot(p_1'+k) + i m\right)\gamma_5 \psi(x_2)\right\} \frac{\partial A_\mu(x_2)}{\partial x_{2\nu}} \left[\frac{-i}{(2\pi)^4} \int d^4q \, \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon}\right]$$

$$\left[\bar{\psi}(x_3)\gamma_5 \psi(x_3)\right] \left[F_{\mu p}'^S + F_{\mu p}'^V \tau_z^{(1)}\right] \tau_{(1)}\cdot\tau_{(2)} \quad \ldots\ldots \quad (A2\text{-}10)$$

Similarly,

$$M_{II} = -f^2 \iint d^4x_2 d^4x_3 \left[\bar{\psi}(x_2)\gamma_5 \left(\gamma\cdot(p_1-k) + i m\right)\sigma_{\mu\nu} \psi(x_2)\right] \frac{\partial A_\mu(x_2)}{\partial x_{2\nu}} \left[\frac{-i}{(2\pi)^4} \int d^4q \, \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon}\right]$$

$$\left[\bar{\psi}(x_3)\gamma_5 \psi(x_3)\right] \tau_{(1)}\cdot\tau_{(2)} \left[F_{\mu p}'^S + F_{\mu p}'^V \tau_z^{(1)}\right] \quad \ldots\ldots \quad (A2\text{-}10a)$$

The dominant term in (10) and (10a) come from the term $4 \neq \mu \neq \nu \neq 4$. This can be verified by direct and tedious calculation. Taking this term and $\gamma\cdot(p_1' \pm k) \approx i\gamma_4 m$ we have from M_I

$$\left[\bar{\Psi}(x_2)\ \sigma_{\mu\nu}\left(\gamma\cdot(p_1'+k)+im\right)\gamma_5\ \Psi(x_2)\right]\ \frac{\partial A_\mu(x_2)}{\partial x_{2\nu}} \approx A_I + B_I$$

$$\left.\begin{aligned} A_I &= m\left[\bar{\Psi}(x_2)\ \gamma_{\mu\neq 4}\ \gamma_{\nu\neq 4\neq\mu}\ \gamma_4\gamma_5\ \Psi(x_2)\right]\ \frac{\partial A_\mu(x_2)}{\partial x_{2\nu}} \\ B_I &= m\left[\bar{\Psi}(x_2)\ \gamma_{\mu\neq 4}\ \gamma_{\nu\neq\mu\neq 4}\ \gamma_5\ \Psi(x_2)\right]\ \frac{\partial A_\mu(x_2)}{\partial x_{2\nu}} \end{aligned}\right\} \cdots (A2\text{-}11)$$

and from M_{II}

$$\left[\bar{\Psi}(x_2)\ \gamma_5\left(\gamma\cdot(p_1-k)+im\right)\sigma_{\mu\nu}\ \Psi(x_2)\right]\ \frac{\partial A_\mu(x_2)}{\partial x_{2\nu}} \approx A_{II} + B_{II}$$

$$\left.\begin{aligned} A_{II} &= m\left[\bar{\Psi}(x_2)\ \gamma_5\gamma_4\ \gamma_{\mu\neq\nu\neq 4}\ \Psi(x_2)\right]\ \frac{\partial A_\mu(x_2)}{\partial x_{2\nu}} \\ B_{II} &= m\left[\bar{\Psi}(x_2)\ \gamma_5\ \gamma_{\mu\neq\nu}\ \gamma_{\nu\neq 4}\ \Psi(x_2)\right]\ \frac{\partial A_{\mu\neq 4}(x_2)}{\partial x_{2\nu}} \end{aligned}\right\} \cdots (A2\text{-}11a)$$

Substituting A_I in (A2-10), integrating over time and extracting the factor $-2\pi i\ \delta$ (Energ.) gives

$$V_{A_I} = \frac{if^2}{4m} \left[\vec{\sigma}_3 \cdot \vec{P}_3 , \left\{ \vec{\sigma}_2 \cdot \vec{P}_2 , \vec{\sigma}_2 \cdot \vec{B}(r_2) f(r_{23}) \right\}_+ \right]_- \left\{ F^{'S}_{\mu p} \tau_{(2)} \cdot \tau_{(3)} + F^{'V}_{\mu p} \tau_t^{(2)} \tau_{(2)} \cdot \tau_{(3)} \right\}$$

where
$$f(r_{23}) = \frac{1}{4\pi} \frac{e^{-\mu |\vec{r}_2 - \vec{r}_3|}}{|\vec{r}_2 - \vec{r}_3|}$$

The term A_{II} substituted in (A4-10a) and treated in the same way gives exactly the hermitian conjugate of V_{A_I}. Similarly, substituting B_I in (A2-10), integrating over time and extracting the factor $-2\pi i\ \delta$ (Energ.) gives

$$V_{B_I} = \frac{if^2}{4m} \left[\vec{\sigma}_3 \cdot \vec{P}_3 , \left[\sigma_2 \cdot P_2 , \vec{\sigma}_2 \cdot \vec{B}(r_2) f(r_{23}) \right]_- \right]_- F^{'V}_{\mu p} \tau_t^{(2)} \tau_{(2)} \cdot \tau_{(3)}$$

Analogously the term B_{II} substituted in (A2-10a) gives exactly the hermitian conjugate of V_{A_I}. So the total result is

$$V_{\mu p}(\vec{B}) = \frac{if^2}{4m} \left[\sigma_3 \cdot P_3 , \left\{ \sigma_2 \cdot P_2 , \vec{\sigma}_2 \cdot \vec{B}(r_2) f(r_{23}) \right\}_+ \right]_- \left\{ 2 F^{'S}_{\mu p} \tau_{(2)} \cdot \tau_{(3)} + F^{'V}_{\mu p} \left\{ \tau_t^{(2)} , \tau_{(2)} \cdot \tau_{(3)} \right\}_+ \right\}$$

$$+ \frac{if^2}{4m} \left[\sigma_3 \cdot P_3 , \left[\sigma_2 \cdot P_2 , \vec{\sigma} \cdot \vec{B} f(r_{23}) \right]_- \right]_- F^{'V}_{\mu p} \left[\tau_t^{(2)} , \tau_{(2)} \cdot \tau_{(3)} \right]_-$$

Finally, if one calculates the commutators one gets the result of equation (17) in Chapter 2.

C. The NN*γ vertex

In this case the starting point is equation (3-8) which is the matrix element corresponding to the diagram of Fig. (8. a) and (8. b), viz.

$$M_I = i \iiint d^4x_1 d^4x_2 d^4x_3 \frac{Gf}{m_\mu} \left[\bar{\Psi}(x_1)\, \gamma_\mu \gamma_5\, S_{\lambda\rho}(x_1 - x_2)\, \Psi(x_2) \right] F_{\lambda\mu}(x_1) \left[\frac{\partial \Delta_F(x_2 - x_3)}{\partial x_{2\rho}} \right]$$

$$\left[\bar{\Psi}(x_3)\, \gamma_5\, \Psi(x_3) \right] \quad \left(\vec{T}'_{(1)} \cdot \tau_{(2)} \right) \qquad \ldots\ldots \quad (A2\text{-}12)$$

$$M_{II} = i \iiint d^4x_1 d^4x_2 d^4x_3 \frac{Gf}{m_\mu} \left[\bar{\Psi}(x_1)\, S_{\rho\lambda}(x_2 - x_1)\, \gamma_\mu \gamma_5\, \Psi(x_1) \right] F_{\lambda\mu}(x_1) \left[\frac{\partial \Delta_F(x_2 - x_3)}{\partial x_{2\rho}} \right]$$

$$\left[\bar{\Psi}(x_3)\, \gamma_5\, \Psi(x_3) \right] \quad \left(\vec{T}'_{(1)} \cdot \tau_{(2)} \right)^* \qquad \ldots\ldots \quad (A2\text{-}12a)$$

The propagator $S_{\lambda\rho}(x_1 - x_2)$ has been given in Chapter 3 and decomposed into six parts as given by equations 3-11a to 3.11f.

Consider first the part given by equation (3.11a) viz.

$$X = \frac{-i}{(2\pi)^4} \int d^4Q \frac{e^{iQ(x_1 - x_2)}}{Q^2 + M^2 - i\varepsilon}\, M \left[\delta_{\mu\nu} - \frac{1}{3} \gamma_\mu \gamma_\nu \right]$$

Substituting this in (A2-12), integrating over d^4x_1 and d^4Q gives

$$M_I(x) = \frac{i}{4\pi} \frac{Gf}{\mu} \iint d^4x_2 d^4x_3 \left[\bar{\Psi}(x_2)\, \gamma_\mu \gamma_5 (-i)\, M \left[\delta_{\lambda\rho} - \frac{1}{3} \gamma_\lambda \gamma_\rho \right] \frac{1}{(P_1' + K)^2 + M^2}\, \Psi(x_2) \right]$$

$$F_{\lambda\mu}(x_2) \left[\frac{(-i)}{(2\pi)^4} \int d^4q\, i q_\rho \frac{e^{iq\cdot(x_2 - x_3)}}{q^2 + \mu^2 - i\varepsilon} \right] \left[\bar{\Psi}(x_3) \gamma_5 \Psi(x_3) \right] \left(\vec{T}'_{(1)} \cdot \vec{\tau}_{(2)} \right)$$

$$\ldots\ldots \quad (A2\text{-}13)$$

From equation (A2-13) on separating the terms with $\lambda = \rho$ and the term with $\lambda = \rho$ but $\mu = \rho$ from the term $\lambda \neq \rho$ and $\mu \neq \rho$, we have

$$M_I^{(a)}(x) = \frac{i}{m}\frac{G}{\mu}\iint d^4x_2 d^4x_3 \left[\bar{\psi}(x_2)\gamma_4\gamma_5 \frac{(-1)}{3}\frac{M}{M^2-m^2}\psi(x_2)\right] F_{\rho 4}(x_2)\left[\frac{(-1)}{(2\pi)^4}\int d^4q\, iq_\rho \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon}\right]$$

$$\left[\bar{\psi}(x_3)\gamma_5\psi(x_3)\right]\left(\vec{T}'_{(2)}\cdot\vec{\tau}_{(3)}\right) \qquad \ldots\ldots \qquad (A2\text{-}14)$$

$$M_I^{(b)}(x) = \frac{i}{m}\frac{G}{\mu}\iint d^4x_2\, d^4x_3 \left[\bar{\psi}(x_2)\gamma_{\lambda\neq 4}\gamma_5 \frac{(-1)}{3}\frac{M}{M^2-m^2}\psi(x_2)\right] F_{\rho\lambda\neq 4}(x_2)\left[\frac{(-1)}{(2\pi)^4}\int d^4q\, iq_\rho \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon}\right]$$

$$\left[\bar{\psi}(x_3)\gamma_5\psi(x_3)\right]\left(\vec{T}'_{(2)}\cdot\vec{\tau}_{(3)}\right) \qquad \ldots\ldots \qquad (A2\text{-}15)$$

$$M_I^{(c)}(x) = \frac{i}{m}\frac{G}{\mu}\iint d^4x_2 d^4x_3 \left[\bar{\psi}(x_2)\gamma_4\gamma_5\gamma_{\lambda\neq\rho \neq 4}\gamma_{\rho\neq 4}\frac{2}{3}i\frac{M}{M^2-m^2}\psi(x_2)\right] i\vec{E}_\lambda(x_2)\left[\frac{(-1)}{(2\pi)^4}\int d^3q \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon} iq_\rho\right]$$

$$\left[\bar{\psi}(x_3)\gamma_5\psi(x_3)\right]\left(\vec{T}'_{(2)}\cdot\vec{\tau}_{(3)}\right) \qquad \ldots\ldots \qquad (A2\text{-}16)$$

$$M_I^{(d)}(x) = \frac{i}{m}\frac{G}{\mu}\iint d^4x_2 d^4x_3 \left[\bar{\psi}(x_2)\gamma_{\mu\neq\rho \neq 4}\gamma_5 \frac{(1)}{3}\gamma_{\lambda\neq\rho}\gamma_{\rho\neq 4}\frac{M}{M^2-m^2}\psi(x_2)\right] F_{\lambda\mu}(x_2)\left[\frac{-1}{(2\pi)^4}\int d^4q\, iq_\rho \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon}\right]$$

$$\left[\bar{\psi}(x_3)\gamma_5\psi(x_3)\right]\left(\vec{T}^{(2)}\cdot\vec{\tau}_{(3)}\right) \qquad \ldots\ldots \qquad (A2\text{-}17)$$

Next substitute the part of the propagator denoted by Y (see equation 3.11b) into equation (A2-12). Integrating over d^4x_1 and then over d^4Q gives

$$M_I(y) = \frac{i}{m}\frac{G}{\mu} f \iint d^4x_2 d^4x_3 \left[\bar{\psi}(x_2)\gamma_\mu\gamma_5 (-i)^2 \gamma (P_1'+K)\left[\delta_{\lambda\rho} - \frac{1}{3}\gamma_\lambda\gamma_\rho\right] \frac{1}{(P_1'+K)^2+M^2} \psi(x_2)\right]$$

$$F_{\lambda\mu}(x_2) \left[\frac{-i}{(2\pi)^4}\int d^4q \; iq_\rho \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon}\right] \left[\bar{\psi}(x_3)\gamma_5\psi(x_3)\right] \left(T_{(2)} \cdot \vec{\tau}_{(3)}\right) +$$

$$+ \frac{i}{m}\frac{G}{\mu} f \iint d^4x_2 d^4x_3 \left[\bar{\psi}(x_2)\gamma_\mu\gamma_5 \frac{(-i)}{3}\left(i\gamma_\lambda (P_1'+k) - \gamma_\rho i (P_1'+K)_\lambda\right) \frac{1}{(P_1'+K)^2+M^2} \psi(x_2)\right] F_{\lambda\mu}(x_2)$$

$$\left[\frac{(-i)}{(2\pi)^4}\int d^4q \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon}\right] \left[\bar{\psi}(x_3)\gamma_5\psi(x_3)\right] \left(\vec{T}'_{(2)} \cdot \tau_{(3)}\right)$$

Comuting $\gamma\cdot(P_1' + K)$ to the left of $\gamma_\mu\gamma_5$ and applying the Dirac equation,

$$M_I(y) = \frac{i}{m}\frac{G}{\mu} f \iint d^4x_2 d^4x_3 \left[\bar{\psi}(x_2)\gamma_\mu\gamma_5 (-i)\left[\delta_{\lambda\rho} - \frac{1}{3}\gamma_\lambda\gamma_\rho\right] \frac{m}{M^2-m^2} \psi(x_2) \; F_{\lambda\mu}(x_2)\right.$$

$$\left[\frac{(-i)}{(2\pi)^4}\int d^4q \; iq_\rho \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon}\right] \left(\bar{\psi}(x_3)\gamma_5\psi(x_3)\right) \left(T'_{(2)} \cdot \tau_{(3)}\right) +$$

$$+ \frac{i}{m}\frac{G}{\mu} f \iint d^4x_2 d^4x_3 \left(\bar{\psi}(x_2)\, 2 P'_{1\mu}\gamma_5\left[\delta_{\lambda\rho} - \frac{1}{3}\gamma_\lambda\gamma_\rho\right] \frac{1}{M^2-m^2} \psi(x_2)\right] F_{\lambda\mu}(x_2)$$

$$\left[\frac{(-i)}{(2\pi)^4}\int d^4q \; iq_\rho \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon}\right] \left[\bar{\psi}(x_3)\gamma_5\psi(x_3)\right] \left(T'_{(2)} \; \tau_{(3)}\right) +$$

$$+ \frac{i}{m}\frac{G}{\mu} f \iint d^4x_2 d^4x_3 \left[\bar{\psi}(x_2)\gamma_\mu\gamma_5 \frac{(-i)}{3}\left(i\gamma_\lambda P'_{1\rho} - \gamma_\rho i P'_{1\lambda}\right) \frac{1}{M^2-m^2} \psi(x_2)\right] F_{\lambda\mu}(x_2)$$

$$\left[\frac{(-i)}{(2\pi)^4}\int d^4q \; iq_\rho \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon}\right] \left[\bar{\psi}(x_3)\gamma_5\psi(x_3)\right] \left(T'_{(2)} \cdot \tau_{(3)}\right) \quad \ldots (A2-18)$$

The first term of equation (A2-18) should be compared with equation (A2-13) They are equal apart from a factor $\frac{M}{m}$. Therefore we have the terms $M_I^{(a)}(Y) \ldots M_I^{(d)}(Y)$ which are equal to the corresponding $M_I^{(a)}(X) \ldots M_I^{(d)}(X)$ apart from this factor

Thus, $$M_I^{(a)}(Y) = \frac{m}{M} M_I^{(a)}(X) \quad \ldots\ldots \quad (A2-19)$$

$$M_I^{(b)}(Y) = \frac{m}{M} M_I^{(b)}(X) \quad \ldots\ldots \quad (A2-20)$$

$$M_I^{(c)}(Y) = \frac{m}{M} M_I^{(c)}(X) \quad \ldots\ldots \quad (A2-21)$$

$$M_I^{(d)}(Y) = \frac{m}{M} M_I^{(d)}(X) \quad \ldots\ldots \quad (A2-22)$$

The remaining two terms in formula (A2-18) give often straightforward but tedious manipulation.

$$M_I^{(e)}(Y) = \frac{i}{m} \frac{G}{\mu} f \iint d^4x_2\, d^4x_3 \left[\bar{\psi}(x_2) \frac{i m}{M^2 - m^2} \gamma_5 \psi(x_2) \right] F_{\rho 4}(x_2) \left[\frac{-i}{(2\pi)^4} \int d^4q\, i q_\rho \frac{e^{iq(x_2 - x_3)}}{q^2 + \mu^2 - i\varepsilon} \right]$$

$$\left[\bar{\psi}(x_3) \gamma_5 \psi(x_3) \right] \left(\vec{T}'_{(2)} \cdot \vec{\tau}_{(3)} \right) \quad \ldots\ldots \quad (A2-23)$$

$$M_I^{(f)}(Y) = \frac{i}{m} \frac{G}{\mu} f \iint d^4x_2\, d^4x_3 \left[\bar{\psi}(x_2) \frac{i m}{M^2 - m^2} \gamma_5 \gamma_{\lambda \neq \rho} \gamma_\rho \psi(x_2) \right] F_{\lambda 4}(x_2)$$

$$\left[\frac{(-i)}{(2\pi)^4} \int d^4q\, i q_\rho \frac{e^{iq(x_2 - x_3)}}{q^2 + \mu^2 - i\varepsilon} \right] \left[\bar{\psi}(x_3) \gamma_5 \psi(x_3) \right] \left(\vec{T}'_{(2)} \cdot \vec{\tau}_{(3)} \right) \quad \ldots \quad (A2-24)$$

Next consider the part of the propagator denoted by Z (see equation 3.11c). Substituting Z into equation (A2-12) we have

$$M_I^{(a)}(Z) = \frac{i}{m}\,\frac{G}{\mu}\,f \iint d^4x_2\, d^4x_3\, \left[\bar{\psi}(x_2)\,\gamma_5\,(1)\left[\frac{m}{3M}\,\frac{m}{M^2-m^2}\right]\gamma_{\rho\neq 4}\,\psi(x_2)\right] F_{4\mu}(x_2)$$

$$\left[\frac{(-i)}{(2\pi)^4}\int d^4q\; i q_\rho\, \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon}\right]\left[\bar{\psi}(x_3)\,\gamma_5\,\psi(x_3)\right]\left(T'_{(1)}\cdot \tau_{(3)}\right) \qquad \cdots\cdots (A2\text{-}25)$$

$$M_I^{(b)}(Z) = \frac{i}{m}\,\frac{G}{\mu}\,f \iint d^4x_2\, d^4x_3\, \left[\bar{\psi}(x_2)\,\gamma_{\mu\neq\rho}\,\gamma_5\,(i)\left(\frac{m}{3M}\,\frac{m}{M^2-m^2}\right)\gamma_{\rho\neq 4}\,\psi(x_2)\right] F_{4\mu}(x_2)$$

$$\left[\frac{(-i)}{(2\pi)^4}\int d^4q\; i q_\rho\, \frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon}\right]\left[\bar{\psi}(x_3)\,\gamma_5\,\psi(x_3)\right]\left(T'_{(1)}\;\tau_{(3)}\right) \qquad \cdots\cdots (A2\text{-}26)$$

Finally substituting the part of the propagator denoted by U (see equation 3.11d) we have

$$M_I^{(a)}(U) = (-)\,\frac{2Gf}{m\mu}\iint d^4x_2\, d^4x_3\, \left[\bar{\psi}(x_2)\,\gamma_\mu\,\gamma_5\,\frac{2i}{3M}\,\psi(x_2)\right] F_{\rho\mu}(x_2)\left[\frac{(-i)}{(2\pi)^4}\int d^4q\; i q_\rho\,\frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon}\right]$$

$$\left[\bar{\psi}(x_3)\,\gamma_5\,\psi(x_3)\right]\left(\vec{T}'_{(1)}\cdot\vec{\tau}_{(3)}\right) \qquad \cdots\cdots\cdots (A2\text{-}27)$$

$$M_I^{(b)}(U) = \frac{i}{m}\,\frac{G}{\mu}\,f\iint d^4x_2\, d^4x_3\, \left[\bar{\psi}(x_2)\,\gamma_{\mu\neq\rho}\,\gamma_5\,\frac{2i}{3M}\,\gamma_{\lambda\neq\rho}\,\gamma_{\rho\neq 4}\,\psi(x_2)\right] F_{\lambda\mu}(x_2)$$

$$\left[\frac{-i}{(2\pi)^4}\int d^4q\; i q_\rho\,\frac{e^{iq(x_2-x_3)}}{q^2+\mu^2-i\varepsilon}\right]\left[\bar{\psi}(x_3)\,\gamma_5\,\psi(x_3)\right]\left(\vec{T}'_{(1)}\;\vec{\tau}_{(3)}\right) \cdots (A2\text{-}28)$$

The parts of the propagator denoted by V(equation 3.11c) and K(equation 3.11f) have also been considered. The terms arising from V have (due to the double derivative) an extra factor $\frac{P}{m}$ compared with the terms stemming from X.... U. The terms arising from K are very small because of the factor $\frac{1}{M^2}$ they contain.

The transition operators corresponding to the matrix elements $M_I^{(a)}$ (X) .... $M_I^{(d)}$ (X), $M_I^{(a)}$ (Y) .... $M_I^{(f)}$(Y), $M_I^{(a)}$(Z), $M_I^{(b)}$(Z), $M_I^{(a)}$ (U) and $M_I^{(b)}$ (U) can be obtained by using the methods already explained in part A of this Appendix. First however the factor $(T_{(1)} \cdot \tau_{(2)})$ which appears in these matrix elements should be replaced by the factor $(T_{(1)} \cdot \tau_{(2)})_{TRV}$ as given by equations (3.10a)* Similar operation with the M_{II} matrix elements arising from the graph of Fig. 8-b gives the hermitian conjugates of these. The complete results are given below.

$$V^{(a)}_{NN^*\gamma}(x) = \frac{G f M}{3 m \mu (M^2 - m^2)} \left[\frac{\sigma_3 \cdot P_3}{2m}, \left\{ \frac{\sigma_2 \cdot P_2}{2m}, \left(\vec{E}(r_1) \cdot (\vec{r}_2 - \vec{r}_3) \mathcal{F}(r_{23}) \right\}_+ \right]_-$$

$$\left[\frac{2}{\sqrt{2}} T_{10} \left(\tau_{(2)} \tau_{(3)} \right) \left(\varepsilon_p^* - \varepsilon_p + \varepsilon_N^* - \varepsilon_N \right) + \frac{2}{\sqrt{3}} \left(\varepsilon_N^* - \varepsilon_N - \varepsilon_p^* + \varepsilon_p \right) \tau_z^{(1)} \right] + (2 \leftrightarrow 3)$$

* See Note at the end of this appendix.

$$V^{(b)}_{NN^*\gamma}(x) = \frac{(-)}{3}\frac{G f M}{m_\mu (M^2-m^2)} \left[\frac{\sigma_3 \cdot p_3}{2m}, \vec{\sigma}_2 \times (r_2 - r_3) \cdot \vec{B}(r_2)\, \mathcal{F}(r_{23}) \right]_-$$

$$\left\{ \frac{2}{\sqrt{2}} T_{20}\left(\tau_{(2)}\,\tau_{(3)}\right) \left(\varepsilon_p^* - \varepsilon_p + \varepsilon_N^* - \varepsilon_N\right) + \frac{2}{\sqrt{3}}\left(\varepsilon_N^* - \varepsilon_N - \varepsilon_p^* + \varepsilon_p\right) \tau_z^{(3)} \right\} + (2 \leftrightarrow 3)$$

$$V^{(c)}_{NN^*\gamma}(x) = \frac{2}{3}\frac{G f M}{m_\mu (M^2-m^2)}\, i \left[\frac{\sigma_3 \cdot p_3}{2m}, \left\{ \frac{\sigma_2 \cdot p_2}{2m}, \sigma_2 \cdot \vec{E}(r_2) \times (r_2 - r_3)\, \mathcal{F}(r_{23}) \right\}_+ \right]_-$$

$$\frac{i}{\sqrt{3}} \left(\varepsilon_p^* - \varepsilon_p - \varepsilon_N^* + \varepsilon_N\right) \left(\tau_{(2)} \times \tau_{(3)}\right)_z$$

$$V^{(d)}_{NN^*\gamma}(x) = \frac{2}{3}\frac{G f}{\mu m}\frac{M}{M^2 - m^2}(-i)\left[\frac{\sigma_3 \cdot p_3}{2m}, \vec{B}(r_2)\cdot(r_2 - r_3)\,\mathcal{F}(r_{23})\right] \frac{i}{\sqrt{3}}\left(\varepsilon_p^* - \varepsilon_p - \varepsilon_N^* + \varepsilon_N\right)$$

$$\left(\tau_{(2)} \times \tau_{(3)}\right)_z$$

The terms $V^{(a)}_{NN^*\gamma}$ (Y) .... $V^{(d)}_{NN^*\gamma}$ (Y) are equal to the ones just listed excepted that they have a factor $\frac{m}{M^2-m^2}$ instead of $\frac{M}{M^2-m^2}$.

$$V^{(e)}_{NN^*\gamma}(y) = \frac{(-)\, G m}{m_\mu (M^2-m^2)} f \left[\frac{\sigma_3 \cdot p_3}{2m}, \left[\frac{\sigma_2 \cdot p_2}{2m}, E(r_2)\;(r_2 - r_3)\, \mathcal{F}(r_{23}) \right] \right]_- \frac{i}{\sqrt{3}}\left(\varepsilon_p^* - \varepsilon_p - \varepsilon_N^* + \varepsilon_N\right)$$

$$\left(\tau_{(2)} \times \tau_{(3)}\right)_z + \quad (2 \leftrightarrow 3)$$

$$V^{(f)}_{NN^*\gamma}(y) = \frac{G m f}{m_\mu (M^2-m^2)}\, i \left[\frac{\sigma_3 \cdot p_3}{2m}, \left[\frac{\sigma_2 \cdot p_2}{2m}, \sigma_2 \cdot \vec{E}(r_2) \times (r_2 - r_3)\, \mathcal{F}(r_{23}) \right]_- \right]_-$$

$$\left\{ \frac{2}{\sqrt{2}} T_{20}\left(\tau_{(2)}\,\tau_{(3)}\right) \left(\varepsilon_p^* - \varepsilon_p + \varepsilon_N^* - \varepsilon_N\right) + \frac{2}{\sqrt{3}}\left(\varepsilon_N^* - \varepsilon_N - \varepsilon_p^* + \varepsilon_p\right) \tau_z^{(3)} \right\} + (2 \leftrightarrow 3)$$

$$V^{(a)}_{NN^*\gamma}(Z) = \frac{Gf}{m_\mu} \frac{m}{3M} \frac{m}{M^2-m^2} \left[\frac{\sigma_3 \cdot P_3}{2m}, \left\{ \frac{\sigma_2 \cdot P_2}{2m}, E(r_2)\cdot(r_2-r_3)\,\mathcal{F}(r_{23}) \right\}\right] \frac{1}{\sqrt{3}} \left(\varepsilon_p^* - \varepsilon_p - \varepsilon_N^* + \varepsilon_N\right)$$

$$\left(\tau^{(2)} \times \tau^{(3)}\right)_z + (2 \leftrightarrow 3)$$

$$V^{(b)}_{NN^*\gamma}(Z) = \frac{Gf}{m_\mu} \frac{m}{3M} \frac{m}{M^2-m^2}\, i \left[\frac{\sigma_3 \cdot P_3}{2m}, \left[\frac{\sigma_2 \cdot P_2}{2m}, \sigma_2 \cdot \vec{E}(r_2) \times (r_2 - r_3)\, \mathcal{F}(r_{23}) \right]\right]$$

$$\left\{ \frac{2}{\sqrt{2}} T_{20}\left(\tau_{(2)}, \tau_{(3)}\right) \left(\varepsilon_p^* - \varepsilon_p + \varepsilon_N^* - \varepsilon_N\right) + \frac{2}{\sqrt{3}} \left(\varepsilon_N^* - \varepsilon_N - \varepsilon_p^* + \varepsilon_p\right) \tau_z^{(3)} \right] + (2 \leftrightarrow 3)$$

and finally

$$V^{(a)}_{NN^*\gamma}(U) = (-)\frac{4}{3} \frac{Gf}{m_\mu M} \left[\frac{\sigma_3 \cdot P_3}{2m}, \left\{ \frac{\sigma_2 \cdot P_2}{2m}, \vec{E}(r_2)\cdot(r_2 - r_3)\, \mathcal{F}(r_{23}) \right\} \right]$$

$$\left\{ \frac{2}{\sqrt{2}} T_{20} \left(\tau_{(2)}, \tau_{(3)}\right)\left(\varepsilon_p^* - \varepsilon_p + \varepsilon_N^* - \varepsilon_N\right) + \frac{2}{\sqrt{3}} \left(\varepsilon_N^* - \varepsilon_N - \varepsilon_p^* + \varepsilon_p\right) \tau_z^{(2)} \right\} +$$

$$+ \frac{4}{3} \frac{Gf}{m_\mu M} \left[\frac{\sigma_3 \cdot P_3}{2m}, \vec{\sigma}_2 \times (\vec{r}_2 - r_3) \cdot \vec{B}(r_2)\, \mathcal{F}(r_{23}) \right] \left\{ \frac{2}{\sqrt{2}} T_{20}\left(\tau_{(2)}, \tau_{(3)}\right)\left(\varepsilon_p^* - \varepsilon_p + \varepsilon_N^* - \varepsilon_N\right) + \right.$$

$$+ \frac{2}{\sqrt{3}} \left(\varepsilon_N^* - \varepsilon_N - \varepsilon_p^* + \varepsilon_p\right) \tau_z^{(2)} + (2 \leftrightarrow 3)$$

$$V^{(b)}_{NN^*\gamma}(U) = \frac{4}{3} \frac{Gf}{m_\mu M}\, i \left[\frac{\sigma_3 \cdot P_3}{2m}, \sigma_2\, \vec{E}(r_2) \times (r_2 - r_3)\, \mathcal{F}(r_{23}) \right] \frac{1}{\sqrt{3}} \left(\varepsilon_p^* - \varepsilon_p - \varepsilon_N^* + \varepsilon_N\right) \left(\tau^{(2)} \times \tau^{(3)}\right)_z +$$

$$+ \frac{4}{3} \frac{Gf}{m_\mu M} (-1) \left[\frac{\sigma_3 \cdot P_3}{2m}, \vec{B}(r_2)\, (r_2 - r_3)\, \mathcal{F}(r_{23}) \right] \frac{1}{\sqrt{3}} \left(\varepsilon_p^* - \varepsilon_p - \varepsilon_N^* + \varepsilon_N\right) \left(\tau^{(2)} \times \tau^{(3)}\right)_z + (2 \leftrightarrow 3)$$

The W_1(B) of equation (12) in Chapter 3 is $V^{(b)}_{NN^*\gamma}(X) + V^{(b)}_{NN^*\gamma}(Y)$ and W_2(B) of equation (13) in Chapter 3 is $V^{(d)}_{NN^*\gamma}(X) + V^{(d)}_{NN^*\gamma}(Y)$ with the commutators evaluated.

NOTE The T.R.I. violating transition operators above have been obtained by replacing the factor $(T_{(2)}\cdot\tau_{(3)})$ by $(T_{(2)}\cdot\tau_{(3)})_{TRV}$ as given by equation (3.10a). If instead of this we replace it by the factor $(T_{(2)}\cdot\tau_{(3)})_{NORM}$ given by equation (3.10b) we would obtain TRI conserving transition operators. These operators would be the contribution of the Lagrangain $\mathcal{L}^{(NORM)}_{N^*N\gamma}$ given by equation (3.5) to the T.R.I. conserving electromagnetic transition, and can be obtained from the T.R.I. violating transition operator by making the following replacements.

(a) From the TRI violating operator which contains the factors

$$\frac{2}{\sqrt{2}}\, T_{20}\, (\tau^{(2)}, \tau^{(3)})\, (\overset{*}{\epsilon}_p - \epsilon_p + \overset{*}{\epsilon}_N - \epsilon_N) + \frac{2}{3}\, (\overset{*}{\epsilon}_N - \epsilon_N - \overset{*}{\epsilon}_p + \epsilon_p)\, \tau_z^{(3)}$$ the corresponding

T.R.I. conserving operator is obtained by replacing this factor by

$$\frac{i}{\sqrt{3}}\, (\overset{*}{\epsilon}_p + \epsilon_p - \overset{*}{\epsilon}_N - \epsilon_N)\, (\tau^{(2)} \times \tau^{(3)})_z$$

(b) From the TRI violating operator which contain the factors

$$\frac{i}{\sqrt{3}}\, (\epsilon_p - \epsilon_p - \epsilon_N + \epsilon_N)\, (\tau^{(2)} \times \tau^{(3)})_z$$ the correspondin g TRI conserving operators is obtained by replacing this factor by

$$\frac{2}{\sqrt{3}}\, T_{20}\, (\tau^{(2)}, \tau^{(3)})\, (\overset{*}{\epsilon}_p + \epsilon_p + \overset{*}{\epsilon}_N + \epsilon_N) + \frac{2}{3}\, (\overset{*}{\epsilon}_N + \epsilon_N - \overset{*}{\epsilon}_p - \epsilon_p)\, \tau_z^{(3)}$$

The operator obtained in this fashion agrees with Chemtob and Rho (1971).

APPENDIX 3

DEVELOPMENT IN MULTIPOLES

The development in multipoles of the transition operators obtained in Chapters 2 and 3 is carried out using the formulae and conventions introduced by Clement (1971). The technique is straightforward and therefore just one example will be given.

Consider the first term of equation 3.12

$$V(B) = \frac{1}{3} \frac{G f i}{\mu(M-m)2m^2} \frac{2}{\sqrt{3}} (\epsilon^*_N - \epsilon_N - \epsilon_p^* + \epsilon_p) \sum_{i<j} \left[B(r_i) \tau_z^{(j)} - B(r_j) \tau_z^{(i)} \right]$$

$$\vec{\sigma}_i \times \vec{\sigma}_j \mathcal{Y}(r_{ij}) \qquad \text{(A3-1)}$$

Taking matrix elements for the emission of a photon we have

$$W(B) = A \sum_{i \leq j} \left[B^*(r_i) \tau_z^{(j)} - B^*(r_j) \tau_z^{(i)} \right] (\vec{\sigma}_i \times \vec{\sigma}_j) \mathcal{Y}(r_{ij}) \qquad \text{(A3-2)}$$

where

$$A = \frac{1}{3} \frac{G f i}{\mu(M-m)2m^2} \frac{2}{\sqrt{3}} (\epsilon^*_N - \epsilon_N - \epsilon_p^* + \epsilon_p)$$

and

$$B^*(r_i) = \langle 1 \text{ photon} \,|\, \vec{B}(r_i) \,|\, 0 \rangle$$

Using the same phases and overall normalisation as Blatt and Weisskopft (1952) and Clement (1971) we can expand the matrix element $B^*(r_i)$ in electric and magnetic multipoles

$$B^{(e)*}_{LM}(r) = i\,K\,C_L\; r \times \nabla\; u^*_{LM} \qquad \text{(A3-3a)}$$

$$B^{(m)*}_{LM}(r) = C_L\; \nabla \times (r \times \nabla)\; u^*_{LM} \qquad \text{(A3-3b)}$$

where

$$C_L = \frac{(2L+1)!!}{K^L (L+1)} \quad \text{and } u_{LM} = j_L(Kr)\, Y_{LM}(\theta\, \varphi)$$

The matrix element for the emission of a photon of the electromagnetic potential $\vec{A}(r)$ and the electric field $\vec{E}(r)$ can also be expanded in electric and magnetic multipoles. (compare with different convention in appendix 4)

$$A^{(e)*}_{LM} = C_L\; \left(\frac{i}{K}\right) \nabla \times (\vec{r} \times \nabla)\; u^*_{LM} \qquad \text{(A3-4a)}$$

$$A^{(m)*}_{LM} = C_L\; \vec{r} \times \nabla u^*_{LM} \qquad \text{(A3-4b)}$$

$$E^{(e)*}_{LM} = C_L \nabla \times (\vec{r} \times \nabla)\; u^*_{LM} \qquad \text{(A3-4c)}$$

$$E^{(m)*}_{LM} = -i\,K\; C_L\; \vec{r} \times \nabla u^*_{LM} \qquad \text{(A3-4d)}$$

To obtain the electric multipoles arising from the operator given by equation (A3-1) we replace $B^*(r_i)$ in equation (A3-2) by the $B^{*(e)}_{LM}(r)$ given by equation (A3-3a). Thus,

$$W(EL) = A \sum_{i<j} i\,K\,C_L \left[\vec{r}_i \times \nabla_i\, u^*_{LM}(r_i)\, \tau_z^{(j)} - \vec{r}_j \times \nabla_j\, u^*_{LM}(r_j)\, \tau_z^{(i)} \right]$$

$$(\sigma_i \times \sigma_j)\, \mathcal{Y}(r_{ij})$$

Then using

$$\vec{r} \times \nabla u^*_{LM} = -i \left[L(L+1) \right]^{\frac{1}{2}} j_L(kr)\ \phi^*_{L,LM}$$

where

$\phi_{J,LM} = [\, Y_J(r) \otimes \vec{e}\,]^{(L)}_M$ and $\vec{e}$ is the unit vector along the co-ordinate axis,

we get

$$V(EL) = A \sum_{i<j} i\, k\, C_L (-i) \left[L(L+1)\right]^{\frac{1}{2}} \left[j_L(k\, r_i)\, \phi^*_{L,LM}(r_i)\, \tau_z^{\,j} - j_L(k\, r_j)\, \phi^*_{L,LM}(r_j)\, \tau_z^{\,i} \right]$$

$$(\sigma_i \times \sigma_j) \mathcal{F}(r_{ij})$$

Further, using

$$[\, \phi^*_{J,LM}(r) \cdot \vec{a}\,] = [\, Y_J(r) \otimes \vec{a}\,]^{*(L)}_M$$

$$V(EL) = A \sum_{i<j} KC_L [(L+1)L]^{\frac{1}{2}} [\, j_L(k\, r_i)\, [\, Y_L(r_i) \otimes (\sigma_i \times \sigma_j)]^{*(L)}_M\, \tau_z^{(j)} - j_L(k\, r_j)$$
$$[\, Y_L(r_j) \otimes (\sigma_i \times \sigma_j)\,]^{*(L)}_M \tau_z^{\,i}\,] \quad \mathcal{F}(r_{ij})$$

Now consider two further approximations. Firstly the familiar long wave approximation

$$J_L(kr) \simeq \frac{(kr)^L}{(2L+1)!!}$$

Secondly the transformation to relative and centre of mass co-ordinates

$\vec{R}_{ij} = \frac{1}{2}\ (\vec{r}_i + \vec{r}_j)$ and $\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$

$$r_i^{\,l}\, Y_{lm}(r_i) = \sum_{\lambda} \left[\frac{4\pi}{2\lambda+1} \quad \frac{(2l+1)!}{(2l+1-2\lambda)!\ (2\lambda)!} \right]^{\frac{1}{2}} R_{ij}^{\,l-\lambda}\ (\tfrac{1}{2}\, r_{ij})^{\lambda}\ [\, Y_{l-\lambda}(R_{ij}) \otimes Y_{\lambda}(r_{ij})\,]^{(l)}_m$$

and similarly for $r_j^{\,l}\, Y_{lm}(r_j)$ which has an additional factor $(-)^{\lambda}$. Since each additional power of r_{ij} introduces and extra factor of the order of $(\mu R)^{-1}$ where R is the nuclear radius (Clement 1971) generally much less than unity, only the lowest power of r_{ij} need be kept. So we take

$$r_i^{\,L}\, Y_{LM}(r_i) \approx R_{ij}^{\,L}\, Y_{LM}(R_{ij})$$

substituting back we get

$$V(EL) \approx A \sum_{i<j} k \left(\frac{L}{L+1}\right)^{\frac{1}{2}} R_{ij}^{\,L} \left[Y_L(R_{ij}) \otimes (\sigma_i \times \sigma_j)\right]^{*L}_{m} \left(\tau_z^{\,j} - \tau_z^{\,i}\right) \mathcal{F}(r_{ij})$$

All other developments are carried out in a similar fashion (see Clement(1971) for more examples).

The usual multipole operators, expanded using the same conventions have the form

$$(EL)_{NORM} = e \sum_i \frac{1}{2}\left(1 + \tau_z^{(i)}\right) r_i^{\,L}\, Y^{*}_{LM}(\underline{r}_i)$$

$$(ML)_{NORM} = \frac{\left[L(2L+1)\right]^{\frac{1}{2}}}{M} \left\{ \frac{e}{L+1} \sum_i \frac{1}{2}\left(1 + \tau_z^{(i)}\right) r_i^{\,L-1} \left[Y_{L-1}(r_i) \otimes l_i \right]^{(L)*}_{M}\right.$$

$$\left. + \frac{1}{2} \sum_i \left[\frac{1}{2}(\mu_n + \mu_p) - \frac{1}{2}(\mu_n - \mu_p)\, \tau_z^{(i)} \right] r_i^{\,L-1} \left[Y_{L-1}(r_i) \otimes \sigma_i \right]^{(L)*}_{M} \right\}$$

APPENDIX 4

A. The Siegert Theorem

This theorem is discussed in a number of important papers for example by Sachs and Austern (1951), Osborne and Foldy (1950), Foldy (1953) and Dalitz (1954).

Let the Hamiltonian of the system under consideration be written

$$H_T = H + \mathcal{H}(\vec{A}) \tag{A4-1}$$

where $\mathcal{H}(\vec{A})$ denotes the interaction between the system represented by the Hamiltonian H and the electromagnetic radiation represented by the vector potential $\vec{A}$ assumed to be in the Coulomb Gauge (i.e. $\nabla \cdot \vec{A} = 0$).

The Siegert theorem refers to the first term $\mathcal{H}_1(\vec{A})$ in an expansion of $\mathcal{H}(A)$ in powers of the coupling constant

$$\mathcal{H}(A) = \mathcal{H}_1(\vec{A}) + \frac{1}{2!}\ \mathcal{H}_2(\vec{A}) + \cdots \tag{A4-2}$$

This first term $\mathcal{H}_1(\vec{A})$ is the interaction responsible for emission or absorption of a single photon (in Chapter 5 $\mathcal{H}_1(\vec{A})$ was decomposed into two parts $\mathcal{H}_1(\vec{A}) = \mathcal{H}_0(\vec{A}) + \mathcal{V}_{t.v.}(\vec{A})$ (see equation 5.5)). The second term $\mathcal{H}_2(\vec{A})$ describes the process where two photons are involved and so on.

The form of $\mathcal{H}(\vec{A})$ is however restricted by the fact that equation (A4-1) must satisfy Gauge invariance which states that if $\vec{A}$ is replaced by $\vec{A} + \vec{\nabla} G$ (where G is an arbitrary function) then there must exist a g such that

$$H + \mathcal{H}(\vec{A} + \vec{\nabla} G) = e^{ig}\ \{ H + \mathcal{H}(\vec{A}) \}\ e^{-ig} \tag{A4-3a}$$

It should be noted that nothing is said at this stage about the form of g (except that it is first order in the coupling constant).

The consequences of equation (A4-3a) have been worked out by Sachs and Austern (1951). Rewriting the equation (A4-3a) with $\vec{A} = 0$, we have

$$H + \mathcal{H}(\nabla G) = e^{ig} \{ H \} e^{-ig} \tag{A4-3b}$$

The right hand side of this equation can be expressed as

$$e^{ig} H e^{-ig} = H + i [g, H] + \frac{1}{2!} [g, [g, H]] + \cdots$$

Substituting for $\mathcal{H}$ (∇G) from equation (A4-2) and equating term of equal order there results

$$\begin{aligned}
&\mathcal{H}_1 \{\nabla G\} = i [g, H] \\
&\mathcal{H}_2 \{\nabla G\} = - [g, [g, H]] \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
&\mathcal{H}_\mu [\nabla G\} = (i)^n [g, [g, \ldots [g, H] \ldots]]
\end{aligned} \tag{A4-4}$$

The first equation of the (A4-4) set will now be rederived using a different method which has the advantage of giving an explicit form for g.

On experimental grounds (see Bohr and Mottelson (1969) page 379 for a discussion) the form of $\mathcal{H}_1$(A) is taken to be

$$\mathcal{H}_1(\vec{A}) = - \int \vec{j} \cdot A \; d^3r \tag{A4-5a}$$

Here $\vec{j}$ is the electromagnetic current of the system described by the Hamiltonian H. (A system of nucleons for the case in which we are interested). We assume current conservation (a direct consequence of Gauge Invariance), thus

$$\operatorname{div} \vec{J} = i\,[\,\rho,\ H] \tag{A4-5b}$$

where ρ is the nuclear charge.

For a system of particles, considered as points, and in particular for a system of nucleons it is a good approximation to take

$$\rho = \sum_i \frac{e}{2}\,(1 + \tau_z^{(i)})\,\delta^3\,(\vec{r}-\vec{r}_i) \tag{A4-5c}$$

It will now be shown that equation (A4-5) imposes a restriction to the form of $\mathcal{H}_1(A)$. Consider, for example, the special case in which $\vec{A}$ is replaced by $\vec{\nabla} G$ (the gradiant of an arbitrary function G), using the identity

$$\nabla \cdot (\vec{j}\ G) = G\,\vec{\nabla} \cdot \vec{j} + \vec{j} \cdot \nabla\ G$$

and finally using equations (A4-5b and c) it follows that

$$\mathcal{H}_1\ (\nabla G) = i\,[g, H] \tag{A4-6}$$

where

$$g = \sum_i \frac{e}{2}\,(1 + \tau_z^{(i)})\ G(r_i) \tag{A4-7}$$

Equation A4-6 is a particular case of the first equation (A4-4), since in the first derivation the form of g is arbitrary, because gauge invariance requires only the existence of a g such that equation A4-3a holds.

Equation (A4-6) has many applications. It can be used not only to check whether a given $H_T = H + \mathcal{H}_1(\vec{A})$ is gauge invariant but as shown in Appendix 9 it can be used to construct $\mathcal{H}_1(\vec{A})$.

Another more important consequence of equation (A4-6) is the Siegert theorem which was used in equation (5. 8). This theorem will now be demonstrated.

The theorem (Siegert (1936)) states that the electric multipoles E(L) derived from the expansion of $\mathcal{H}_1(\vec{A})$ can be written as

$$E(L) = i\left[H_0, D_L(K)\right] \qquad \text{(A4-8)}$$

where

$$D_L(K) = \sum_i \frac{e}{2}\left(1 + \tau_z^{(i)}\right) \frac{iX_L^*}{K} \sqrt{\frac{L+1}{2L}} \quad r_{(i)}^L \, C_{LM}^* \, (\hat{r}_{(i)}) \qquad \text{(A4-9)}$$

The starting point of the demonstration is equation (A4-3a) viz.

$$\mathcal{H}_1(\vec{A}) = -\int \vec{j} \cdot \mathbb{A} \, d^3r$$

The electromagnetic vector potential $\vec{A}$ (in a cubic box of side L) is expressed as follows (Brink and Rose (1967))

$$\vec{A}(\vec{r}) = \sum_{k\eta} \left(\frac{2\pi}{L^3 k}\right)^{\frac{1}{2}} \{\vec{\epsilon}_\eta \, e^{i\vec{k}\vec{r}} \, a_{k\eta} + \epsilon_\eta \, e^{-i\vec{k}\cdot\vec{r}} \, a_{k\eta}^+\} \qquad \text{(A4-10)}$$

In equation (A4-10) $a_{k\eta}^+$ and $a_{k\eta}$ are creation and annihilation operators for a photon with wave number $\vec{k}$ and polarisation ϵ_η (ϵ_1 and ϵ_2 are two orthogonal polarisation vectors).

The field $\vec{A}(\vec{r})$ can be expanded in electric (e) and magnetic (m) multipoles (Brink and Rose (1967))

$$\epsilon_q \, e^{i\vec{k}\cdot\vec{r}} = -\frac{1}{\sqrt{2}} \sum_{LM} \left(q \, \vec{A}_{LM}^{(m)} + \vec{A}_{LM}^{(e)}\right) D_{Mq}^L(R) \qquad \text{(A4-11)}$$

In equation (A4-10) $D^L_{Mq}(R)$ is an element of the irreducible (2L+1) dimensional representation of the rotation group (Brink and Satchler (1968)). The rotation R brings the z-axis to the direction of $\vec{k}$. The functions $\vec{A}^{(m)}_{LM}$ and $\vec{A}^{(e)}_{LM}$ are given in the long wave approximation (i. e. kr >>1) by

$$\vec{A}^{(e)}_{LM} = \frac{i X_L}{k} \sqrt{\frac{L+1}{L}}\ i\vec{\nabla} (r^L C_{LM}) \tag{A4-12}$$

$$\vec{A}^{(m)}_{LM} = \frac{i X_L}{\sqrt{L(L+1)}} \vec{\nabla} (r^L C_{LM}) \times \vec{r} \tag{A4-13}$$

where

$$X_L = \frac{(ik)^L}{(2L-1)!!}$$

Substituting (A4-11) in (A4-10) it results

$$\vec{A}(r) = -\frac{1}{\sqrt{2}} \sum_{LMq} \sum_{k\eta} \left(\frac{2\pi}{L^3 k}\right)^{\frac{1}{2}} \{ (q\, \vec{A}^{(m)}_{LM} + i\vec{A}^{(e)}_{LM})\, a_{k\eta} + \text{h. c.} \}\ D^L_{Mq}\, (R) \tag{A4-14}$$

Therefore $\mathcal{H}_1(A)$ can be separated as follows

$$\mathcal{H}_1(\vec{A}) = \sum_{LMq} \mathcal{H}^{(e)}_1(LM) + \sum_{LMq} \mathcal{H}^{(m)}_1(LM \tag{A4-15}$$

where the first term, which gives rise to the electric multipoles EL is

$$\mathcal{H}^{(e)}_1\ (LM) = \int d^3 x\ \vec{J} \cdot \sum_{kq} \frac{1}{\sqrt{2}} \left(\frac{2\pi}{L^3 k}\right)^{\frac{1}{2}} \{ \frac{i X_L}{L} \sqrt{\frac{L+1}{L}}\ \vec{\nabla} (r^L C_{LM}) a_{kq} + \text{h. c.} \}\ D^L_{Mq}(R)$$

Taking matrix elements for emission of a photon with momentum $\vec{K}$ when the nucleus decays from a state $|I_i m_i>$ to $|I_f m_f>$, the electric part $\sum_{LMq} \mathcal{H}^{(e)}_1$ (LM) of (A4-15) gives

$$<I_f m_f \vec{K}| \sum_{LMq} \mathcal{H}_1^{(e)} (LM) | I_i m_i > = \left(\frac{2\pi}{L^3 k}\right)^{\frac{1}{2}} <I_f m_f| \sum_{LMq} \int d^3r \; \vec{j} \cdot \nabla (r^L C_{LM})^*$$

$$\frac{(i)\, X^*_L}{K} \sqrt{\frac{L+1}{2L}} \; | \; I_i \, m_i > D^L_{Mq}(R) \qquad \text{(A4-16a)}$$

The electric multipole EL is by definition (Brink and Rose 1967)

$$(EL) = \int d^3r \; \vec{j} \cdot \nabla (r^L \, C^*_{LM}) \; \frac{(i)\, X^*_L}{K} \sqrt{\frac{L+1}{2L}} \qquad \text{(A4-16b)}$$

Equation (A4-16b) has the same form as equation (A4-5a) with $\vec{A}$ having the form $\vec{\nabla} G$ and we can therefore use the result of eq. (A4-6) so that

$$EL = i \; [H_0, \, D_L(\,)] \qquad \text{(A4-8)}$$

where

$$D_L(K) = \sum_i \frac{e}{2} \, (1 + \tau_z^{(i)}) \; \frac{i\, X^*_L}{K} \sqrt{\frac{L+1}{2L}} \; r^L_{(i)} \; C^*_{LM}(\hat{r}_i) \qquad \text{(A4-9)}$$

This form of $D_L(k)$ depends on equation (A4-5c) being true, i. e. , if the system in interaction with the electromagnetic field is a collection of point particles. If this is not the case however, equation (A4-8) still holds although then $D_L(k)$ has a different form from the one given by the equation (A4-9).

B. <u>Critical Analyses of the Siegert Theorem</u>

The demonstration of the Siegert Theorem given in part A of this Appendix will now be examined critically. The purpose is to show that the theorem is an exact result only in the limit where the energy of the emitted (or absorbed) photon k vanish (i. e. when $k \rightarrow 0$).

The paper by Brennan and Sachs (1952) was the first one to show a breakdown of the Siegert Theorem for high energy photons. They pointed out that when the long wave approximation ($kr << 1$) is no longer valid (which occurs at high energy) the equation (A4-12) should be replaced by

$$A^{(e)}_{LM} = (k\sqrt{L(L+1)})^{-1} \nabla \times \vec{L}\, \phi_{LM} \tag{A4-17}$$

where $\phi_{LM} = i^{L}(2L+1)\, j_L(kr)\, C_{LM}(\hat{r})$ and $j_L(kr)$ is the spherical Bessel function. Therefore equation (A4-16) is no longer valid and we have instead

$$(EL) = \int d^3r\ \vec{j}\cdot (K\sqrt{L(L+1)})^{-1} \nabla \times L\ \phi^{*}_{LM} \tag{A4-18}$$

Since equation (A4-18) is not of a form suitable for the application of the result given by equation (A4-6) it follows that the Siegert theorem no longer holds.

It will now be shown that the Siegert Theorem also does not hold for those parts of the interaction Hamiltonian between the electromagnetic radiation and the system which depends directly on $\vec{E}$ (the electric field) and $\vec{B}$(the magnetic field) rather than on $\vec{A}$ (the vector potential). It should be pointed out however that since both $\vec{B}$ and $\vec{E}$ vanish when $k \to 0$ those terms contribute very little to the emission (or absorption) of low energy photons compared with the terms which depend on A.

Examples of transition operators depending on E and B have been given in Chapters 2 and 3. Since however all the operators given there are T.R.I. violating we give below two examples of T.R.I. preserving operators of this type.

As an example of an operator depending on $\vec{B}$ we take the second term of the usual electromagnetic transition operator.

$$\mathcal{H}_0(\vec{A}) = \sum_i \frac{e}{2m} \frac{1}{2} (1 + \tau_z^{(i)}) \; (p_i \cdot \vec{A}(r_i) + \vec{A}(r_i) \cdot p_i) + \sum_i \frac{e}{2m} \; \mu_i \, \sigma_i \times \vec{B}(r_i) \qquad \text{(A4-19)}$$

where

$$\mu_i = \frac{1}{2} \; (\mu_u + \mu_p) - \frac{1}{2} (\mu_u - \mu_p) \tau_z^{(i)}$$

As an example of a transition operator depending on $\vec{E}$ we take the case of the transition operator stemming from a four particle vertex (NN$\pi\gamma$) as in Figure A.1

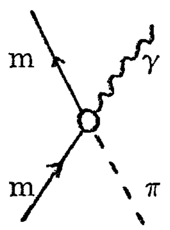


Fig. A1

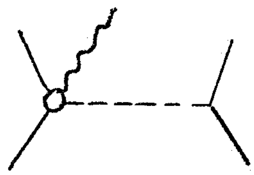


Fig. A 2

Following Clement and Heller (1971) the phenomenological Lagrangian corresponding to this vertex is taken to be

$$\mathcal{L}_2(x) = \frac{\lambda}{m\mu} \; \vec{\psi}(x) \; \vec{\tau} \cdot \vec{\phi}(x) \, . \, i \gamma^5 \; \sigma_{\mu\nu} \; \psi(x) \; F_{\mu\nu}(x) \qquad \text{(A4-20)}$$

The transition operator is found by calculating the diagram of Fig. A2 using as Lagrangian for the right hand side usual NNπ vertex the expression.

$$\mathcal{L}_1(x) = i \, G \; \vec{\psi}(x) \;\; \gamma^5 \tau \;\; \cdot \phi \, (x) \; \psi(x)$$

The resulting transition operator is found to be

$$\mathcal{H} \, (r_1 r_2 \, \vec{E}) = \frac{\lambda F}{2\pi m \mu^2} \;\; (\tau_{(1)} \cdot \tau_{(2)}) \; [\, \sigma_1 \cdot \vec{E}(r_1) \, \sigma_2 \cdot (r_1 - r_2) + \vec{\sigma}_2 \cdot \vec{E} \, (r_2) \, \sigma_1 \cdot (\vec{r}_1 - \vec{r}_2)] \, \mathcal{Y}(r_{12})$$

(A4-21)

where

$$F = \frac{\mu G}{2m} \quad \text{and} \; \mathcal{G}(r_{12}) = -\frac{1}{4\pi}\left(\frac{\mu}{|\vec{r}_1-\vec{r}_2|^2} + \frac{1}{|\vec{r}_1-\vec{r}_2|^3}\right) e^{-\mu|r_1-r_2|}$$

The $\vec{B}$ dependent transition operator given by the second term of equation (A4-19) does not contribute to the electric multipoles in the approximation given by the equation (A4-12). This is so because we can write $\vec{B} = \vec{\nabla} \times \vec{A}$ and of course $\nabla \times \nabla (r^L C_{LM}) = 0$ However using equation (A4-17) instead of (A4-12) it is possible to get a small contribution to the electric multipoles not of the form (A4-8).

The contribution to the electric and magnetic multipoles of the $\vec{E}$ dependent transition operator given by equation (A4-21) can be easily obtained by substituting $\vec{E}$ in equation (A4-21) by

$$E^{(e)}_{LM} = \frac{k}{i} \vec{A}^{(e)}_{LM} = X_L \sqrt{\frac{L+1}{L}} \; \nabla (r^L C_{LM}) \qquad \text{(A4-22)}$$

and

$$E^{(m)}_{LM} = -ik \, \vec{A}^{(m)}_{LM} = \frac{k X_L}{L(L+1)} \vec{\nabla} (r^L C_{LM}) \times \vec{r} \qquad \text{(A4-23)}$$

respectively and proceding like in equations (A4-16a) and (A4-16b). As can be easily checked from the result of the manipulations outlined above, the transition operator $\mathcal{K}(r_1 r_2 \vec{E})$ given by A4-21 contributes an electric multipole operator which cannot be expressed in the form (A4-8). Note however that due to the fact that $\mathcal{K}(r_1 r_2 E)$ depends on $\vec{E}$, the matrix elements of the electric multipole arising from it will have an extra factor (KR_0) (introduced in equation A4-22) compared to the matrix elements of the electric multipoles arising from a transition operator depending on A. Therefore for low energy transitions the contribution of such an operator is negligible.

C. Comments on the use of the Siegert Theorem

The Siegert Theorem has many applications some of which are explained below.

The more important of these applications is that the Siegert Theorem can be used to replace the electric multipoles (EL) given by equation (A4-8) by a simpler, effective operator $(EL)_{eff}$ defined by

$$(EL)_{eff} = i\,(E_f - E_i)\,D_L(\kappa) \qquad \text{(A4-24)}$$

where E_f and E_i are the energies of the final and initial states of the transition concerned. It is this effective multipole operator which is usually found in the literature denoted as electric multipoles. (However the $(EL)_{eff}$ found in the literature may differ from (A4-24) in phase and overall normalisation. This point is treated by Brink and Rose (1967) in detail).

There are both advantages and disadvantages in using $(EL)_{eff}$ in place of the more general (EL) given by (A4-8). One disadvantage is that $(EL)_{eff}$ transforms differently from (EL) under Hermitian configuration and unless one is very careful this can produce errors in phase in the calculations (Brink and Rose (1967)). The main advantage of using $(EL)_{eff}$ is explained below.

Consider the effect of a (small) additional potential V_{ad}, added to the usual strong Hamiltonian H_0. Suppose that in order to maintain gauge invariance an extra term $\mathcal{V}_{ad}(\vec{A})$ is necessary, thus

$$H_T = H_0 + V_{ad} + \mathcal{H}_0(\vec{A}) + \mathcal{V}_{ad}(\vec{A}) \qquad \text{(A4-25)}$$

Let $(EL)_0$ and $(EL)_{ad}$ be the electric multipole resulting from the expansion of $\mathcal{K}_0(A)$ and $\mathcal{V}_{ad}(\vec{A})$ respectively in multipoles. The Siegert theorems requires that

$$(EL)_0 + (EL)_{ad} = i \left[H_0 + V_{ad}, D_L(k) \right] \qquad (A4\text{-}26)$$

Therefore the effective electric multipole corresponding to (A4-26) is

$$\left((EL)_0 + (EL)_{ad} \right)_{eff} = i \, (E_f - E_i) \, D_L(k) \qquad (A4\text{-}27)$$

and it has the same form as the operator given by equation (A4-24) (note however that the operator given by equation (A4-27) must be used with eigenstates of $H_0 + V_{ad}$ and that E_i and E_f are the corresponding eigenvalues).

The fact that (A4-27) has the same form as (A4-24) is usually expressed in the literature by saying that $\mathcal{V}_{ad}(\vec{A})$ has no effect on the electric multipoles. This statement however is misleading in that $(EL)_{ad}$ has effects.

One must always keep in mind that $(EL)_{eff}$ can not replace (EL) in all the circumstances. This point is illustrated with the comments on the paper by Michell(1965) on the effect of Parity violating nuclear forces in a gamma transition given below. Michell considers the effects of a small one body Parity violating potential ($V_{pv} = G'' \vec{\sigma} \cdot \vec{p}$) on a system consisting of a particle moving in a potential V(r). (This system is intended to be a rough model for an odd A nucleus). The Hamiltonian is therefore taken to be (see equation (20) in Michell (1965))

$$H = H_0 + G'' \vec{\sigma} \cdot \vec{p} \qquad (A4\text{-}28)$$

where

$$H_0 = \frac{p^2}{2m} + V(r) \tag{A4-29}$$

Next he observes that $s = m\, G'' \vec{\sigma} \cdot \vec{r}$ is such that (to first order in G)

$$H = e^{iS} H_0 e^{-iS} \tag{A4-30}$$

Therefore the matrix elements of any operator A between eigenstates ψ of H is equal to the matrix elements of the operator $e^{-iS} A e^{iS}$ between the corresponding eigenstates ϕ of H_0 so that

$$< \psi_f | A | \psi_i > = < \phi_f | e^{-iS} A\, e^{iS} | \phi_i > \approx A < \phi_f | A | \phi_i > - i < \phi_f | [S, A] | \phi_i > + 0(G''^2) \tag{A4-31}$$

Using (A4-31) he concludes that electric multipole transitions are not affected because $(EL)_{eff}$ comutes with S. This reasoning however is not correct since the use of $(EL)_{eff}$ in this case is not appropriate.

However it should also be noted that Michell (see also the paper by Walborn (1964)) does not take Gauge invariance into account. This follows from the fact that the Hamiltonian (A4-28) as it stands is not Gauge Invariant. We should therefore replace equation (A4-28) by the following gauge invariant Hamiltonian

$$H = H_0 + G'' \sigma \cdot p + \mathcal{H}_0(\vec{A}) + e\, G'' \vec{\sigma} \cdot \vec{A} \frac{1}{2} (1 + \tau_z) \tag{A4-32}$$

where

$$\mathcal{H}_0(\vec{A}) = \frac{e}{2m} [\vec{p} \cdot \vec{A} + \vec{A} \cdot \vec{p}\,] \frac{1}{2} (1 + \tau_z) + \frac{e}{2m} \mu \sigma \cdot \nabla \times \vec{A} \tag{A4-33}$$

and

$$\mu = \frac{1}{2}(\mu_n + \mu_p) - \frac{1}{2}(\mu_n - \mu_p)\,\tau_z$$

The effects of the extra parity violating transition operator in equation (A4-32) (namely $\mathcal{V}^{pv}(\vec{A}) = e\,G''\vec{\sigma}\cdot\vec{A}\,\frac{1}{2}(1+\tau_z)$) will be calculated below. In particular we will show that by taking $\mathcal{V}^{pv}(\vec{A})$ into account it is possible to show that there is no parity violating effects in the electric electromagnetic transition. There are however other effects that modify some of the results obtained by Michell.

The easiest way to show that there is no parity violating effects in the electric transitions for the Hamiltonian given by (A4-32) is to apply the Siegert Theorem. However, in order to consider at the same time the contribution of $\mathcal{V}^{pv}(\vec{A})$ to magnetic multipoles we shall follow a longer path.

The electric and magnetic multipoles stemming from $\mathcal{V}^{pv}(\vec{A})$ will be denoted by $(EL)^{pv}$ and $(ML)^{pv}$ respectively. The total electric and magnetic multipoles are now

$$(EL)_{TOTAL} = (EL)_{NORM} + (EL)^{p.v.}$$

$$(ML)_{TOTAL} = (ML)_{NORM} + (ML)^{p.v.}$$

where $(EL)_{NORM}$ and $(ML)_{NORM}$ stands for the usual parity conserving electric and magnetic multipoles.

The effects of $(EL)^{pv}$ and $(ML)^{pv}$ is taken into account by replacing equation (A4-31) by (see section 5-2 for a similar calculation)

$$\langle \psi_f | (EL)_{TOTAL} | \psi_i \rangle = \langle \phi_f | (EL)_{NOR} | \phi_i \rangle - i \langle \phi_f | [S, (EL)_{NOR}] | \phi_i \rangle + \langle \phi_f | (EL)^{pv} | \phi_i \rangle \qquad \text{(A4-34)}$$

$$< \psi_f | (ML)_{TOTAL} | \psi_i > = < \phi_f | (ML)_{NOR} | \phi_i > - i < \phi_f | [S, (ML)_{NOR}] | \phi_i > +$$

$$< \phi_f | (ML)^{pv} | \phi_i > \qquad (A4-35)$$

To show that there is no parity violating effects to the electric electromagnetic transitions it is sufficient to consider the first part of $\mathcal{H}_0(A)$ given by equation (A4-33) namely

$$\mathcal{H}_{01}(A) = \frac{e}{2m} [\vec{p} \cdot \vec{A} + A \cdot p] \frac{1}{2} (1 + \tau_z) \qquad (A4-36)$$

This is so because as we have seen in part B of this appendix the second term of equation (A4-33) (namely $\mathcal{H}_{02}(A) = \frac{e}{2m} \mu \sigma \cdot \nabla x \vec{A}$) do not contribute (in the long wave approximation) to the electric multipoles.

By using the results given by equations (A4-34) and (A4-35) we have

$$< \psi_f | \mathcal{H}_{01}(A) + e\, G'' \vec{\sigma} \cdot \vec{A} \frac{1}{2} (1 + \tau_z) | \psi_i > = < \phi_f | \mathcal{H}_{01}(\vec{A}) | \phi_i > -$$

$$< \phi_f | - i [S, \mathcal{H}_{01}(\vec{A})] | \phi_i > < \phi_f | e\, G'' \vec{\sigma} \cdot \vec{A} \frac{1}{2} (1 + \tau_z) | \phi_i > \qquad (A4-37)$$

but

$$i [S, \mathcal{H}_{01}(\vec{A})] = e\, G'' \vec{\sigma} \cdot \vec{A} \frac{1}{2} (1 + \tau_z) \qquad (A4-38)$$

and therefore to first order in G

$$< \psi_f | \mathcal{H}_{01}(\vec{A}) + \mathcal{H}_{02}(\vec{A}) + e\, G'' \vec{\sigma} \cdot \vec{A} \frac{1}{2} (1 + \tau_z) | \psi_i > = < \phi_f | \mathcal{H}_{01}(\vec{A}) + \mathcal{H}_{02}(\vec{A}) | \phi_i >$$

$$- i < \phi_f | [S, \mathcal{H}_{02}(\vec{A})] | \phi_i >$$

Therefore the only parity violating effect remaining is produced by $\mathcal{H}_{02}(\vec{A})$ which as mentioned before do not contribute to the electric multipoles.

To calculate the parity violating effects in a particular magnetic multipole one has to expand $\mathcal{H}_{02}(\vec{A})$ in multipoles. Denoting $(ML)_{02}$ the magnetic L-multipole stemming from $\mathcal{H}_{02}(\vec{A})$ we have

$$(ML)_{02} = \frac{e}{2m} \mu \vec{\sigma} \cdot \nabla (r^{L} Y_{LM}) \tag{A4-39}$$

and therefore the effect is given by

$$<\psi_f | (ML)_{01} + (ML)_{02} + (ML)^{pv} | \psi_i> = <\phi_f | (ML)_{01} + (ML)_{02} | \phi_i> +$$

$$+ \frac{G''e}{2m} < \phi_f | \mu \vec{r} \cdot [\vec{\sigma} \times \vec{\nabla} (r^L C_{LM})] | \phi_i> \tag{A4-40}$$

This result differs from that obained by Michell (see equation (29) in the paper by Michell (1965)) since he includes the term $-i [S, (ML)_{01}]$ which, as we have seen, cancels with the magnetic multipole generated by $\mathcal{V}_{pv}(\vec{A})$.

APPENDIX 5

THE N* INCLUDED IN THE WAVE FUNCTIONS

In Chapter 3 the effects of a possible T.R.I. violation in the $N^*N\gamma$ vertex have been calculated by introducing the N* nucleon resonance as an intermediate state in a set Feymann graphs. Recently however (see Green and Schucan (1971) for a review) it has been argued that the effects of the nucleon resonance N* should be calculated by introducing it directly into the nuclear wave function. Thus the nuclear wave function is written

$$\Psi = (1 - \alpha^2)^{\frac{1}{2}} \; \psi(N) + \alpha \;\; \psi(N-1, N^*) \qquad \text{(A5-1)}$$

where $\psi(N)$ is the wave function of N nucleons in the ground state and $\psi(N-1, N^*)$ represents the state where one of the nucleons is in an excited state N*. (Of course one should also consider terms of the wave function where the N* resonance appear more than once. In practice however one considers that these are negligible).

In equation (A5-1) $\psi(N)$ and $\psi(N-1, N^*)$ can be written in a non relativistic approximation. However such a wave function is clearly foreign to the usual formalism of nuclear physics and therefore ambiguities are likely to appear. It is the purpose of this Appendix to discuss these.

We begin by studying the forms of $\psi(N)$ and $\psi(N-1, N^*)$. The first is the usual many nucleons wave function and therefore is a well known object. We write $\psi(N-1, N^*)$ as follows.

$$\psi(N-1,N^*) = \phi(x_1 \ldots x_{N-1})\ \xi(x_{N^*})\ \begin{pmatrix}\alpha\\ \beta\\ \gamma\\ \delta\end{pmatrix}\ \begin{pmatrix}a\\ b\\ c\\ d\end{pmatrix} \tag{A5-2}$$

where $\phi(x_1 \ldots x_{N-1})$ is a usual N-1 nucleon wave function, $\xi(x_{N^*})$ is the radial part of the wave function of the N* resonance and the matrices $\begin{pmatrix}\alpha\\ \beta\\ \gamma\\ \delta\end{pmatrix}$ and $\begin{pmatrix}a\\ b\\ c\\ d\end{pmatrix}$ are respectively the spin and i-spin matrices of the $N^*(J = {}^3/2,\ T = {}^3/2\ M = 1230\ \text{MeV})$.

The wave function $\psi(N)$ and $\psi(N-1,N^*)$ are eigenstates of the Hamiltonian

$$H_0 = \sum_{i=1}^{N-1} \frac{p_i^2}{2m} + \frac{p_{N^*}^2}{2M} + \sum_{i<j} V_{NN,NN}(x_i, x_j) + \sum_i V_{NN^*,NN^*}(x_i, x_{N^*}) \tag{A5-3}$$

where $V_{NN,NN}(x_i, x_j)$ is the usual internucleon potential and $V_{NN^*,NN^*}(x_i, x_{N^*})$ is the potential energy between the N* resonance and the i^{th} nucleon.

The potential $V_{NN^*,NN^*}(x_i, x_{N^*})$ can be derived from the graphs of Figures A5-1 and A5-2 by using techniques similar to the ones described in section (2-3).

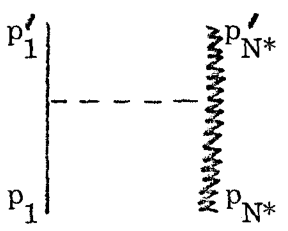


Fig. A5-1

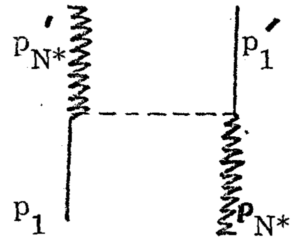


Fig. A5-2

Once we have $V_{NN,NN}(x_i, x_j)$ and $V_{NN^*,NN^*}(x_i, x_{N^*})$ we should be able in principle to solve the Hamiltonian (A5-3) and obtain the two sets of eigenstates $\psi(N)$

and ψ (N-1, N*). (We know however that this is only possible to a reasonable accuracy for nuclear matter or very light nuclei such as ^3H or ^3He).

However the total Hamiltonian of the system will include also the term W_{N^*N} that is in fact a transition operator in the sense that it transforms a two nucleon state into a state with an N* and a nucleon, thus

$$H = H_0 + W_{N^*N} \tag{A5-4}$$

It is W_{N^*N} that admixes $\psi(N)$ to ψ(N-1, N*) as in equation (A5-1) and therefore we concentrate now on how W_{N^*N} is derived and how it should be treated.

The transition operator W_{N^*N} can be derived from the graphs of Fig. (A5-3) given below.

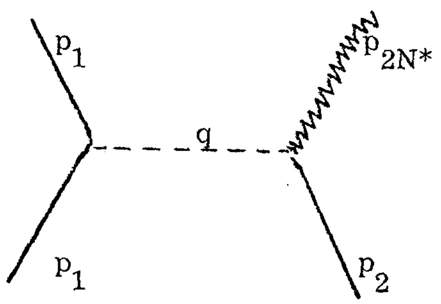


Fig. A5-3a

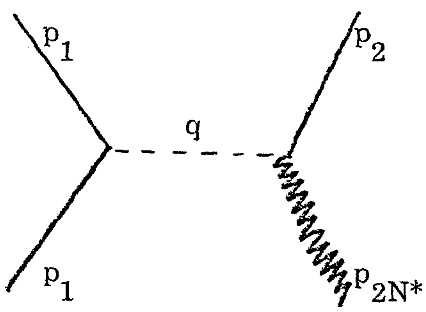


Fig. A5-3b

Using the Lagrangians and the notation given in Chapter 3 the matrix element corresponding to the process shown in Fig. A5-3 can be written in co-ordinate space as follows.

$$M = (-i)^2 \iint d^4 x_1 \, d^4 x_2 \, [-i f \bar{\psi}(x_1)\gamma_5 \vec{\tau}_{(1)} \psi(x_1)] \, \frac{\partial \Delta(x_1 - x_2)}{\partial x_{2\rho}} \cdot [\frac{G}{\mu} \bar{\psi}_\rho(x_2) \vec{T}_{(2)} \psi(x_2)]$$

(A5-5)

Integrating over time and extracting a factor $-2\pi i\,\delta$ (Energies) gives

$$W_{N^*N} = \frac{i\,f\,G}{\mu}\iint d^3r_1 d^3r_2\,[\bar{\psi}(r_1)\,\gamma_5\vec{\tau}_{(1)}\,\psi(r_1)]\left\{\frac{1}{(2\pi)^3}\int d^3q(-i)\,\vec{q}_{\rho\neq 4}\right.$$

$$\left.\frac{e^{iq\cdot(r_1-r_2)}}{q^2-(p'_{20}-p_{20})^2+\mu^2-i\epsilon}\right\}\cdot\;[\bar{\psi}_\rho(r_2)\;\vec{T}_{(2)}\,\psi(r_2)\,] \qquad \text{(A5-6)}$$

To obtain equation (A5-6) we have specialised to the rest frame of the nucleon resonance and therefore eliminating the time component of Rarita-Schwinger spinor ψ_ρ. (This is not necessary but simplifies the "non-relativistic" reduction).

At this stage there is a simplification which is used in the literature. This is to expand the denominator in equation (A5-6) in powers of $(p'_{20}-p_{20})^2$ and to keep only the first term, viz.

$$W_{NN^*} = \frac{i\,f\,G}{\mu}\iint d^3r_1\;d^3r_2\,[\,\bar{\psi}(r_1)\,\gamma_5\vec{\tau}_{(1)}\,\psi(r_1)]\left\{\frac{1}{(2\pi)^3}\int\;d^3q(-i)\,\vec{q}_{\rho\neq 4}\right.$$

$$\left.\frac{e^{i\vec{q}\cdot\,(r_1\,-\,r_2)}}{\vec{q}^2+\mu^2\;-\;i\;\epsilon}\right\}\cdot\,[\,\bar{\Psi}_\rho\,\vec{T}_{(2)}\,\psi(r_2)\,] \qquad \text{(A5-7)}$$

This approximation is not good since $(p'_{20} - p_{20}) \sim (M-m)$ is not small. The justification found in the literature for using such an approximation is to consider that the N* present in the nucleus is a virtual one (off the mass shell). This idea is pursued by both Riska and Brown (1970) and by Green and Schucan (1971). The factor $\left(\frac{m + M}{M}\right)^{\frac{1}{2}}$ that appears in Brown and Riska's work is intended to take into account this virtual aspect of the resonance. An alternative will be outlined later in this appendix but first the approximation given by equation (A5-6) will be accepted in order to illustrate the non relativistic reduction techniques to be used in connection with the Rarita-Schwinger spinors.

The first bracket of equation (A5-7) involves only nucleons and therefore the non-relativistic reduction can be carried out as in Appendix 2. The second bracket involves the Rarita-Schwinger spinors (see Lurie (1968) and Broadhurst (1971) for details)

$$U_\mu(p,\lambda) = \sum_{\lambda_1(\lambda_2)} \epsilon_\mu(p,\lambda_1)\; U(p,\lambda_2)\; \langle 1\tfrac{1}{2}\,\lambda_1\lambda_2 | \tfrac{3}{2}\,\lambda \rangle \qquad \text{(A5-8)}$$

$$(\lambda = \tfrac{3}{2}, \ldots\ldots -\tfrac{3}{2})$$

and the usual Dirac Spinor.

$$U(k,\sigma) = \left(\frac{E_k + m}{2E_k}\right)^{\frac{1}{2}} \begin{pmatrix} \xi_\sigma \\ \dfrac{\sigma\cdot k}{E_k+m}\,\xi_\sigma \end{pmatrix} \qquad \text{(A5-9)}$$

$$(\sigma = \tfrac{1}{2}, -\tfrac{1}{2})$$

In equation (A5-8) the factor $\langle 1\frac{1}{2}\lambda_1\lambda_2 | \frac{3}{2}\lambda \rangle$ is a Clebsch-Gordan coefficient and $\epsilon_\mu(p,\lambda)$ are polarisation vectors which in the rest frame become

$$\vec{\epsilon}(0,1) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} \qquad \vec{\epsilon}(0,-1) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix} \qquad \vec{\epsilon}(0,0) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

In equation (A5-9) $E_k = +(\vec{k}^2 + m^2)^{\frac{1}{2}}$ and

$$\xi_{\frac{1}{2}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \xi_{-\frac{1}{2}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The non relativistic reduction is carried out separately for each change of spin (i. e. for each pair λ,σ). The result is expressed in terms of the (4 x 2) vector matrix $[\vec{S}]_{S^*s}$ and of the wave function ψ_4 defined below in equation (A5-10) and (A5-11) respectively.

$$[\vec{S}_\rho]_{S^*s} = \vec{\epsilon}_\rho\,(0,\lambda) \;\; < 1\, \frac{1}{2} \lambda\, s \,|\, \frac{3}{2}\, S^* > \qquad \text{(A5-10)}$$

$$\psi_4 = \xi\,(X_{N^*}) \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \qquad \text{(A5-11)}$$

The following result is obtained.

$$W_{NN^*} = \frac{f\,G}{\mu}\, i \iint d^3r_1\, d^3r_2\; \psi^+(r_1)\, \Psi_4^+(r_2) \left[\frac{\sigma_1 \cdot p_1}{2m}\,,\; (\vec{r}_1-\vec{r}_2)\cdot \vec{S}\,\mathcal{J}(r_{12})\right]$$

$$\vec{\tau}_{(1)} \cdot \vec{T}_{(2)}\, \psi_L(r_1)\; \psi_L(r_2) \qquad \text{(A5-12a)}$$

from which the following transition operator results

$$W_{NN^*} = \left[\frac{f\,G}{\mu}\; \frac{1}{2m}\; \sigma \cdot \vec{S}\,\mathcal{J}(r_{12}) + \frac{f\,G}{\mu}\; \frac{1}{2m}\; (\vec{r}_1-\vec{r}_2)\cdot \vec{S}\,(\vec{r}_1-\vec{r}_2)\cdot \sigma_1\, \frac{1}{|\vec{r}_1-\vec{r}_2|}\right.$$

$$\left.\frac{d\,\mathcal{J}(|r_1-r_2|)}{d\,(\,|r_1-r_2|\,)}\right] \vec{\tau}_{(1)} \cdot \vec{T}_{(2)} + (1 \leftrightarrow 2) + \text{h. c.} \qquad \text{(A5-12b)}$$

The hermitian conjugate (h. c.) comes of course from the graph of Fig. A5-3b and

$$\mathcal{J}\,(|r_1-r_2|) = -\,\frac{1}{4\pi} \left(\frac{\mu}{|\vec{r}_1-\vec{r}_2|^2} + \frac{1}{|\vec{r}_1-\vec{r}_2|^3}\right) e^{-\mu |r_1-r_2|}$$

The operator given by equation (A5-12) is, except for a factor $\left(\frac{m+M}{2M}\right)^{\frac{1}{2}}$ already mentioned, the same as that obtained by Brown and Riska (1970).

An alternative to the approximation used in obtaining the equation (A5-7) will now be given. Writting $m_I^2 = (p'_{20} - p_{20})^2 - \mu^2$ we can re-write equation (A5-7) as

$$W_{NN*} = \frac{i\,f\,G}{\mu} \iint d^3r_1\, d^3r_2\, [\bar{\psi}(r_1)\,\gamma_5\,\vec{\tau}_{(1)}\,\psi(r_1)] \left\{ \frac{1}{(2\pi)^3} \int d^3q(-i)\,\vec{q}_{\rho\neq 4} \frac{e^{i\vec{q}\cdot(\vec{r}_1-\vec{r}_2)}}{\vec{q}^2-(m_I)^2 - i\varepsilon} \right\} \cdot [\bar{\Psi}_\rho\, \vec{T}_{(2)}\, \psi(r_2)] \qquad \text{(A5-13)}$$

By performing the integration over d^3q we have

$$W_{NN*} = \frac{i\,f\,G}{\mu} \iint d^3r_1\, d^3r_2\, [\bar{\psi}(r_1)\,\gamma_5\,\vec{\tau}_{(1)}\,\psi(r_1)] \left\{ \frac{(-1)}{|r_1-r_2|}\, \frac{d}{d(|r_1-r_2|)}\, \frac{e^{-im_I|r_1-r_2|}}{|r_1-r_2|} \right. (\vec{r}_1-\vec{r}_2)_\rho \quad [\bar{\Psi}_\rho\, \vec{T}_{(2)}\, \psi(r_2)] \qquad \text{(A5-14)}$$

The non relativistic reduction can be carried out as before and there results

$$W_{NN*} = \frac{fG}{\mu}\, i \iint d^3r_1\, d^3r_2\, \psi_L^+(r_1)\, \Psi_4^+(r_1) \left[\frac{\sigma_1 \cdot \vec{p}_1}{2m}\, ,\ (\vec{r}_1-\vec{r}_2)\cdot \vec{S}\, Z\,(r_{12}) \right] \vec{\tau}_{(1)} \cdot \vec{T}_{(2)}\, \psi_L(r_1)\, \psi_L(r_2) \qquad \text{(A5-15)}$$

where

$$Z\,(r_{12}) = \frac{(-1)}{|r_1-r_2|}\, \frac{d}{d(|r_1-r_2|)}\, \frac{1}{4\pi}\, \frac{e^{-im_I|r_1-r_2|}}{|\vec{r}_1-\vec{r}_2|}$$

The "retardation factor" $e^{-im_I|r_1-r_2|}$ depends through m_I on the initial and final energies of the system. Since however we are considering the final and initial velocities to be small we can replace $m_I^2 = (p^1_{20}-p_{20})^2 - \mu^2$ in equation (A5-15) by

$m_{II}^2 = (M-m)^2 - \mu^2$ and therefore.

$$W_{NN*} = \left[\frac{fG}{\mu} \frac{1}{2m} \sigma_{(1)} \cdot S_{(2)} K(r_{12}) + \frac{fG}{\mu} \frac{1}{2m} (\vec{r}_1 - \vec{r}_2) \cdot \vec{S} (\vec{r}_1 - \vec{r}_2) \cdot \sigma_1 \frac{1}{|\vec{r}_1 - \vec{r}_2|} \frac{dK(|\vec{r}_1 - \vec{r}_2|)}{d|\vec{r}_1 - \vec{r}_2|} \right] \vec{\tau}_{(1)} \cdot \vec{T}_{(2)} + (1 \longrightarrow 2) \qquad \text{(A5-16)}$$

where

$$K(r_{12}) = \frac{(-1)}{|\vec{r}_1 - \vec{r}_2|} \frac{d}{d(|\vec{r}_1 - \vec{r}_2|)} \frac{1}{4\pi} \frac{e^{-im_{II}|\vec{r}_1 - \vec{r}_2|}}{|\vec{r}_1 - \vec{r}_2|}$$

The "retardation factor" in equation (A5-16) appears quite frequently in this type of calculation. We refer to the Chapter 6 of the book by Akhiezer and Berestetskii (1965) for several examples and specific ways of dealing with them.

Once W_{N*N} is obtained we can in principle solve the Hamiltonian H given by equation (A5-4), although in practice this task is beyond present capabilities (except for nuclear matter or very light nuclei). However in order to describe how the effects of an assumed T.R.I. violation in the $N^*N\gamma$ vertex can be calculated we shall assume that we have been able to solve the Hamiltonian H_0 given by equation (A5-3) for $\psi(N)$ and $\psi(N-1, N^*)$. Furthermore we assume also that we can treat W_{N*N} in perturbation theory. Therefore the final state Ψ_f and the initial state Ψ_i involved in a given electromagnetic transition can be written

$$\Psi_f = \psi_f(N) + \sum_{k \neq f} \frac{\langle \psi_f(N) | W_{NN*} | \psi_k(N-1, N^*) \rangle}{E_{\psi_f(N)} - E_{\psi_k(N-1, N^*)}} | \psi_\mu(N-1, N^*) \rangle$$

$$\Psi_i = \psi_i(N) + \sum_{k \neq i} \frac{\langle \psi_i(N) | W_{NN*} | \psi_k(N-1, N^*) \rangle}{E_{\psi_i(N)} - E_{\psi_k(N-1, N^*)}} | \psi(N-1, N^*) \rangle$$

The effects of a T.R.I. violating $N^*N\gamma$ vertex can now be calculated. First, by using the method described in section 2-3 a one body transition operator $H(N^*N\gamma)$ can be extracted from the graphs of Figure A5-3a and A5-3b. If we use the Lagrangian $\mathcal{L}^{TRV}_{N^*N\gamma}$ given by equation 4-3, the operator $H(N^*N\gamma)$ will be TRI violating and its effect is given by

$$\langle\Psi_f|\, H(N^*N\gamma)\,|\psi_i\rangle = \sum_{k\neq f} \frac{\langle\psi_k(N-1,N^*)\,|\,H(N^*N\gamma)\,|\,\psi_i(N)\rangle\,\langle\psi_k(N-1,N^*)\,|\,W_{NN^*}\,|\,\psi_f(N)\rangle}{E_{\psi_f(N)} - E_{\psi_k(N-1,N^*)}}$$

$$+ \sum_{k\neq i} \frac{\langle\psi_f(N)\,|\,H(NN^*\gamma)\,|\,\psi_k(N-1,N^*)\rangle\,\langle\psi_i(N)\,|\,W_{NN^*}\,|\,\psi_k(N-1,N^*)\rangle}{E_{\psi_i(N)} - E_{\psi_k(N-1,N^*)}} \qquad \text{(A5-17)}$$

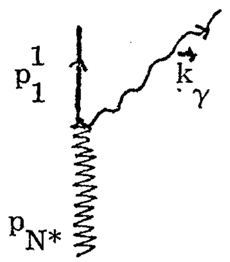


Fig. A5-3a

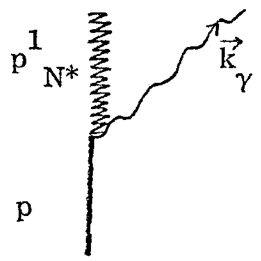
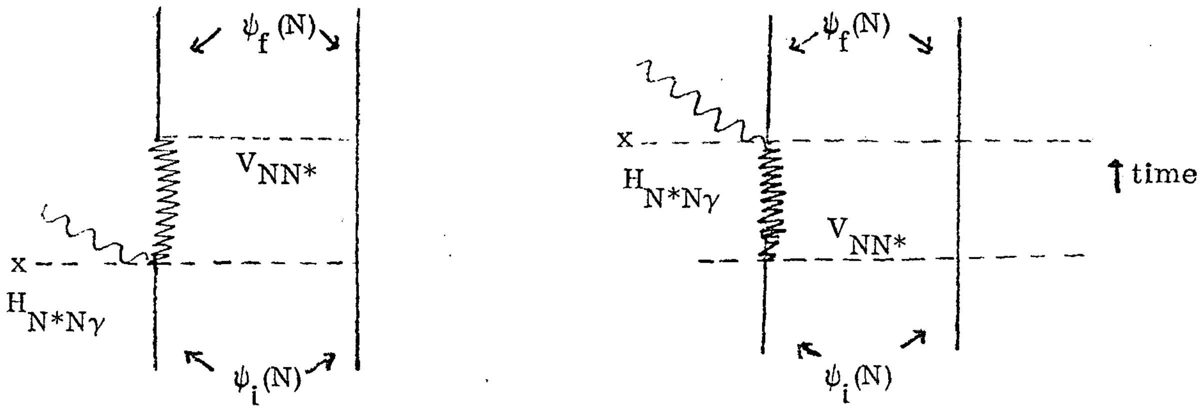


Fig. A5-3b

The equation (A5-17) corresponds of course to the two graphs of Fig. A5-6



(a) (b)

Fig. A5. 6

Now it is clear that the approach described in this Appendix is similar to the one used in Chapter 3. There are however differences. Firstly we note that the approach used in this appendix does not take into account exchange contribution like the one given by Fig. A5-7 which are included in the method of Chapter 3.

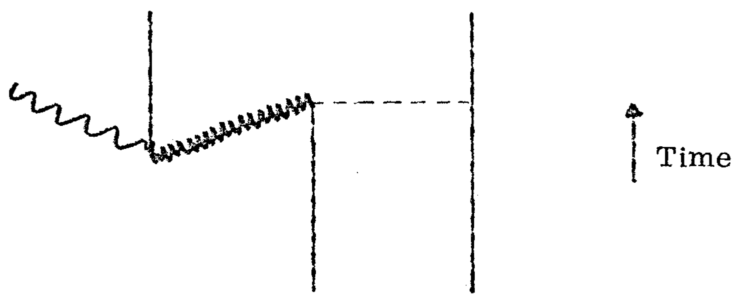


Fig. A5-7

Secondly, there is a small normalisation correction which the approach in Chapter 3 does not take into account, namely the factor $(1 - \alpha^2)^{\frac{1}{2}}$ in equation (1). Thirdly the approach in this Appendix would take account of interaction between the N* and the nucleons by using the correct ψ_n (N-1, N*). Since this is impossible to calculate in heavy nuclei, we feel justified in using the simpler approach of Chapter 3.

APPENDIX 6

ISOSPIN EXPANSION

This Appendix is intended to prove equations (3.10a) and (3.10b) namely

$$(T_{(1)}\cdot\vec{\tau}_{(2)})_{TRV} = \frac{1}{\sqrt{2}}\, T_{20}\,(\tau_{(1)}\tau_{(2)})\,(\epsilon_p^{*}-\epsilon_p+\epsilon_N^{*}-\epsilon_N) + \frac{i}{2\sqrt{3}}(\epsilon_p^{*}-\epsilon_p-\epsilon_N^{*}+\epsilon_N)(\tau_{(1)}\times\tau_{(2)})_z$$

$$+\frac{1}{\sqrt{2}}(\epsilon^{*}-\epsilon_N-\epsilon_p^{*}+\epsilon_p)\,\tau_z^{(2)} \qquad \text{(A6-1)}$$

$$(T_{(1)}\cdot\vec{\tau}_{(2)})_{NOR} = \frac{1}{\sqrt{2}}\, T_{20}\,(\tau_{(1)}\tau_{(2)})\,(\epsilon_p^{*}+\epsilon_p+\epsilon_N^{*}+\epsilon_N) + \frac{i}{2\sqrt{3}}(\epsilon_p^{*}+\epsilon_p-\epsilon_N^{*}-\epsilon_N)(\tau_{(1)}\times\tau_{(2)})_z$$

$$+\frac{1}{\sqrt{3}}(\epsilon_N^{*}+\epsilon_N-\epsilon_p^{*}-\epsilon_p)\,\tau_z^{(2)} \qquad \text{(A6-2)}$$

The matrix ϵ (see equations 3.3 and 3.4) will be written

$$\epsilon = \begin{pmatrix} 0 & 0 \\ a & 0 \\ 0 & b \\ 0 & 0 \end{pmatrix} \qquad \text{(A6-3)}$$

so that taking $a = \epsilon_p - \epsilon_p^{*}$ and $b = \epsilon_N - \epsilon_N^{*}$ we have the T.R.I. violating case and analogously taking $a = \epsilon_p + \epsilon_p^{*}$ $b = \epsilon_N + \epsilon_N^{*}$ we have the normal case.

The task is to calculate (see equation 3.9)

$$\vec{T}' = \epsilon^{*}\,\vec{T}_{(1)} \qquad \text{(A6-4)}$$

APPENDIX 7

RECOUPLING OF OPERATORS

The purpose of this Appendix is to illustrate the standard techniques used to decouple the operators given by equations 3.16 and 3.17 in Chapter 3 and to extract from them the operators of section 4.3-B. Let us take as an example the second part of the operator given by equation 3.16, viz.

$$V(L) = (-)\,\frac{1}{3}\,\frac{G\,f}{\mu(M-m)2m}\,\frac{2}{\sqrt{3}}\;i\,(\epsilon^{*}_{N}-\epsilon_{N}-\epsilon^{*}_{p}+\epsilon_{p})\,\left[(2L+1)L\right]^{\frac{1}{2}}\sum_{i<j} R_{ij}^{L-1}\Big[Y_{L-1}(R_{ij})\otimes$$

$$\Big\{\left[\sigma_i \times (\vec{r}_i-\vec{r}_j)\right]\sigma_j\cdot(r_i-r_j)\,\tau_z^{\,j} + \left[\sigma_j \times (\vec{r}_i-\vec{r}_j)\right]\sigma_i\cdot(\vec{r}_i-\vec{r}_j)\,\tau_z^{\,i}\Big\}\Big]^{*L}_{M}\;\frac{K(|\vec{r}_i-\vec{r}_j|)}{|\vec{r}_i-\vec{r}_j|} \qquad \text{(A7-1)}$$

by using

$$\left[\sigma_i \times (r_i-r_j)\right] = (-i)\sqrt{2}\,\left[\sigma_i \otimes (r_i-r_j)\right]^{(1)}$$

$$\left[\sigma_i\cdot(\vec{r}_i-\vec{r}_j)\right] = -\sqrt{3}\,\left[\sigma_j \otimes (\vec{r}_i-\vec{r}_j)\right]^{(0)}$$

$$\hat{r}^{(1)}_{m} = \sqrt{\frac{4\pi}{3}}\;Y_{1m}$$

$$\Big\{\left[\sigma_i \times(\vec{r}_i-\vec{r}_j)\right]\sigma_j\cdot(\vec{r}_i-\vec{r}_j)\,\tau_z^{\,j} + \left[\sigma_j \times (\vec{r}_i-\vec{r}_j)\right]\,\sigma_i\cdot(\vec{r}_i-\vec{r}_j)\,\tau_z^{\,i}\Big\} =$$

$$= i\sqrt{6}\;\frac{4\pi}{3}\;|\vec{r}_i-\vec{r}_j|^2\,\Big\{\Big[\left[\sigma_i \otimes Y_1(\hat{r}_{ij})\right]^{(1)}\otimes\left[\sigma_j \otimes Y_1(\hat{r}_i)\right]^{(0)}\Big]^{(1)}\,\tau_z^{\,j}$$

$$+\Big[\left[\sigma_j \otimes Y_1(\hat{r}_{ij})\right]^{(1)}\otimes\left[\sigma_i \otimes Y_1(\hat{r}_{ij})\right]^{(0)}\Big]^{(1)}\,\tau_z^{\,i}\Big\}$$

where $\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$

Now

$$\left[[\sigma_i \otimes Y_1(\hat{r}_{ij})]^{(1)} \otimes [\sigma_j \otimes Y_1(\hat{r}_{ij})]^{(0)} \right]^1_M =$$

$$\sum_{LL'} \left[(2L+1)(2L'+1)3 \right]^{\frac{1}{2}} \begin{Bmatrix} 1 & 1 & L \\ 1 & 1 & L' \\ 1 & 0 & 1 \end{Bmatrix} \left[[\sigma_i \otimes \sigma_j]^{(L)} \otimes [Y_1(\hat{r}_{ij}) \otimes Y_1(\hat{r}_{ij})]^{L'} \right]^{(1)}_M$$

which, using

$$[Y_1(\hat{r}_{ij}) \otimes Y_1(\hat{r}_{ij})]^{(l)}_m = \left(\frac{3}{4\pi}\right)^{\frac{1}{2}} \sqrt{3} \begin{pmatrix} 1 & 1 & l \\ 0 & 0 & 0 \end{pmatrix} Y_{lm}(\hat{r}_{ij})$$

is

$$= \sum_{LL'} \left(\frac{3}{4\pi}\right)^{\frac{1}{2}} \sqrt{3} \begin{pmatrix} 1 & 1 & L' \\ 0 & 0 & 0 \end{pmatrix} [(2L+1)(2L'+1)3]^{\frac{1}{2}} \begin{Bmatrix} 1 & 1 & L \\ 1 & 1 & L' \\ 1 & 0 & 1 \end{Bmatrix} \left[[\sigma_i \otimes \sigma_j]^{(L)} \otimes Y_{L'}(r_{ij}) \right]^{(1)}_M$$

$$= \left(\frac{3}{4\pi}\right)^{\frac{1}{2}} \sqrt{3} \frac{\begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}}{3} \left[[\sigma_i \otimes \sigma_j]^{(1)} \otimes Y_0(\hat{r}_{ij}) \right]^{(1)}_M +$$

$$+ \left(\frac{3}{4\pi}\right)^{\frac{1}{2}} (-) \, [5 \times 3]^{\frac{1}{2}} \begin{Bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \end{Bmatrix} \begin{pmatrix} 1 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \left[[\sigma_i \otimes \sigma_j]^{(1)} \otimes Y_2(\hat{r}_{ij}) \right]^{(1)}_M$$

$$+ \left(\frac{3}{4\pi}\right)^{\frac{1}{2}} [5 \times 5]^{\frac{1}{2}} (-) \begin{Bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \end{Bmatrix} \begin{pmatrix} 1 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \left[[\sigma_i \otimes \sigma_j]^{(2)} \otimes Y_2(\hat{r}_{ij}) \right]^{(1)}_M$$

Substituting in (A7-1) we get

$$V(1) = (-) \frac{1}{3} \frac{G f i}{\mu(M-m)2m^2} \frac{2}{\sqrt{3}} (\epsilon^*_N - \epsilon_N - \epsilon^*_p + \epsilon_p) \sum_{i<j} \left(\frac{1}{4\pi}\right)^{\frac{1}{2}} (-i)\sqrt{2} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} [\sigma_i \otimes \sigma_j]^{*1}_M (\tau_z^j - \tau_z^i)$$

$$|r_i - r_j| \; K(|r_i - r_j|) \; +$$

$$+ \frac{1}{3} \frac{G f i}{\mu(M-m)2m^2} \frac{2}{\sqrt{3}} (\epsilon_N^* - \epsilon_N - \epsilon_p^* + \epsilon_p) \sum_{i<j} [6 \times 3 \times 5]^{\frac{1}{2}} (-i) \begin{Bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \end{Bmatrix} \begin{pmatrix} 1 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\left[[\sigma_i \otimes \sigma_j]^{(1)} \otimes Y_2 \; (\hat{r}_{ij}) \right]^{*1}_{M} (\tau_z^{\,j} - \tau_z^{\,i}) \; +$$

$$+ \frac{1}{3} \frac{G f i}{\mu(M-m)2m^2} \frac{2}{\sqrt{3}} (\epsilon_N^* - \epsilon_N - \epsilon_p^* + \epsilon_p) \sum_{i<j} [6 \times 5 \times 5]^{\frac{1}{2}} (i) \begin{Bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \end{Bmatrix} \begin{pmatrix} 1 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\left[[\sigma_i \otimes \sigma_j]^{(2)} \otimes Y_L(\hat{r}_{ij}) \right]^{*1}_{M} (\tau_z^{\,i} + \tau_z^{\,j}).$$

The first two parts of V(1) are called $W_1^{(b)}$ (M· 1) and $W_1^{(c)}$(M· 1) respectively in Chapter 4.

APPENDIX 8

FORMULAE OF ANGULAR CORRELATION

In section 4-2 the angular correlation function W(1,2) of two γ-rays emitted in succession and with no perturbation of the intermediate state was defined and decomposed as follows (eq. 4-3)

$$W(1,2) = W^{(0)}(1,2) + W^{(1)}(1,2) + \dots + W^{(l)}(1,2) \qquad \text{(A8-1)}$$

$$0 < l \leq 2\, I_i$$

where each term $W^{(k)}$ (1,2) is proportional to the corresponding statistical tensor $R^{(k)}_{0}$.

Equations(4-4) and (4-5) give the first two terms $W^{(0)}(1,2)$ and $W^{(1)}(1,2)$ respectively. The purpose of this appendix is to give the general term $W^{(l)}(1,2)$ (we refer to Coutinho and Ridley (1970) for details) in terms of the angles defined in Fig. 12.

(a) l-even

$$W^{(l)} = [\,(2I_i+1)(2l+1)]^{\frac{1}{2}}\, R^{(l)}_{0} \sum_{\substack{k,p \\ \text{(even)}}} A^{k}_{lp}\,(L_1 L_1{+}1\; I_i)\; A_p(L_2 L_2{+}1\; I_f I)$$

$$\sum_{N} (-)^{N} \begin{pmatrix} P & k & l \\ |N| & -|N| & 0 \end{pmatrix} 2\, P_k^{|N|}(\cos\beta_1)\; P_p^{|N|}(\cos\beta_2)\, \cos|N|\,\phi \qquad \text{(A8-2)}$$

(b) l-odd

$$W^{(l)} = \left[(2l+1)(2I_i+1) \right]^{\frac{1}{2}} R_0^{(l)} \sum_{\substack{k,p \\ (\text{even})}} A_{lp}^{k} (L_1 L_1{+}1\, I I_i)\, A_p (L_2 L_2{+}1\, I_f I)\ x$$

$$x \sum_N (-)^N \begin{pmatrix} p & k & l \\ |N| & -|N| & 0 \end{pmatrix} 2\, P_k^{|N|} (\cos \beta_1)\ P_p^{|N|} (\cos \beta_2) \sin |N| \phi \qquad \text{(A8-3)}$$

where $\phi = \phi_2 - \phi_1$ and

$$A_{lp}^{k} (L_1 L_1{+}1 I I_i) = \left[1 + |\delta (1)|^2 \right]^{-1} \left[F_{lp}^{k} (L_1 L_1 I I_i) + (\delta (1) + (-)^l \delta^*(1))\, F_{lp} (L_1 L_1{+}1\, I I_i) \right.$$

$$\left. + |\delta (1)|^2 F_{lp}^{k} (L_1{+}1\ L_1 + 1\ I I_i) \right] \qquad \text{(A8-4)}$$

$$A_p (L_2 L_2{+}1\ I_f I) = \left[1 + |\delta (2)|^2 \right]^{-1} \left[F_p (L_2 L_2 I_f I) - (\delta (2) + \delta (2)^*)\, F_p (L_2 L_2{+}1\, I_f I) + \right.$$

$$\left. + |\delta (2)|^2 F_p (L_2{+}1\ L_2{+}1\ I_f I) \right] \qquad \text{(A8-5)}$$

$$F_{g_1 g_3}^{g_2} (LL' j_1 j_0) = (-)^{L-1} \{ (2j_1 + 1)^{\frac{1}{2}} (2j_0+1)^{\frac{1}{2}} (2L+1)^{\frac{1}{2}} (2L' +1)^{\frac{1}{2}} \} \langle LL' 1 - 1 | g_2 0 \rangle$$

$$\begin{Bmatrix} j_0 & j_0 & g_1 \\ L & L' & g_2 \\ j_1 & j_1 & g_3 \end{Bmatrix} \qquad \text{(A8-6)}$$

$$F_k (LL' I_i I) = (-)^{I_i + I - 1} \left[(2L+1)\ (2L'+1)\ (2I+1)\ (2K+1) \right]^{\frac{1}{2}} \begin{pmatrix} L & L' & K \\ 1 & -1 & 0 \end{pmatrix} \begin{Bmatrix} L & L' & K \\ I & I & I_i \end{Bmatrix}$$

(A8-7)

APPENDIX 9

DETERMINATION OF $\mathcal{V}_{t.v.}(\vec{A})$ GIVEN $V_{t.v.}$

The general problem of constructing a gauge invariant combination $V + \mathcal{V}(\vec{A})$ from a given potential V will be considered in this appendix. We shall limit ourselves to the case where $\mathcal{V}(\vec{A})$ is linearly dependent on A and therefore according to equation (A4-6), $V + \mathcal{V}(\vec{A})$ is gauge invariant if

$$\mathcal{V}(\nabla G) = i\,[g, V] \tag{A9-1}$$

where

$$g = \sum_i \frac{e}{2}(1 + \tau_z^{(i)})\, G(r_i) \text{ and } V = \sum_{i \neq j} V_{ij}$$

As already explained in section 5-2 the usual replacement $\vec{p} \to \vec{p} - e\vec{A}\frac{1}{2}(1 + \tau_z)$ is not sufficient to ensure gauge invariance of a potential which contains an isospin exchange term (e.g. $V = \sum_{i \neq j} V_\tau(|r_i - r_j|)\,\tau_i \cdot \tau_j$). This type of potential is clearly not gauge invariant since it contributes to the commutator in equation (A9-1).

However gauge invariance alone is not sufficient to determine $\mathcal{V}(\vec{A})$ uniquely from V. In fact the only way of obtaining $\mathcal{V}(\vec{A})$ uniquely is by going back to a field theoretical basis and using the Feynman Graphs from which V itself has been extracted. This has been done in the case of parity violating potentials by Fischback and Tadic (1971) (see also Tadic and Eman (1971)). Some of their results will be used to compare with the phenomenological procedure described below.

The procedure is based on equation (A9-1) and is defined as follows.

(i) Calculate the commutator

$$C = i\left[\frac{e}{2}(1+\tau_z^{(i)})\,G(r_i) + \frac{e}{2}(1+\tau_z^{j})\,G(r_j),\ V_{ij}\right]$$

(ii) The result will have terms of the form

$$[\nabla_i G(r_i) \pm \nabla_j G(r_j)] \text{ and } [G(r_i) - G(r_j)]$$

(iii) Replace $[\nabla_i\ G(r_i) \pm \nabla_j\ G(r_j)]$ by $[\vec{A}(r_i) \pm \vec{A}(r_j)]$ and $[G(r_i) - G(r_j)]$ by an arbitrary functional $\mathbb{F}(r_i r_j\ A)$ such that $\mathbb{F}(r_i r_j\ \nabla G) = G(r_i) - G(r_j)$ so leading to a term in the Hamiltonian H $(A(r_i),\ A(r_j))$, depending on A.

(iv) $\mathcal{V}(\vec{A}) = \sum_{i \neq j} H(A(r_i), A(r_j))$ since by construction it satisfies (A9-1).

The functional $\mathbb{F}(r_i r_j A)$ referred above could be for example

$$\mathbb{F}(r_i r_j A) = \int_{\vec{r}_i}^{\vec{r}_j} \vec{A}(r)\cdot d\vec{r}$$

where the integration is over an arbitrary path. Another example for F is

$$\mathbb{F}(r_i r_j \cdot \vec{\mathbb{A}}) = \int d^3x\, \vec{\xi}\,(\vec{x}-\vec{r}_i,\ \vec{x}-\vec{r}_j)\cdot \vec{A}(x)$$

where $\vec{\xi}(x-r_i,\ x-r_j)$ falls off rapidly with $\vec{x}$ and

$$\mathrm{div}_x\ \vec{\xi} = \delta^3(\vec{x}-\vec{r}_i) - \delta^3(\vec{x}-\vec{r}_j)$$

As an example of this procedure, consider the following parity violating potential (Tadic and Eman - 1971)

$$V_\pi = \frac{k}{2m}\left[[\sigma_1 \cdot \vec{p}_1,\ f(1,2)] - [\sigma_2 \cdot \vec{p}_2, f(1,2)] \right] T^-$$

where

$$f(1,2) = \frac{1}{4\pi}\ \frac{e^{-\mu|\vec{r}_1-\vec{r}_2|}}{|\vec{r}_1 - \vec{r}_2|}$$

We have

$$i\ [g, V_\pi] = i\ e\ [\ G(r_1)\frac{1}{2}(1+\tau_z^{(1)}) + G(r_2)\frac{1}{2}(1+\tau_z^{(2)}),\ V_\pi]_- =$$

$$= \frac{-ek}{2m}[\sigma_1 \cdot \nabla G(r_1) + \sigma_2 \cdot \nabla G(r_2)]\, f(1,2)\ T^{(+)} + \frac{i\,ek}{2m}[(\sigma_1 \cdot p_1 - \sigma_2 \cdot p_2), [G(r_1)-G(r_2)]f(1,2)]_-\, T^+$$

and therefore

$$\mathcal{V}_\pi(\vec{A}) = \frac{-ek}{2m}[\sigma_1 \cdot A(r_1) + \sigma_2 \cdot A(r_2)]f(1,2)T^+ + \frac{i\,e\,k}{2M}[(\sigma_1 \cdot p_1 - \sigma_2 \cdot p_2), F(r_1 r_2 \vec{A})f(1,2)]\ T^+$$

(A9-2)

The potential V_π was derived from field theory from the diagram in Fig. A9-1

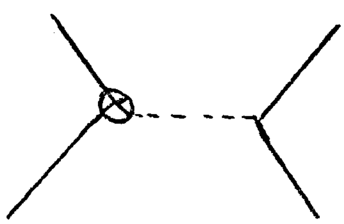


Fig. A9-1

where the crossed bubble is the p. v. vertex.

Tadic and Fischbach (1971) show that gauge invariance is obtained by considering the graphs of Fig. (A9-2).

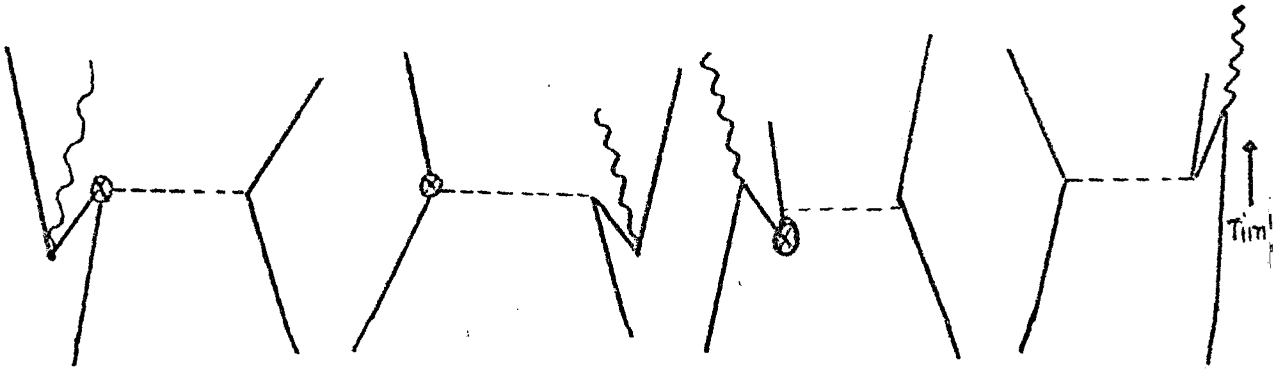


Fig. (A9-2)

which gives

$$V_{\pi}^{(a)}(\vec{A}) = \frac{-e\,K}{2m}\,(\sigma_1 \cdot \vec{A}(r_1) + \sigma_2 \cdot \vec{A}(r_2))\, f(1,2)\, T^{+} \qquad \text{(A9-3)}$$

and the diagram of Fig. (A9-3)

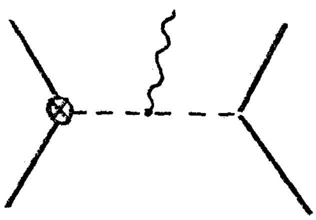


Fig. (A9-3)

which gives

$$V_{\pi}^{(b)}(\vec{A}) = -\,iek \left\{ \left[\frac{\sigma_1 \cdot p_1}{2m}, \phi(1,2)\right]_{-} T^{(+)} - \left[\frac{\sigma_2 \cdot p_2}{2m}, \phi(1,2)\right] T^{(+)} \right\} \qquad \text{(A9-4)}$$

where

$$\phi(1,2) = \int d^3 r_3 \;\left[f(3,2)\,(\nabla_3 f(1,3)) - f(1,3)\,(\nabla_3 f(3,2))\right] \cdot \vec{A}(r_3)$$

Comparison of equations (A9-2) to (A9-3) and (A9-4) shows that the phenomenological approach agrees with the field theoretical approach if we choose

$$\mathbb{F}\,(r_1 r_2 \vec{A}) = -\int d^3 r_3 \left[\frac{f(3,2)}{f(1,2)} \quad (\nabla_3\, f(1,3)) - \frac{f(1,3)}{f(1,2)}\,(\nabla_3 f(3,2)\right] \cdot A(r_3)$$

This would of course be quite impossible to guess in the absence of a detailed theory. However for T. R. I. violating forces where no detailed theory exists one might use a phenomenological treatment and choose a simple form for $\mathbb{F}$

APPENDIX 10

AVERAGE PROCEDURE

This Appendix describes how to obtain a one body operator that is equivalent to a given two body T. R. I. violating operator in the sense that it has the same matrix element between two Slater determinants differing by just one orbital.

The T. R. I. violating potential considered is due to Huffman (1970) and was given in equation (5-29), namely

$$V(i,j) = G \frac{\vec{r}_i - \vec{r}_j}{|\vec{r}_i - \vec{r}_j|} (\vec{p}_i - \vec{p}_j)\, Q(\mu\, |\vec{r}_i - \vec{r}_j|)(\sigma_i \cdot \sigma_j)(\tau_{(i)} \cdot \tau_{(j)} - \tau^z_{(i)} \cdot \tau^z_{(j)}) + \text{h. c.} \tag{A10-1}$$

where $Q(\mu\,|r_i - r_j|)$ is a short range function given in the Huffman paper.

The matrix element of a two body operator between Slater determinants differing by just one orbital (denoted below by u and ν) is given by equation (5-20), namely

$$< V(i,j) > \; = \; V(\text{direct}) + V(\text{exchange}) \tag{A10-2}$$

where V(direct and V(exchange) is given by equations (5-21) and (5-22).

We consider first the V(direct) term i. e.

$$V(\text{direct}) = \sum_k \int d(1)\, d(2)\; U^*(1)\; W^*_K(2)\, V(1,2)\, \nu(1)\, W_K(2)$$

Since $\tau_1 \cdot \tau_2 - \tau^z_{(1)} \tau^z_{(2)} = \tau^{(1)}_{(+)} \tau^{(2)}_{(-)} + \tau_{(-)} \tau_{(2)} = T^{(+)}$ it is easy to see that this matrix element vanishes.

The exchange matrix element is

$$V(\text{exchange}) = - \sum_k \int d(1)\, d(2)\;\; U^*(1)\; W_K^*(2)\; V(1,2)\; \nu\,(2)\; W_K(1)$$

which is first written in the form $V(1,2) = V_x(1,2)\; V_\sigma(1,2)\; V\;\; (1,2)$ separating the space, spin and i-spin parts. Thus,

$$V(\text{exchange}) = - \left[\sum_{\sigma_k} \chi_{\sigma_u}(1)\;\; \chi_{\sigma_k}(2)\; V_\sigma(1,2)\;\; \chi_{\sigma_\nu}(2)\;\; \chi_{\sigma_k}(1) \right] \;\; x$$

$$\left[\sum_{\tau_k} \chi^*_{\tau_u}(1)\;\; \chi^*_{\tau_k}(2)\; V_\tau(1,2)\;\; \chi_{\tau_\nu}\;\; \chi_{\tau_k}(1) \right] \;\; x \left[\sum_k \iint d(1)\; d(2)\; U^*(1)\;\; W_k^*(2)\; V_x(1,2) \right.$$

$$\left. \nu\,(2)\;\; W_k(1) \right]$$

We now treat each part separately, the aim being to obtain

$$V_{eq} = -V_{eq}(\text{spin}) \times V_{eq}(\text{i-spin}) \times V_{eq}(\text{space}) \tag{A10-3}$$

Consider first the spin part. We have

$$\sum_{\sigma_k} \chi^*_{\sigma_u}(1)\; \chi^*_{\sigma_k}(2)\;\; V_\sigma(1,2)\;\; \chi_{\sigma_\nu}(2)\;\; \chi_{\sigma_k}(1) =$$

$$= \frac{1}{2} \sum_{\sigma_k} \chi^*_{\sigma_u}(1)\;\; \chi^*_{\sigma_k}(2)\;\;\; \left[V_\sigma(1,2)\; P_\sigma + P^*_\sigma\, V_\sigma(2,1) \right] \chi_{\sigma_\nu}(1)\;\; \chi_{\sigma_k}(2)$$

where

$$P_\sigma = \frac{1}{2}(1 + \sigma_1 \cdot \sigma_2)$$

is the usual spin exchange operator. In the Huffman case $V_\sigma(1,2) = \sigma_1 \cdot \sigma_2$. We therefore define

$$V_{eq}(\text{spin}) = \frac{1}{2}\,\Sigma_{\sigma_k}\, \chi^*_{\sigma_k}(2)\ \left[V_\sigma(1,2)\,P_\sigma + P^*_\sigma\, V_\sigma(2,1)\right]\chi_{\sigma_k}(2) = 3\ \mathbb{1} \qquad \text{(A10-4)}$$

where $\mathbb{1}$ is the unit 2 x 2 matrix. Similarly

$$\sum_{\tau_k} \chi^*_{\tau_u}(1)\ \chi^*_{\tau_k}(2)\ V_\tau(1,2)\ \chi_{\tau_\nu}(2)\ \chi_{\tau_k}(1) =$$

$$= \frac{1}{2}\sum_{\tau_k} \chi^*_{\tau_u}(1)\ \chi^*_{\tau_k}(2)\ \left[V_\tau(1,2)\ P_\tau + P^*_\tau V_\tau(2,1)\right]\ \chi_{\tau_\nu}(1)\ \chi_{\tau_k}(2)$$

where $P_\tau = \frac{1}{2}(1 + \tau_1 \cdot \tau_2)$. We therefore define

$$V_{eq}(\text{i-spin}) = \frac{1}{2}\sum_{\tau_k} \chi^*_{\tau_k}(2)\ \left[V_\tau(1,2)\,P_\tau + P^*_\tau\, V_\tau(2,1)\right]\ \chi_{\tau_k}(2) =$$

$$\sum_{\tau_k} \chi^*_{\tau_k}(2)\ \frac{1}{2}\left(1 - \tau^z_{(1)}\ \tau^z_{(2)}\right)\ \chi_k(2)$$

The last result is approximated (see Michell 1965) by

$$V_{eq}(\text{i-spin}) = \frac{1}{2}\left(1 - \frac{N-Z}{A}\ \tau^z_{(1)}\right) \qquad \text{(A10-5)}$$

Finally the space part is calculated in the following way. First write

$$\sum_k \iint d(1)\ d(2)\ U^*(1)\ W_k(2)\ V_x(1,2)\ \nu(2)\ W_k(1) =$$

$$\int d(1)\ U^*(1)\ \Sigma \int d(2)\ W^*_k(2)\ \frac{1}{2}\ \left[V_x(1,2)\ P_x + P^*_x\ V_x(2,1)\right] W_k(2)\ \nu(1) \quad .$$

The operator P_x is given by (Sachs - 1948)

$$P_x = \sum_n \frac{1}{n!} \left((\vec{r}_1 - \vec{r}_2) \cdot \nabla_2 + (r_2 - r_1) \cdot \nabla_1 \right)^n$$

The first two terms of P_x are

$$P_x = 1 - i\, (\vec{r}_1 - \vec{r}_2) \cdot (\vec{p}_1 - \vec{p}_2) + \ldots .$$

Because the potential (A10-1) is a short range one we can approximate $P_x \approx 1$.

Therefore

$$V_{eq}(\text{space}) = G \sum_k \int d(2)\, W^*_k(2) \left[\frac{\vec{r}_1 - \vec{r}_2}{|r_1 - r_2|} \cdot (\vec{p}_1 - \vec{p}_2)\, Q(\mu |\, r_1 - r_2|) + \text{h. c.} \right] W_k(2)$$

and therefore

$$V_{eq}\ (\text{space}) = G\, \vec{P}_1 \cdot \left(\sum_k \int d(2)\, W^*_k(2)\, \frac{\vec{r}_1 - \vec{r}_2}{|r_1 - r_2|}\, Q(\mu |\, r_1 - r_2 |)\, W_k(2) \right) + \text{h. c.}$$

or (see Blin-Stoyle 1955)

$$V_{eq}(\text{space}) = G\, \vec{P}_1 \cdot \vec{r}_1 \left(\sum_k \int d(2)\, W^*_k(2) \left(1 - \frac{\vec{r}_2 \cdot \vec{r}_1}{|r_1|^2} \right) \frac{Q(\mu |\vec{r}_1 - \vec{r}_2|)}{|\vec{r}_1 - \vec{r}_2|}\, W_k(2) \right) + \text{h. c.}$$

(A10-6)

Now is we put (A10-4), (A10-5) and (A10-6) in (A10-3) we get

$$V_{eq}(\text{exchange}) = -\, G\, (\vec{P}_1 \cdot \vec{r}_1) \left\{ 3 \sum_k \int d(2)\, W^*(2) \left(1 - \frac{\vec{r}_2 \cdot \vec{r}_1}{|r_1|^2} \right) \frac{Q(\mu |r_1 - r_2|)}{|r_1 - r_2|}\, W_k(2) \right\}$$

$$\frac{1}{2} \left(1 - \tau^z_{(1)} \left(\frac{N-Z}{A} \right) \right) + \text{h. c.} \qquad \text{(A10-7)}$$

Now (see Blin-Stoyle - 1955) define

$$\mathcal{H}(r_1) = 3 \sum_k \int d(2)\, W_k^*(2) \left(1 - \frac{\vec{r}_2 \cdot \vec{r}_1}{|r_1|^2}\right) \frac{Q(\mu|r_1 - r_2|)}{|r_1 - r_2|}\, W_k(2)$$

$$= -3 \sum_{nlm} \int d\vec{r}_2\, \psi^*_{nlm}(r_2) \left(\frac{\vec{r}_2 \cdot \vec{r}_1}{|r_1|^2} - 1\right) \frac{Q(\mu|r_1 - r_2|)}{|r_1 - r_2|}\, \psi_{nlm}(r_2) \qquad \text{(A10-8)}$$

$$\rho(r, r') = 3 \sum_m \psi^*_{nlm}(r_1') \left(\frac{r' \cdot r}{|r|^2} - 1\right) \psi_{nlm}(r') \qquad \text{(A10-9)}$$

$$J(|r - r'|) = \frac{Q(\mu|r - r'|)}{|r - r'|} \qquad \text{(A10-10)}$$

so that

$$\mathcal{H}(r) = -\int J(|r - r'|)\, \rho(r, r')\, d\vec{r}'$$

and now using Blin-Stoyle results.

$$\mathcal{H}(r) = \sum_{n,l} \frac{-2\,(2l+1)}{3}\, \frac{3}{2} \left(\int J(s)\, s^4\, ds\right) \frac{1}{r} \frac{d}{dr} \left[f_{n,l}^{\,2}(r)\right] \qquad \text{(A10-11)}$$

where $f_{nl}(r)$ is the radial function for a single particle in the state (nl) and is related to the mean density distribution of particles $\rho_{nl}(r)$ by

$$\rho_{nl}(r) = 2\, f_{nl}^{\,2}(r)\, \frac{(2l+1)}{4\pi} \quad \text{so}$$

$$\mathcal{H}(r) = -\frac{4\pi}{3}\, \frac{3}{2} \sum_{nl} \left(\int J(s)\, s^3\, ds\right) \frac{1}{r} \frac{d\rho_{nl}}{dr}$$

or

$$\mathcal{H}(r) = \frac{k}{r}\, \frac{d\,\rho(r)}{dr} \qquad \text{(A10-12)}$$

where

$$k = -2\pi \int Q\,(\mu s)\; S^3\, ds \tag{A10-13}$$

Now if we substitute A10-12 and A10-13 in A10-7 we have equation 5-30a.

REFERENCES

Akhiezer and Berestetski (1965) - Quantum electrodynamics, John Wiley and Sons, N.Y. (1965).

Barshay (1966) - Phys. Rev. Lett. 17 (1966) 46.

Bernstein et. al. (1965) - Phys. Rev. 139 (1965) B1650

Bincer (1960) - Phys. Rev. 118, 855 (1960)

Blatt and Weisskopf - Theoretical Nuclear Physics, John Wiley and Sons

Blin-Stoyle (1955) - Phys. Mag. 7 (1955) 973

Blin-Stoyle and Nair (1966) - Advances in Physics 15, (1966) 493

Bohr and Motelson (1969) - Nuclear Structure Vol. I., W. A. Benjamin Inc. N.Y. (1969)

Brennan and Sachs (1952) - Phys. Rev. 88, (1952) 824

Brink and Rose (1967) - Rev. of Mod. Phys. (1967)

Brink and Satchler (1968) - Angular Momentum - Oxford University Press.

Broadhurst (1971) - D. Ph. Thesis- University of Sussex; Nuclear Physics B20 (1970) 603

Brown and Riska (1970) - Nuclear Phys. A153 (1970), 8

Chemtob (1968) - Nuclear Phys. A123 (1969) 449

Chemtob (1969) - Ph. D. Thesis, University of Paris

Chemtob and Rho (1971) - Nucl. Phys. A163 (1971) 1

Chollet et. al. (1969) - CERN preprint 69-7

Clement and Heller (1971) - Phys. Rev. Lett. 27 (1971)

Clement (1971) - Harwell preprint T.P. 460.

Coutinho and Ridley - University of Sussex Internal Report (1972)

Cronin (1968) - 14th International Conference on High Energy Physics, Vienna, p. 281

Dalitz (1954) - Phys. Rev. 95 (1954) 799

Das (1968) - Phys. Rev. Letts. 9 (1968) 409

Drell and Zachariasen (1961) - Electromagnetic Structure of nucleons - Oxford University Press

Elliott and Flowers (1955) - Proc. of the Royal Soc. 229 (1955) 536.

Fano (1951) - National Bureau of Standards Report n° 1214

Foldy (1953)-Phys. Rev. 92 (1953) 178

Frauenfelder and Steffen (1965) - see Siegbahn (1965)

Glashow (1965) - Phys. Rev. Letts. 14 (1965) 35

Gourdin (1966) - Diffusion des Electrons de Haute Energie, Masson Cie Editeurs - Paris

Green (1971) - Phys. Lett. 34B (1971) 451

Green and Schucan (1971) - Nucl. Phys. A188 (1972) 289

Hamilton (1971) - Private communication

Hannon and Trammel (1968) - Phys. Rev. Letts. 16 (1968) 726

Henley and Jacobson (1958) - Phys. Rev. 113 (1959) 225 and 224

Henley and Jacobson (1966) - Phys. Rev. Letts. 16 (1966) 706

Henley (1969) - Annual Review of Nuclear Science 19 (1969) 367

Herczeg (1965)-Nuclear Phys. 75 (1966) 655.

Holmes (1972) - To be published in Nuclear Physics

Huffman (1970) - Phys. Rev. D 1 (1970) 882

Kuo and Brown (1966) - Nucl. Phys. 85 (1966) 40

Lee (1965a) - Phys. Rev. 140 (1965) B 959

Lee (1965b) - Phys. Rev. 140 (1965) B967

Lipshutz (1967) - Phys. Rev. 158 (1967) 1491

Lobov (1969) - Atomic Energy Review vol. VII no. 3

Lurie (1968) - Particles and Fields - John Wiley and Sons

Maqueda and Blin-Stoyle (1967) - Nucl. Phys. A91 (1967) 460

Michell (1965) - Phys. Rev. 133 (1964) B329

Nishijima and Sank (1966) - Phys. Rev. 146 (1966) 1161

Nishijima and Sank (1967a) - Nucl. Phys. B3 (1967a) 553

Oakes (1968) - Phys. Rev. Letts. 20 (1968) 1539

Osborne and Foldy (1950) - Phys. Rev. 79 (1950) 795

Rarita and Schwinger (1941) - Phys. Rev. 60 (1941) 61.

Sachs (1948) - Phys. Rev. 74 (1948) 433

Sachs and Austern (1951) - Phys. Rev. 81 (1951) 705

Sakurai (1964) - Invariance Principles and Elementary Particles, Princeton University Press.

Salin (1963) - Nuovo Cimento 28 (1963) 1294 and 27 (1963) 193

Siegbahn (1965) - Alpha, Beta and Gamma Ray Spectroscopy - North-Holland

Siegert (1936) - Phys. Rev. 52 (1937) 787

Sugawara (1953) - Prog. Theor. Phys. 10 (1953) 199.

Tadic and Fischbach (1971) - Preprint of Pardue University

Tadic and Eman (1971) - Phys. Rev. C 4 (1971) 661

Takahashi and Umezawa (1953) - Prog. Theor. Phys. 9 (1953) 1

Walborn (1964) - Phys. Rev. 138 (1965) B530

Warburton et. al. (1967) - Phys. Rev. 155 (1967) 1164

Wheeler (1936) - Phys. Rev. 50 (1936) 643

Wolfenstein (1964) - Phys. Rev. Letts. 13 (1964) 562