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A mathematical model for metastable condition determination in highly flashing liquid flows through expansion devices

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ABSTRACT

The determination of the metastability condition in fluid flows through singularities (expansion devices) when flashing occurs is the key to determine the mass flow rate going through devices when there is a great pressure difference between upstream and downstream. An application of the evaporation waves considered together with a maximizing condition for the pressure jump through the wave to determine the metastable state is presented. The model results are compared to several outflows reported in the literature indicating values within engineering standards for those flows in which fluids are highly superheated (highly expanded flashing liquid jets).

1. Introduction

The liquid fluid transport through piping systems containing valves as singularities and constraints is quite usual. In some of these cases and for certain fluid types in particular circumstances, thermodynamic conditions that may cause liquid–vapor phase changes can be achieved during the draining.

The most common processes of phase changes by reducing the pressure without heating are known as cavitation and flashing. In nuclear power plants, in typical refinery processes, and also in oil and liquefied gas industrial production and pumping, as well as in many other applications (such as refrigeration cycles), the arrangement shown schematically in Fig. 1 is usual and indicates some system examples in which the change from liquid to vapor phase and the shocked flow phenomenon are possible. They are: a control valve (Kiesbauer, 2001), a relief valve (Darby et al., 2001; Darby, 2004; Lenzing et al., 1998; Leung, 2004; Osakabe and Isono, 1996; Kim and No, 2001) or relief line (Attou et al., 2000), an orifice tube (capillary-tube expansion device Escanes et al., 1995) and an orifice or nozzle (Simões-Moreira et al., 2002). The mass flow rate determination when the differential pressure upstream and downstream of the restrictions are known is so important that, according to some researchers (Weisman and Tentner, 1978), the discharge flow rate largely determines the decompressor rate of the reactor system, the containment pressurization rate and the forces on the reactor vessel, structure and accompanying piping.

The fluid passage through the restriction may lead to phase change if the pressure in the constraint (P_G) is less than or equal to the liquid saturation pressure at the flow temperature (P_σ). Comparing the pressure at the singularity exit (P_3) to the saturation pressure, it is possible to say whether the phase change will occur as cavitation, for the cases in which $P_3 > P_\sigma$, or as a flashing, in situations which $P_3 \leqslant P_\sigma$. In the cavitation process, vapor bubbles implode when there is no flow blocking and they face pressures above the saturation pressure shortly after their formation. In fast evaporation, a series of more complex effects, mainly related to the thermodynamic behavior of the substance, makes the analysis much more difficult. Especially for the cases in which there are two-phase choked flows, the physical phenomena are not completely revealed (Kim and No, 2001), making the accurate determination of the mass flow rate through the singularities (be they relief valves, control, or sudden enlargements in pipes or orifices) still a challenge when flashing is present.

Several reports (Lenzing et al., 1998; Bolle et al., 1996) highlighted the large deviations of the mathematical models results in certain circumstances involving flashing flows. This article aims to establish a mathematical model to determine the metastability condition and, therefore, the mass flow rate through a throttling device with initially sub-cooled liquid. Based on the evaporation wave theory (Simões-Moreira and Shepherd, 1999), together with an operating condition, the choked flow condition will be determined. The evaporation wave implementation in such cases is not new and was originally suggested by Simões-Moreira and Bullard (2003). However, the numerical investigation proposed by these researchers did not include the possibility of metastability

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Nomenclature

A	nozzle exit section area
C_D	discharge coefficient
h	specific enthalpy
J	superficial mass flux (mass flux per unit of area)
L	length of metastable liquid core
$\dot{m}$	mass flow rate
P	pressure
R_b	nozzle exit section radius
r	cylindrical radial coordinate
T	temperature
V_j	jet velocity
v	specific volume
W	evaporation wave normal component velocity
x	vapor mass quality
z	cylindrical axial coordinate
ρ	density
Π	metastability degree

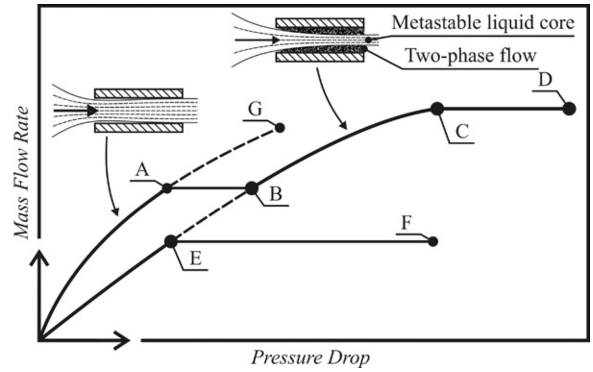

Fig. 2. Flow characteristics of subcooled and saturated liquid through a short tube (Pasqua, 1953).

determination, which directly influences the mass flow rate estimative.

The model is based on the idea that equivalent short nozzles can approximate the behavior of more complex devices (Attou et al., 2000) featuring the main characteristics of the phase shift dynamics and of the flow patterns. In these cases, the flow pattern depends on the metastability degree the liquid can reach by crossing the restriction and on the existence or absence of heterogeneous nucleation on the surface of the orifice or injection nozzle.

The proposed model is also able to determine conditions associated with some types of nuclear power plant accidents related to the most critical events, such as LOCA (Loss of Cooling Accidents), which is generated by the rupture of pressurized water lines (Edwards and O'Brien, 1970; Forrest and Stern, 1993). The model uses mainly the metastable liquid core that is formed in the evaporative jet flow to compare numerical and experimental results and to validate the theory. Considering the thermodynamic conditions reached by the substances along the flow and the apparent independence of the liquid metastability condition, the results are extrapolated to other flows on restrictions.

2. Metastability in the expansion devices

The superheated or metastable state corresponds to the case in which the liquid reaches its boiling or saturation point without the occurrence of a phase transition (Angelo and Simões-Moreira, 2007).

Bailey (1951) studied the water flow close to saturation or saturated through orifices, nozzles and short tubes of reduced diameter, focusing on determining the mass flow rate through these devices as a function of difference between the water downstream and upstream pressures. He discussed some basic concepts of the behavior of such flows and cited the occurrence of liquid overheating and choked flow. According to Bailey (1951), the traditional formulation (which considers the beginning of phase changing when these flows reach the saturation pressure corresponding to their temperature) has showed a significant gap towards the experimental data. In this situation, the values of discharge coefficient oscillate between 2 and 7.5. One explanation is the delayed bubbles formation, evidencing the metastability condition of such flows. Pasqua (1953) got the same results using the R12 coolant fluid in short tubes. There are indications (Pasqua, 1953) that the metastable state cannot be measured due to practical difficulties; however, the mass flow rate can easily be determined revealing that there is a metastable state in the flow.

The flow patterns obtained by the cited researchers (Bailey, 1951; Pasqua, 1953) and also by Kim (1993) are represented in Fig. 2 in orifice tubes (capillary expansion device) of air conditioning systems. The liquid inlet orifice as a sub-cooled liquid line corresponds to OABCD line and the saturated liquid to the OEF line. The abscissa axis refers to the difference between inlet and outlet orifice tube pressures, while the ordinate corresponds to the mass flow rate.

The OA segment characterizes the tube completely filled with fluid, AB is the transition of the flow that completely fills the tube to the equivalent flow through an orifice (which has the same diameter of the capillary-tube), BC is the flow that would occur through an

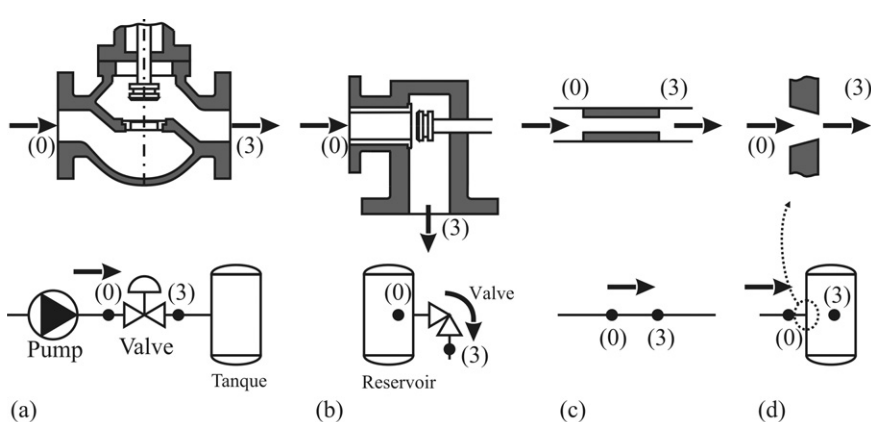

Fig. 1. Expansion devices: (a) control valve, (b) relief valve, (c) capillary-tube and (d)nozzle.

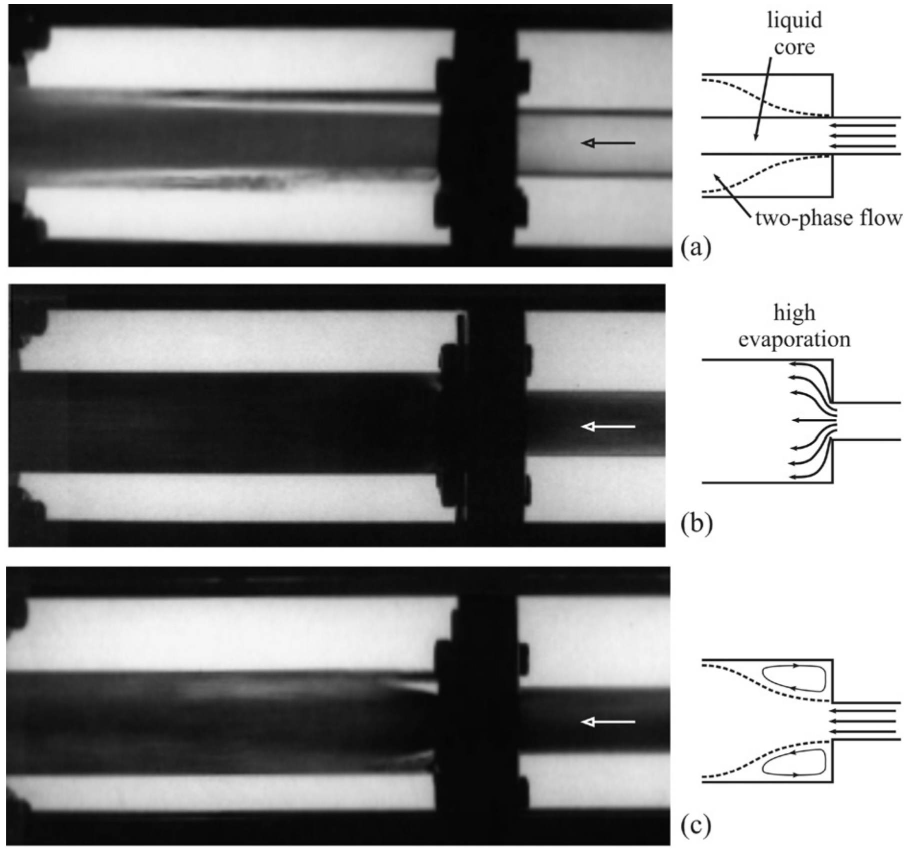

Fig. 3. Flow characteristics of relief line (Attou et al., 2000). (a) Confined quasi free jet pattern,(b) Expanded jet pattern and (c) Submerged jet pattern.

orifice and CD is a level indicating a constant mass flow rate. Point A is a function of the initial temperature and pressure. At point B, the fluid is completely separated from the tube wall and the discharge pressure corresponds to the saturation pressure at the initial temperature (input) of the fluid. Between points A and B, the mass flow alternates between the AG and EB statistical mean curves.

Between points B and C, the phase change occurs only on the surface of the liquid core, which is metastable, and the annular space between the tube and metastable core is filled with steam. The lower the pressure in the outlet orifice tube, the greater the metastability degree achieved up to point C, where the choked flow occurs. The choked flow is characterized by a non-increase in the mass flow rate, despite the decrease in the downstream pressure of the device. The concept of metastable liquid flow is not new and was introduced by Silver and Mitchell (1951) who considered that the flow through a nozzle could be pictured as a metastable liquid core surrounded by a vapor annulus. They asserted that the phase change occurs only on the surface of this core. It is also known (Pasqua, 1953) that the liquid core may be disturbed by vibrations or any external influence leading to phase change.

The abrupt increase of pipe cross-sectional areas was studied by some researchers (Attou et al., 2000). Water flows were studied under three experimental conditions at the expansion upstream: highly subcooled liquid, liquid near saturation, and biphasic mixture (with short mass volume fraction). Three distinct patterns were obtained and can be seen in Fig. 3.

For the case of highly subcooled fluid in the section upstream to the expansion, the onset of the rapid evaporation occurs soon after the fluid leaves the expansion. A liquid core is formed, surrounded by steam. According to the authors (Attou et al., 2000), the observed pattern is not influenced by the downstream pressure and measurements indicate that the pressure at the base of the expansion (pressure in the separation region) remains equal to the average pressure.

In the case of liquid near the saturation region, at the singularity entrance, the authors of this study pointed out that the central flow current lines in the vicinity of the enlargement strongly diverge from the duct axis. The pressure at the downstream, for these conditions, has great influence on the pattern of the expanded jet. For the case of two-phase flow downstream, the abrupt increase in area, there is the formation of a recirculation zone similar to that observed in fluid flows without phase change.

Other authors (Fraser and Abdelmessih, 2002a,b) studied the effect of the beginning of two-phase flow in pipes, as a water line, maintained at high pressure and temperature when they were exposed to atmospheric pressure by a rupture, for example. From the rupture point, due to the exposure to a pressure below the saturation pressure at the temperature flow, the fluid becomes metastable and begins the phase change. The breakpoint location in the pipeline, therefore, determines the onset of two-phase flow inside the tube. In their latter work, the authors put a thin ring (where rapid evaporation starts) inside the pipe and found the relationship between the critical mass flow rate (shocked flow) and the ring position in the tube (simulating the rupture point).

3. Evaporation waves in highly flashing liquid jets

The phase change of a substance can happen through various distinct processes (Simões-Moreira, 1994). When it comes to evaporation, or change from liquid to vapor, it can occur through: explosive evaporation, evaporation waves, and surface evaporation. The evaporation mode depends on the type of substance, its overheating degree, the medium where it is located, and how the evaporation process begins.

If the substance has a thermodynamic state close to the mechanical stability limit, given by the maximum overheating degree allowed, the phase change can happen through an abrupt and sudden change of properties, culminating, in some situations, in explosions. As we decrease the overheating degree, other phenomena appear as evaporation waves and surface evaporation.

In cases in which overheating is very slight or nonexistent, simple evaporation is observed. It is characterized by low or no pressure change, case widely discussed in textbooks since it is a classical problem of thermodynamics equilibrium. The phenomenon of interest will occur when the liquid reaches a certain degree of overheating during the flow.

Rapid evaporation is a milder and more controlled phenomenon as compared to the explosive evaporation and may occur through evaporation waves or surface evaporation. Evaporation waves are phase-change phenomena characterized by a sudden evaporation of the superheated liquid within a discrete and observable interface. They will be used in this paper to model the phase change.

As in other papers (Reitz, 1990; Kurschat et al., 1992; Athans, 1995; Vieira, 2005), the existence of a metastable liquid core inside the singularities is assumed in the proposed model, a fact supported by experimental evidence obtained by high shutter speed photographic techniques, especially in highly expanded flashing jets. Only high shutter speed photographic techniques have been able to verify an inner intact liquid metastable core downstream the nozzle exit section. The evaporation wave remains stationary in the metastable liquid core. The highly expanded flashing jet underlies and exemplifies the proposed model application, especially for the quantity and quality of several experimental studies indicated in the scientific literature.

The pictorial diagram of Fig. 4a summarizes the significant details of the flashing process studied in this paper. As revealed in previous studies (Weisman and Tentner, 1978; Angelo and Simões-Moreira, 2007), the evaporative jet analysis shows that flashing takes place on the surface of the liquid core through a normal evaporation wave process ($1 \rightarrow 2$), which usually produces a sonic two-phase flow, but can also be subsonic in less severe conditions (higher backpressure). This sonic state (2) is also a point of maximum mass flow rate given by the Chapman–Jouguet condition. Next, the freshly sonic two-phase flow expands to a high supersonic flow ($2 \rightarrow 2a$) to eventually terminate the supersonic expansion process throughout a shock wave structure ($2a \rightarrow 2b$) that subsonically compresses the fluid up to the pressure chamber ($2b \rightarrow 3$) in the far field.

The proposed mathematical model predicts the occurrence of a sequence of cylindrical and stationary evaporation waves on the surface of the metastable liquid core shortly after leaving the injection nozzle (see Fig. 4b–d). The radial position of the evaporation waves decreases until the complete metastable liquid evaporation.

This model is significantly different from that presented by Angelo and Simões-Moreira (2007) which only considers an oblique evaporation wave of conical shape involving the entire liquid core. In the Angelo and Simões-Moreira (2007) model, the liquid core has a homogeneous thermodynamic condition, that is, the entire liquid core experiences the same pressure and temperature. Angelo and Simões-Moreira (2007) did not intend to determine the mass flow rate once a differential pressure condition upstream and downstream of the injection nozzle was given. In fact, they aimed to the dynamics of the two-phase mixture expansion inside the low pressure chamber, i.e., post-evaporation wave, also capturing subsequent effects as shock wave formation

According to the evaporation wave theory, the mass, momentum and energy conservation equations must be solved to calculate the evaporation properties jump across the metastable liquid two-phase interface. Assuming uniform thermodynamic properties within a phase domain, the downstream state (2) is at

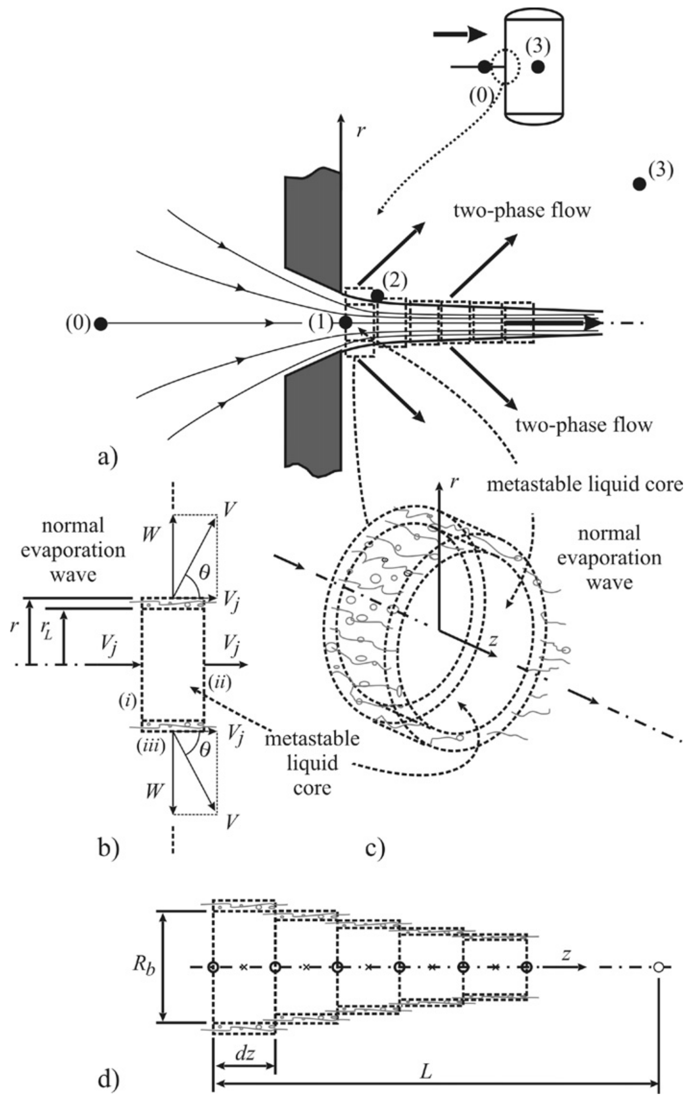

Fig. 4. Nozzle. (a) Schematic of the jet stressing the main thermodynamic states, (b) Control volume detail, (c) Perspective control volume detail and (d) Length of metastable liquid core.

thermodynamic equilibrium (thermal and mechanical) and the process is at steady state; then the 1-D mass, momentum and energy conservation equations are respectively (Simões-Moreira et al., 2002; Simões-Moreira and Shepherd, 1999; Simões-Moreira and Bullard, 2003; Angelo and Simões-Moreira, 2007; Simões-Moreira, 1994; Kurschat et al., 1992; Vieira, 2005) (for one evaporation wave only):

$$[J] = 0 \tag{1}$$

$$[P + WJ] = 0 \tag{2}$$

$$\left[h + \frac{W^2}{2}\right] = 0 \tag{3}$$

The square brackets indicate a jump in the enclosed value, that is, $[f] = f_2 - f_1$. For a given metastable liquid upstream state (1), there is a corresponding singular solution for low downstream pressure values (2). This solution is the Chapman–Jouguet (C–J) condition (Simões-Moreira et al., 2002; Simões-Moreira and Shepherd, 1999; Simões-Moreira and Bullard, 2003; Angelo and Simões-Moreira, 2007; Simões-Moreira, 1994; Vieira, 2005). In the C–J solution, the two-phase normal velocity (W_2) is sonic.

The Simões-Moreira research (Simões-Moreira, 1994) indicates details of the evaporation wave calculation. Summarizing the procedure, the state downstream to the evaporation wave is obtained from the solution of the conservation equations presented, i.e., the solution of Eqs. (1)–(3). Traditionally, these three equations are combined to get only two other equations, known in the literature as Rayleigh (R) and Rankine–Hugoniot (R–H) equations.

These equations involve only thermodynamic properties, allowing a graphical analysis in the pressure versus specific volume diagram. This analysis favors the understanding of thermodynamic processes, promoting the interpretation of the physical phenomena and of the relationship between the upstream and downstream properties of the evaporation wave. The Rayleigh and Rankine–Hugoniot equations are respectively:

$$J^2 = -\frac{[P]}{[\nu]} \tag{4}$$

$$[h] = [P]\frac{\nu_1 + \nu_2}{2} \tag{5}$$

In mathematical terms, the condition of maximum mass flow rate (Simões-Moreira, 1994) (C–J condition) can be expressed as $dJ^2 = 0$, which applied to Eq. (5) leads to:

$$\frac{d\nu_2}{dT_2} = -\frac{1}{J^2_{C-J}}\frac{dP_2}{dT_2} \tag{6}$$

For a set of normal evaporation waves, the implementation of the conservation equations, assuming a quasi one-dimensional model and considering the metastable liquid velocity constant and equal to V_j and also that there is no pressure gradient inside the liquid core, the mass and energy conservation equations applied to the control volumes shown in Fig. 4d are, respectively:

$$(\rho_L V_j \pi r^2)_i - (\rho_L V_j \pi r_L^2)_{ii} - [\rho_b V_j \pi(r^2 - r_L^2)]_{ii} - (\rho_b W 2\pi r\, dz)_{iii} = 0 \tag{7}$$

$$\left(\left(h_L + \frac{V_j^2}{2}\right)\pi r^2\right)_i - \left(\left(h_L + \frac{V_j^2}{2}\right)\pi r_L^2\right)_{ii} - \left(\left(h_b + \frac{V_j^2}{2}\right)\pi(r^2 - r_L^2)\right)_{ii} - \left(\left(h_b + \frac{W^2}{2}\right)2\pi r\, dz\right)_{iii} = 0 \tag{8}$$

The *i*, *ii*, and *iii* index show the control volume faces. Index *b* indicates the two-phase flow and r and r_L dimensions are outlined in Fig. 4b. The momentum conservation equation is necessary only if the hypothesis of zero pressure gradient within the liquid core is removed.

4. Metastable thermodynamic condition and mass flux determination

Experimental (Attou et al., 2000; Simões-Moreira et al., 2002; Simões-Moreira and Bullard, 2003; Kurschat et al., 1992; Athans, 1995; Vieira, 2005; Oza Rajshekhar and Sinnamon James, 1983) and numerical (Simões-Moreira and Bullard, 2003; Angelo and Simões-Moreira, 2007) works did not show a clear methodology to determine the metastable state reached during the flow through the singularity. Even using the evaporation wave theory, as described above, it is not possible to obtain the metastability degree achieved in the liquid flow (the post-CJ evaporation wave condition is only found if a known metastable state is admitted).

The metastability degree experienced during draining between the saturation condition and the mechanical stability limit is expected to preferably follow a physical condition, a natural condition.

There are infinite solutions which satisfy the inequality $P_\sigma < P_1 < P_{lim}$ (where P_1 is the metastable pressure and the superheated (Reid, 1976) or metastable-limit is found by using the $(\partial P \partial \nu)_T = 0$ criterion). For each one of these conditions there is also a C–J post-evaporation wave condition.

Extensive tests indicated that among the infinite possible metastable conditions, one maximizes the pressure decrease through the evaporation wave; thus, the pressure difference upstream and downstream the evaporation wave $\Delta P = P_1 - P_2$ is maximized. Fig. 5 schematically shows some possible solutions to the evaporation wave and the condition of maximizing the pressure jump through the wave (condition in which the metastable pressure is indicated by $P_{1Solution}$). The metastability degree is determined by $\Pi = (P_1 - P_\sigma)/P_\sigma$ and is indicated next to the pressure axis in Fig. 5. In all tested cases, it was always possible to find the condition for maximizing the pressure jump for the evaporation wave C–J solutions type family.

The mass flow rate through the singularity, in this case a convergent short nozzle (Simões-Moreira et al., 2002; Angelo and Simões-Moreira, 2007), can be obtained by the following expression:

$$\dot{m} = 2A^2 C_D^2 \frac{P_0 - P_1}{\nu_1} \tag{9}$$

The previous equation is the modified energy conservation law applied to a streamline from the reservoir stagnant condition (0) to the nozzle section exit (1). The temperature of state (1) was approximated by the injection temperature value assuming an isothermal process between state (0) and state (1), thus $T_1 \approx T_0$.

The sequence for determining the mass flow rate through the singularity is shown schematically in the simplified block diagram of Fig. 6.

Applying this methodology to other singularities, such as valves, depends on a minor adjustment. In such cases, the nozzle area should be replaced by the smaller passage area, located between the seat and the valve stem, so that the flow has the same or a similar behavior to a convergent divergent nozzle type.

In the capillary tube case, it is possible to estimate the position of the evaporation wave through the relationship with the load loss in the tube.

The mass flow rate determination through the evaporation wave technique can only be used in cases of high pressure gradients. If this condition is lacking, the occurrence of other phase change effects significantly deviates the mathematical solution from the actual results.

The proposed mathematical model is essentially dependent on thermodynamic conditions: the injection conditions are enough to determine the metastable and the choked conditions. The solution is a set of two conditions, the maximum pressure jump through the evaporation wave and the sonic solution of C–J post discontinuity. A comparison of the proposed model to others in the literature indicates that several researchers use parameters strongly dependent geometry (Abuaf et al., 1983; Chen et al., 2009; Nilpueng and Wongwises, 2010) and adjusted, for these cases, by empirical constants which clearly do not diminish their validity; however, they remain absolutely private solutions. It is possible to cite an approach involving electronic expansion valves application using single-phase incompressible flow coefficient and an empirical correlation for the metastable pressure determination (Chen et al., 2009). In other cases, elaborate and complex models was employed, per example, a hyperbolic two-phase flow involving five

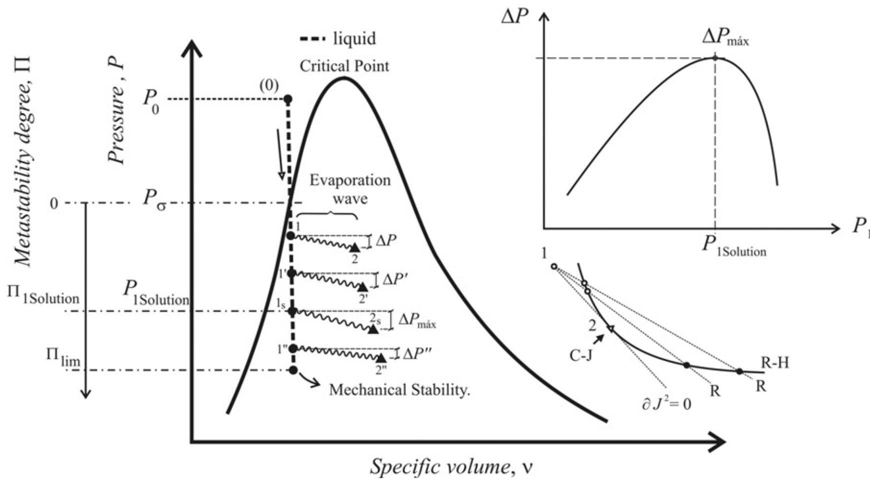

Fig. 5. Evaporation wave thermodynamic states and maximum difference pressure condition.

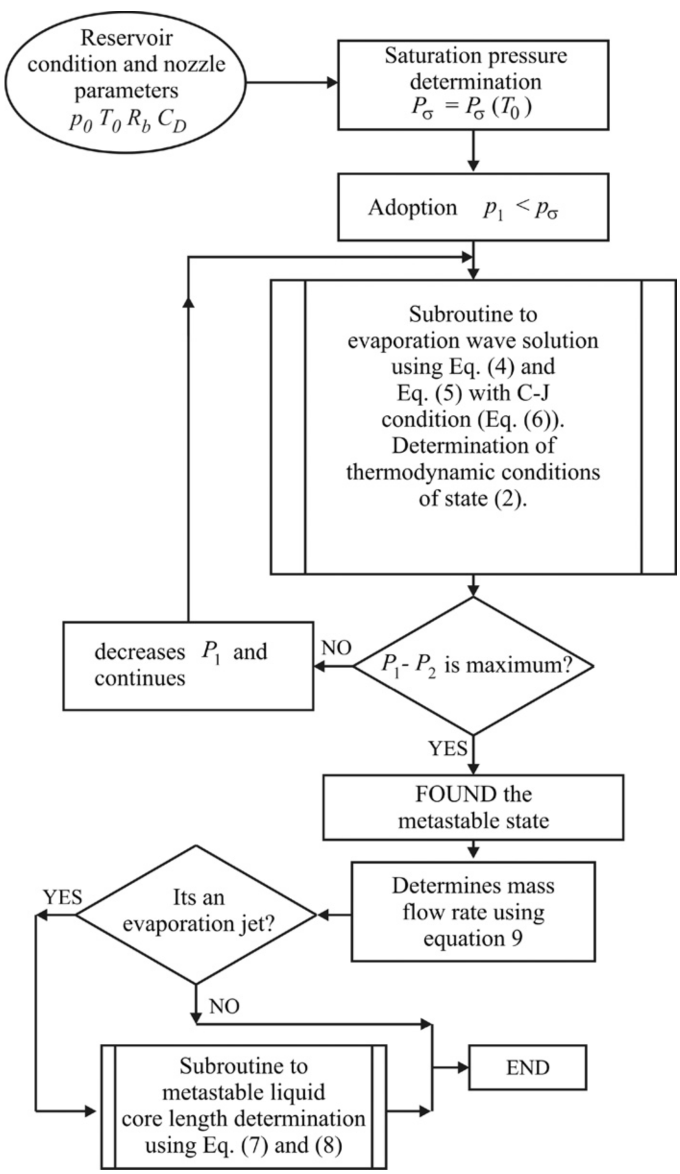

Fig. 6. Metastable condition determination road-map.

differential equation constructed for liquid–gas interface (Saurel et al., 2008).

All the thermodynamic properties of the substances were calculated by Lee–Kesler (Lee and Kesler, 1975) conventional equation of state, with the exception of water, calculated by using a methodology based on modified Lee–Kesler equation (Xiang and Deiters, 2008).

5. Results of numerical solution

To demonstrate the maximum differential pressure condition across the evaporation wave, three (Vieira, 2005) experimental conditions for a convergent nozzle were used. Fig. 7 shows three curves of differential pressure results for three different injection temperatures (and practically equal injection pressures) as a function of the metastable pressure. The complete experimental conditions of the studied cases and the model results are in Table 1. In all curves, as well as for all the simulated cases, there is a clear condition in which the pressure decrease through the evaporation wave reaches a maximum value.

Keeping the injection pressure constant and varying only the stagnation temperature, it is possible to see that the points corresponding to the maximum pressure drop through the evaporation wave for several cases are aligned following a clear trend, Fig. 8.

The experimental cases reported in Table 1 were obtained for tests with convergent nozzles in experiments using C_8H_{18} (Athans, 1995; Vieira, 2005) and C_6F_{14} (Kurschat et al., 1992) as test substances. The values of the experimental mass flow rates and those

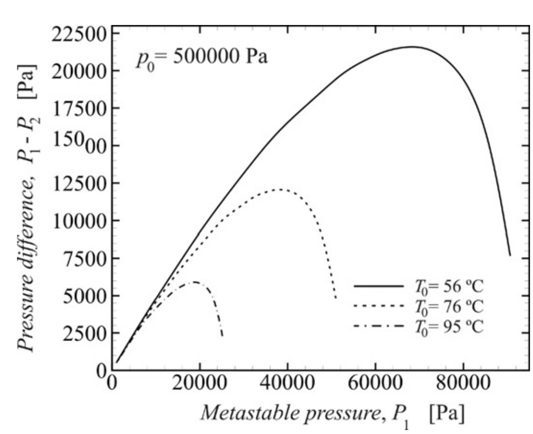

Fig. 7. Pressure difference versus metastable pressure (series 7, 8 and 9 – Table 1).

Table 1
Numerical results.

Series	Stagnation cond.		Experimental data						Metastable state		C–J condition					Model			
	P_0 kPa	T_0 °C	P_0 kPa	T_0 °C	P^- kPa	m g/s	L mm	C_D [1]	P_1 kPa	v_1 m^3/kg	P_2 kPa	T_2 °C	x_2 %	v_2 m^3/kg	W_2 m/s	m g/s	Error %	L mm	P_1 kPa
1		56	122.1	56.4	[d]	0.84469	6.9	0.9124	6.175	0.001511	3.412	11.847	28.467	1.8466	71.458	0.796	5.8	9.16	19.529
2	125[a]	76	122.7	75.2	[d]	0.83396	5.2	0.9165	7.743	0.001551	4.1559	15.274	38.898	2.0949	86.728	0.728	12.7	5.43	35.362
3		95	123.5	94.8	[d]	0.80225	3.1	0.9195	14.713	0.001597	7.3981	27.479	45.039	1.4134	101.737	0.576	28.2	2.45	67.423
4		56	250.2	56.5	[d]	1.217	7.2	0.9216	14.388	0.001511	9.1018	31.806	16.473	0.4316	47.854	1.206	0.9	13.70	19.723
5	250[a]	76	251.5	76.7	[d]	1.218	6	0.94	17.879	0.001551	10.4017	34.533	28.5	0.6523	69.925	1.167	4.2	8.18	37.642
6		95	251.3	95.5	[d]	1.071	4.7	0.926	59.673	0.0016	40.774	70	19.012	0.1241	48.755	1.046	2.3	4.32	69.256
7		56	501.6	56.6	[d]	1.746	11	0.9288	23.23	0.001511	19.0755	49.348	5.081	0.0667	16.846	1.758	0.7	19.97	19.796
8	500[a]	76	496.3	76.7	[d]	1.695	7.5	0.9319	35.6939	0.001551	20.0027	50.417	18.271	0.2272	59.921	1.696	0.1	11.91	37.642
9		95	503.4	95.4	[d]	1.635	6.3	0.9339	64.97	0.0016	42.9521	71.437	17.88	0.1114	49.889	1.629	0.4	6.69	69.507
10		56	751.1	56.4	[d]	2.1605	14.1	0.933	25.834	0.001511	20.38	50.747	3.962	0.0498	16.73	2.177	0.8	24.68	19.769
11	750[a]	76	752.5	77	[d]	2.091	10.2	0.926	42.674	0.00156	34.1349	62.322	10.699	0.089	27.815	2.103	0.6	14.52	38.264
12		95	752.3	96.4	[d]	2.029	7.3	0.9374	80.973	0.0016	60.1258	81.696	11.332	0.0527	33.681	2.047	0.9	8.13	72.004
13a			2397	204	5.73	21.4	9[e]	$0.9 \le C_D \le 0.76$	450.46	0.00201	263.2947	136.812	67.143	0.0741	119.488	18.459	13.7	4.04	731.079
13b																22.100	3.3	4.81	
14a	2400[c]	200	2338	200	8.25	21.5	9[e]	$0.9 \le C_D \le 0.76$	435.87	0.00198	256.9434	135.672	63.745	0.0721	115.251	18.550	13.7	4.41	672.143
14b																22.210	3.3	5.82	
15a			2293	200	9.87	21.4	9[e]	$0.91 \le C_D \le 0.76$	430.81	0.00198	250.7164	134.584	64.488	0.0746	117.604	18.290	14.5	4.35	672.143
15b																21.900	2.3	5.20	
16	1500[b]	20	1500	20	1	6.85	12[e]	0.95[e]	10.9	0.000533	6.687	−3.8	15.854	0.1555	25.656	7.23	5.5	20.3	17.746

[a] Vieira (2005) – with iso-octane (C_8H_{18}) and nozzle exit radius of 0.154 mm.
[b] Kurschat et al. (1992) – with C_6F_{14} and nozzle exit radius of 0.18 mm.
[c] Athans (1995) – with iso-octane (C_8H_{18}) and nozzle exit radius of 0.432 mm.
[d] Near absolute vacuum.
[e] Estimated.

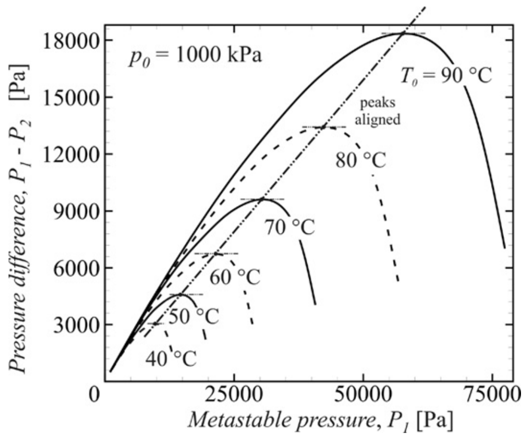

Fig. 8. Pressure difference versus metastable pressure.

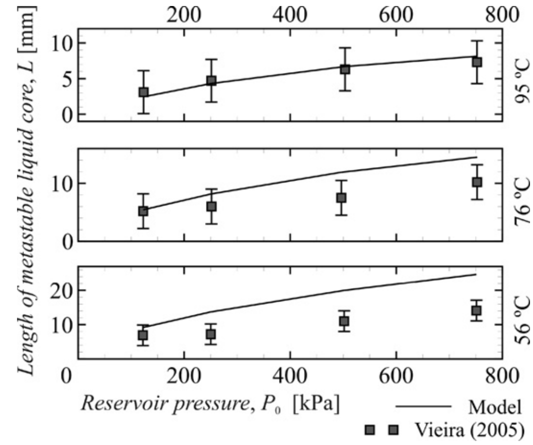

Fig. 10. Comparison of deviations in the determination of metastable liquid core length experimental data of Vieira (2005) and the model.

obtained from mathematical processing are confronted and a deviation is calculated. It is worth remarking that the experimental determination has an uncertainty degree of ±2% for Vieira cases (Vieira, 2005), while it is not reported by Athans (1995) or by Kurschat et al. (1992).

The best model results are those that present situations in which there is a greater pressure jump through the evaporation wave, especially in circumstances related to higher temperature and stagnation pressure conditions.

Fig. 9 graphically demonstrates the error values for the conditions tested of Vieira's cases (Vieira, 2005). Vieira's results consistently indicate, for the series of injection pressures of 250 kPa, 500 kPa and 750 kPa, a clear trend of mass flow rate decreasing with the injection temperature increasing, a trend that was not found in the range of 125 kPa. It is necessary to check the values reported for the 125 kPa sequence through other tests to certify the results of this experimental sequence, since other researchers, such as Kurschat et al. (1992), have already reported the behavior latter confirmed by Vieira (2005) in the 250 kPa, 500 kPa and 750 kPa series, not citing any behavior that is anomalous or out of the standard defined for other pressures.

For the cases reported by Vieira (2005), the deviations between the values of the liquid core length determined by the mathematical model and by the experiments were also compared, Fig. 10.

The experimental values of the liquid core length were measured by Vieira (2005) directly on the photographies, in an experimental procedure that may contain a significant error mainly caused by the photographic technique used and the difficulty in determining the exact limits of the liquid core. In such flows, there is a large number of droplets surrounding the liquid core which may preclude an accurate determination of the correct dimensions.

Results show major consistency for responses of higher injection temperature cases. Apparently, in these cases, there is a greater amount of stored energy (internal energy) which is used to the phase change (properties jump across the evaporation wave). The liquid superheating degree increases with the inlet temperature increase and decreases with the mass flow rate increase (exactly as reported in experimental studies of Vieira, 2005 and Huerta et al., 2007).

Fig. 11 presents the model results for the Athans cases (Athans, 1995). Athans's experiments (differ by a small variation of reservoir pressure) were also conducted with iso-octane, the same substance chosen by Vieira (2005). However, the experimental conditions and the diameter of the injection nozzle were quite different, indicating that the proposed model also gave satisfactory results in these cases. The sharp evaporation seems to favor the application of the C–J condition.

Some tests were performed with the experimental conditions of Attou et al. (2000) for the pure water substance. Its experimental

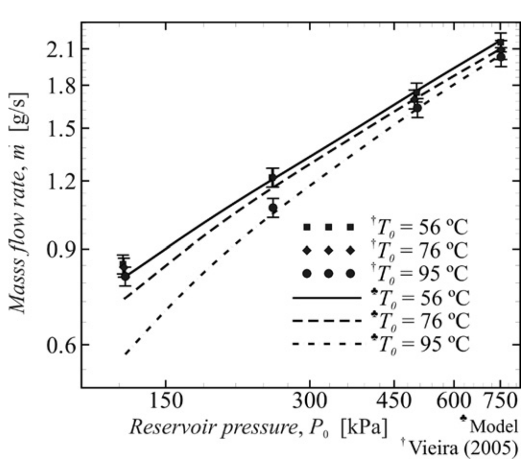

Fig. 9. Comparison of deviations in the determination of mass flow of the experimental data of Vieira (2005) and the model.

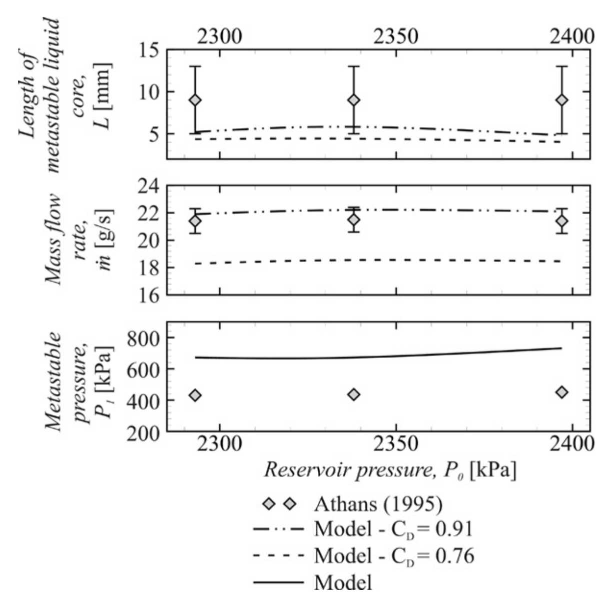

Fig. 11. Comparison of deviations – experimental data of Athans (1995) and the model.

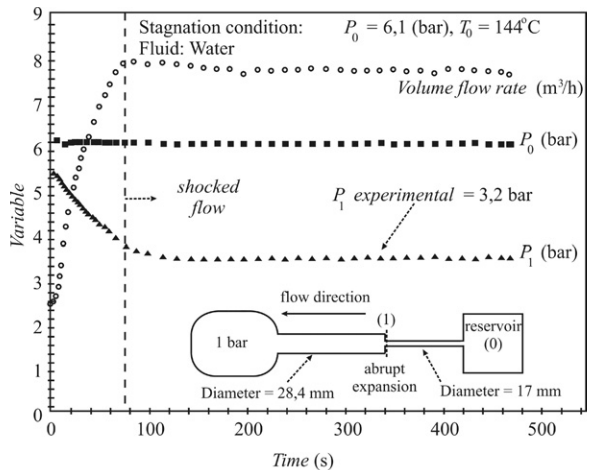

Fig. 12. Schematic diagram of the experiment and some results of Attou et al. (2000).

scheme can be seen in Fig. 12, together with a tested situation and the collected data.

The experimental setup had pressure gauges placed at strategic places, useful for comparing the results with the proposed mathematical model. The interesting situation was that one in which, a fluid undergoes abrupt expansion. The discharge coefficient of expansion is not reported in the study; the experimental situation was thus simulated and the result was compared with the metastable pressure. The model response for the metastable pressure was 3.14 bar, i.e. a difference of 1.9% as compared with the measured value of 3.2 bar.

Another simulated case for the Attou et al. (2000) conditions used the same flow geometry (pipes and fittings) schematically shown in Fig. 12. The test conditions were set as follows: stagnation pressure of 6.2 bar, stagnation temperature of 120°C, and experimental metastable temperature of 1.8 bar. The model response for the metastable pressure was 1.48 bar, i.e. a deviation of 17.8%.

A set of simulations indicated the injection pressure plays a less important role in the metastable pressure value than the injection temperature. This is justified by the small difference between the thermodynamic properties of liquids, even under distinct conditions.

The presented results confirms the interpretation of evaporation fronts as expansion stationary waves of the equilibrium system (Saurel et al., 2008), and suggests that the condition of maximizing the pressure jump across evaporation wave is a plausible hypothesis for the mathematical model closure.

6. Limitations and observations about the model and the phenomenon

The model presented for metastable condition determination on the highly expanded flashing liquid flows through expansion devices, has limitations due to assumptions and approximations in the mathematical representation of the physical problem. The main limitations are: (a) The metastable liquid core is an experimental evidence; however, the liquid evaporation ratio is unknown and was assumed to be constant; (b) The boundary layer is not considered in the flow inner nozzle (expansion device), as its effect may produce instabilities in the nozzle exit flow (Wildgen and Straub, 1989); (c) internal nucleation and bubble growth inside the expansion device are possible; (d) The mechanism of the evaporation front instability still remains to be completely clarified; (e) The constant properties inside the metastable liquid core is a model assumption without experimental evidence and, (f) The homogeneous two-phase equilibrium model downstream evaporation wave could be an inaccurate model, in the cases of moderate evaporation beyond evaporation wave.

7. Conclusion

Under certain circumstances, liquid flows through singularities, such as nozzles, valves and restrictions may cause phase change due to pressure drop. Among the phase change processes, there is one known as flashing that can lead the flow, in extreme situations, to a condition known as choked flow in which the mass flow rate is no longer affected by a pressure decrease in sections downstream the singularity.

It is also widely reported in the literature that during the flow, a liquid may reach a thermodynamic condition in which phase change should occur; however, the fluid remains in a liquid metastable condition. Nevertheless, several theoretical, numerical, and experimental studies failed to show a reliable correlation to determine the metastability degree reached by a liquid during the flow through a singularity.

This paper presented a mathematical model, resulting from the evaporation wave theory and a condition for maximizing the pressure jump across the discontinuity, to determine the metastability degree of flows in expansions.

The numerical simulations considered several different experimental conditions reported in the scientific literature by many researchers with completely different fluids such as iso-octane (C_8H_{18}), water (H_2O) or perfluorhexane (C_6F_{14}). When some parameters, such as liquid core length and mass flow rate through the devices, were compared to the experimental data, consistent results were obtained within acceptable error levels for engineering.

Limitations of the model were observed for flow cases in which the evaporation waves have very low intensity, related to conditions of low temperature and stagnation pressure.

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