Analysis of $\gamma\gamma$ directional correlation data taken with a multi-detector system

J. Y. Zevallos-Chávez
Instituto de Física da Universidade de São Paulo Rua do Matão, Travessa R, 187, 05508-900 São Paulo, SP, Brazil

F. A. Genezini and C. B. Zamboni
Instituto de Pesquisas Energéticas e Nucleares, IPEN/CNEN-SP, Av. Prof. Lineu Prestes, 2242, 05508-000 São Paulo, SP, Brazil

M. T. F. da Cruz, M. N. Martins,[a] V. R. Vanin,
Z. O. Guimarães-Filho, and P. R. Pascholati
Instituto de Física da Universidade de São Paulo Rua do Matão, Travessa R, 187, 05508-900 São Paulo, SP, Brazil

(Received 22 December 2004; accepted 29 April 2005; published online 22 June 2005)

When performed with a multi-detector system, $\gamma\gamma$ directional correlation measurements need to be corrected differently for each independent detector pair, especially when those detectors have different active volumes and subtend different solid angles with respect to the radioactive source. We show here a method, which is suitable for setups containing dissimilar detectors. Successful tests were made for four $\gamma\gamma$ cascades, from ^{60}Co, ^{133}Ba and ^{152}Eu. © *2005 American Institute of Physics.* [DOI: 10.1063/1.1938527]

I. INTRODUCTION

The various setups for $\gamma\gamma$ directional correlation measurements described in the literature may be classified according to the number of Ge detectors they have and their geometry: (i) the so-called balls, arrays with tens of detectors, like NORDBALL[1] with 20 detectors, and (ii) those systems in which the detectors have their axes contained in a plane,[2] usually with less than ten detectors, frequently used in residual activity measurements.

The use of a multi-detector coincidence system, where n detectors are associated as $N=n(n-1)/2$ independent pairs, represents an improvement, due to the reduction of the acquisition time when compared to the setups with only two detectors. The time reduction factor is N and a careful choice of the angles prevents the redundancy due to the symmetries of the angular correlation function. Nevertheless, the use of several detectors demands a more complex analysis procedure, in order to take into account the different detector efficiencies.

Due to the fact that N increases rapidly with the number of detectors, the approach to be followed in the analysis will depend on the experimental setup. Ekström and Nordlund,[1] for the NORDBALL, show a method suitable to treat directional correlations from oriented nuclei (DCO) with a large number of detectors; Yamada, Sharshar, and Okano[2] use a four-detector system assembled in a plane and present an analysis that requires coincidences and singles measurements at two different sets of angles for each detector combination; Asai *et al.*[3] follow the method of Yamada and co-workers, and normalize the correlation data with the help of characteristic x rays emitted by the source; Vénos and Tlustý[4] use a seven-detector system and present an alternative method to determine efficiency corrections, based on the random coincidence counting rates. In each case, the analysis of the two-fold coincidences shows some particularities: Ekström's method is limited to DCO measurements performed with nearly identical detectors; Yamada's two-step measurement may introduce undesirable changes in the efficiency of a detector when it is displaced; the method devised by Asai *et al.* circumvents those efficiency problems, but is restricted to measurements in which the decay is followed by the emission of x rays; and the method of Vénos and Tlustý requires that the measured spectra present accidental peaks over the whole energy range of interest, which are not desirable in coincidence experiments.

Any method of analysis must deal with the differences in the detection efficiencies among the detectors of the setup and with the solid angle corrections. We present here a method for the analysis of $\gamma\gamma$ directional correlations performed with very dissimilar detectors (see the experimental setup below) suitable for setups with several detector pairs.

II. $\gamma\gamma$ DETECTOR EFFICIENCY AND THE ANGULAR CORRELATION FUNCTION

Differences among the efficiencies of detector pairs must be taken into account, and this prevents the raw coincidence data from matching a continuous model function, $W(\theta)$ (see Figs. 2–5). The coincidence efficiencies must be introduced into the model function to be used for the fits. The expression

[a]Author to whom correspondence should be addressed; electronic mail: martins@if.usp.br

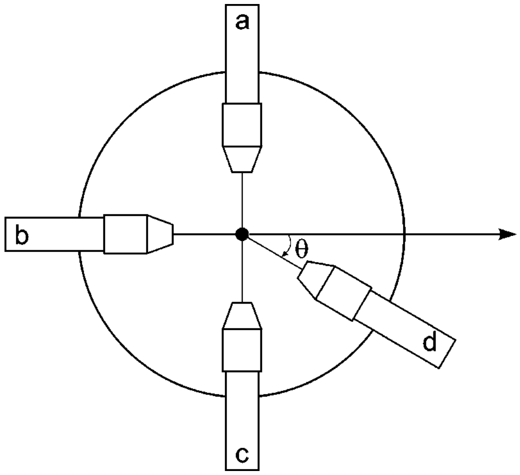

FIG. 1. Schematic drawing of the four-detector system used to acquire the data.

for the angular correlation between two nuclear radiations emitted in a rapid succession, without observing their polarization, is

$$W_{\mathrm{ab}}^{\mathrm{exp}}(\theta) = \frac{Sy_{12}}{4} \sum_{k_{\mathrm{even}}=0}^{4} A_{kk}[J_k^{\mathrm{a}}(E_1) J_k^{\mathrm{b}}(E_2)] P_k(\cos\theta), \tag{1}$$

where $W_{\mathrm{ab}}^{\mathrm{exp}}(\theta)$ is the number of counts obtained with the detector pair ab (the ab subscript of W^{exp} means that the first photon of the cascade was detected by detector a and the second one by detector b); E_1 and E_2 are the photon energies, detected with the detector pair ab; θ is the angle formed by the axes of the detectors. S is the number of nuclear disintegrations that took place during the acquisition time; y_{12} is the emission probability of the pair $\gamma_1\gamma_2$; and J_k is defined by

$$J_k(E_\gamma) = \int d\Omega\, \epsilon(E_\gamma, \vartheta) P_k(\cos\vartheta), \tag{2}$$

where ϵ is the coincidence efficiency of each detector and ϑ is polar angle measured from the detector axis. Note that, for $k=0$, $J_0=\epsilon(E_\gamma)$. Thus, we can write

$$W_{\mathrm{ab}}^{\mathrm{exp}}(\theta) = C_{\mathrm{ab}}\left[1 + \sum_{k_{\mathrm{even}}=2}^{4} A_{kk} Q_{kk}^{\mathrm{ab}} P_k(\cos\theta)\right] \tag{3}$$

with

TABLE I. Multipole mixing ratios (or A_{kk} values where indicated) obtained in the fits of the multi-detector data, together with δ values from Ref. 9, and A_{kk} for ^{60}Co from Ref. 10.

Decaying nuclide and gamma cascade	δ or A_{kk} (This work)	δ or A_{kk} [a,b]
^{60}Co		
1173−1333 keV	$A_{22}=0.102(15)$	$A_{22}=0.1020$
	$A_{44}=0.009(15)$	$A_{44}=0.0091$
^{133}Ba		
81−356 keV	−0.137(7)	−0.151(2)
^{152}Eu		
344−778 keV	0.03(4)	0.002(6)
344−1089 keV	29(29)	29^{+42}_{-11}

[a]Reference 9.
[b]Reference 10.

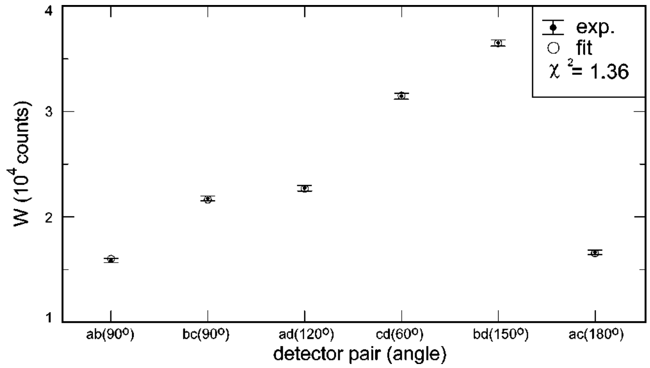

FIG. 2. Fit results for the 1173−1333 keV cascade from ^{60}Co decay. Plot of the experimental values of W against the detector pair (full circles with error bars), together with the corresponding fit results (open circles). The inset shows the reduced chi square of the fit.

$$C_{\mathrm{ab}} = \frac{Sy_{12}}{4} J_0^{\mathrm{a}}(E_1) J_0^{\mathrm{b}}(E_2) = \frac{Sy_{12}}{4} \epsilon^{\mathrm{a}}(E_1)\epsilon^{\mathrm{b}}(E_2), \tag{4}$$

where $Q_{kk}^{\mathrm{ab}}=Q_k^{\mathrm{a}}(\gamma_1)Q_k^{\mathrm{b}}(\gamma_2)$ with $Q_k=J_k/J_0$ being the (k dependent) solid angle corrections, and P_k the Legendre polynomials.

Considering that for each detector pair we actually have two independent sets of data for the same nuclear cascade (the second one associated with the detection of photon γ_1 by detector b and photon γ_2 by a), we will employ the symbol [ab] on parameters that are invariant under the exchange of detectors a and b, or photons γ_1 and γ_2, the same not holding for the symbol ab (i.e., ab$\neq$ba). So, in order to write Eq. (3) in this invariant form, we add up the two independent data, $W_{[\mathrm{ab}]}^{\mathrm{exp}}(\theta)=W_{\mathrm{ab}}^{\mathrm{exp}}(\theta)+W_{\mathrm{ba}}^{\mathrm{exp}}(\theta)$

$$W_{[\mathrm{ab}]}^{\mathrm{exp}}(\theta) = C_{[\mathrm{ab}]}\left[1 + \sum_{k_{\mathrm{even}}=2}^{4} A_{kk} Q_{kk}^{[\mathrm{ab}]} P_k(\cos\theta)\right] \tag{5}$$

with

$$C_{[\mathrm{ab}]} = \frac{Sy_{12}}{4}[\epsilon^{\mathrm{a}}(E_1)\epsilon^{\mathrm{b}}(E_2) + \epsilon^{\mathrm{b}}(E_1)\epsilon^{\mathrm{a}}(E_2)] \tag{6}$$

and $Q_{kk}^{[\mathrm{ab}]}$ are now the symmetric solid angle corrections that appear in Ref. 5.

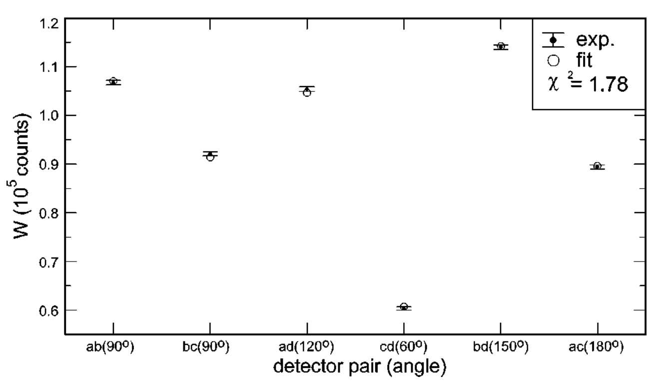

FIG. 3. Same as in Fig. 2, but for the 81−356 keV cascade from ^{133}Ba decay.

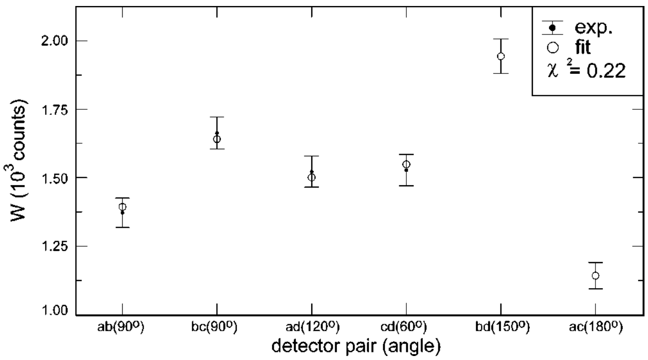

FIG. 4. Same as in Fig. 2, but for the 344–778 keV cascade from ^{152}Eu decay.

III. FIT OF THE FUNCTION $W(\theta)$

To fit the experimental data with a model function, we must take into account that the A_{kk} depend on the spins of initial, intermediate, and final states, and on the multipole mixing ratios, $\delta_1 = \delta(\gamma_1)$ and $\delta_2 = \delta(\gamma_2)$, of the cascade. The structure of the angular correlation function in Eq. (5) represents a model that can be fitted to the experimental data by a least squares method. This function, $W^{\text{theo}}(\theta)$, may be put into matrix form as follows: the column vector containing the model estimates is written

$$\mathbf{W}^{\text{theo}} = C_0 \cdot \boldsymbol{\epsilon} \cdot \mathbf{R} \cdot \mathbf{X}, \tag{7}$$

where the coefficient

$$C_0 = \frac{Sy_{12}}{4} \tag{8}$$

is one of the quantities to be fitted, and $\boldsymbol{\epsilon}$ represents the efficiency matrix. Matrix $\mathbf{R}$ is given by the product of the Legendre polynomials and the solid angle correction coefficients. If the spin sequence of the cascade is known, then the column vector $\mathbf{X}$ contains the correlation coefficients expressed as functions of the multipole mixing ratios. Details about the matrices are given in the Appendix.

The minimization procedure involves the fit of correlated data, the covariances being due to the detection efficiencies. We then minimize the quantity

$$\chi^2 = \boldsymbol{\Delta}^t \mathbf{M}^{-1} \boldsymbol{\Delta} \tag{9}$$

by varying C_0, δ_1 and δ_2.

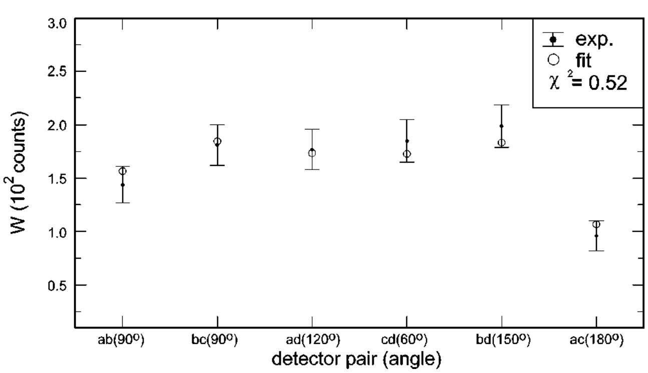

FIG. 5. Same as in Fig. 2, but for the 344–1089 keV cascade from ^{152}Eu decay.

In Eq. (9), $\boldsymbol{\Delta}$ is the column vector containing the differences

$$\boldsymbol{\Delta} = \mathbf{W}^{\text{exp}} - \mathbf{W}^{\text{theo}}, \tag{10}$$

and $\mathbf{M}$ is the complete covariance matrix of $\mathbf{W}_{\text{exp}}$, see the Appendix for details.

For the minimization of the chi square in Eq. (9) we have generalized the linear least squares equations of Ref. 6, to be used in a modified version of routine CURFIT[6] with correlated input data.

If the spin sequence of the cascade is not known *a priori*, one should apply the method described above to each possible spin sequence, seeking the lowest chi square.

IV. EXPERIMENTAL SETUP

The multi-detector system used at the Linear Accelerator Laboratory of the Instituto de Física da Universidade de São Paulo is composed of four Ge detectors with active volumes ranging from 50 to 190 cm^3. The detectors were arranged with their axes on a horizontal plane, radially set at the coordinate angles 30°, 90°, 180° and 270°, as shown in Fig. 1. They were mounted inside lead collimators, to reduce coincidence events of photons scattered from one detector into another.

Signals from the detectors were fed into usual fast-slow electronics, where the decision functions were performed by an in-house built CAMAC module. If all logic conditions were met, a CAMAC controller, directly coupled to the bus of an IBM-compatible personal computer (PC), managed the data transfer to the PC, through an ISA bus lengthener.[7] Both devices, the controller and the lengthener, were designed and built in house, and allow a high data throughput. The data were recorded on event-by-event mode on hard disk. An acquisition and display program was used for visualization of the data.[8] The angles formed by the six independent pairs were, respectively, 60°, 120°, 150°, 90°, 180°, and 90°. Due to the symmetries of the angular correlation function all data points may be represented in the second quadrant as: 90° (twice), 120° (twice), 150° and 180°.

V. DISCUSSION

Reliability tests of the method were successfully performed for four cascades, belonging to three different radioisotopes usually employed as calibration sources for γ-ray spectroscopy: one cascade from the decay of ^{60}Co, one from ^{133}Ba and two from ^{152}Eu. Table I shows the results for the multipole mixing ratios obtained from the fits, compared to the accepted values of the literature[9,10] with excellent agreement.

Figures 2–5 show the data points versus the point number. The plot is shown in this fashion since different detector pairs, with different efficiencies, do not have matching results for the coincidence counts acquired at the same angles. The insets in these plots indicate the reduced chi square of the fit. So, our method allows one to obtain multipole mixing ratios without the need to perform two-step measurements or x-ray normalization to take into account the different detector efficiencies.

The successful application of this method requires that the detector efficiencies be determined using the very same electronics used in the experiment. One must include the effects of the overall dead time and of the fast electronics.

We used both covariant and noncovariant least squares methods in the analysis. The inclusion of covariances in the process does not change the estimated values, but affects their standard deviation.

ACKNOWLEDGMENTS

The authors acknowledge the partial support of Fundação de Amparo à Pesquisa do Estado de São Paulo, FAPESP, Conselho Nacional de Desenvolvimento Científico e Tecnológico, CNPq, and Coordenação de Aperfeiçoamento de Pessoal de Nível Superior, CAPES.

APPENDIX

The angular correlation function to be fitted [Eq. (7)], is composed by the product of three other matrices, described below.

$\boldsymbol{\epsilon}$ represents the diagonal efficiency matrix, written as

$$\boldsymbol{\epsilon} = \begin{bmatrix} \epsilon^{\mathrm{a}}(E_1)\epsilon^{\mathrm{b}}(E_2) + \epsilon^{\mathrm{b}}(E_1)\epsilon^{\mathrm{a}}(E_2) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \epsilon^{\mathrm{c}}(E_1)\epsilon^{\mathrm{d}}(E_2) + \epsilon^{\mathrm{d}}(E_1)\epsilon^{\mathrm{c}}(E_2) \end{bmatrix}. \quad \text{(A1)}$$

The order N of this matrix is the number of independent detector pairs. Matrix $\boldsymbol{\epsilon}$ was prepared with the absolute photopeak efficiencies obtained experimentally, as described in the beginning with Eq. (A5) below.

The matrix $\mathbf{R}$ is given by

$$\mathbf{R} = \begin{bmatrix} 1 & P_2(\cos\theta_{\mathrm{ab}})Q_{22}^{[\mathrm{ab}]} & P_4(\cos\theta_{\mathrm{ab}})Q_{44}^{[\mathrm{ab}]} \\ \vdots & \vdots & \vdots \\ 1 & P_2(\cos\theta_{\mathrm{cd}})Q_{22}^{[\mathrm{cd}]} & P_4(\cos\theta_{\mathrm{cd}})Q_{44}^{[\mathrm{cd}]} \end{bmatrix}. \quad \text{(A2)}$$

The solid angle correction coefficients Q_{kk} were calculated by Monte Carlo simulation of each detector geometry, using the EGS4 code.[11] Several million events were generated, and variances were negligible.

The column vector $\mathbf{X}$ contains the correlation coefficients expressed as functions of the multipole mixing ratios

$$\mathbf{X} = \begin{bmatrix} 1 \\ A_{22}(\delta_1,\delta_2) \\ A_{44}(\delta_1,\delta_2) \end{bmatrix}. \quad \text{(A3)}$$

The matrix $\mathbf{M}$ [Eq. (9)] is the complete covariance matrix of $\mathbf{W}_{\mathrm{exp}}$ and is composed of two independent contributions

$$\mathbf{M} = \mathbf{M}_{W\mathrm{exp}} + \mathbf{M}_{\epsilon}, \quad \text{(A4)}$$

where $\mathbf{M}_{W\,\mathrm{exp}}$ is the usual, diagonal matrix, containing the standard deviations of the counts, while $\mathbf{M}_{\epsilon}$ includes the efficiency contributions, built with the help of the Jacobian algebra of the transformation $\epsilon \to W$.[12]

The coincidence photopeak efficiency can be written as

$$\epsilon = \epsilon_t \,.\, \epsilon_\gamma \,.\,, \quad \text{(A5)}$$

where ϵ_t arises from the dead time associated with the data acquisition system, time to digital converter (TDC) and other coincidence circuits, and is defined to be independent of the γ-ray energy; ϵ_γ is the detection efficiency component, which depends on the γ-ray energy.

The experimental efficiencies were measured through the coincidence electronics gated by the corresponding fast discriminators, and the live time was determined by a pulse generator. The data were fitted by the function

$$\epsilon_\gamma = [a_1 \exp(-a_2 E + a_3 \exp(-a_4 E)\ ]\exp\left(-\frac{\mu}{\rho}x\right), \quad \text{(A6)}$$

where a_1, a_2, a_3 and a_4 are parameters to be fitted, μ/ρ is the germanium mass attenuation coefficient, and x is a parameter associated with the detector dead layer, also to be fitted.

A fit of the efficiency function for detector a is presented in Fig. 6.

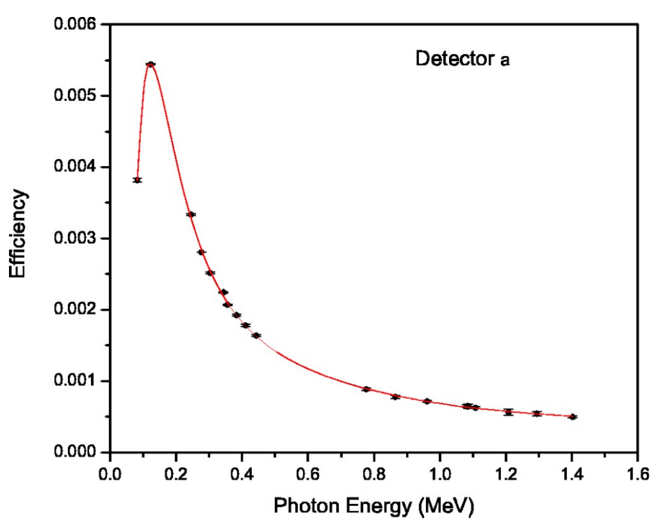

FIG. 6. Absolute γ-ray detection efficiency of detector a determined with ^{60}Co, ^{133}Ba and ^{152}Eu sources.

[1] L. P. Ekström and A. Nordlund, Nucl. Instrum. Methods Phys. Res. A **313**, 421 (1992).
[2] S. Yamada, T. Sharshar, and K. Okano, Nucl. Instrum. Methods Phys. Res. A **316**, 276 (1992); S. Yamada, A. Taniguchi, K. Okano, and K. Aoki,

Eur. Phys. J. A **7**, 327 (2000).
[3] M. Asai, K. Kawade, M. Shibata, Y. Kojima, A. Osa, M. Koizumi, and T. Sekine, Nucl. Instrum. Methods Phys. Res. A **463**, 205 (2001).
[4] D. Vénos and P. Tlustý, Nucl. Instrum. Methods Phys. Res. A **336**, 567 (1993).
[5] D. C. Camp and A. L. van Lehn, Nucl. Instrum. Methods **76**, 192 (1969).
[6] P. R. Bevington and D. K. Robinson, *Data Reduction and Error Analysis for the Physical Sciences*, 3rd ed. (McGraw-Hill, Boston, 2003).
[7] D. Bärg-Filho, R. C. Neves, and V. R. Vanin, *ISA-to-CAMAC Interface. Proceedings of the XX Brazilian Workshop on Nuclear Physics*, edited by S. R. Souza, O. D. Gonçalves, C. L. Lima, L. Tomio, V. R. Vanin (World Scientific, Singapore, 1998), p. 424.
[8] D. Bärg-Filho, M.Sc. thesis, IPEN/CNEN-SP, 1999 (unpublished).
[9] R. B. Firestone and V. S. Shirley, *Table of Isotopes* (Wiley, New York, 1996).
[10] H. Frauenfelder and R. M. Steffen, in *Alpha-, Beta- and Gamma-Ray Spectroscopy* (North–Holland, Amsterdam, 1965), Vol. 2, p. 1032.
[11] W. R. Nelson, H. Hirayama, and D. W. O. Rogers, The eGS4 Code System, SLAC Report No. 265, December 1985.
[12] L. Lyons, *Statistics for Nuclear and Particle Physicists* (Cambridge University Press, Cambridge, 1986).