Transactions of the **13th International Conference on Structural Mechanics in Reactor Technology** (SMiRT 13), Escola de Engenharia - Universidade Federal do Rio Grande do Sul, Porto Alegre, Brazil, August 13-18, 1995

Evaluation of the overall collapse of a ring-stiffened cylindrical shell

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ABSTRACT: The possible collapse modes of ring-stiffened cylindrical shells under external pressure are: local inter frame collapse of the shell between ring frames, which is an interaction between elastic-plastic buckling and axisymmetric membrane shell yielding at mid-bay; overall collapse of the shell-and-rings combination between rigid sections such as flat heads. This mode is associated with ring frames which are weak to resist either: out-of-circularity bending leading to premature yielding; sideways tripping of the ring frames which may then precipitate the out-of-circularity bending.

The collapse pressures of these modes are sensitive to the so-called "imperfections" related to the geometry of the structure, boundary conditions, materials and loads (including residual stresses). For the overall modes of failure the collapse pressures are rather sensitive to shape imperfections and so their prediction is less certain than for inter frame collapse.

The presence of imperfections can cause the shell to reach yield and collapse at pressure lower than for a geometrically perfect ring-stiffened cylinder.

In this paper we investigate the overall collapse of a imperfect ring-stiffened cylindrical shell under external pressure using a combination of analytical and numerical methods and considering the imperfections as a sinus lobe with the amplitude being a percentage of the inner radius of the shell and a discrete number of waves in the axial and circumferential directions of the shell.

1 INTRODUCTION

Ring-stiffened cylindrical shells under external hydrostatic pressure can have the following principal failure modes: local inter frame collapse of the shell between ring frames, and overall collapse of the shell-and-rings combination between rigid sections such as flat heads.

The local inter frame collapse of the shell between ring frames is an interaction between elastic-plastic buckling (called lobar buckling) and axisymmetric membrane shell yielding at midbay.

Lobar buckling is an elastic-plastic instability of the shell between ring frames and is characterized by inward and outward lobes, which may or may not develop around the entire periphery of the cylindrical shell. The failure may occur in one or more ring spaces. This mode of failure indicates that the rings have greater resistance to buckling than the shell between them.

Axisymmetric shell yielding is initiated by elastic yielding at the extreme fibers at both the outer surface of the shell midway between ring frames and the inner surface of the shell at the stiffeners. The yield leads to elastic-plastic collapse that is characterized by an accordion type of pleat extending around the periphery of the cylinder. This failure may occur in one or more spaces between the rings.

Overall collapse of the shell-and-rings combination occurs between rigid sections such as flat heads or heavy stiffeners. It is associated with rings which are weak to resist widely spaced hard sections and an out-of-circularity bending leading to premature yielding. Sideways tripping of the rings, when precipitates out-of-circularity bending, may cause overall collapse. This failure mode is also characterized by inward and outward lobes, but the lobes are fewer (usually just two or three) than the number of lobes in lobar buckling.

The collapse pressures of these modes are sensitive to the so-called "imperfections" related to the geometry of the structure, boundary conditions, materials and loads (including residual stresses). The presence of imperfections can cause the shell to reach yield and collapse at pressure lower than for a geometrically perfect ring-stiffened cylinder.

For the overall modes of failure the collapse pressures are rather sensitive to shape imperfections and so their prediction is less certain than for inter frame collapse. It is usual to design so that the collapse is precipitated by inter ring failure of the shell since this mode is least affected by shape imperfections. The stiffening is usually designed so that inter frame collapse leads to overall collapse if the pressure is maintained. This is because ring frames have to be extremely heavy to maintain circularity after shell plating has collapsed.

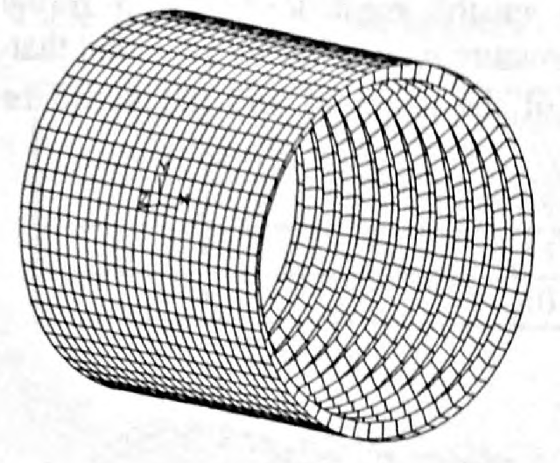
In the overall failure the buckling is initiated by ring yielding and the collapse pressures are greatly reduced by out-of-circularities which create an eccentric load path for the compressive hoop force producing bending moments in the ring.

In this paper we investigate the collapse of an imperfect ring-stiffened cylindrical shell under external hydrostatic pressure using a combination of analytical and numerical methods. Analytical formulae are used to predict the pressures that cause the yielding in central frames that characterize the beginning of the overall collapse. Linear and nonlinear finite element analyses (FEA) are used to predict the pressures that correspond to the elastic instability, the yielding in central frames and the elastic-plastic collapse. It is important to mention that in the elastic-plastic collapse evaluation it is also considered the geometric nonlinearities.

2 THE RING-STIFFENED CYLINDRICAL SHELL UNDER ANALYSIS

The general dimensions of the ring-stiffened cylindrical shell under investigation in this paper are shown in Figure 1, along with a typical model of half length.

This shell was designed, using analytical formulae, in such a way that the collapse pressures corresponding to the three modes of failure have about the same value. It is important to notice that the overall collapse is associated with an amplitude of the out-of-circularity of 0,3% of the inner radius of the shell in the central ring, with a sinusoidal shape having half wave and 3 waves in the axial and circumferential directions of the shell, respectively.



shell mean radius = 2500 mm
shell thickness = 24 mm
ring thickness = 24 mm
ring height = 223 mm
distance between rigid sections = 9265 mm

Figure 1: General dimensions and finite element model of the ring-stiffened cylindrical shell

3 ANALYTICAL FORMULAE

The bending stress σ_b in the ring frame is given by

(1) $\sigma_b = E \kappa e$

where E is the Young's modulus, κ is the curvature and e is the distance from neutral axis.

The relation between deflection and curvature is

(2) $$\kappa = \frac{w}{R_c^2}(n^2 - 1)$$

where w is the deflection, R_c is the radius of the neutral axis of the bending cross section of the ring and n is the number of waves over the circumference.

From Franitza (1988),

(3) $w = w_0\, p / (p_g - p)$

where w_0 is the amplitude of the out-of-circularity, p is the applied hydrostatic pressure and p_g is a reference pressure given by

(4) $$p_g = \frac{E\, t\, \beta_1^4}{(n^2 - 1 + \beta_1^2/2)(n^2 + \beta_1^2)^2 R} + \frac{E\, I\, (n^2 - 1)}{R_c^3\, l_c (n^2 - 1 + \beta_2^2/2)}$$

where $\beta_1 = \pi R / L$, $\beta_2 = \pi R / l_c$, t is the shell thickness, R is the mean radius of the shell, L is the distance between the hard sections, I is the moment of inertia of the rings including the effective width of the shell, and l_c is the distance between the rings.

It is important to notice that the out-of-circularity is assumed as a sinusoidal lobe with n waves over the circumference, half wave over the length (between rigid sections) and an amplitude w_0.

Considering that the beginning of the yielding in the most loaded ring frame characterize the overall collapse, the corresponding pressure p_O is obtained provided that $\sigma_b = \sigma_y$ in equation (1) (σ_y is the yielding stress of the material) and the adequate substitutions in equations (2) and (3) be performed

(5) $p_O = (A / (w_0 + A))\, p_g$

and

(6) $A = \sigma_y R_c^2 / (E\, e\, (n^2 - 1))$

Using equation (5) it is possible to estimate the overall collapse pressure for different values of out-of circularity.

4 FINITE ELEMENT ANALYSES

The shell and the ring frames are modeled using quadrilateral shell finite elements with 4 nodes and 6 degrees of freedom (dof's) per node. A typical model is shown in Figure 1.

The material properties used in the finite element analyses were: 205000 MPa as the Young's modulus, 0,3 as the Poisson's ratio, and the stress-strain data of Table 1. Three geometries were investigated, with 2, 3, and 4 circumferential waves.

Table 1: Stress-strain data

Stress (MPa)	254,20	321,77	344,92	356,30	399,86
Strain (‰)	1,24	2,70	4,70	8,50	46,50

From data of Table 1, the yielding stress is 337,30 MPa.

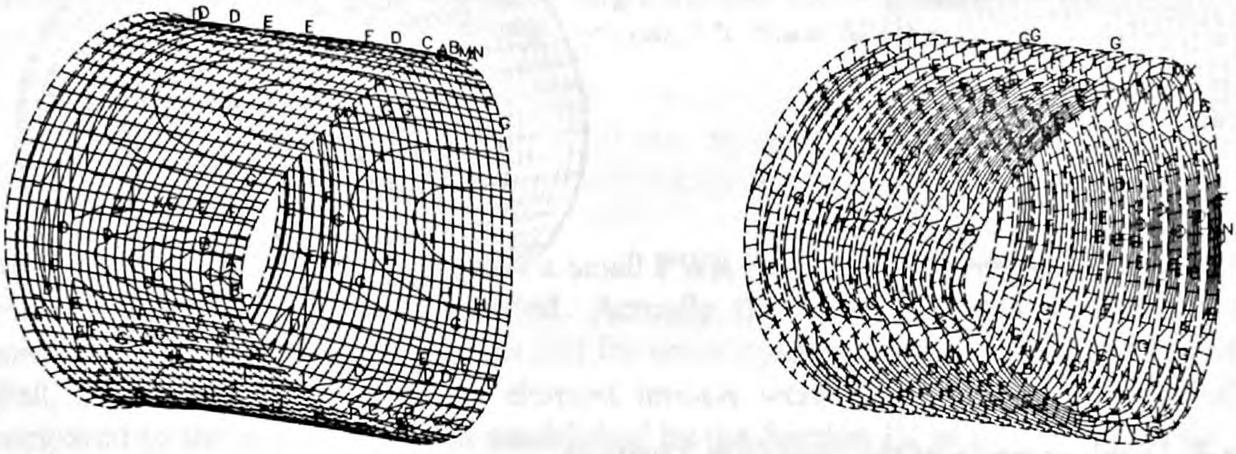
The analyses were performed with the finite element program ANSYS (SASI, 1992). For each geometry, two approaches were used: elastic instability and elastic-plastic collapse evaluations. Both were used to determine the collapse pressures and the failure modes.

5 RESULTS

The collapse pressures and the failure modes of the structure were obtained using the methodologies above described. The several configurations already mentioned, with the same amplitude of the out-of-circularity (0,3 % of the inner radius of the shell) and different number of circumferential waves, were analyzed. The results are shown in Table 2. The collapse pressures from finite elastic-plastic analyses correspond to failure of the shell in the central position. This can be seen in Figure 2 for n = 3 waves and the same occurs for n = 2 and 4 waves. If it is possible to maintain the applied hydrostatic pressure, the shell failure precipitates the overall collapse with the yielding of the rings. In Figure 2, it can be seen that the stresses in the central frames are near the yielding stress of the material.

Table 2: Collapse pressures (MPa)

n (number of waves)	From equation (5)	From elastic-plastic FEA	From elastic instability FEA
2	5,05	5,11	22,74
3	4,35	4,75	22,45
4	4,40	5,08	22,39



A=227 MPa to I = 357 MPa
(a) Shell

A = 114 MPa to I = 341 MPa
(b) Ring frames

Figure 2: Von Mises Stress Intensities (MPa)

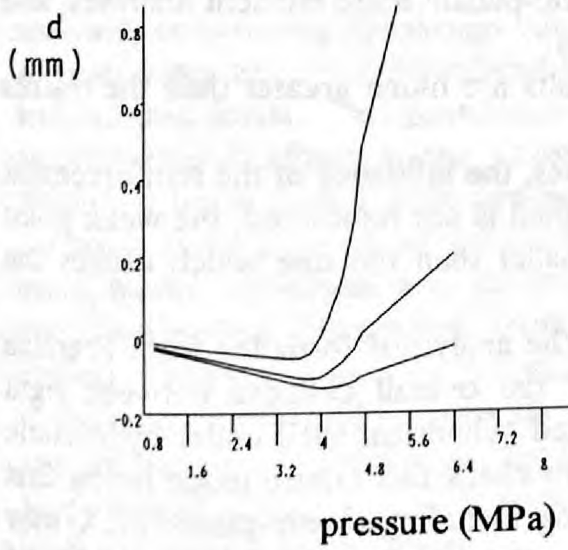
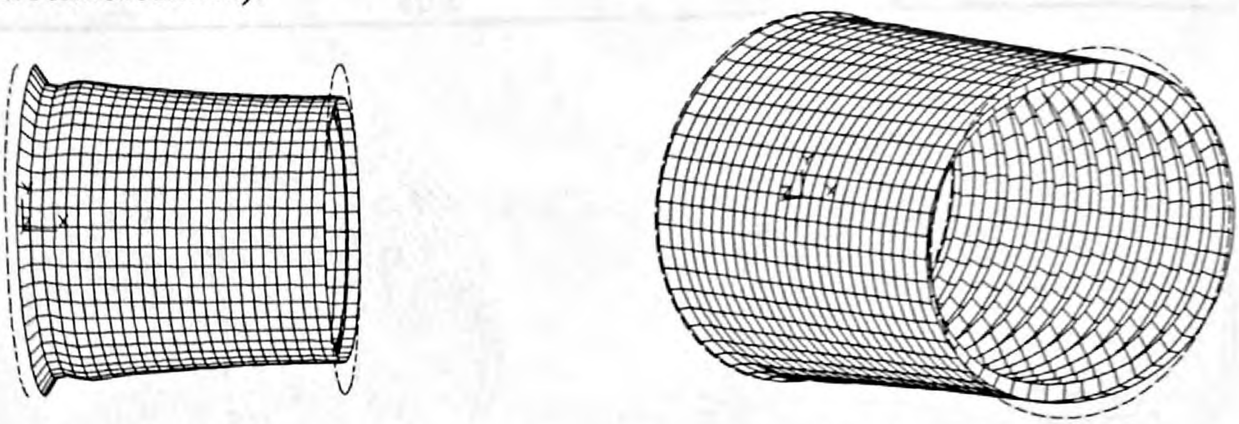


Figure 3: Radial displacement (mm) x pressure (MPa) of three nodes

The tendency of the shell structural behavior towards the overall collapse can be observed in Figures 2 and 3. Figure 3 shows the radial displacement versus pressure of three nodes of the shell (in the middle, in an intermediate position and near the flat head). The chosen nodes have the maximum out-of-circularity in the sections they belong to. From Figure 3, it can be seen that in the linear range the displacements decrease and, where the nonlinear behavior starts, the displacements tend to increase characterizing the large bending of the ring frames.

If the overall collapse pressure is characterized by the beginning of yielding in the most loaded ring, then one may say that the pressures corresponding to this failure mode are slightly greater than the collapse pressures from the elastic-plastic finite element collapse analyses.

It is important to mention that in the elastic-plastic finite element analyses it was necessary to reinforce the shell near the flat head (rigid sections on extremes) in order to force that the failure occurs in the center of the shell. Figure 4 shows the displacement field, just before the collapse, for the structure with n = 2 waves, without and with a reinforcement of 25 % of the thickness of the shell over the extended 2 and a half ring spaces from the discontinuity. In the first case the collapse occurs near the flat head at a pressure of 4,82 MPa and, in the other, in the central position of the shell at a pressure of 5,11 MPa. With reinforcement, the overall collapse of the shell is almost characterized in the elastic-plastic finite element analyses for the 3 cases studied in this paper (see Figure 4, with reinforcement).



(a) without reinforcement (b) with reinforcement

Figure 4: Displacements of the shell (n = 2 waves)

6 CONCLUSIONS

For the shell under study, Table 2 shows that the analytical formulae from Franitza (1988) results agree very well with those from elastic-plastic finite element analyses, and that the first is conservative with respect to the latter.

The elastic instability finite element analyses results are much greater than the results above cited, around 4 to 5 times.

From elastic-plastic finite element collapse analyses, the influence of the reinforcement near the rigid sections is well characterized. If the shell is not reinforced, the weak point is in the extremes and the collapse pressure is smaller than the one which causes the failure in the central position of the shell.

Based on the results obtained one can say that the analytical formulae from Franitza (1988) give a good estimate of the pressure for the overall collapse between rigid sections of an imperfect equally spaced ring-stiffened cylindrical shell under hydrostatic external pressure. Thus, the formulae can be used to check this failure mode in the first design phase of this type of structure. Further studies, including elastic-plastic FEA, may be necessary to conclude the design considering rings not equally spaced, the inclusion of reinforcement near discontinuities and a more precise definition of imperfections.

REFERENCES

Franitza, S. 1988. A contribution to an easy approximation of the elastic general instability pressure of ring-stiffened shells including deep frames. *WARSHIP 88 - International Symposium on Conventional Naval Submarines*, Vol II: paper 24, London: RINA.

SASI 1992. *ANSYS User's manual for revision 5.0*. Houston, PA, USA: SASI.

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Stress analysis of a research PWR pressure vessel: a general description

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ABSTRACT. The stress analyses of a small PWR research pressure vessel for prospective construction in Brazil are presented. Actually the stress analyses of the vessel were concluded for the design conditions and for some dynamic postulated loads. Axisymmetric shell, 3-D shell, and beam finite element models were used. The obtained results were compared to the allowable limits established by the Section III of the ASME code.

1 INTRODUCTION

This paper intends to present a general description of the stress analyses of a small PWR type research reactor that is being designed for prospective construction. The stress analyses, considering the design conditions, were already done for the vessel taking into account some impulsive postulated loads, internal pressure, dead weight, bolt-tightening, and seismic loads. The postulated loads are of impulsive type which produces non-axisymmetric loadings in the vessel. Such loads were modeled by Fourier series. A simplified beam model with concentrated masses, a 3-D model, and an axisymmetric harmonic finite element model were employed for the stress analyses. In the simplified beam model the vessel and its internals were taken into account and the postulated impulsive loading was applied. From the results of the analyses, an acceleration level to be applied in the vessel as a static loading was established. The reactions over the vessel due to the connected structures was also acknowledged. The axisymmetric model was used to analyze the vessel components far from the gross structural discontinuities, including its support skirt. The 3-D models were used to analyze the vessel head with its holes and a localized region in its lower part. The nozzles, in the design condition phase, were analyzed with simplified methods as that proposed by Mershon et al. (1987).

2 GENERAL DESCRIPTION AND REQUIREMENTS

Typically a nuclear reactor pressure vessel consists of a cylindrical body with a spherical bottom head, and a removable torispherical upper head which is flanged to the cylindrical body. Body and upper head are tightened by bolt sets. The cylindrical body of the vessel is provided with inlet and outlet nozzles. The upper head has appropriate openings designed for the control rod drive mechanisms and for the in-core instrumentation tubes. According