EFFECT OF DIFFERENCES IN GAS-DYNAMIC BEHAVIOR ON THE SEPARATION PERFORMANCE OF ULTRACENTRIFUGES CONNECTED IN PARALLEL.

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Contains:

11 Pages,
4 Tables,
4 Figures.

ABSTRACT

This paper is concerned with the degradation of separation factors occurred when groups of ultracentrifuges having different gas-dynamic behavior are connected in parallel arrangements.

Differences in the gas-dynamic behavior were traduced in terms of different tails pressures for the same operational conditions, that are feed flow rate, product pressure and cut number.

A mathematical model describing the ratio of the tails flow rates as a function of the tails pressure ratios and the feed flow rate was developed using experimental data collected from a pair of different ultracentrifuges connected in parallel. The optimization of model parameters was made using Marquardt's algorithm.

The model developed was used to simulate the separation factors degradation in some parallel arrangements containing more than two ultracentrifuges. The obtained results were compared with experimental data collected from different groups of ultracentrifuges. It was observed that the calculated results were in good agreement with experimental data.

This màthematical model, which parameters were determined in a two-ultracentrifuges parallel arrangement, is useful to simulate the effect of quantified gas-dynamic differences in the separation factors of groups containing any number of different ultracentrifuges and, consequently, to analyze cascade losses due to this kind of occurrence.

Keywords: Isotope Separation, gas centrifugation, uranium isotopes.

1. Introduction.

The relation between the separation factor, defined as the ratio between the ^{235}U abundance ratio measured in the product stream and the ^{235}U abundance ratio measured in the tails stream and the feed flow rate for a given cut number and a given product stream pressure in an isolated ultracentrifuge is easily determined experimentally.

When two or more ultracentrifuges are connected forming parallel arrangements, however, it is observed that, for the same operating conditions as for the isolated ultracentrifuge, the resulting separation factor may be smaller than the expected value.

In groups of ultracentrifuges for which this reduction of the separation factor occurs, if we could analyze the individual behavior of each centrifuge, we could see that each of them operates with a different cut number. This fact is due to differences in the gas-dynamic behavior for each different ultracentrifuge.

This kind of difference may be quantified, for example, submitting each ultracentrifuge of the group to the same feed flow rate, cut number and product stream pressure, and measuring the resulting tails stream pressure.

In tests with two different ultracentrifuges connected in parallel, as indicated in Fig. 1, it is observed that, for a given flow rate, increasing the difference between their tails stream pressures, or decreasing the ratio between them, the resulting separation factor of the parallel arrangement decreases, and that this effect increases as the feed flow rate decreases.

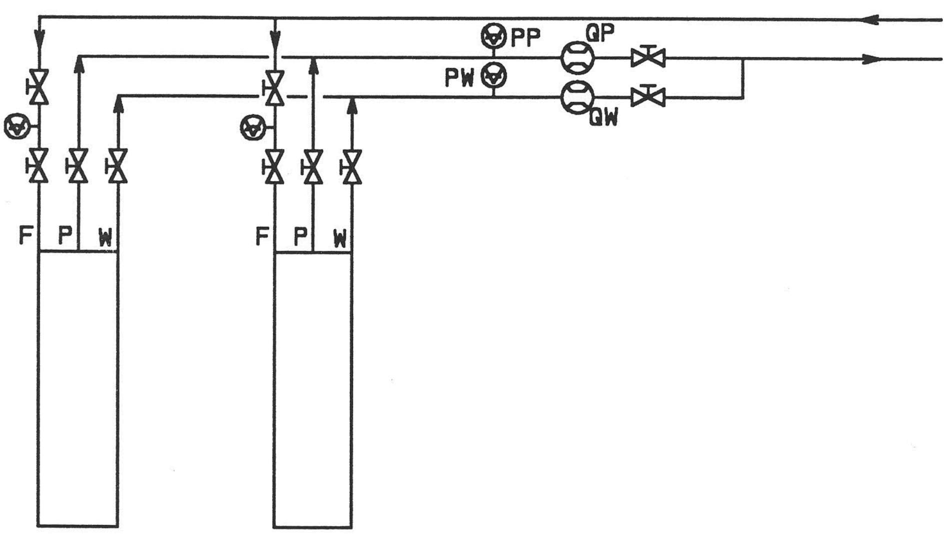

Figure 1 - Experimental ultracentrifuges arrangement.

This can be seen in Fig. 2, and easily understood, because it's easier to throw out the tails stream for an ultracentrifuge with a higher tails stream pressure than for an ultracentrifuge with a lower tails stream pressure. When these two ultracentrifuges are connected in parallel, the one with the highest tails stream pressure will disturb the other one behavior, forcing it to operate with a higher cut number. This ultracentrifuge, then, will disturb the throw out of the first one product stream, forcing it to operate with a lower cut number.

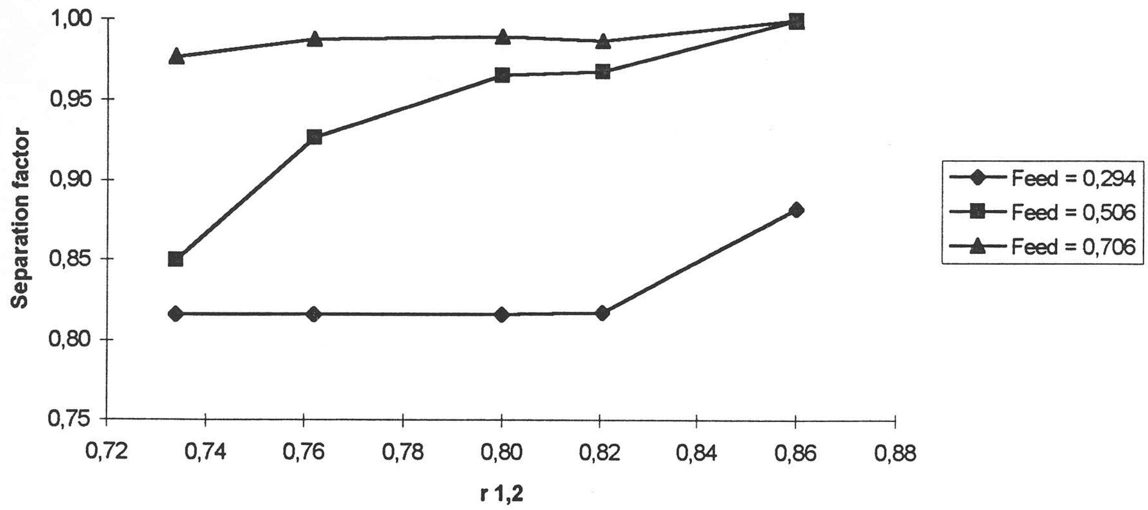

Figure 2 - Normalized separation factor for different ratios of tails stream pressures.

Operation of the group with low or high cut numbers also causes reduction of the global separation factor, as it can be seen in Fig. 3, for the same reason discussed above.

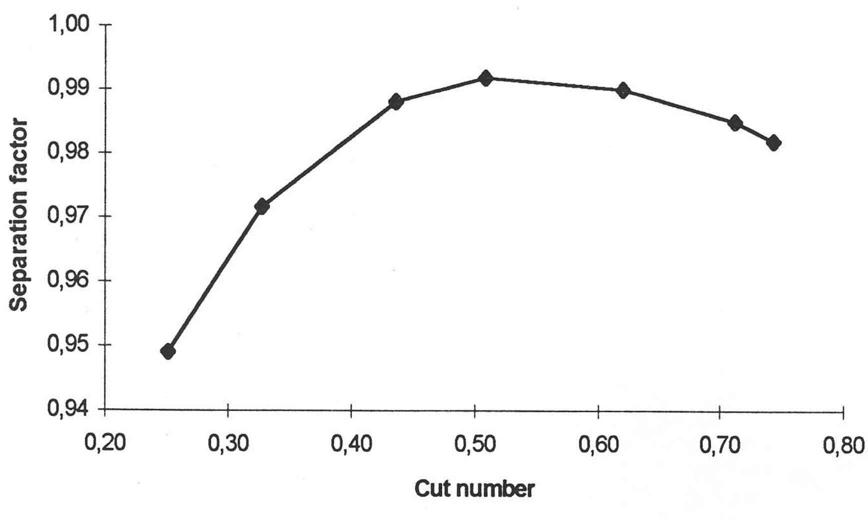

Figure 3 - Normalized separation factor for different cut numbers.

The relation between the global separation factor and the feed flow rate for a given tails stream pressures ratio can be seen in Fig. 4.

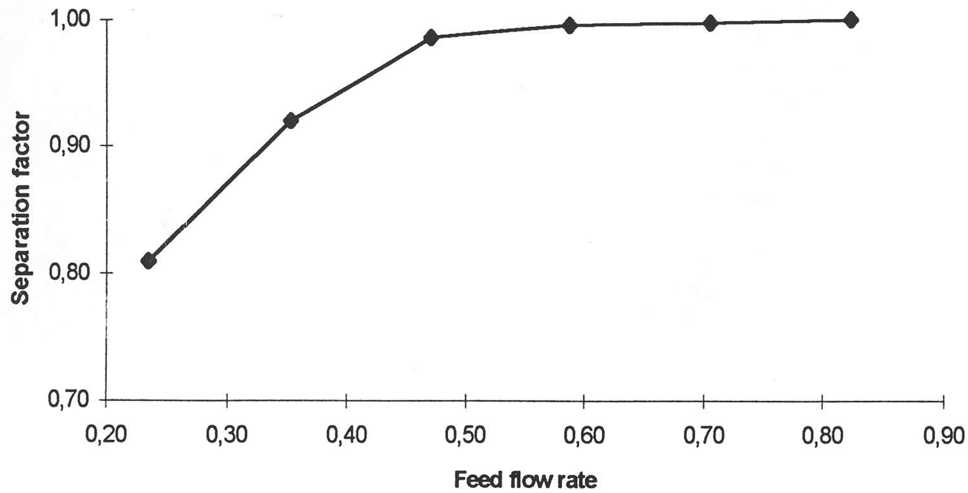

Figure 4 - Normalized separation factor for different feed flow rates.

2. The model.

In a group composed of two different ultracentrifuges connected in parallel, submitted to a given feed flow rate 2F, a given cut number θ and a given product stream pressure p_P , if we could know the relation between the ratio of their tails flow rates, the ratio of their tails stream pressures and their feed flowrate:

$$\frac{W_1}{W_2} = f\left(\frac{p_{w1}}{p_{w2}}, F\right),$$

we could obtain the relation between their real cut numbers as:

$$\frac{1-\theta_1}{1-\theta_2} = f\left(\frac{p_{w1}}{p_{w2}}, F\right).$$

Knowing that, for the global arrangement:

$$\theta = \frac{P}{2F} = \frac{\theta_1 F + \theta_2 F}{2F} = \frac{\theta_1 + \theta_2}{2},$$

we could calculate θ_1 and θ_2 and, consequently, the two ultracentrifuges real heads and tails separation factors:

$$\beta_1 = \frac{R_{p1}}{R_f} = \frac{\alpha}{\theta_1(\alpha - 1) + 1}; \qquad \gamma_1 = \frac{\alpha}{\beta_1},$$

$$\beta_2 = \frac{R_{p2}}{R_f} = \frac{\alpha}{\theta_2(\alpha - 1) + 1}; \qquad \gamma_2 = \frac{\alpha}{\beta_2},$$

where α is the separation factor of one isolated ultracentrifuge submitted to the same feed flow rate F and the same product stream pressure p_p.

The global heads and tails separation factors could be calculated for:

$$\bar{\beta} = \frac{\bar{R}_p}{R_f} = \frac{\theta_1 F \beta_1 R_f + \theta_2 F \beta_2 R_f}{(\theta_1 F + \theta_2 F) R_f} = \frac{\theta_1 \beta_1 + \theta_2 \beta_2}{\theta_1 + \theta_2},$$

$$\bar{\gamma} = \frac{R_f}{\bar{R}_w} = \frac{(1-\theta_1)F + (1-\theta_2)F}{(1-\theta_1)F\frac{R_f}{\gamma_1} + (1-\theta_2)F\frac{R_f}{\gamma_2}} = \frac{\alpha[2-\theta_1-\theta_2]}{(1-\theta_1)\beta_1 + (1-\theta_2)\beta_2}.$$

The same calculation scheme could be used to simulate the behavior of more than two ultracentrifuges connected in parallel arrangements.

Experimental data presented in Fig. 1 to 4, collected from pairs of ultracentrifuges, were used to model the relation between the ratio of tails flow rates, the ratio of tails stream pressures and the feed flow rate, as described above.

Many models for this relation were tried and the one for which calculated results were in best agreement with experimental data is:

$$\frac{W_1}{W_2} = r_{1,2} + \left[k_1 F^{-k_2}\right]^{r_{1,2}^{-k_3}} \ln(1 - r_{1,2} + r_{1,2}^2) \qquad \text{if } r > r_0,$$

$$\frac{W_1}{W_2} = 0 \qquad \text{if } r \le r_0,$$

with,

$$r_0 + \left[k_1 F^{-k_2}\right]^{r_0^{-k_3}} \ln(1 - r_0 + r_0^2) = 0,$$

$$r_{1,2} = \frac{p_{w1}}{p_{w2}},$$

being p_{w1} the lowest tails stream pressure.

Using Marquardt's algorithm(*) to optimize the parameters k_1, k_2 and k_3, we obtained:

$k_1=682$,
$k_2=1.34$,
$k_3=2.40$.

As it can be seen in Table 1, the average deviation between the calculated and the measured values for $\bar{\beta}/\sqrt{\alpha}$ is 0.0073 ± 0.5118.

(*) Main program elaborated by GIUDICI, R. - Laboratório de Simulação e Controle de Processos - Departamento de Engenharia Química - Escola Politécnica da Universidade de São Paulo.

3. Model verification for arrangements with more than two ultracentrifuges.

This model can be used to simulate the degradation of the heads separation factor in arrangements with more than two ultracentrifuges connected in parallel.

POINT	MEASURED VALUE	CALCULATED VALUE	DEVIATION (%)
1	0.9345	0.9318	-0.2918
2	0.9078	0.9033	-0.4975
3	0.9033	0.9033	+0.0000
4	0.9033	0.9033	+0.0000
5	0.9033	0.9033	+0.0000
6	0.9972	0.9915	-0.5777
7	0.9778	0.9811	+0.3387
8	0.9782	0.9766	-0.1629
9	0.9658	0.9508	-1.5560
10	0.9178	0.9246	+0.7383
11	0.9980	0.9963	-0.1682
12	0.9901	0.9927	+0.2645
13	0.9905	0.9910	+0.0540
14	0.9934	0.9914	-0.1998
15	0.9862	0.9874	+0.1203
16	0.8994	0.8994	+0.0000
17	0.9529	0.9669	+1.4720
18	0.9854	0.9850	-0.0449
19	0.9875	0.9867	-0.0826
20	0.9908	0.9904	-0.0333
21	0.9931	0.9919	-0.1211
22	1.0030	1.0120	+0.8749
23	1.0080	1.0090	+0.0960
24	1.0010	0.9987	-0.2496
25	0.9911	0.9892	-0.1871
26	0.9723	0.9731	+0.0819
27	0.9568	0.9593	+0.2619
28	0.9539	0.9546	+0.0733

Table 1 - Deviation between calculated and measured values of $\bar{\beta} / \sqrt{\alpha}$ for parallel arrangements with two ultracentrifuges.

In a group of **n** ultracentrifuges connected in parallel, if we know the ratios $\frac{p_{w1}}{p_{w2}}, \frac{p_{w1}}{p_{w3}}, \cdots, \frac{p_{w1}}{p_{wn}}$, being p_{w1} the lowest value of tails stream pressure in the group, we can calculate $\frac{W_1}{W_2}, \frac{W_1}{W_3}, \cdots, \frac{W_1}{W_n}$, and $\theta_1, \theta_2, \cdots, \theta_n$ as functions of θ_1 :

$$\theta_2 = 1 - \frac{1-\theta_1}{f\left(\frac{p_{w1}}{p_{w2}}, F\right)}, \cdots, \theta_n = 1 - \frac{1-\theta_1}{f\left(\frac{p_{w1}}{p_{wn}}, F\right)},$$

Knowing that:

$$\theta = \frac{P}{nF} = \frac{\theta_1 F + \theta_2 F + \cdots + \theta_n F}{nF},$$

we can calculate $\theta_1 + \theta_2 + \cdots + \theta_n$ values. The values of the heads and tails separation factors can be, then, calculated for:

$$\beta_i = \frac{\alpha}{\theta_i(\alpha - 1) + 1}, \quad \text{and} \quad \gamma_i = \frac{\alpha}{\beta_i},$$

$$\overline{\beta} = \frac{\theta_1\beta_1 + \cdots + \theta_n\beta_n}{\theta_1 + \cdots + \theta_n}, \quad \text{and} \quad \overline{\gamma} = \frac{\alpha[n - (\theta_1 + \cdots + \theta_n)]}{(1 - \theta_1)\beta_1 + \cdots + (1 - \theta_n)\beta_n},$$

$$\overline{\alpha} = \overline{\beta}\overline{\gamma}.$$

This calculation scheme was used to simulate the resulting separation factors of groups with 3,4,5 and 6 ultracentrifuges, and compared with the corresponding experimental data. As it can be seen in Table 2, the average deviation between the calculated and the measured values for $\overline{\beta} / \sqrt{\alpha}$ is -0.0756 $\pm$ 0.4521.

Statistical tests indicate that this average value is equal to that obtained for the model points.

POINT	MEASURED VALUE	CALCULATED VALUE	DEVIATION (%)
1	0.9901	0.9886	-0.1501
2	1.0020	0.9988	-0.3128
3	0.9948	0.9922	-0.2683
4	0.9930	0.9904	-0.2544
5	0.9983	0.9964	-0.1916
6	0.9896	0.9874	-0.2237
7	0.9687	0.9818	+1.3520
8	0.9807	0.9830	+0.2416
9	0.9884	0.9872	-0.1282
10	0.9848	0.9817	-0.3062
11	0.9921	0.9893	-0.2798
12	0.9891	0.9871	-0.1945
13	0.9981	0.9954	-0.2662

Table 2 - Deviation between calculated and measured values of $\overline{\beta} / \sqrt{\alpha}$ for parallel arrangements with more than two ultracentrifuges.

4. Model verification in cascade arrangements.

This model can be easily implemented in cascade calculation programs.

Calculated results for a little cascade composed of 14 ultracentrifuges with known tails stream pressures, distributed in six stages as shown in Table 3, in six different operating conditions are presented in Table 4.

These tests were performed in this cascade arrangement and, as it can be seen in Table 4, 58% of the obtained deviations for $\bar{\beta} / \sqrt{\alpha}$ are contained in the interval d ± s, defined for the model points, and 100% of the obtained deviations are contained in the interval d ± 2s.

Calculating the average deviation for $\bar{\beta} / \sqrt{\alpha}$ using those data, we obtained -0.4176 ± 0.3314. Comparing this interval with that obtained for the model points, we can see that the average deviation was shifted, but the standard deviation was of the same order.

STAGE	N. OF ULTRAC.	TAILS PRESSURE RATIOS
1	1	-
2	3	$r_{1,2}$ = .977143 $r_{1,3}$ = .955307
3	4	$r_{1,2}$ = .971098 $r_{1,3}$ = .971098 $r_{1,4}$ = .943820
4	3	$r_{1,2}$ = .976879 $r_{1,3}$ = .884817
5	2	$r_{1,2}$ = .959064
6	1	-

Table 3 - Tails pressure ratios in the cascade stages.

TEST	STAGE	FEED FLOW RATE	CUT NUMBER	CALCULATED VALUES		MEASURED VALUES		RELATIVE DEVIATIONS (%)		
				$\bar{\beta}/\sqrt{\alpha}$	$\bar{\gamma}/\sqrt{\alpha}$	$\bar{\beta}/\sqrt{\alpha}$	$\bar{\gamma}/\sqrt{\alpha}$	$\bar{\beta}/\sqrt{\alpha}$	$\bar{\gamma}/\sqrt{\alpha}$	R_p
1	2	0.3545	0.5210	0.9914	1.0078	0.9913	1.0081	+0.0101	-0.0298	-1.8
	3	0.4621	0.5540	0.9870	1.0127	0.9889	1.0160	-0.1921	-0.3248	
	4	0.4649	0.3830	1.0115	0.9797	1.0183	0.9842	-0.6678	-0.4572	
	5	0.3391	0.4520	1.0038	0.9947	1.0118	1.0025	-0.7907	-0.7781	
2	2	0.3549	0.4470	1.0050	0.9939	1.0067	0.9953	-0.1689	-0.1407	-2.4
	3	0.4065	0.5320	0.9899	1.0094	0.9910	1.0113	-0.1110	-0.1879	
	4	0.4069	0.4130	1.0047	0.9827	1.0139	0.9899	-0.9074	-0.7273	
	5	0.3496	0.4920	0.9966	1.0022	1.0035	1.0104	-0.6876	-0.8116	
3	2	0.3734	0.4460	1.0053	0.9937	1.0060	0.9942	-0.0696	-0.0503	-1.3
	3	0.3994	0.5070	0.9942	1.0050	0.9943	1.0053	-0.0101	-0.0298	
	4	0.4120	0.4620	0.9979	0.9923	1.0016	0.9959	-0.3694	-0.3615	
	5	0.5148	0.5870	0.9824	1.0176	0.9864	1.0249	-0.4055	-0.7123	
4	2	0.3687	0.4180	1.0105	0.9885	1.0119	0.9893	-0.1384	-0.0809	-1.6
	3	0.3967	0.5300	0.9901	1.0092	0.9932	1.0138	-0.3121	-0.4537	
	4	0.4185	0.4500	1.0000	0.9902	1.0020	0.9919	-0.1996	-0.1714	
	5	0.4077	0.4930	0.9969	1.0023	1.0053	1.0123	-0.8356	-0.9878	
5	2	0.3582	0.4480	1.0049	0.9940	1.0066	0.9955	-0.1689	-0.1507	-1.5
	3	0.3983	0.5210	0.9918	1.0074	0.9931	1.0094	-0.1309	-0.1981	
	4	0.4119	0.4510	0.9995	0.9901	1.0058	0.9960	-0.6264	-0.5924	
	5	0.4112	0.5060	0.9945	1.0048	1.0029	1.0154	-0.8376	-1.0439	
6	2	0.3581	0.4600	1.0025	0.9964	1.0048	0.9986	-0.2289	-0.2203	-2.2
	3	0.3923	0.5060	0.9944	1.0048	0.9969	1.0081	-0.2508	-0.3273	
	4	0.3845	0.4440	0.9989	0.9873	1.0089	0.9966	-0.9912	-0.9332	
	5	0.3096	0.4200	1.0098	0.9880	1.0193	0.9959	-0.9320	-0.7933	

Table 4 - Calculated and measured values of $\bar{\beta} / \sqrt{\alpha}$ and $\bar{\gamma} / \sqrt{\alpha}$ for the cascade arrangement.

Variance analysis indicates that, probably, there is a systematic sampling error associated with the cascade interstage points. Eliminating this systematic error, 100% of the obtained deviations would be contained in the interval d ± s defined for the model points.

Considering that this cascade has 4 enriching stages and that it is expected a difference between the calculated and the real values of $\overline{\beta} / \sqrt{\alpha}$, for each stage, in the order of (0.0073 ± 0.5118)%, the deviations obtained for the product ^{235}U abundance ratios are justified.

5. Conclusion.

It was demonstrated that the model developed using data collected from an arrangement containing two ultracentrifuges connected in parallel can be used to simulate the degradation of the global separation factor in the stages of cascade arrangements. We couldn't forget, however, that the error obtained for the model points has to be propagated through the cascade stages to the top composition, increasing the deviation between the calculated and the real value as the number of enriching stages is increased.

6. Acknowledgment.

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7. Notation.

F : ultracentrifuge feed flow rate;
P : ultracentrifuge product flow rate;
W : ultracentrifuge tail flow rate;
Q_p : group product flow rate;
Q_w : group tails flow rate;
p_p : product stream pressure;
p_w : tails stream pressure;
θ : cut number;
α : separation factor;
β : heads separation factor;
γ : tails separation factor;
R_p : product ^{235}U abundance ratio;
R_f : feed ^{235}U abundance ratio;
R_w : tails ^{235}U abundance ratio;
$r_{i,j}$: ratio of i and j ultracentrifuge tails stream pressures;
d : medium deviation between calculated and measured values;
s : standard deviation of the deviations between calculated and measured values.

8. References.

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