

**A GENERAL STUDY OF UNDERSAMPLING PROBLEMS IN  
MONTE CARLO CALCULATIONS**

**A Dissertation  
Presented for the  
Doctor of Philosophy  
Degree**

**The University of Tennessee, Knoxville**

**Wilson José Vieira**

**December, 1989**

A GENERAL STUDY OF UNDERSAMPLING PROBLEMS IN  
MONTE CARLO CALCULATIONS

A Dissertation

Presented for the

Doctor of Philosophy

Degree

The University of Tennessee, Knoxville



Wilson José Vieira

December 1989

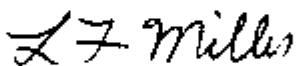
To the Graduate Council:

I am submitting herewith a dissertation written by Wilson José Vieira entitled "A General Study of Undersampling Problems in Monte Carlo Calculations." I have examined the final copy of this dissertation for form and content and recommend that it be accepted in partial fulfilment of the requirements for the degree of Doctor of Philosophy, with a major in Nuclear Engineering.



Paul N. Stevens  
Paul N. Stevens, Major Professor

We have read this dissertation  
and recommend its acceptance:



Accepted for the Council:

  
Lammel  
Vice Provost  
and Dean of The Graduate School

---

## **DEDICATION**

to Ligia and Daniel.

## **ACKNOWLEDGEMENTS**

I would like to thank all the people that contributed in some way for the realization of this work. In particular, the Conselho Nacional de Desenvolvimento Científico e Tecnológico of Brazil, which solely provided financial funds.

The members of the committee to whom I am particular indebted for the experience and knowledge that each one provided for my education.

To my advisor Prof. Paul N. Stevens who actually suggested the theme and with his encouragement and expertise provided important guidelines throughout this work. However, whatever errors remain in my work, are solely my responsibility. Also, I would like to thank Mrs. Barbara Stevens for her continuous encouragement.

## ABSTRACT

Various techniques devised to flag undersampling conditions were investigated. Undersampling conditions can often lead to underestimation and a solution which can be significantly smaller than the true solution but the estimate of the standard deviation may seem acceptably small. In an attempt to identify undersampling in Monte Carlo calculations, the estimation of F values, the coefficient of variation of the standard deviation, the figure of merit, and a particle contribution distribution histogram were incorporated into the MORSE code.

It was found for the problems considered that the F tests were not conclusive because the distribution of contributions was not normally distributed. The calculation of the coefficient of variation turned out to require significantly more computational effort than that necessary to directly achieve a very small standard deviation because of its dependency upon the kurtosis of the distribution which takes much longer than the variance to achieve a stable value. If the kurtosis is not too large, this coefficient can be used in problems which demand high degrees of precision such as criticality calculations.

The figure of merit  $FOM = 1/\sigma^2 t$  is a function of the variance of the population which becomes stable faster than its coefficient of variation. Therefore, the  $FOM$  provides a more reliable guarantee of a stable solution - also, because it tends to a constant value, it is more easily analyzed. However, in severe undersampling conditions the  $FOM$  may become apparently constant over a large range of sample sizes and then abruptly changing with the sampling of rare particles. Also, a sudden increase in the variance may not have an accompanying significant change in the mean while the figure of merit experiences a jump. Under this condition, the solution may still be a perfectly acceptable estimate.

The creation of the particle contribution distribution which is output at the end of each

batch provides a very effective way of detecting undersampling. If only a few particles account for a large fraction of the response, the estimates of both mean and standard deviation should be regarded as unreliable and, therefore, the sample size should be increased.

Although not thoroughly investigated, the utilization of the statistical tools implemented into the MORSE code was demonstrated to be useful in the study of the behavior of particle distributions when subjected to various biasing and/or estimation procedures.

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Theoretical Background</b>	<b>4</b>
2.1	Measures of Error . . . . .	4
2.2	Estimators for the Mean and Variance . . . . .	4
2.3	An F-test for the Equality of the Group Means . . . . .	7
2.4	Coefficients of Variation . . . . .	10
2.5	The Figure of Merit . . . . .	12
2.6	Sampling of Rare Events . . . . .	13
<b>3</b>	<b>Modifications Introduced into the MORSE Code</b>	<b>15</b>
<b>4</b>	<b>Analysis of Results</b>	<b>20</b>
4.1	Sample Problem 1 . . . . .	20
4.1.1	Next-Event Surface Crossing Estimator Solution . . . . .	21
4.1.2	Point-Detector Estimator Solution . . . . .	25
4.2	Sample Problem 2 . . . . .	37
4.3	Sample Problem 3 . . . . .	37
<b>5</b>	<b>Conclusions</b>	<b>41</b>
<b>BIBLIOGRAPHY</b>		<b>43</b>

<b>APPENDICES</b>	<b>45</b>
<b>A Batch Output</b>	<b>46</b>
A.1 Sample Problem 1 . . . . .	46
A.1.1 Next Event Surface Crossing Estimator . . . . .	46
A.1.2 Point Detector Estimator . . . . .	52
A.2 Sample Problem 2 . . . . .	63
A.3 Sample Problem 3 . . . . .	69
A.3.1 Without Source Step Biasing . . . . .	69
A.3.2 With Source Step Biasing . . . . .	75
<b>B Another Approach to the Monte Carlo Point-Estimator Concept</b>	<b>81</b>
<b>C Modified and Added Subroutines</b>	<b>84</b>
<b>D Derivation of Some Equations of Chapter 2</b>	<b>120</b>
D.1 Derivation of Equation 2.38 . . . . .	120
D.2 Derivation of Equations 2.46 and 2.47 . . . . .	122
D.3 Derivation of Equation 2.48 . . . . .	124
<b>VITA</b>	<b>126</b>

# List of Figures

3.1	Particle Contribution Histogram for Detector 2 - Batch 1. . . . .	16
3.2	Present MORSE Output after a Batch is Completed. . . . .	17
3.3	Layout of the New Storage Areas in the Blank Common. . . . .	18
4.1	Neutron Flux, <i>FSD</i> , <i>CV(SD)</i> , and <i>FOM</i> Behavior for Detector 2 - NESXE. . . . .	22
4.2	Neutron Flux, <i>FSD</i> , <i>CV(SD)</i> , and <i>FOM</i> Behavior for Detector 6 - NESXE. . . . .	23
4.3	Neutron Flux, <i>FSD</i> , <i>CV(SD)</i> , and <i>FOM</i> Behavior for Detector 10 - NESXE. . . . .	24
4.4	Neutron Flux, <i>FSD</i> , <i>CV(SD)</i> , and <i>FOM</i> Behavior for Detector 2 - PDE. . . . .	27
4.5	Neutron Flux, <i>FSD</i> , <i>CV(SD)</i> , and <i>FOM</i> Behavior for Detector 6 - PDE. . . . .	28
4.6	Neutron Flux, <i>FSD</i> , <i>CV(SD)</i> , and <i>FOM</i> Behavior for Detector 10 - PDE. . . . .	29
4.7	Three-Dimensional Behavior of <i>FSD</i> and <i>CV(SD)</i> for Sample Problem 1 - PDE. . . . .	32
4.8	Behavior of <i>FSD</i> and <i>CV(SD)</i> for Sample Problem 1 — PDE with New Starting Random Number. . . . .	33
4.9	Behavior of <i>FSD</i> and <i>CV(SD)</i> for Sample Problem 1 — with PATH = 0.5. . . . .	34
4.10	Behavior of <i>FSD</i> and <i>CV(SD)</i> for Sample Problem 1 — using Russian Roulette, Splitting, and PATH = 0.5. . . . .	35
4.11	Three-Dimensional Behavior of <i>FSD</i> and <i>CV(SD)</i> for Sample Problem 2. . . . .	38
4.12	<i>FOM</i> and <i>CV(SD)</i> Behavior for Sample Problem 3. . . . .	40
B.1	Schemes of the New Point Detector Estimator. . . . .	83

# List of Tables

2.1 A Standard Analysis-of-Variance Table. . . . .	5
4.1 Energy Structure for Sample Problem 1. . . . .	21
4.2 F Values for Sample Problem 1 with Next-Event Estimator. . . . .	26
4.3 F Values for Sample Problem 1 with Point-Detector Estimator. . . . .	36
B.1 Comparison of the Calculated Results. . . . .	82

# Chapter 1

## Introduction

Monte Carlo calculations are possible in almost every field which involve mathematical modeling. All such Monte Carlo analyses comprise of the generation of sequences of random variables and have as solutions estimates of means and variances. The problems can range from the estimation of the mean of a small set of Monte Carlo generated data to the estimation of detailed radiation particle flux distributions which represent the solutions of the familiar integro-differential Boltzmann transport equation.

The quality of a Monte Carlo calculation involves two basic concerns: accuracy and precision. Accuracy is a measure of how close the Monte Carlo estimate is from the true value and is related with the amount of *bias*. Precision is a measure of the statistical uncertainty associated with the estimate and is usually expressed in terms of the standard deviation. Many factors can introduce bias into the Monte Carlo solution of a problem which will affect accuracy as well as the behavior of the estimator of precision. Bias can be caused by inadequacies of the model such as may occur in geometry or cross section descriptions and through the use of some sampling schemes such as point-detector estimators whose expected values are not the true solutions. One interesting way to understand the role of the parameters involved in a statistical estimation is to address the following question: *How can the quality of an estimate be guaranteed?* This raises another question: *Does the population sample represent the true population?* The answer to these questions is the major objective of this work which is to analyze the statistical features of the mean, variance, and

population sample and to devise schemes to utilize more of the information generated during the calculation.

Undersampling occurs in problems where the effect of interest is determined primarily by very rare events and a very large population sample is required to achieve a good estimate. In a completely analog Monte Carlo particle transport calculation, the distribution of the scores is binomial and the effect of interest is simply the probability of scoring a success. Therefore the scores of a problem with a small scoring probability would be mostly zeroes and would require on the order of 400 successes (rare events) to achieve a 5% fractional standard deviation – approximately  $1/\sqrt{n}$  where  $n$  is the number of successes. Undersampling has been recognized as a major source of concern. The inexperienced user may not recognize this condition and accept a solution that can be orders of magnitude too low even though the standard deviation indicates good precision. This problem was pointed out by Gelbard [1] and Cramer et al. [2], both of whom considered that many of the problems associated with statistical uncertainty were still unresolved. Dubi et al. [3] showed that the one-particle method yields more reliable estimates of the variance, but also recognized the usefulness of the batch method. Lux et al. [4] devised a correction scheme for the estimates of the mean and variance and emphasized the importance of distinguishing between rare events and the unimportant background.

The problem of *undersampling* is particularly important when point-detector estimators are used because the contributions to the outer detectors from collisions near the source region are by their nature very small and can be essentially of the same magnitude. If a sufficient number of particles are not sampled to include enough particles which experience rare events, the final results will be unrealistically small while their standard deviations may indicate seemingly acceptable results. In this work, this condition will be called *underestimation*. The computational evolution of a statistically acceptable estimate of an effect of interest, in a deep penetration problem using a point-detector estimator, presents three different stages. The first stage is when undersampling is so severe that no major-contributing

particle is sampled. A major-contributing particle is not only a particle that yields a high contribution relative to background values but also the value of its contribution has to be sufficiently high to significantly move the estimated solution up to values around the true mean. One way of detecting the undersampled condition during the first stage would be to analyze the statistics throughout the region between the source and the point of interest. This procedure is based on the fact that the standard deviation should increase with the distance from the source. If it decreases, this condition indicates that the contributions are due to collisions far from the detector and that there were no important collisions (rare events) near the detector. The second stage is when a few major-contributing particles are sampled. The results as they evolve during this stage will experience jumps in the estimated fluxes and also in their variances. Finally, the third stage is reached when a sufficient number of major-contributing particles are sampled and unbiased estimates are achieved. This behavior is more or less obvious and depends upon factors such as the distance in mean free paths between the source and the detector and the utilization of the various variance reduction techniques.

In Chapter 2 some theoretical considerations about the interpretation of Monte Carlo results are presented. Chapter 3 describes the modifications made to a standard version of the MORSE code [5] to accomplish the calculation and output of the additional information. This modified version of MORSE will be designated as the MORSE/STAT package.

Chapter 4 describes the problems studied and the techniques used in their solution, such as type of estimators and variance reduction techniques used. The results for each problem are analyzed in Chapter 4 with respect to the benefits realized by the user through the proper interpretation of the additional information compiled by the bookkeeping procedures implemented into the MORSE code.

## **Chapter 2**

# **Theoretical Background**

The mathematical and statistical bases which underlie this study are presented in this chapter.

### **2.1 Measures of Error**

The variance and mean square error are central to the characterization of the error associated with a sampling distribution. The population variance of an estimator is a measure of the dispersion of the distribution around the mean, and the mean square error which is a measure of the dispersion around the true value of the parameter. The mean square error can be defined as the sum of the variance plus the square bias i.e.

$$MSE = E(x - \bar{x})^2 + E(\bar{x} - \mu)^2. \quad (2.1)$$

Bias is another concept of error and is defined as the difference between the estimated mean and the true value of the parameter. If the bias is equal to zero, then the mean square error is given by the variance alone.

### **2.2 Estimators for the Mean and Variance**

The statistical behavior of estimators commonly used in Monte Carlo calculations can be better understood with the consideration of some *Analysis of Variance* theory. Consider a

Table 2.1: A Standard Analysis-of-Variance Table.

Group						
1	2	...	$i$	...	$I$	
$x_{11}$	$x_{21}$	...	$x_{i1}$	...	$x_{I1}$	
$x_{12}$	$x_{22}$	...	$x_{i2}$	...	$x_{I2}$	
:	:	:	:	:	:	:
$x_{1j}$	$x_{2j}$	...	$x_{ij}$	...	$x_{Ij}$	
:	:	:	:	:	:	:
$x_{1J}$	$x_{2J}$	...	$x_{iJ}$	...	$x_{IJ}$	
$\bar{x}_1$	$\bar{x}_2$	...	$\bar{x}_i$	...	$\bar{x}_I$	$\bar{x}$

population sample of  $I$  groups each with  $J$  elements as shown in Table 2.1. Summing the  $x_{ij}$  over  $j$ , the mean corresponding to the  $i$ th group is given by

$$\bar{x}_i = \frac{1}{J} \sum_{j=1}^J x_{ij}. \quad (2.2)$$

An estimator for the total mean  $\mu$  is the average of the  $I$   $\bar{x}_i$  estimates

$$\bar{x} = \frac{1}{I} \sum_{i=1}^I \bar{x}_i, \quad (2.3)$$

which is equivalent to the estimator

$$\bar{x} = \frac{1}{N} \sum_{n=1}^N x_n, \quad (2.4)$$

where  $N = I \times J$ . Equation 2.4 effectively considers all elements to belong in just one group, i.e.

$$\bar{x} = \frac{1}{IJ} \sum_{i=1}^I \sum_{j=1}^J x_{ij}. \quad (2.5)$$

It is possible to derive various estimators for the population variance  $\sigma^2$ . The first one  $S_w^2$  is called the variance of *mean square within*, which can be calculated using

$$S_w^2 = \frac{1}{J-1} \sum_{j=1}^J (x_{ij} - \bar{x}_i)^2 \quad (2.6)$$

so that

$$S_w^2 = \frac{1}{I} \sum_{i=1}^I S_i^2. \quad (2.7)$$

This estimator is used when results from several runs are combined and represents the population variance of the grand mean.

Another estimator for the population variance  $S_b^2$  is called variance of the *mean square between*, which is based on the fact that for a sufficient number of elements, the group means are normally distributed with a variance equal to the population variance divided by  $J$

$$\frac{\sigma^2}{J} = \frac{1}{J-1} \sum_{i=1}^J (\bar{x}_i - \bar{x})^2, \quad (2.8)$$

and

$$S_b^2 = \frac{J}{J-1} \sum_{i=1}^J (\bar{x}_i - \bar{x})^2. \quad (2.9)$$

This is the *batch* estimate of the population variance and is the procedure used in standard versions of the MORSE Monte Carlo code.

Finally, a third estimator for the population variance is designated as  $S_t^2$  and is called the variance of the *mean square total*,

$$S_t^2 = \frac{1}{N-1} \sum_{i=1}^I \sum_{j=1}^J (x_{ij} - \bar{x})^2. \quad (2.10)$$

This is the *one particle* estimate of the population variance and is the procedure used in most Monte Carlo programs.

Any of the equations 2.11, 2.12, or 2.13 can be used to calculate an estimate for the population variance  $S^2$  and estimates for the variance of mean  $S_{\bar{x}}^2$  are given by:

$$S_{\bar{x}}^2 = \frac{1}{N} S_w^2, \quad (2.11)$$

$$S_{\bar{x}}^2 = \frac{1}{N} S_b^2, \quad (2.12)$$

and

$$S_{\bar{x}}^2 = \frac{1}{N} S_t^2. \quad (2.13)$$

Equations 2.12 and 2.13 represent the *batch* estimator and the *one particle* estimator of the variance of the mean, respectively.

The fractional standard deviation (*FSD*) is the estimate of precision calculated in the MORSE code and is given by:

$$FSD = \frac{\sqrt{S_w^2}}{\bar{x}}, \quad (2.14)$$

Equations 2.7, 2.9, and 2.10 can be written as

$$S_w^2 = \frac{SSW}{I(J-1)}, \quad (2.15)$$

$$S_b^2 = \frac{SSB}{I-1}, \quad (2.16)$$

$$S_t^2 = \frac{SST}{N-1}. \quad (2.17)$$

Remembering the *sum-of-squares* identity

$$\sum_{i=1}^I \sum_{j=1}^J (x_{ij} - \bar{x})^2 = J \sum_{i=1}^I (\bar{x}_i - \bar{x})^2 + \sum_{i=1}^I \sum_{j=1}^J (x_{ij} - \bar{x}_i)^2, \quad (2.18)$$

the following relationship follows

$$SST = SSB + SSW. \quad (2.19)$$

Therefore, just two of the three sums have to be calculated and the other can be obtained from Equation 2.19.

### 2.3 An F-test for the Equality of the Group Means

One way to guarantee sufficient accuracy in the Monte Carlo estimate is to observe the individual solutions of the groups which comprise the total solution. It is expected that the group means are normally distributed, but suppose that one of the groups yielded a much larger value for its mean. This would strongly suggest the existence of undersampling in all the batches — since the batches were drawn from the same population, individual groups having equal numbers of samples should have essentially the same estimates for their means and variances. Therefore, it would be necessary to increase the group size so that the distribution of the group means becomes more normal and also the variance of the grand

mean becomes smaller. Before proceeding further, expressions for the expected values of  $S_b^2$  and  $S_w^2$  are derived. From a linear model, each element  $x_{ij}$  from the  $i$ -th group is written as

$$x_{ij} = \mu_i + \epsilon_{ij}, \quad (2.20)$$

where  $\mu_i$  is the mean of the  $i$ -th group and  $\epsilon_{ij}$  is the random error of the individual samples which is assumed to have a mean zero and a variance  $\sigma^2$ . The  $x_{ij}$  can be expressed in terms of the the group mean  $\mu_i$  and the grand mean  $\mu$

$$x_{ij} = \mu + \alpha_i + \epsilon_{ij}, \quad (2.21)$$

where

$$\mu_i = \mu + \alpha_i. \quad (2.22)$$

Therefore, it will be possible to distinguish between the variance production due to the variation of the group means  $\mu_i$  and that due to the variation of the elements of the population  $\epsilon_{ij}$ . The *fixed effect* model is characterized by  $\sum_{i=1}^I \alpha_i = 0$ , otherwise the *random effect* model [6] applies. However, since both models utilize the same F-test, the fixed effect model will be used — which should be appropriate for the present analysis.

The expected value of  $S_b^2$  can be calculated by substituting

$$\bar{x}_i = \mu + \alpha_i + \bar{\epsilon}_i \quad (2.23)$$

and

$$\bar{x} = \mu + \bar{\epsilon} \quad (2.24)$$

into Equation 2.9, which yields

$$E(S_b^2) = \frac{J}{J-1} \sum_{i=1}^I E(\alpha_i + \bar{\epsilon}_i - \bar{\epsilon})^2. \quad (2.25)$$

Since  $\sum_{i=1}^I \alpha_i = 0$  and  $E(\epsilon_{ij}) = 0$ , Equation 2.25 can be rewritten as

$$E(S_b^2) = \frac{J}{J-1} \sum_{i=1}^I E(\alpha_i^2 + \bar{\epsilon}_i^2 + \bar{\epsilon}^2). \quad (2.26)$$

With the fact that

$$E(\bar{\epsilon}_i^2) = \sigma^2/J, \quad (2.27)$$

and

$$E(\bar{\epsilon}^2) = \sigma^2/IJ, \quad (2.28)$$

Equation 2.26 becomes

$$E(S_b^2) = \sigma^2 + \frac{J}{I-1} \sum_{i=1}^I \alpha_i^2, \quad (2.29)$$

which can be written as

$$E(S_b^2) = \sigma^2 + J\bar{\alpha}^2. \quad (2.30)$$

The expected value of  $S_w^2$  can be derived by substituting  $E(\bar{x}_i) = \mu$  into Equation 2.7.

That is

$$E(S_w^2) = E\left(\frac{1}{I} \sum_{i=1}^I \frac{1}{J-1} \sum_{j=1}^J (x_{ij} - \mu)^2\right), \quad (2.31)$$

which yields

$$E(S_w^2) = E\left(\frac{1}{I} \sum_{i=1}^I \sigma^2\right) = \sigma^2. \quad (2.32)$$

A statement of the F-test for the equality of the group means would be:

$$\begin{aligned} H_0 &: \mu_1 = \mu_2 = \dots = \mu_I \\ H_1 &: \text{at least one is different,} \end{aligned} \quad (2.33)$$

and it is performed by calculating

$$F_{(I-1)(N-I)} = \frac{S_b^2}{S_w^2}, \quad (2.34)$$

which has as expected value

$$\frac{\sigma^2 + J\bar{\alpha}^2}{\sigma^2}. \quad (2.35)$$

When the null hypothesis is true, the test yields a value of one. The ability to perform such tests is one of the most important features of the batch method.

From the above discussion there are some important observations. First,  $S_b^2$  is greater than or at least equal to  $S_w^2$ . Therefore,  $S_w^2$  and  $S_b^2$  are more reliable because they provide

tighter bounds for the statistical error. Second, as demonstrated above,  $S_t^2$  can be used together with  $S_w^2$  to calculate F values that may detect anomalous differences between the group means — which can be an effect associated with undersampling. Another interesting fact is that because both  $S_w^2$  and  $S_t^2$  take in account the variations within the elements of the population they are approximately equal — which also can be implied from the fact that they have the same expected value  $\sigma^2$ . This means that  $S_w^2$  can be used instead of  $S_t^2$  which is a good way to avoid computational round-off errors.

## 2.4 Coefficients of Variation

Another procedure that provides measures of reliability of the various estimators is the concept of relative error. The coefficient of variation  $V$  is defined as the square root of the relative variance of the population  $V^2$ , which is given by:

$$V^2 = \frac{S^2}{\bar{x}^2} = \frac{1}{N-1} \sum_{i=1}^N \frac{(x_i - \bar{x})^2}{\bar{x}^2}. \quad (2.36)$$

Analogously, the relative variance of the variance is given by:

$$V_{S^2}^2 = \frac{\sigma_{S^2}^2}{(\sigma^2)^2}. \quad (2.37)$$

Hansen, Hurwitz, and Madow [7] (see Appendix D) demonstrated that Equation 2.37 can be expressed as

$$V_{S^2}^2 = \frac{1}{N} \left( \beta - \frac{N-3}{N-1} \right), \quad (2.38)$$

where

$$\beta = \frac{\mu_4}{(\sigma^2)^2}, \quad (2.39)$$

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2, \quad (2.40)$$

$$\mu_4 = \frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^4 \quad (2.41)$$

$$= \bar{x}_4 - 4\bar{x}\bar{x}_3 + 6\bar{x}^2\bar{x}_2 - 3\bar{x}^4, \quad (2.42)$$

and

$$\bar{x}_r = \frac{1}{N} \sum_{i=1}^N x_i^r. \quad (2.43)$$

For sufficiently large  $N$ , Equation 2.38 becomes:

$$V_{S^2}^2 = \frac{\beta - 1}{N}, \quad (2.44)$$

and for sufficiently large  $\beta$

$$V_{S^2}^2 = \frac{\beta}{N}. \quad (2.45)$$

For  $I$  random groups of  $J$  elements, it can be shown that

$$V_{S^2}^2 = \frac{1}{I} \left( \beta_J - \frac{I-3}{I-1} \right), \quad (2.46)$$

where

$$\beta_J = \frac{\beta}{J} + 3 \frac{J-1}{J}. \quad (2.47)$$

The derivations of Equations 2.46 and 2.47 can be found in reference [7] (see Appendix D).

The coefficient of variation of the standard deviation  $V_S$  is related with the coefficient of variation of the variance  $V_{S^2}$  by

$$V_S = \frac{V_{S^2}}{2}. \quad (2.48)$$

The proof of Equation 2.48 can also be found in reference [7] (see also Appendix D). A reasonable value for  $V_S$  is a subject for concern. The confidence limits for the standard deviation do not need to be the same as the confidence limits for the mean. Because  $V_S$  depends upon the kurtosis of the distribution, the significance of the confidence limits of the standard deviation varies for different distributions. A distribution with a large kurtosis is characterized by a large value for  $V_S$  which can be more reliable than an undersampled distribution with a small kurtosis and a much smaller  $V_S$ .

According to the central limit theorem, as  $J$  increases the distribution of the group means becomes more normal and  $\beta_J$  approaches the value of 3. If a normal distribution for the group means is assumed and if a 0.1 value for  $V_S$  is desired, at least 51 groups would

be necessary as indicated by Equation 2.46. Therefore, to achieve a coefficient of variation of the standard deviation on the order of 0.1, a reasonable number of groups ( $I > 50$ ) and a sufficiently large value of  $J$  are required so that the group means would be normally distributed. However, the results obtained with Equation 2.38 were consistently smaller than those obtained with Equation 2.46, which may be explained by the same argument that the expected value of the mean square within is smaller than the expected value of the mean square between. Because of this reasoning, Equation 2.38 is used instead of Equation 2.46. Also, it is shown in Chapter 4 that to achieve a stable value of  $\beta$ , a sample size larger than the one necessary to achieve a sufficiently reliable estimate of  $S^2$  is required.

## 2.5 The Figure of Merit

The figure of merit  $\sigma_x^2 T$  is widely accepted as a measure of the calculational efficiency and also as an indicator that the solution has achieved the asymptotic  $1/\sqrt{N}$  behavior as predicted by the central limit theorem. The figure of merit can also be expressed as

$$FOM = \frac{1}{\sigma_x^2 T}, \quad (2.49)$$

where  $\sigma_x^2$  is the variance of the mean and  $T$  is the total computation time. A larger  $FOM$  indicates a more efficient calculation.

Considering  $t$  as the average computation time per particle, Equation 2.49 can be rewritten as

$$FOM = \frac{N}{\sigma^2 N t} = \frac{1}{\sigma^2 t}. \quad (2.50)$$

Therefore, when  $\sigma^2$  becomes constant, i.e. a sufficient number of particles have been sampled, the figure of merit also becomes constant. However, for highly skewed distributions with a small proportion of very large contributions, the behavior of  $\sigma_x^2$  is typically not  $1/\sqrt{N}$  and may assume either faster or slower rates of convergence. The figure of merit will become stable only after a sufficient number of particles have been sampled. Also, if undersampling is severe, the figure of merit will appear essentially constant over a wide range of sample sizes thus giving false indications about the solution.

## 2.6 Sampling of Rare Events

In deep penetration particle transport problems, the utilization of some form of importance sampling is necessary. For example, an analog solution of a deep penetration problem with a transmission factor of  $10^{-12}$  will require an average of  $10^{12}$  particles to score a success. It is obvious that if a reasonable number of successes are not scored, the results will be worthless. If the parameter being estimated is already known, the sampling scheme can be modified so that every sample yields the same contribution – which results in what is called a zero-variance calculation. It is possible to use any previously known information about the population distribution to enhance the probability of scoring. This procedure is commonly referred to as importance sampling.

Importance sampling may be accomplished by variance reduction techniques such as Russian-roulette, splitting, survival biasing, stratified sampling, weight cut-offs, exponential transform etc. The variance can also be reduced by using estimators or samplers such as the last-flight and the next-event estimators. The utilization of importance sampling requires the use of weight corrections so that the bias introduced by the sampling scheme is eliminated thereby preserving the fair game. However, some techniques such as the point-detector estimator introduce some bias that is not properly corrected by weight correction alone [8]. Importance sampling schemes in general will result in reliable (unbiased) estimates if a sufficiently large number of particles are sampled and provided of course that all sources of bias are corrected. It is also true that improper utilization of importance sampling techniques can result in a calculation that is less efficient.

A counter was introduced into the MORSE code that records the number of particles that have the value of their contributions within a given range or channel. Therefore, it is possible to know how many particles account for specific fractions of the total response. This counter is collapsed from 188 intervals to 10 percentage intervals which would indicate if a small number of particles are responsible for a large percentage of the total value of the

response. This provision turn out to be very useful in the analysis and identification of the undersampled condition because of its ability to isolate the contribution background.

## **Chapter 3**

# **Modifications Introduced into the MORSE Code**

Modifications were introduced into the MORSE code in order to calculate the statistics on a per particle basis according to equations 2.4 and 2.13. The original MORSE input was maintained and the output of the fractional standard deviations for the total responses calculated on a batch basis with Equation 2.12 was also retained. Another feature introduced was a table that shows in terms of percentage bins the distribution of all particle contributions according to their increasing values. This table is created from a 188 multichannel-type histogram as shown in Figure 3.1. This histogram is shown in a scale which enhances the importance of particles in the higher channels. Figure 3.2 shows this table in the output of the same batch that generated the histogram in Figure 3.1. Analyzing Detector 2 in Figure 3.2, 69700 particles account for the first 20% of the total response, 14634 particles account for the next 10% fraction of the response, and so on until the 5 particles of highest weight which account for the last 10% of the total response.

Besides the addition of the histogram mentioned above, this new version of the MORSE code also includes the output of the total responses and their standard deviations for each batch and for the accumulated estimates. The output also presents the estimates of the coefficient of variation and figure of merit, based on the accumulated statistics after each batch is completed.

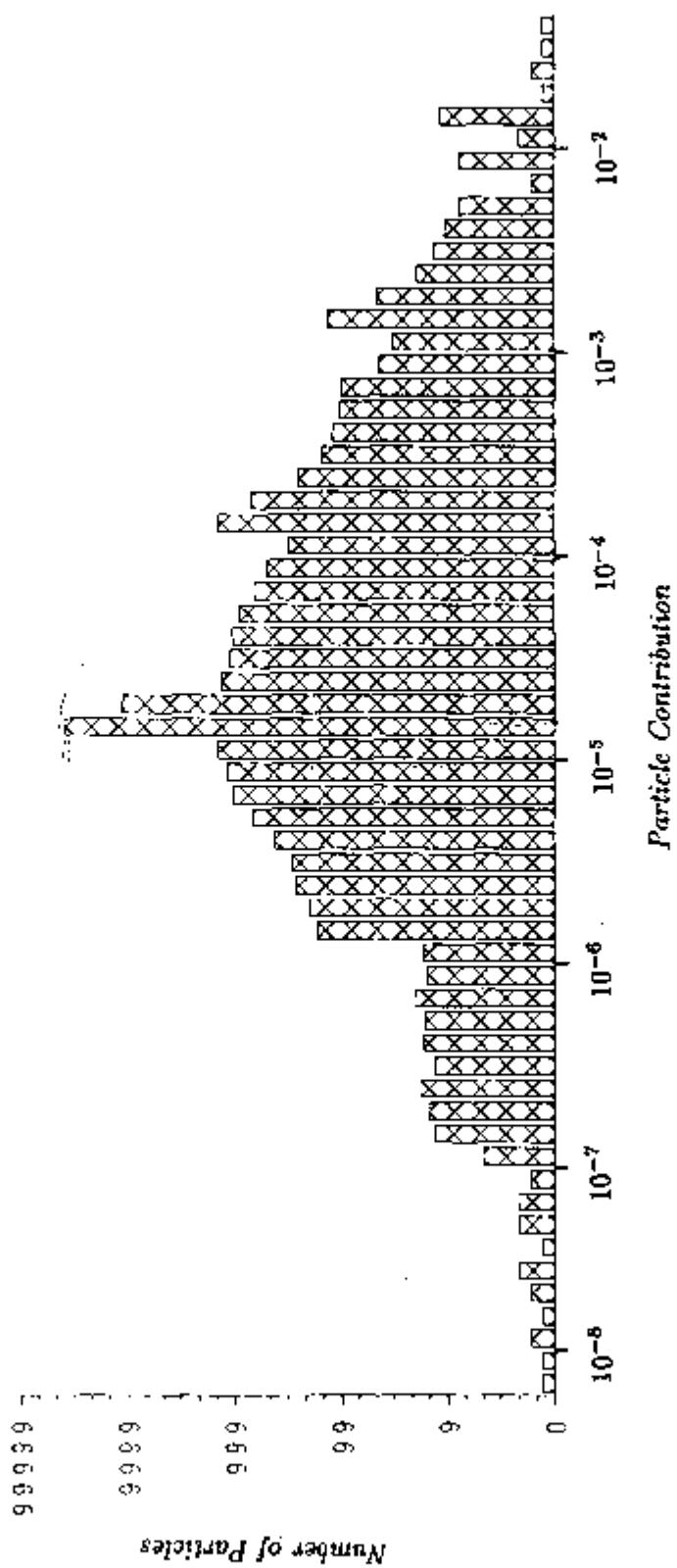


Figure 3.1: Particle Contribution Histogram for Detector 2 - Batch 1.

\*\*\*START BATCH 1

RANDOM-13579BDFDR97

SOURCE DATA  
YOU ARE USING THE DEFAULT VERSION OF SOURCE WHICH SETS WATE TO DDP AND PROVIDES AN ENERGY IG.  
YOU ARE USING THIS DEFAULT VERSION OF OMNIBL WHICH ASSUMES GEOMETRY AND XSECT MEDIA ARE IDENTICAL.

WAVE TYPE WAVE WAVE XAVE YAVE ZAVE AGRAVE  
1.00E+05 1.00 -0.0022 -0.0005 0.0016 0.0 0.0 0.0

PARTICLES PER CH\*\*2 PER SECOND

DETECTOR	PATCH	RESPONSE(FSD)	RESPONSE(ACCUMLD)	NEUTRON FLUX	FSD	CY(SD)	FOM
1	3.8195D-04	0.02420	3.8195D-04	0.02420	0.18380	9.6140D+01	
2	4.0657D-05	0.03220	4.0657D-05	0.03220	0.17042	5.4278D+01	
3	1.0318D-05	0.18977	1.0318D-05	0.18977	0.45750	1.5631D+00	
4	2.2010D-06	0.10484	2.2010D-06	0.10484	0.24872	5.1215D+00	
5	5.801BD-07	0.16399	5.801BD-07	0.16399	0.24418	2.0932D+00	
6	1.3670D-07	0.13992	1.3670D-07	0.13992	0.20058	2.6754D+00	
7	1.0456D-07	0.65115	1.0456D-07	0.65115	0.49116	1.3276D+01	
8	3.6075D-08	0.39992	3.6075D-08	0.39992	0.31527	3.5196D-01	
9	4.4715D-09	0.26794	4.4715D-09	0.28794	0.26414	6.7894D-01	
10	8.1909D-10	0.22073	0.1909D-10	0.22073	0.24052	1.1553D+00	

DETECTOR

	0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100.0
1	62140	0	0	0	0	6767	7358	7117	735	251	49
2	63700	0	14634	7215	3000	1316	971	68	103	0	5
3	78956	13447	1962	1753	289	144	23	13	1	0	0
4	81208	12326	2213	405	198	43	21	6	5	0	0
5	90177	4818	1100	165	76	10	7	3	1	1	1
6	B8529	6765	865	246	37	34	2	4	2	0	0
7	96335	200	15	3	1	0	0	0	0	0	0
8	96700	30	0	1	1	1	0	0	0	0	0
9	96124	619	63	10	0	1	2	0	1	0	0
10	96140	703	86	21	0	3	2	0	1	0	0

PARTICLE DISTRIBUTION IN PERCENTAGE BINS RELATIVE TO INCREASING VALUE OF CONTRIBUTIONS

NUMBER OF COLLISIONS OF TYPE MCOLL  
SOURCE SPLIT(D) FISHN GAMGEN RREALCOLL ALBEDO BDRYX ESCAPE  
100000 0 0 188986 0 98169 46 99952  
TIME REQUIRED FOR THE PRECEDING BATCH WAS 17 MINUTES, 45 SECONDS.

TIME CUT TIME KILL R R KILL R R SURV GANLOST

Figure 3.2: Present MORSE Output after a Batch is Completed.

LOCSD	
1	SD
2	SSD
3	SSD2
4	SD
5	SSD
6	SSD2
7	SSD
8	SSD2
9	SSD
10	SSD2
11	SSD3
12	SSD4
13	SSD5
14	SSD6
LOCQE	15 SUD and SSD units

Figure 3.3: Layout of the New Storage Areas in the Blank Common.

To perform the necessary additional bookkeeping the *user* part of the *blank common* was separated, augmented, and put in double precision (COMMON /SSS/). The particle bank now can accept only one source particle and any number of secondaries particles (NMOST). Therefore, the memory requirements are substantially reduced and the utilization of double precision arithmetic was required to work with numbers of particles on the order of several millions. Figure 3.3 shows the lay out of the new storage areas.

In Appendix C, a listing of all added and modified subroutines is included. All major characteristics of these subroutines are presented in the following list:

1. FLUXST - modified to perform additional bookkeeping in COMMON /SSS/.
2. FTEST - calculates the F values.
3. GSTORE - modified to include only one source particle in the particle bank and at most (NMOST) secondary particles.
4. INPUT1 - minor modification in the input format of CARD B.

5. MAIN - modified to include COMMON /SSS/.
6. MORSE - modified to perform additional calculations for the per particle analysis.
7. MSOUR - modified to help the interim batch information output.
8. NHATCH -- now performs only the total response in the per batch method.
9. NPART - called at the end of each history to do the sums needed in the per particle analysis.
10. NRNPRI ~ perform all calculations in the per particle basis and outputs the new batch information.
11. NRUN modified to output the differential responses in the per particle basis.
12. OUTPT - modified to accomplish the interim batch information output.
13. RELCOL - this subroutine was modified according to Appendix B and it is shown only because of this purpose.
14. SCORIN ~ modified to create the new bookkeeping area in COMMON /SSS/.
15. STBTCH - now performs only the zeroing of the per batch bookkeeping areas.
16. STPART - provides the zeroing of the proper bookkeeping areas of the per particle analysis.
17. TESTW - modified in order to handle the the new allocation of secondary particles.
18. VAR2 - modified to include the per particle estimate of the *FSD*.
19. VAR3 - modified to perform only estimates of the *FSD* in the per particle basis.
20. VAR4 - provides the estimates of the *FSD* and of the *CV(SD)*.

# **Chapter 4**

## **Analysis of Results**

At the beginning of this work it was necessary to identify a problem which has a solution with an underestimated mean and a small standard deviation. The search for such a problem yielded what became the main objective of this work — which was to provide the user with more information, and thereby to better understand the behavior of the solution and the associated sampling distribution. This information will be the key for a successful calculation.

Because of the extensive amounts of output generated, the solutions to most of the sample problems were put in Appendix A. The reader can more easily refer to these data for more refined comparisons.

### **4.1 Sample Problem 1**

Sample problem 1 consists of a 1-meter radius concrete sphere with 10 detectors positioned at 10-cm intervals from the center of the sphere. An isotropic monoenergetic (14-Mev) neutron point source is located at the center of the sphere. All responses are for the first group only. Table 4.1 shows the group structure for the cross sections.

This problem offers several degrees of difficulty for the effects of interest calculated. For example, the response of Detector 1 is very easy to calculate as compared with that of Detector 10. Also, the utilization of point detectors makes the flux estimation of Detector 10 very difficult to accomplish because of the three-dimensional nature of point detectors.

Table 4.1: Energy Structure for Sample Problem 1.

Group	Upper Energy	Group	Upper Energy	Group	Upper Energy
1	1.50E+07	2	1.22E+07	3	1.00E+07
4	8.18E+06	5	6.36E+06	6	4.96E+06
7	4.06E+06	8	3.01E+06	9	2.46E+06
10	2.35E+06	11	1.83E+06	12	1.11E+06
13	5.50E+05	14	1.11E+05	15	3.35E+03
16	5.83E+02	17	1.01E+02	18	2.90E+01
19	1.07E+01	20	3.06E+00	21	1.12E+00
22	4.14E-01				

#### 4.1.1 Next-Event Surface Crossing Estimator Solution

The utilization of a next-event surface crossing estimator (NESXE) posed no problems in the solution of sample problem 1. Figures 4.1, 4.2, and 4.3 show the evolution of the flux, *FSD*, and *FOM* with increasing number of particles for detectors 2, 6, and 10 respectively. It is useful to observe the asymptotic behavior of these statistics over a wide range of sample sizes. In this problem the solutions are free from undersampling because it was possible to process a sufficient number of particles and the statistical quality of these solutions are described by the central limit theorem. There is no reason to question the statistics calculated in this problem. Even *FSD*'s much higher than 0.5 are likely to yield fluxes within  $3\sigma$  of the expected values most of the time.

An important fact suggested from Figure 4.1 and from the data in Appendix A.1.1 is that the coefficient of variation of the standard deviation *CV(SD)* generally follows but is almost always higher than the standard deviation. Also, the amount by which the *CV(SD)* changes is often much more peaked than that experienced by the standard deviation because of the changes in the kurtosis of the distribution. An increase in the *CV(SD)* may reflect either a small increase or even a decrease in the standard deviation. Another interesting aspect of these results is that the *FOM* accepts the solution of Detector 10 much earlier than the solution of Detector 2 which indicates that this parameter may cause the rejection of perfectly acceptable solutions.

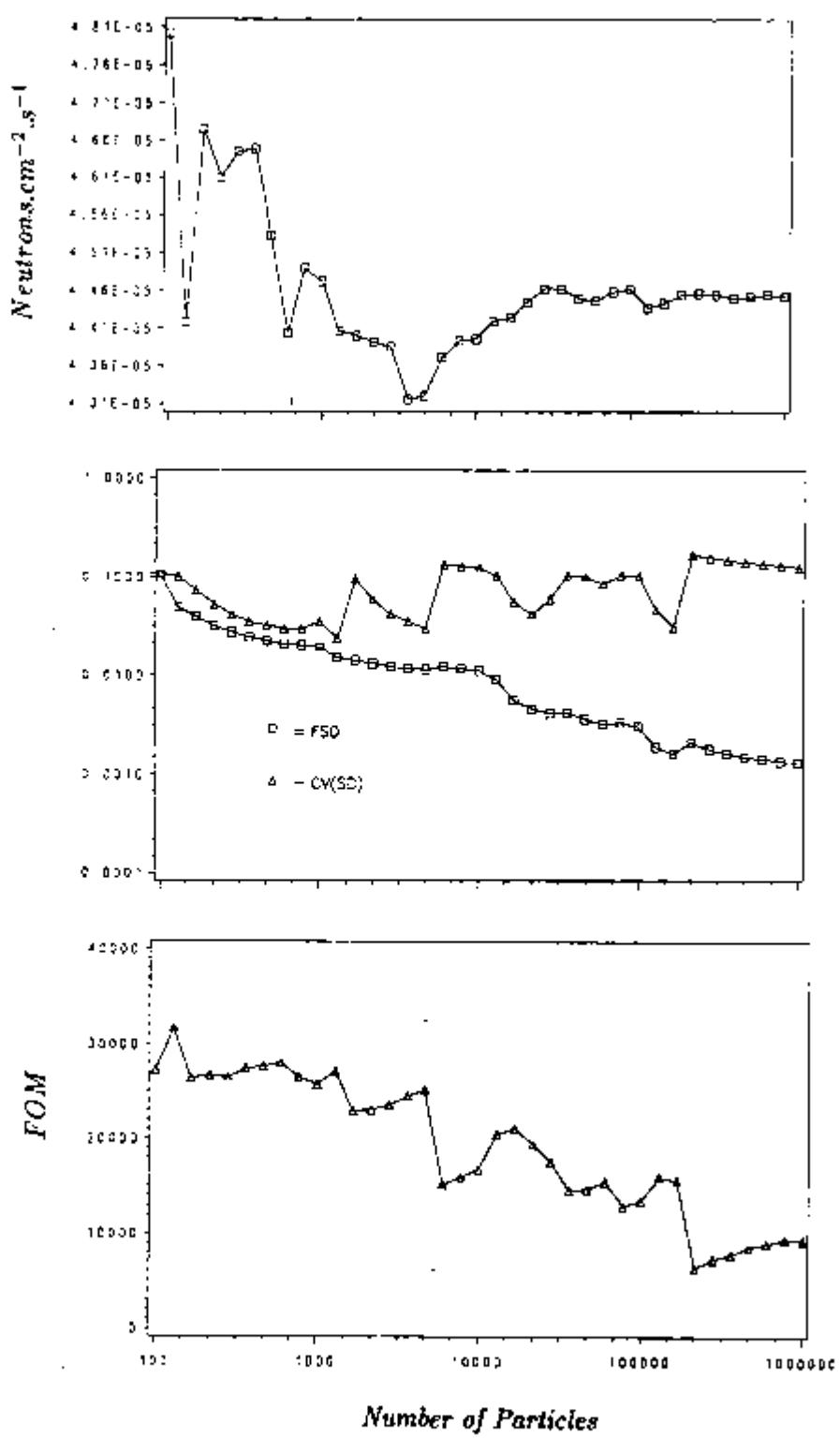


Figure 4.1: Neutron Flux,  $FSD$ ,  $CV(SD)$ , and  $FOM$  Behavior for Detector 2 – NESXE.

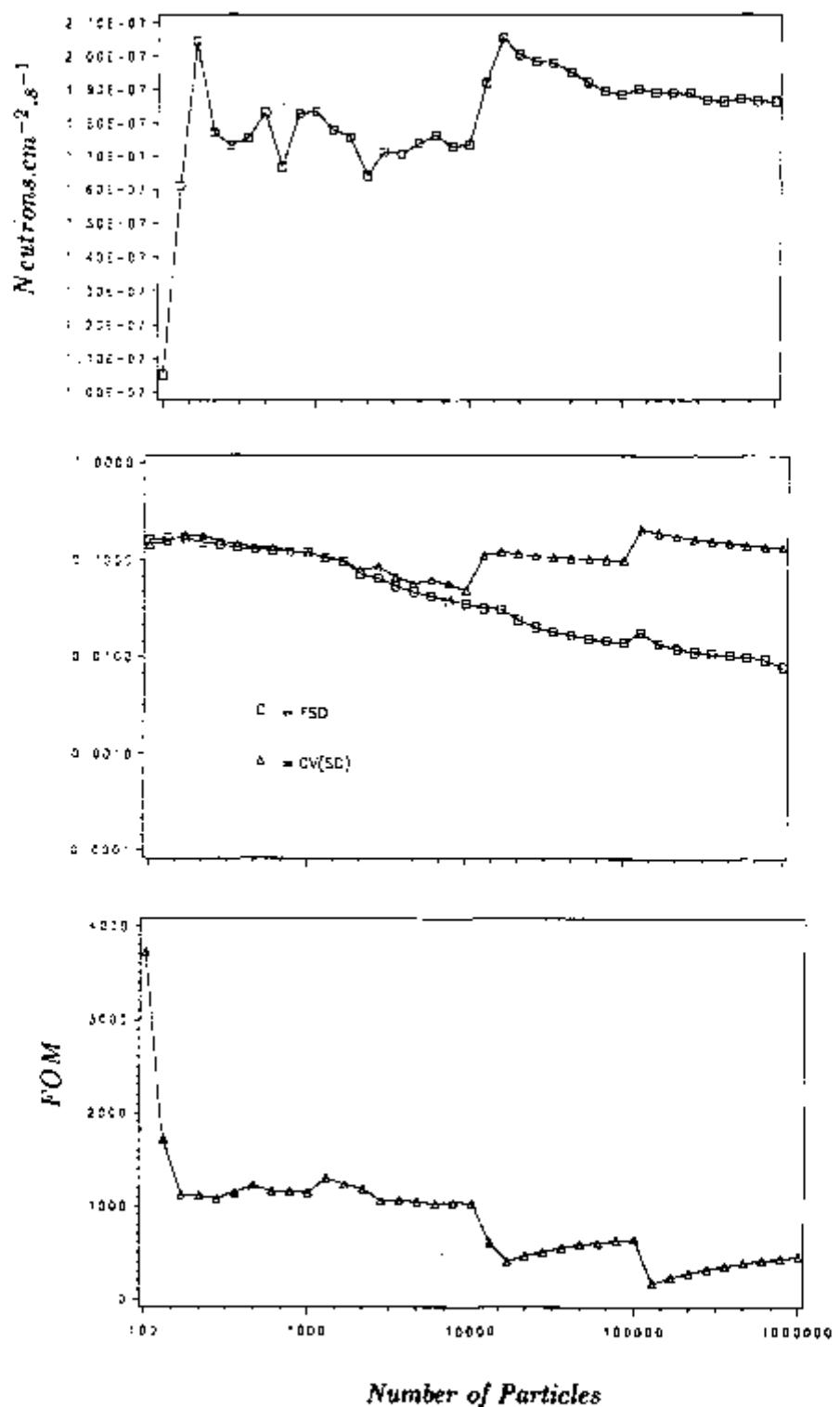


Figure 4.2: Neutron Flux,  $FSD$ ,  $CV(SD)$ , and  $FOM$  Behavior for Detector 6 – NESXE.

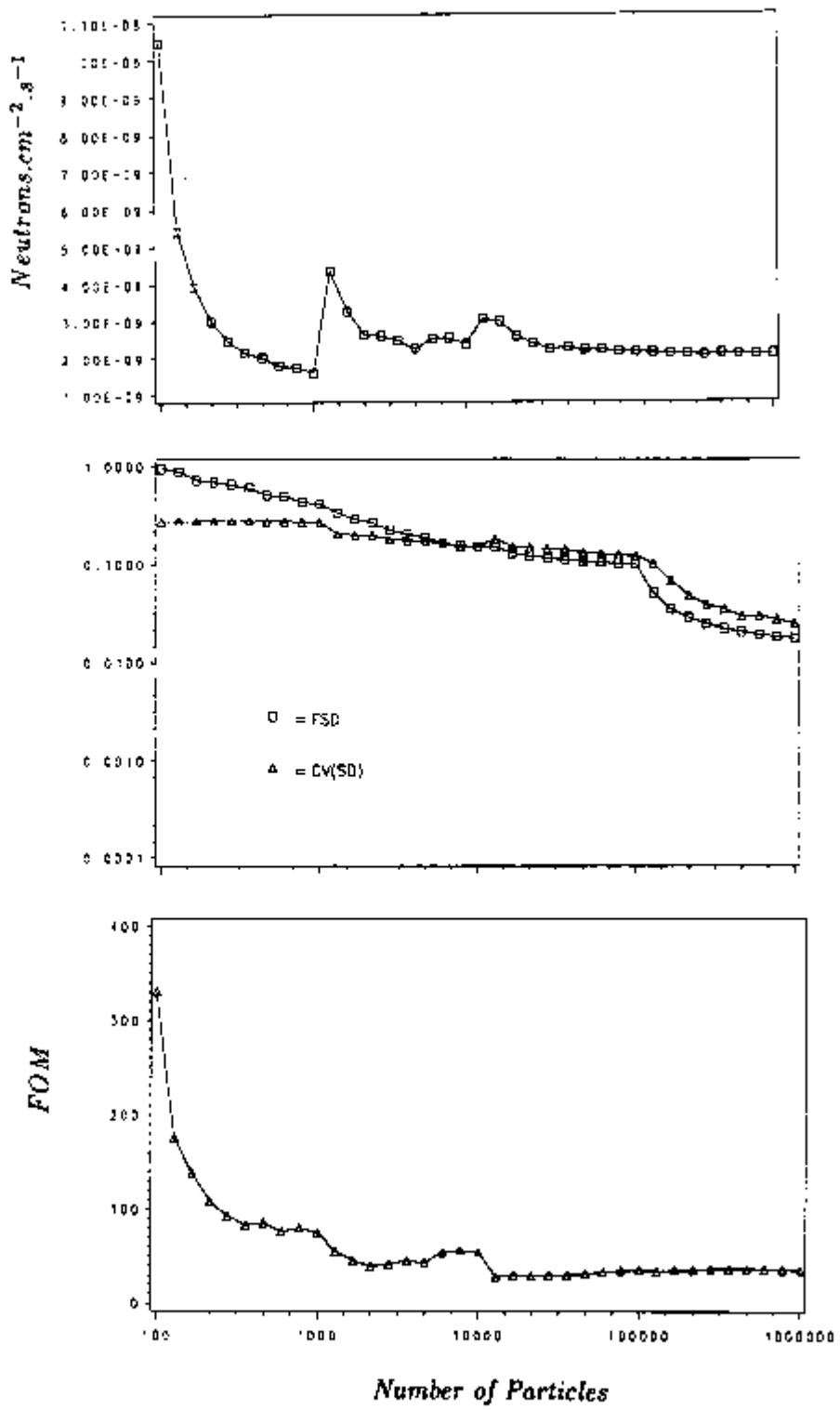


Figure 4.3: Neutron Flux,  $FSD$ ,  $CV(SD)$ , and  $FOM$  Behavior for Detector 10 – NESXE.

Table 4.2 shows the results for three phases of the calculation. The three phases consisted of 10 batches each with  $10^3$ ,  $10^4$ , and  $10^5$  particles per batch respectively. The F tests present some high values that indicate major differences between the group means. For example, the F value is 1.99945 for Detector 6 in the range of  $10^4$  to  $10^5$  particles. It is possible to see in Figure 4.2 that effectively there are large differences between the group means. However, it should be noted that the solution has converged to the true solution and the standard deviation is also sufficiently small. In the case of Detector 7 in the range of  $10^5$  to  $10^6$  particles the standard deviation is definitely small enough to guarantee the results — which means that this parameter can also reject perfectly acceptable solutions.

#### 4.1.2 Point-Detector Estimator Solution

The utilization of point-detector estimators (PDE) significantly increases the degree of difficulty in the solution of this problem. In fact, the effects of undersampling can be easily seen when the point detector solutions are compared with the solutions described in Section 4.1.1.

Figures 4.4, 4.5, and 4.6 show the behavior of the neutron flux,  $FSD$  and  $CV(SD)$ , and the figure of merit with increasing values for the sample size for detectors 2, 6, and 10 respectively. In the case of Detector 2 the undersampling condition yielded underestimation for sample sizes up to 200,000 particles and for Detector 6 for sample sizes in the range of 2,000 to 500,000 particles. For both detectors the  $FOM$  exhibits stable plateaus in the undersampled region. In the case of Detector 10, the simulation is so undersampled that is possible to see the peaks in Figure 4.6 which are caused by contributions from individual particles. It is interesting to note from Appendix A.1.2 that the particle contribution distribution for Detector 6 contained more particles in the higher channels in batch 5 than in batch 10. This is an effect of the sampling of a few important particles that shifted the distribution and minimized the undersampling condition.

Table 4.2: F Values for Sample Problem 1 with Next-Event Estimator.

Number of Particles	Detector	Total Response	FSD Batch	FSD Accum.	F Value
$10^3 - 10^4$	1	3.9502D-04	0.01518	0.02069	0.53761
	2	4.3966D-05	0.01400	0.01480	0.89466
	3	8.6957D-06	0.01311	0.02216	0.34984
	4	2.0963D-06	0.03622	0.03055	1.40545
	5	6.2572D-07	0.02918	0.05013	0.33860
	6	1.7343D-07	0.04160	0.05967	0.48560
	7	5.7541D-08	0.11905	0.09939	1.43532
	8	2.0712D-08	0.12643	0.13236	0.91234
	9	8.9238D-09	0.26682	0.23243	1.31807
	10	2.3464D-09	0.27192	0.25963	1.09688
$10^4 - 10^5$	1	3.8398D-04	0.00394	0.00327	1.45084
	2	4.4619D-05	0.00422	0.00526	0.64334
	3	8.9053D-06	0.00626	0.00679	0.84945
	4	2.2075D-06	0.01124	0.01134	0.98125
	5	6.2544D-07	0.00921	0.01497	0.37831
	6	1.8826D-07	0.03401	0.02405	1.99945
	7	6.0974D-08	0.02165	0.03847	0.31681
	8	2.0435D-08	0.04259	0.04535	0.88189
	9	6.3437D-09	0.05786	0.06413	0.81423
	10	2.1429D-09	0.11009	0.10191	1.16697
$10^5 - 10^6$	1	3.8319D-04	0.00079	0.00122	0.42173
	2	4.4523D-05	0.00168	0.00200	0.69934
	3	8.9761D-06	0.00289	0.00343	0.70744
	4	2.2398D-06	0.00626	0.00637	0.96701
	5	6.2468D-07	0.00535	0.00572	0.87482
	6	1.8613D-07	0.00883	0.00907	0.94718
	7	5.8928D-08	0.01453	0.01125	1.66750
	8	1.9024D-08	0.01464	0.01860	0.62002
	9	6.0442D-09	0.01903	0.02137	0.79305
	10	2.0664D-09	0.02113	0.03236	0.42638

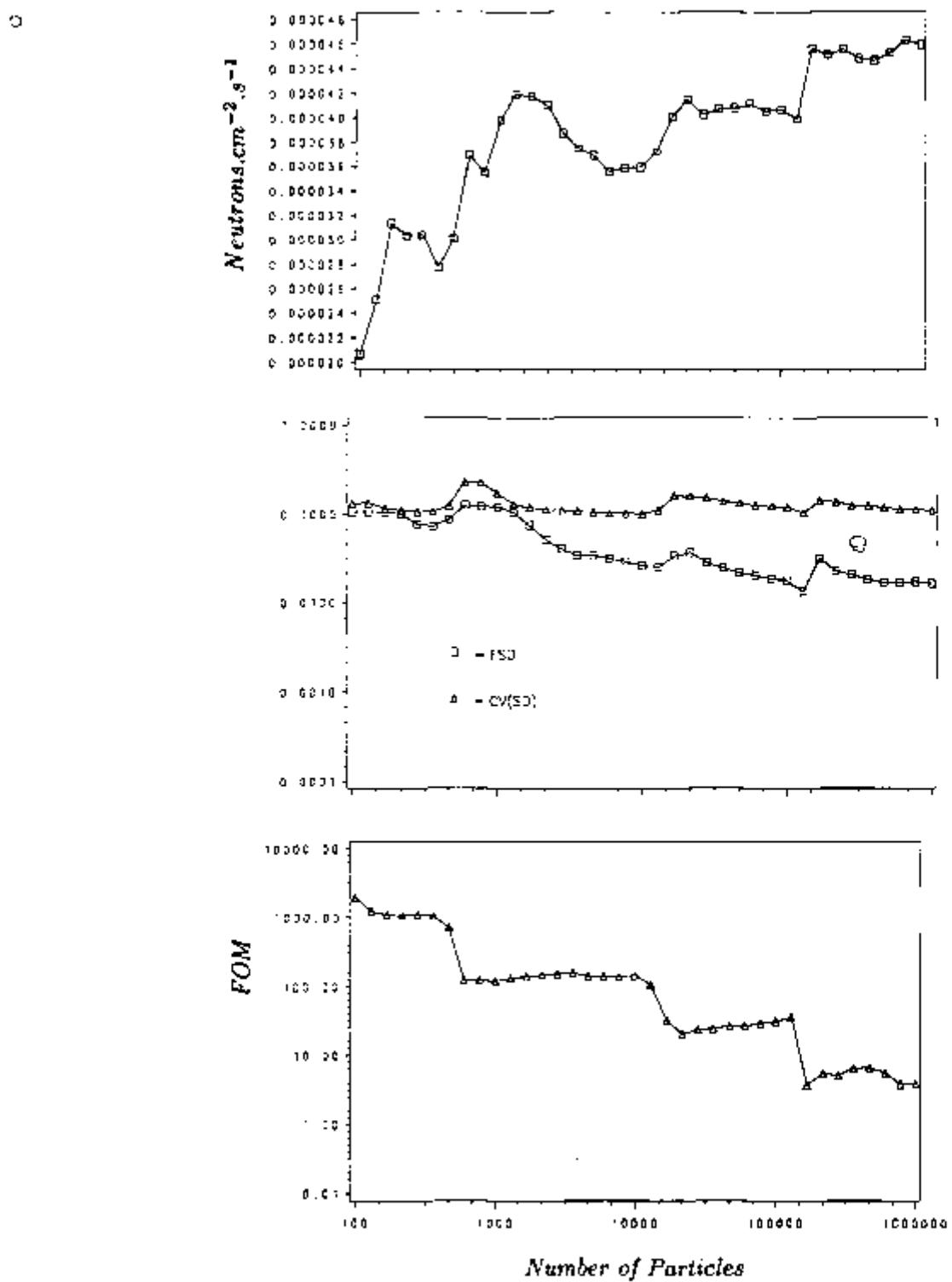


Figure 4.4: Neutron Flux,  $FSD$ ,  $CV(SD)$ , and  $FOM$  Behavior for Detector 2 – PDE.

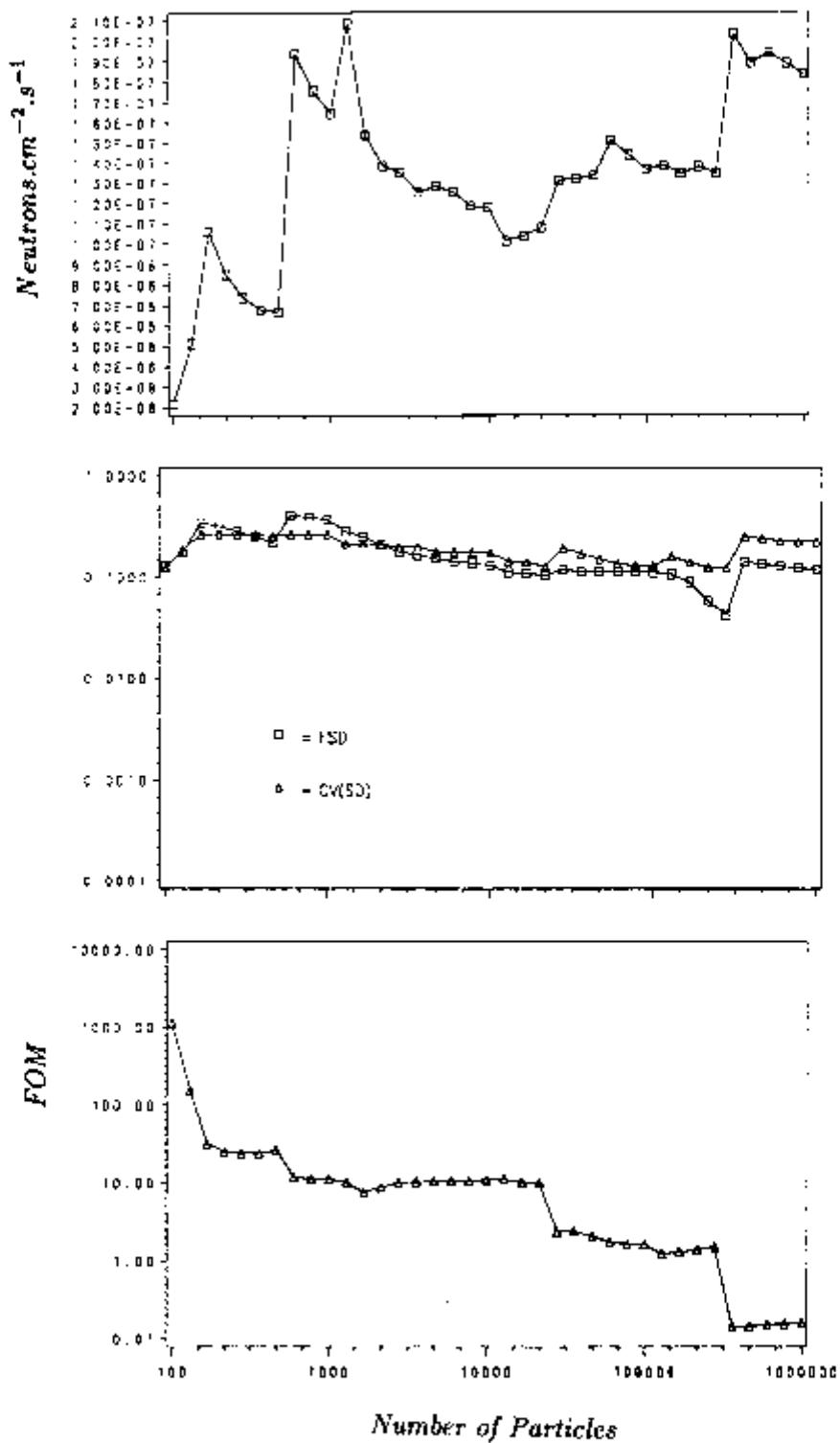


Figure 4.5: Neutron Flux,  $FSD$ ,  $CV(SD)$ , and  $FOM$  Behavior for Detector 6 – PDE.

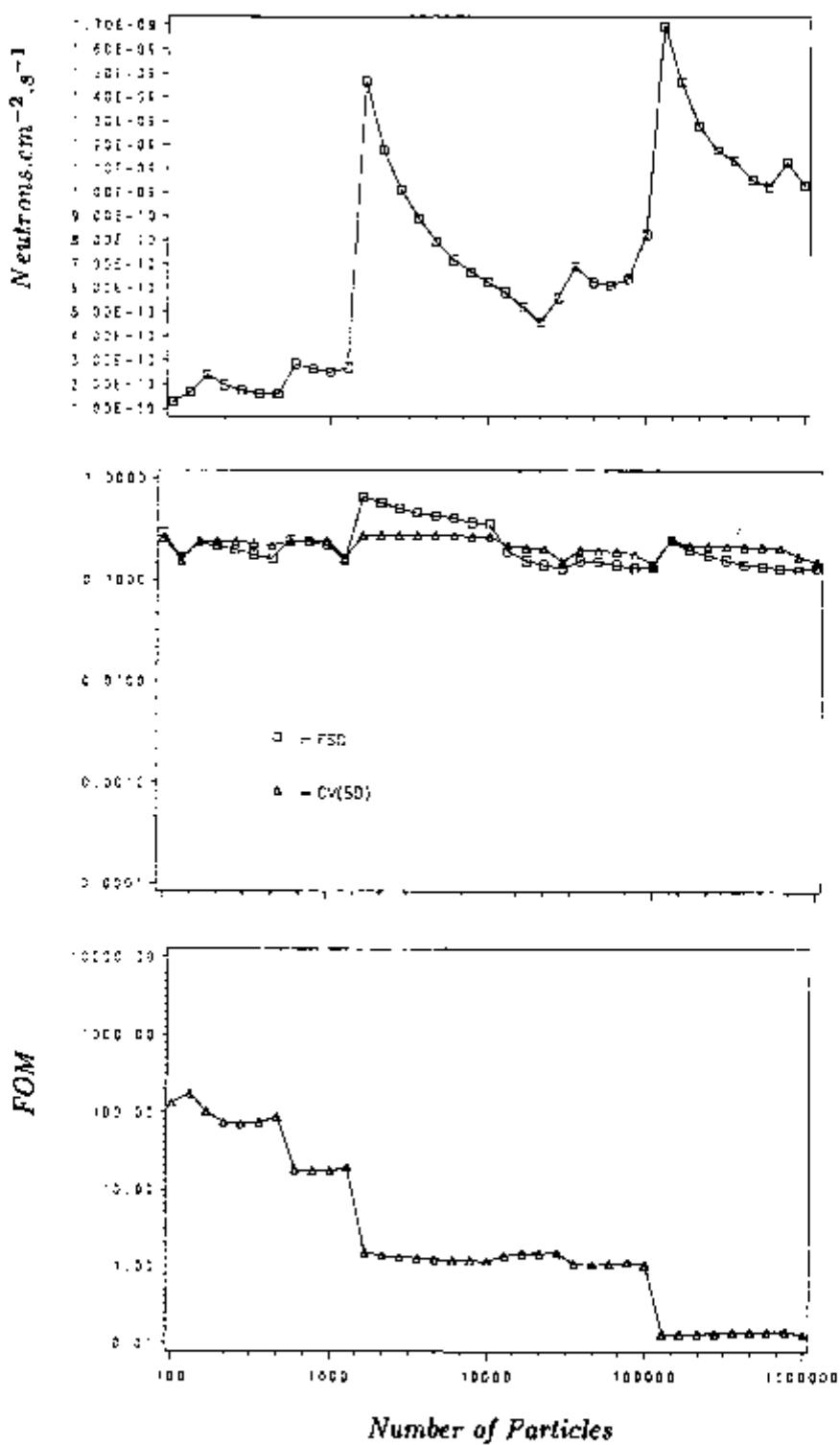


Figure 4.6: Neutron Flux,  $FSD$ ,  $CV(5D)$ , and  $FOM$  Behavior for Detector 10 - PDE.

Appendix A.1.2 shows the output for 10 batches of  $10^5$  particles each. For the first batch, detectors 1 and 2 are clearly acceptable solutions on the basis of the behaviors of their standard deviations alone. Detectors 3 and 4 present peaks in their standard deviation and  $CV(SD)$  and are very close to the true solutions<sup>1</sup> although they exhibit somewhat large standard deviations. The distribution of particle contributions for all detectors beyond Detector 2, shows that the number of particles accounting for large fractions of the solutions is becoming too small.

In Batch 4, the peaks in the standard deviation and  $CV(SD)$  shifted to Detector 5. The distribution of particle contributions for this detector shows a relatively small number of particles accounting for a large fraction of the response which usually characterizes the undersampling condition. Finally, in Batch 10 the peak shifted to Detector 6 — again, a relatively small number of particles account for a large fraction of the response.

Based on these results and the discussion above, the solutions for detectors 1, 2, 3, 4 and 5 can be considered reliable. Detector 6 represents the beginning of what will be called the *breakdown region*, which is associated with a sample size that provides either only a few or no important rare event making the standard deviations — which can be very small — experience large increases. Obviously, all detectors beyond this point will be undersampled unless additional particles are processed. It is interesting to observe the behavior of the solution at Detector 7 which initially overestimates the true solution because of a highly contributing particle that was sampled early in the calculation.

An important fact, as seen in Appendix A.1.2, is that the particle contribution histogram always included a small number of particles which accounted for large fractions of the solutions for the undersampled detectors. Therefore, the histogram is able to detect the undersampling condition for both the breakdown region and the region of severe undersampling. In this problem, the explanation for this behavior is that even in the severe undersampling condition, there are some particles that make contributions only large

---

<sup>1</sup> True solutions are considered the solutions obtained in Section 4.1.1 with  $10^6$  particles.

enough to make them stand out from the background contributions, and as the sample size increases these contributions will eventually move to the intermediary channels.

Figure 4.7 shows a three-dimensional view of the standard deviation and of the coefficient of variation of the standard deviation. It can be seen that the plot of the coefficient of variation is much more structured than that of the standard deviation providing a qualitative rather than quantitative indicator of the behavior of the solution.

Figure 4.8 shows the same calculation as in Figure 4.7 but with the last random number of that calculation used as the initial one in this calculation. A  $FSD$  comparison shows good agreement up to Detector 6 and a  $CV(SD)$  comparison shows good agreement only up to Detector 3 which demonstrates that this indicator is too sensitive to be taken as a quantitative measure of precision. Also, from figures 4.7 and 4.8, it can be assumed that Detector 10 represents the end of the breakdown region or the beginning of the region of severe undersampling. Figure 4.9 shows the calculations using the exponential transform technique with  $PATH = 0.5$  and Figure 4.10 shows the results using source angular biasing, Russian roulette, splitting, and exponential transform. It is possible to see that the variance reduction techniques significantly reduce the variability of  $CV(SD)$  making it less structured. Because this behavior may indicate the effectiveness of a given variance reduction technique and also that the sampling distribution has become sufficiently stable, it was concluded that the use of this coefficient needs further study.

Table 4.3 shows the calculated F values after 10 batches are completed. The two values that flag greater differences in the batch means are for Detector 2 and Detector 6. These results illustrate the weakness of the test in discerning acceptable from unacceptable solutions. The test detected that there are significant differences in the group means of Detector 6. Therefore, the results should be checked with other indicators to determine if they are acceptable. Otherwise it is necessary to increase the batch size.

In the case of Detector 2 the explanation can be based on the fact that although it is unlikely, there are values in the tails of the normal distribution. And the null hypothesis

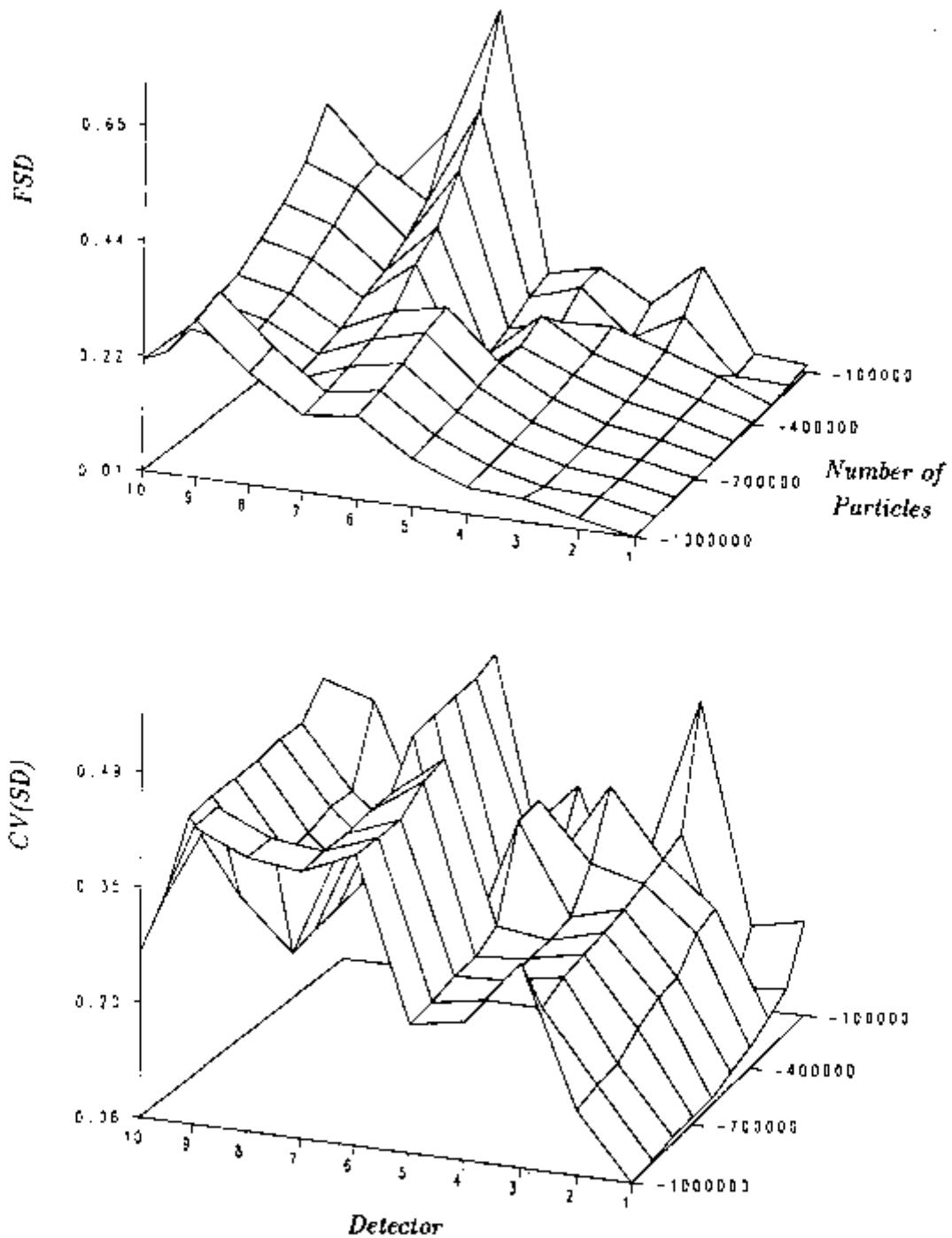


Figure 4.7: Three-Dimensional Behavior of  $FSD$  and  $CV(SD)$  for Sample Problem 1 – PDE.

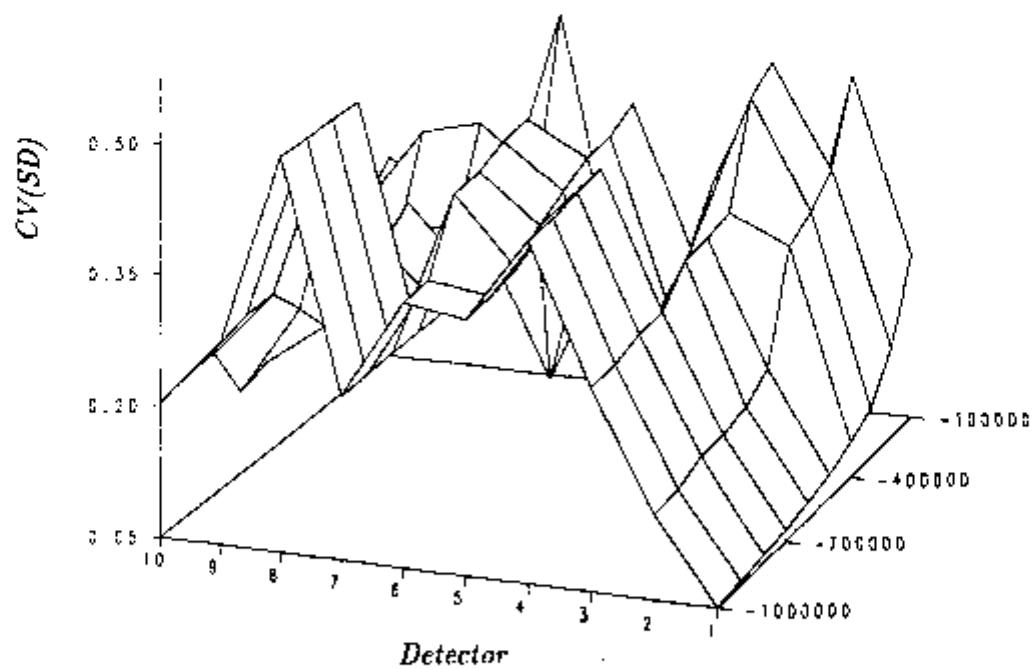
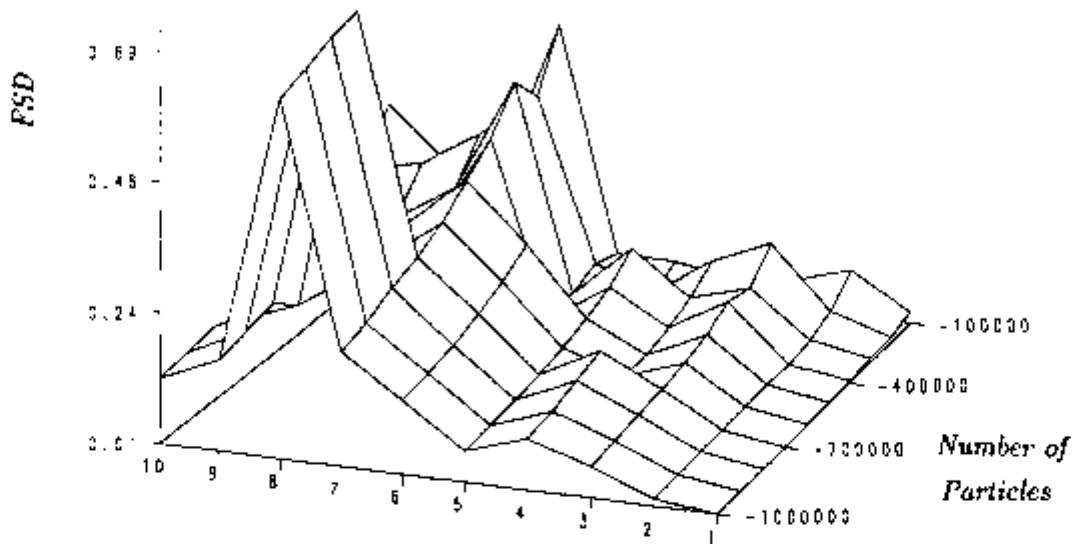


Figure 4.8: Behavior of  $FSD$  and  $CV(SD)$  for Sample Problem 1 - PDE with New Starting Random Number.

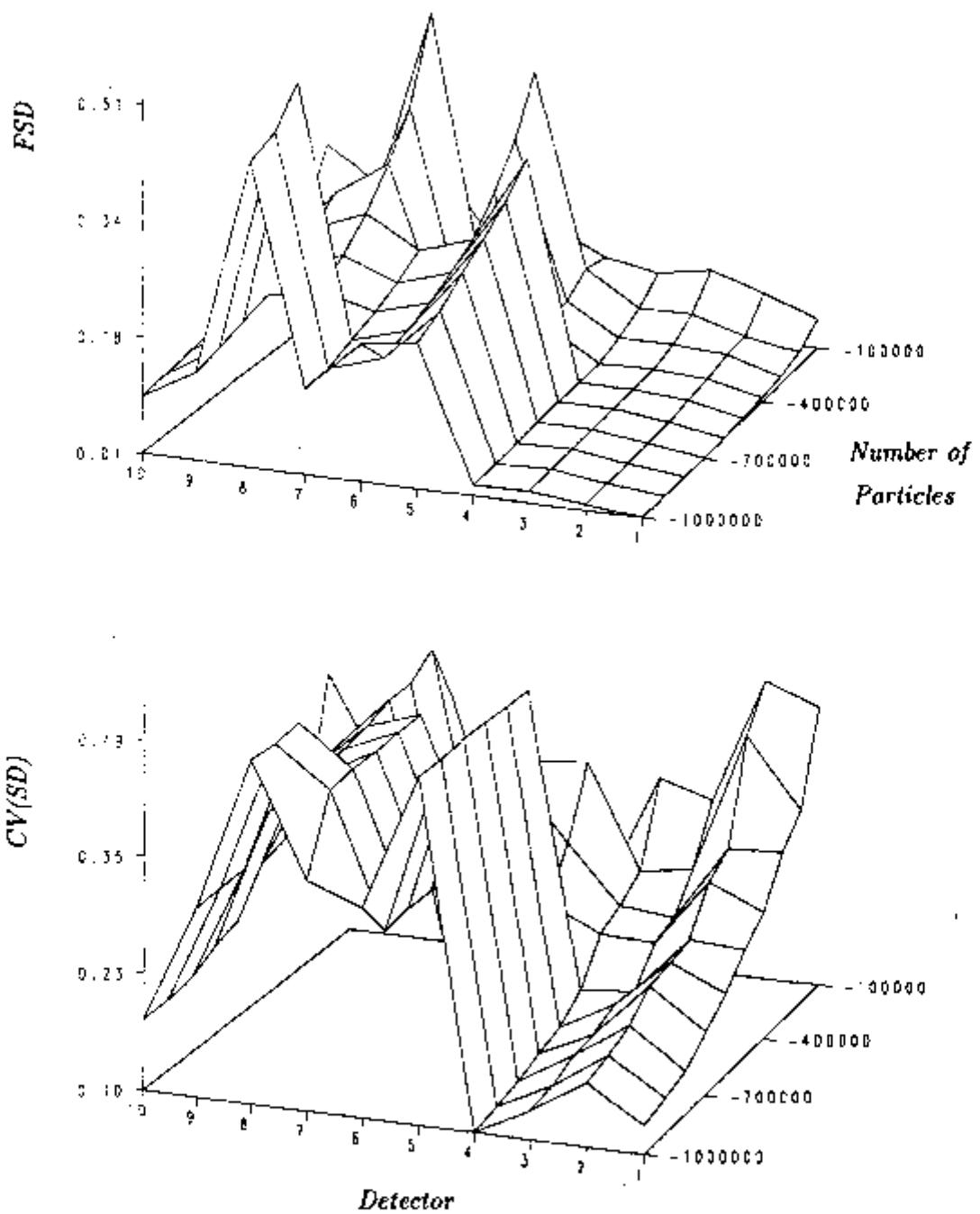


Figure 4.9: Behavior of *FSD* and *CV(SD)* for Sample Problem 1 – with *PATH* = 0.5.

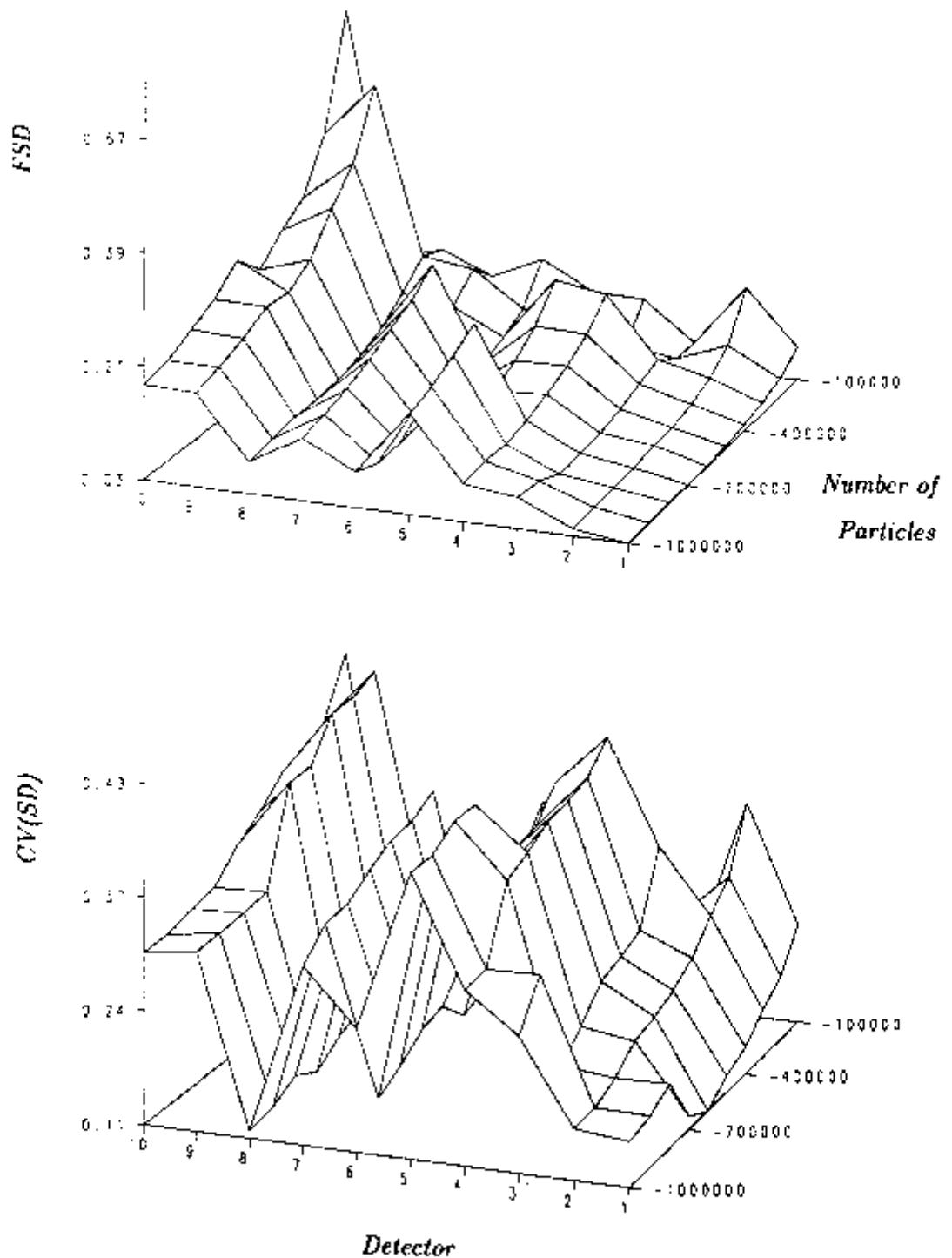


Figure 4.10: Behavior of  $FSD$  and  $CV(SD)$  for Sample Problem 1 – using Russian Roulette, Splitting, and  $PATH = 0.5$ .

Table 4.3: F Values for Sample Problem 1 with Point-Detector Estimator.

Number of Particles	Detector	Total Response	FSD Batch	FSD Accum.	F Value
$10^3 - 10^4$	1	3.5122D-04	0.02451	0.03081	0.63240
	2	3.5908D-05	0.05013	0.04783	1.09872
	3	1.4941D-05	0.30124	0.30203	0.99466
	4	3.4210D-06	0.38488	0.39837	0.93329
	5	1.0802D-06	0.56001	0.57999	0.93216
	6	1.1766D-07	0.16116	0.20402	0.62366
	7	3.4278D-08	0.34404	0.38522	0.79743
	8	1.5081D-08	0.68582	0.69850	0.96390
	9	2.6313D-09	0.59333	0.60736	0.95422
	10	6.2342D-10	0.57851	0.59306	0.95140
$10^4 - 10^5$	1	3.8195D-04	0.01798	0.02420	0.55218
	2	4.0657D-05	0.03121	0.03220	0.93898
	3	1.0318D-05	0.19179	0.18977	1.02138
	4	2.2010D-06	0.12374	0.10484	1.39315
	5	5.8018D-07	0.16108	0.16399	0.96486
	6	1.3670D-07	0.14838	0.13992	1.12463
	7	1.0456D-07	0.71689	0.65115	1.21213
	8	3.6075D-08	0.48367	0.39992	1.46273
	9	4.4715D-09	0.30049	0.28794	1.08911
	10	8.1909D-10	0.26011	0.22073	1.38867
$10^5 - 10^6$	1	3.9006D-04	0.00957	0.00912	1.10126
	2	4.5916D-05	0.04192	0.02979	1.97938
	3	9.0757D-06	0.04628	0.05148	0.80815
	4	2.1204D-06	0.05785	0.05826	0.98588
	5	7.0483D-07	0.09354	0.09736	0.92293
	6	1.8347D-07	0.22707	0.16347	1.92943
	7	5.0838D-08	0.16119	0.15096	1.14010
	8	1.7408D-08	0.20189	0.21587	0.87468
	9	5.5855D-09	0.26034	0.29830	0.76171
	10	1.0233D-09	0.22022	0.21594	1.04000

actually claims a uniform distribution for the group means. For the last four detectors which are undersampled and exhibit some underestimation, the test fails to show any differences between the means because in the undersampling regime the group means are likely to be very close to each other. As the sample size increases and some heavy contributing particles are sampled larger differences in the group means are observed. Also, the application of F tests as mentioned in Chapter 2 requires that the sampling distribution should be normally distributed. If the kurtosis is too high the tests will tend to be too small [6] and the hypothesis of equality of the means can not be tested.

## 4.2 Sample Problem 2

This problem belongs to a set of sample problems that comes with the MORSE code package. It comprises of a point fission source in air and uses a surface crossing estimator in the analysis. Appendix A.2 shows the output for the problem using variance reduction techniques with the same parameters as in the MORSE manual. It is interesting to note that most of the  $CV(SD)$  values are well within commonly acceptable values. Also, the distribution of particle contributions exhibits a more uniform dispersion of particles in the percentage bins.

Figure 4.11 shows the behavior of both  $FSD$  and  $CV(SD)$ . This figure also shows that although the kurtosis of the distribution eventually becomes constant, that happens long after an acceptable solution has been achieved.

## 4.3 Sample Problem 3

This problem was solved to demonstrate the behavior of the solutions in a more complex geometry. The configuration consists of a concrete cylinder of 150.2-cm height and a 150.0-cm radius with a cylindrical duct of 7.62-cm radius placed along the main axis. A 14-Mev neutron source is positioned along the bottom side of the cylinder and emits neutrons with

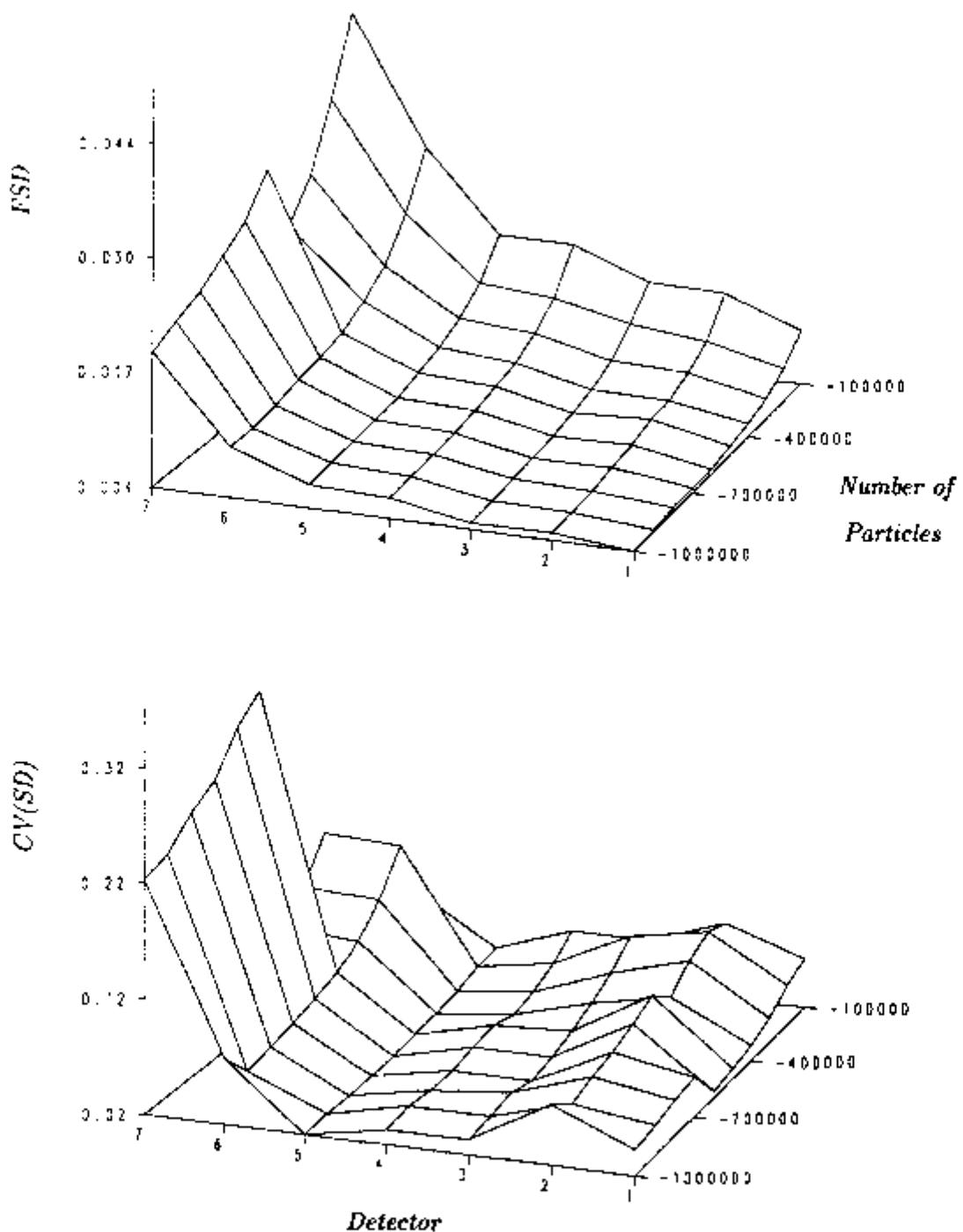


Figure 4.11: Three-Dimensional Behavior of  $FSD$  and  $CV(SD)$  for Sample Problem 2.

directions uniformly distributed in the hemisphere facing the cylinder. Also, only the top fourteen groups are analyzed.

Because of the characteristics of the problem, source biasing provides large variance reductions which is accomplished using a step importance function for the position of emission which samples particles within a 10-cm radius 1000 times more often than outside this radius.

Appendix A.3.1 shows the data without source biasing and Appendix A.3.2 with source biasing. The effect of the source biasing is to decrease the size of the background contributions. It is interesting to see the behavior of the *FOM* in this problem. Figure 4.12 shows that although the step biasing procedure increased the efficiency of the calculation, the figure of merit of the problem without step biasing has a much more constant structure.

Figure 4.12 also shows that an increase in efficiency in the calculation may not reflect a smaller  $CV(SD)$  which again presents very little information about the behavior of the solution.

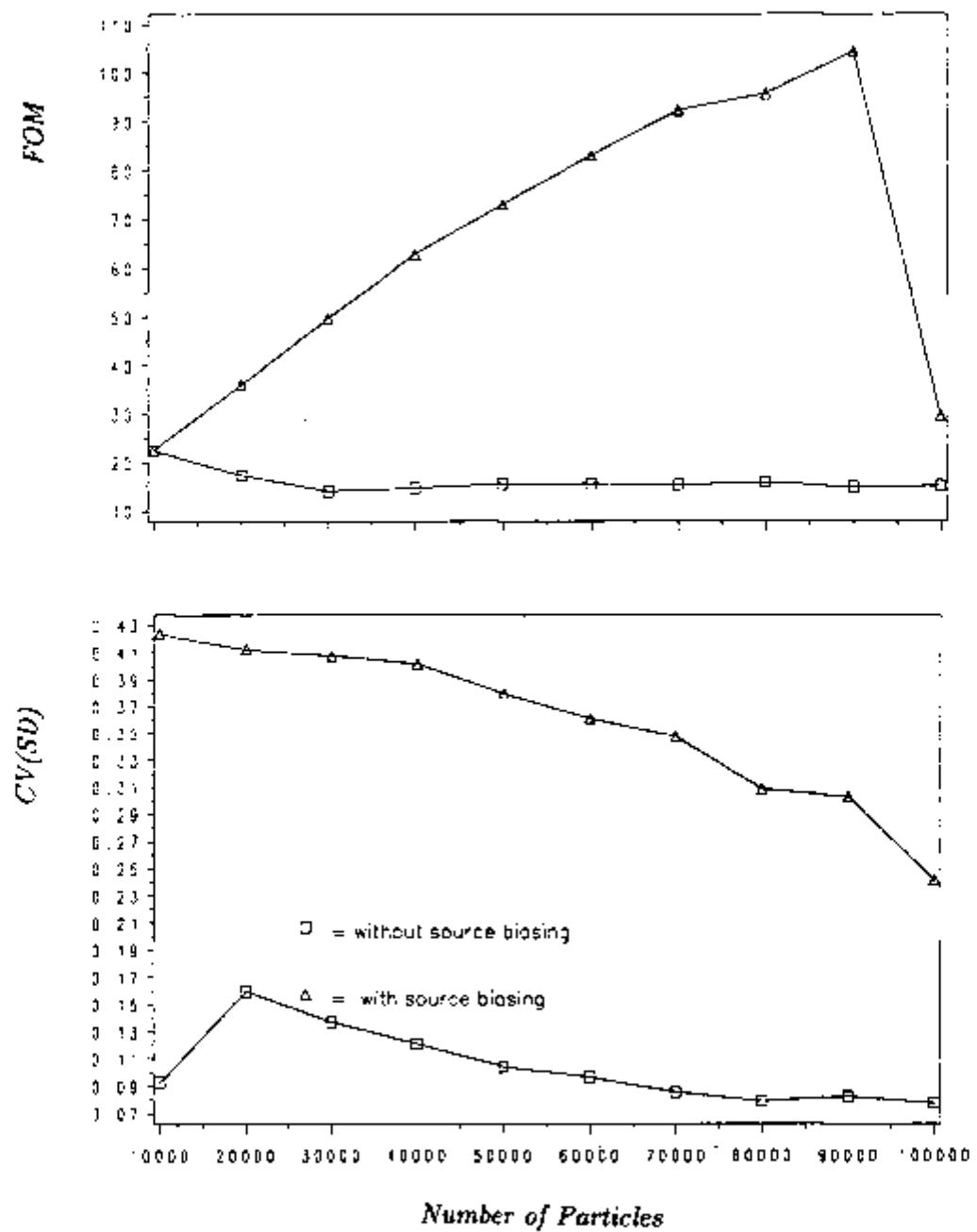


Figure 4.12: *FOM* and *CV(SD)* Behavior for Sample Problem 3.

## Chapter 5

# Conclusions

In this work several potential techniques to detect undersampling conditions were investigated.

The utilization of F tests failed to be conclusive because the contributions were not normally distributed. However, as a recommendation for future work, it could be used if the samples were drawn from a population of group means which have a more normal distribution. Also, problems that can be solved with a purely analog Monte Carlo calculation would have binomial distribution of the scores. In this case, F tests could be applied for other purposes rather than to detect undersampling conditions since this condition rarely occurs in such problems. For the reasons explained above, the utilization of F tests is not recommended without more study.

The calculation of the coefficient of variation of the standard deviation which provides confidence limits for the standard deviation estimate was found to be too expensive for deep penetration calculations because of its strong dependence upon the kurtosis of the contribution distribution. It also yields smaller values in the presence of undersampling which eventually increase with the sampling of important particles. Therefore, its utilization was found to be not too productive.

The benefits of calculating the figure of merit are twofold, first it allows an easy comparison of the efficiencies of different schemes and also is an indicator that the variance of the population has become stable. Although the *FOM* may stay essentially constant in

---

severe undersampling conditions, once some important particles are processed it becomes an effective means to verify the stability of the estimates. Therefore, its usage is recommended.

The particle contribution distribution histogram was found to be the most effective way of detecting undersampling. In the breakdown region and more important — under severe undersampling conditions — the number of particles accounting for large fractions of the response is very small. This condition is easily identified from the histogram. This analysis does not heavily depend upon previous observations and is much faster and more reliable than any other means. It also provides a measure of the effectiveness of variance reduction techniques — which generally have the net effect of spreading out the values of the contributions producing a smoother distribution. Its utilization is highly recommended.

From the discussion above, it became evident that the most effective means to guarantee the reliability of an estimate in problems subject to undersampling is to know and characterize the population distribution. Therefore, the ability of the Monte Carlo method to simulate detailed descriptions of the population distribution must be exploited by the user — this is greatly facilitated by the new version of the MORSE code, MORSE/STAT — developed during the course of this research. This version of the code provides much more insight into the statistical quality of the solution obtained. The observation of the behavior of the new parameters as the solution evolves is a powerful means of determining if the undersampling condition exists. Because the standard version of the code provides only the standard deviation at the end of a complete run, the user loses all the interim information that the new version now provides.

The use of graphics for the presentation of the evolution of the statistics being estimated would enormously facilitate the observation of the behavior of the solution of the problem. Therefore, for future work, the coupling of a graphics package is highly recommended.

## **BIBLIOGRAPHY**

## Bibliography

- [1] E. L. Gelbard. Unfinished Monte Carlo business. In *Proceedings ANS/ENS Topical Meeting of Advances in Mathematical Methods for the Solution of Nuclear Engineering Problems*, 1981.
- [2] S. N. Cramer, J. Gonnord, and J. S. Hendricks. Monte Carlo techniques for analyzing deep-penetration problems. *Nuclear Science and Engineering*, 92, 1986.
- [3] A. Dubi. On analysis of the variance in Monte Carlo calculations. *Nuclear Science and Engineering*, 72, 1979.
- [4] I. Lux and Z. Zatmary. Combined estimation of a common mean from few sample sets and from sets of rare events. *Nuclear Science and Engineering*, 89, 1985.
- [5] RSIC Computer Code Collection CCC-474. *MORSE-CGA - A General Purpose Monte Carlo Multigroup Neutron and Gamma-Ray Transport Code System with Array Geometry Capability*. Oak Ridge National Laboratory, October 1987.
- [6] H. R. Lindman. *Analysis of Variance in Complex Experimental Designs*. W. H. Freeman and Company, San Francisco, 1974.
- [7] M. H. Hansen, W. N. Hurwitz, and W. G. Madow. *Sample Survey Methods and Theory*. John Wiley and Sons, Inc., New York, 1953.
- [8] A. Dubi, T. Elperin, and H. Rief. On confidence limits and statistical convergence of Monte Carlo point-flux estimators with unbounded variance. *Annals of Nuclear Energy*, 9, 1982.

## **APPENDICES**

# **Appendix A**

## **Batch Output**

### **A.1 Sample Problem 1**

#### **A.1.1 Next Event Surface Crossing Estimator**

\*\*\*START BATCH 1

RANDOM-13579HDFB97

SOURCE DATA

YOU ARE USING THE DEFAULT VERSION OF SOURCE WHICH SETS RATE TO DOP AND PROVIDES AN ENERGY IC.  
 YOU ARE USING THE DEFAULT VERSION OF GTM6 WHICH ASSUMES GEOMETRY AND XSELECT MEDIA ARE IDENTICAL.

WAVE	WAVE	XAVE	YAVE	ZAVE	RATE	AGRAVE
1.000E+05	1.00	0.0009	0.0004	0.0013	0.0	0.0
PARTICLES PER CM**2 PER SECOND						

47 DETECTOR

DETECTOR	BATCH RESPONSE	RESPONSE(DETECTOR)	NEUTRON FLUX	P(SD)	CV(SD)	PSD
1	3.8398D-04	0.00127	3.8398D-04	0.00327	0.17907	3.5055D+04
2	4.4619D-05	0.00526	4.4619D-05	0.00526	0.10571	1.3518D+04
3	8.9053D-06	0.00679	8.9053D-06	0.00679	0.04823	8.1330D+03
4	2.2075D-06	0.01134	2.2075D-06	0.01134	0.12660	2.9112D+03
5	6.2544D-07	0.01497	6.2544D-07	0.01497	0.05511	1.6710D+03
6	1.8826D-07	0.02405	1.8826D-07	0.02405	0.10014	6.4760D+02
7	6.0974D-08	0.03847	6.0974D-08	0.03847	0.21179	2.5318D+02
8	2.0435D-08	0.04535	2.0435D-08	0.04535	0.06699	1.8214D+02
9	6.3437D-09	0.06413	6.3437D-09	0.06413	0.09174	9.1108D+01
10	2.1429D-09	0.10191	2.1429D-09	0.10191	0.17910	3.6075D+01

47 DETECTOR PARTICLE DISTRIBUTION IN PERCENTAGE BINS RELATIVE TO INCREASING VALUE OF CONTRIBUTIONS

DETECTOR	0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100.0
1	66060	0	0	0	0	10600	10902	1821	5249	3102	186
2	56420	0	18868	7335	6219	9127	0	0	0	1586	445
3	53727	16856	18827	0	3611	2010	1460	1905	1544	60	
4	65088	17609	7038	4878	3029	0	1162	642	297	757	
5	79683	11774	4679	1275	1088	1026	0	313	102	60	
6	B3737	11062	3458	453	612	286	337	0	0	55	
7	93476	4674	1255	156	103	170	76	81	0	9	
8	96354	2566	602	225	83	47	48	26	28	1	
9	98235	1351	188	100	62	26	15	6	12	3	
10	99334	521	36	52	36	0	6	7	5	1	

NUMBER OF COLLISIONS OF TYPE NCOLL  
 SOURCE SPLIT(D) FISHN GAGEN REACCOLL ALBEDO BDRX ESCAPE E-CUT TIMERILL R R KILL R R SURV GAMOST  
 100000 0 0 0 169483 0 98727 46 99954 0 0 0 0

TIME REQUIRED FOR THE PRECEDING BATCH WAS 2 MINUTES, 40 SECONDS.

\*\*\*START BATCH 2

RANDOM=24F04D19FDAB

SOURCE DATA	JAVE	UAVE	VAVE	WAVE	XAVE	YAVE	ZAVE	AGEAVE
WTAVE 1.00E+05	1.00	-0.0003	0.0039	0.00400	0.0	0.0	0.0	0.0
PARTICLES PER CM**2 PER SECOND								

DETECTOR	BATCH RESPONSE	RESPONSE(FSD)	ACUMULATED RESPONSE	NEUTRON FLUX	FSD	CV(5D)	POM
1	3.8273D-04	0.00311	3.8335D-04	0.00226	0.11038	3.6803D-04	
2	4.4120D-05	0.00439	4.4370D-05	0.00343	0.06929	1.5921D-04	
3	8.8814D-06	0.00746	8.8933D-06	0.00504	0.06061	7.3797D-03	
4	2.2191D-06	0.01225	2.2133D-06	0.00835	0.08322	2.6908D-03	
5	6.0900D-07	0.01549	6.1722D-07	0.01077	0.06376	1.6179D-03	
6	1.9154D-07	0.06106	1.8990D-07	0.03302	0.38768	1.7202D-02	
7	5.7283D-08	0.03704	5.9128D-08	0.02675	0.14203	2.6218D-02	
8	1.0250D-08	0.04534	1.0443D-08	0.03340	0.05806	1.6809D-02	
9	5.6615D-09	0.06451	6.0026D-09	0.04563	0.06415	9.0066D-01	
10	2.1078D-09	0.10720	2.1253D-09	0.07393	0.10690	3.4318D-01	

DETECTOR PARTICLE DISTRIBUTION IN PERCENTAGE BINS RELATIVE TO INCREASING VALUE OF CONTRIBUTIONS

	0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100.0
1	132184	0	0	0	21311	12256	17050	10461	6386	346	
2	112869	0	26190	26281	12466	18104	0	0	3205	885	
3	107469	33677	37648	0	7256	4132	2632	3807	3068	111	
4	130292	35146	34137	9807	5953	0	2273	1274	589	535	
5	159478	23730	9132	2533	2137	2073	0	614	0	303	
6	170370	20745	5437	913	1206	650	575	0	68	36	
7	185365	10233	3218	0	551	246	192	177	0	18	
8	192784	5137	1154	502	152	88	42	51	51	4	
9	196529	2694	269	334	156	0	49	41	22	6	
10	198697	1028	67	106	58	0	16	13	12	3	

NUMBER OF COLLISIONS OF TYPE NCOLL  
SOURCE SPLIT(D) PISHN GAMGEN REACOLL ALBEDO BDRYX ESCAPE  
100000 0 0 189052 0 98516 53 99947 0 0 0 0 0

TIME REQUIRED FOR THE PRECEDING BATCH WAS 2 MINUTES, 39 SECONDS.

\*\*\*START BATCH 0

RANDOM-3E1275B60730

SOURCE DATA	XAVE	YAVE	ZAVE	XAVE	YAVE	ZAVE	AGEAVE
1.000E105	1.00	-0.0005	-0.0004	0.0013	0.0	0.0	0.0

PARTICLES PER CM\*\*2 PER SECOND

DETECTOR	BATCH RESPONSE	RESPONSES (DYNAMIC) USD	NEUTRON FLUX ACUMULATED RESPONSE	FSD	CV( SD )	POW
1	3.0373D-04	0.00305	3.8317D-04	0. JU141	0.16056	2.3771D+04
2	4.4566D-05	0.00523	4.4517D-05	0. JU0230	0.20914	6.9237D+03
3	9.0082D-06	0.00783	8.9607D-06	0. JU0282	0.05729	5.9278D+03
4	2.2733D-06	0.01736	2.2431D-06	0. JU0770	0.35579	7.9330D+02
5	6.2939D-07	0.01752	6.2615D-07	0. JU0657	0.11378	1.0905D+03
6	1.9336D-07	0.02525	1.8713D-07	0. JU0666	0.24013	4.1411D+02
7	6.4693D-08	0.03233	5.3507D-08	0. JU1278	0.06407	2.6792D+02
8	1.9984D-08	0.04316	1.9247D-08	0. JU2064	0.16855	1.1039D+02
9	6.4010D-09	0.06625	6.1403D-09	0. JU2373	0.03558	8.3492D+01
10	2.0340D-09	0.10733	2.0756D-09	0. JU3504	0.05307	3.6615D+01

DETECTOR	PARTICLES DISTRIBUTION IN PERCENTAGE BINS RELATIVE TO INCREASING VALUE OF CONTRIBUTIONS
0.0	0.0 10.0 20.0 30.0 40.0 50.0 60.0 70.0 80.0 90.0 100.0
1	529853 0 0 0 0 0 0 0 0 0 0 0
2	452633 0 150167 58858 84241 87323 30172 41805 25352 1353
3	430779 133766 150331 0 28621 16754 20115 11242 12810 3681
4	521317 140381 55955 39103 24168 0 9209 5238 7919 472
5	638110 93931 37016 10163 8619 8565 0 2442 3340 1269
6	681110 76996 27975 3767 4779 2127 2837 0 707 447
7	740819 41476 12958 0 2239 959 769 0 0 449
8	770BB2 20727 4371 2370 552 395 390 177 0 80
9	786102 10870 1101 960 445 208 102 75 104 33
10	794864 3347 937 398 277 0 85 43 31 16

SOURCE SPLIT(D)	FISHN GAMGEN REALCOLL	ALBEDO	BDRYX	ESCAPE	E-CUT TIMEKILL R R KILL R SURY	GAMMOT
1000000 0	0 0 109577	0	99451	47	99953 0 0 0 0	0 0 0 0

TIME REQUIRED FOR THE PRECEDING BATCH WAS 2 MINUTES, .39 SECONDS.

\*\*\*START BATCH 0

RANIXM-27BFA3F26.19F

SOURCE DATA

WAVE

WAVE  
2.000E+05  
1.00 0.0022 -0.0000 0.0022 0.0 0.0 0.0 0.0 0.0

PARTICLES PER CM\*\*2 PER SECOND

DETECTOR	PARTICLE	RESPONSES(DETECTOR)		NEUTRON FLUX		CY(SD)	FOM
		BATCH RESPONSE	FSD RESPONSE	ACCUMULATED RESPONSE	FSD		
1	3.8472D-04	0.00348	3.8334D-04	0.00131	0.14715	2.4404D+04	
2	4.4803D-05	0.00469	4.4549D-05	0.00211	0.19622	9.4277D+03	
3	8.9128D-06	0.00690	8.9554D-06	0.00262	0.05259	6.0939D+03	
4	2.2128D-06	0.00949	2.2398D-06	0.00693	0.34775	8.6942D+02	
5	6.1276D-07	0.01534	6.2466D-07	0.00609	0.10566	1.1265D+03	
6	1.7981D-07	0.02118	1.8632D-07	0.00976	0.22722	4.3688D+02	
7	5.5462D-08	0.03434	5.9057D-08	0.01200	0.05833	2.9038D+02	
8	1.7885D-08	0.05078	1.9095D-08	0.01923	0.15596	1.1298D+02	
9	5.4812D-09	0.07087	6.0670D-09	0.02250	0.03368	8.2513D+01	
10	1.8839D-09	0.10804	2.0543D-09	0.03402	0.04968	3.6114D+01	

DETECTOR

PARTICLE DISTRIBUTION IN PERCENTAGE BINS RELATIVE TO INCREASING VALUE OF CONTRIBUTIONS

50

	0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100.0
1	595879	0	168798	66221	94801	98081	33904	47112	28712	1510	
2	509025	0	169496	56059	81339	0	0	0	14422	4145	
3	484478	150329	0	32261	18786	22543	12673	6895	528		
4	586235	158142	63076	43886	27236	0	10347	5890	3764	1424	
5	717728	105839	41604	11408	9752	9643	0	2725	803	488	
6	754266	98561	31573	4224	4062	3646	3163	0	0	505	
7	833392	46723	11032	3572	2491	1078	849	777	0	86	
8	867238	23362	4922	2639	600	378	422	198	204	177	
9	884445	12186	1241	957	593	231	113	84	113	37	
10	894297	37331	1022	446	306	0	90	49	35	21	

NUMBER OF COLLISIONS OF TYPE NOKI  
SOURCE SPLIT(D) FISHN GANGEN REACCOLL ALBEDO HDRX ESCAPE CY-55  
100000 0 0 189798 0 98936 0 0 0 0 0 0

TIME REQUIRED FOR THE PRECEDING PATCH WAS 2 MINUTES, 39 SECONDS.

\*\*\*START DATA 10

RANDOM=LOC(132.274)0P

SOURCE DATA  
 ATAVE 1.00 UAVE 0.0008 VAVE 0.00011 WAVE 0.0 XAVE 0.0 ZAVE 0.0  
 AGEAVE 0.0

PARTICLES PER CM\*\*2 PER SECOND

DETECTOR	BATCH RESPONSE	RESPONSE(DETECTOR)		NEUTRON FLUX		FSD	CV(SD)	FOM
		FSD	ACUMULATED RESPONSE	FSD	ACUMULATED RESPONSE			
1	3.6186D-04	0.00322	3.8119D-04	0.00122	0.13722	2.5230D+04		
2	4.4207D-05	0.00650	4.4523D-05	0.00200	0.17752	9.3759D+03		
3	9.1627D-06	0.02447	8.57761D-06	0.00343	0.22880	3.1964D+03		
4	2.2404D-06	0.01259	2.2398D-06	0.00637	0.33420	9.2849D+02		
5	6.2480D-07	0.01638	6.2468D-07	0.00572	0.09747	1.1512D+03		
6	1.8444D-07	0.02180	1.8613D-07	0.00907	0.21428	4.5682D+02		
7	5.7758D-08	0.03139	5.8926D-08	0.01125	0.05431	2.9727D+02		
8	1.0387D-08	0.06864	1.9024D-08	0.01860	0.13948	1.0877D+02		
9	5.8103D-09	0.06809	6.0412D-09	0.02137	0.03183	6.2390D+01		
10	2.1751D-09	0.10446	2.05664D-09	0.03236	0.04605	3.5921D+01		

DETECTOR

PARTICLE DISTRIBUTION IN PERCENTAGE BINS RELATIVE TO INCREASING VALUE OF CONTRIBUTIONS

	0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100.0
1	662277	0	0	0	105337	108947	37615	52237	31913	1673	
2	565774	0	187608	73382	62248	90329	0	0	15975	4583	
3	538490	167125	186134	0	35685	20868	25121	14123	9865	589	
4	651487	17594	70015	48009	30237	0	11539	6571	4183	1565	
5	797553	117404	46254	12758	10858	10711	0	3035	0	1427	
6	638013	109550	35143	46666	4526	4028	3504	0	0	570	
7	925919	52009	12262	3931	2774	1203	950	857	0	95	
8	963540	26067	5450	2954	657	412	461	221	218	20	
9	982734	13539	1486	1154	547	255	126	97	120	42	
10	993666	4160	1130	489	333	0	103	54	41	24	

NUMBER OF COLLISIONS OF TYPE NCOLL  
 SOURCE SPLIT(D) PISHN GAMGEN REACCOLL ALBEDO HULLX ESCAPP  
 100000 0 0 0 188934 0 98992 53 99947

TIME REQUIRED FOR THE PRECEDING BATCH WAS 2 MINUTES, 39 SECONDS.

### **A.1.2 Point Detector Estimator**

\*\*\*START BATCH 1

RANDOM=13579BDDB97

SOURCE DATA

YOU ARE USING THE DEFAULT VERSION OF SOURCE WHICH SETS WAVE TO 0.00 AND PROVIDES AN ENERGY IG.  
YOU ARE USING THE DEFAULT VERSION OF GEOMD WHICH ASSUMES GEOMETRY AND XSECT MEDIA ARE IDENTICAL.  
WAVE YAVE WAVE YAVE XAVE ZAVE AGEAVE  
1.00E+05 1.00 0.0022 -0.0005 0.0018 0.0 0.0 0.0 0.0

PARTICLES PER CM\*\*2 PER SECOND

DETECTOR	HATCH	RESPONSE(DETECTOR)		NEUTRON FLUX	FSN	POM
		RESPONSE	FSD			
1	3-B195D-04	0.02420	3.8195D-04	0.02420	0.18360	9.6140D+01
2	4-0657D-05	0.03220	4.0657D-05	0.03220	0.17042	5.4278D+01
3	1-0318D-05	0.18977	1.0318D-05	0.18977	0.45750	1.5631D+00
4	2-2010D-06	0.10484	2.2010D-06	0.10484	0.24872	5.1215D+00
5	5-B013D-07	0.16399	5.8018D-07	0.16399	0.24418	2.0932D+00
6	1-3670D-07	0.13992	1.3670D-07	0.13992	0.20058	2.8754D+00
7	1-0456D-07	0.65115	1.0456D-07	0.65115	0.49116	1.3276D+01
8	3-6075D-08	0.39992	3.6075D-08	0.39992	0.31527	3.5196D+01
9	4-4715D-09	0.28794	4.4715D-09	0.28794	0.26414	6.7894D+01
10	6-1909D-10	0.22073	8.1909D-10	0.22073	0.24052	1.1553D+00

PARTICLE DISTRIBUTION IN PERCENTAGE BINS RELATIVE TO INCREASING VALUE OF CONTRIBUTIONS

DETECTOR	0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100.0
1	82140	0	0	0	0	6767	7358	717	735	251	49
2	69700	0	14634	7215	3000	1316	971	68	103	5	
3	78956	13447	1962	1753	289	144	23	13	1	0	
4	81208	12320	2213	405	198	43	21	6	5	0	
5	90177	4818	1100	165	76	18	7	3	1	1	
6	88529	6765	885	246	37	34	2	4	2	0	
7	96335	288	15	3	1	0	0	0	0	0	
8	96700	30	0	1	1	1	0	0	0	0	
9	96124	619	63	10	0	1	2	0	1	0	
10	96140	703	86	21	0	3	2	0	1	0	

NUMBER OF COLLISIONS OF TYPE NCOLL

SOURCE SPLIT(D) FISHN GAMGEN REALCOLL ALBEDO BDRYX ESCAPE E-CUT TIMEKILL R R KILL R R SURV D GAMLOSS

100000 0 0 0 188986 0 98169 46 99952 0 0 0 0 0

TIME REQUIRED FOR THE PRECEDING BATCH WAS 17 MINUTES, 45 SECONDS.

\*\*\*START BATCH 1

RANDOM=EGGGR42396547

SOURCE DATA	TAVE	WAVE	WAVE	WAVE	XAVS	YAVS	ZAVS	AGEAVE
1.000E+05	1.46	0.0012	0.0013	-0.0041	0.0	0.0	0.0	0.0

PARTICLES PER CM\*\*2 PER SECOND

DETECTOR	BATCH	RESPONSE(DETECTOR)		NEUTRON FLUX		PSD	CV(%)	FOM
		RESPONSE	FSD	ACCUMULATED	RESPONSE			
1	3.0376D-04	0.02215	3.0286D-04	0.01640	0.11907	1.0453D+02	5.9872D+01	
2	3.9041D-05	0.02885	3.9849D-05	0.02167	0.11323			
3	1.0062D-05	0.16319	1.0190D-05	0.12539	0.30288	1.7883D+00		
4	1.9870D-06	0.08756	2.0940D-06	0.06900	0.18091	5.9046D+00		
5	6.1231D-07	0.26090	5.9624D-07	0.15592	0.35056	1.1564D+00		
6	1.4004D-07	0.20385	1.3837D-07	0.12417	0.28457	1.8236D+00		
7	3.9161D-08	0.29061	7.1859D-08	0.48030	0.47789	1.2188D-01		
8	1.6222D-08	0.28986	2.6147D-08	0.29016	0.28698	3.3394D-01		
9	6.8172D-09	0.54999	5.6443D-09	0.35118	0.43483	2.2798D-01		
10	2.5654D-09	0.60292	1.6923D-09	0.16012	0.45654	1.3280D-01		

DETECTOR

PARTICLE DISTRIBUTION IN PERCENTAGE BINS RELATIVE TO INCREASING VALUE OF CONTRIBUTIONS

54

	0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100.0
1	164540	0	0	0	0	13448	14540	1483	1387	617	71
2	139619	0	29245	12846	7456	2592	1562	563	156	40	
3	158315	26555	3992	3536	554	274	50	19	2	0	
4	156810	29433	4799	1350	415	145	64	7	12	0	
5	180516	10233	1614	356	126	37	11	4	2	0	
6	178712	12469	1426	381	90	51	4	6	2	1	
7	191744	1422	153	21	4	2	1	3	0	0	
B	193308	223	18	0	1	2	1	0	0	0	
9	193073	643	21	10	5	2	1	1	0	0	
10	193791	173	5	5	1	1	0	0	0	0	

NUMBER OF COLLISIONS OF TYPE NCOLL

SOURCE SPLIT ID	FISHN	GAMGEN	REACCOL	ALRDOO	ADRYX	ESCAPE	E-CUT TIME	KILL R	KILL R SURV	GAMLOSS
100000	0	0	0	0	98732	45	99935	0	0	0

TIME REQUIRED FOR THE PRECEDING BATCH WAS 17 MINUTES, 48 SECONDS.

\*\*\*START BATCH 3

RANDON=6A 5839CPC0C6.3

SOURCE DATA	XAVE	YAVE	ZAVE	WAVE	XAVE	YAVE	ZAVE	AGEAVE
1.00E+05	1.00	-0.0026	-0.0011	-0.0049	0.0	0.0	0.0	0.0

PARTICLES PER CM\*\*2 PER SECOND

DETECTOR	BATCH RESPONSE	RESPONSE(FST)	RESPONSE(DETECTOR)	NEUTRON FLUX	FSD	CV( SD )	FOM
1	3.6996D-04	0.01681	ACTIONIALIZED RESPONSE	3.7856D-04	0.01264	0.10199	1.1718D+02
2	5.7066D-05	0.12868		4.5568D-05	0.03516	0.23556	6.1556D+00
3	6.3213D-06	0.06395		9.5679D-06	0.09094	0.29039	2.2647D+00
4	2.7789D-06	0.29318		2.3223D-06	0.12408	0.37712	1.2165D+00
5	6.1005D-07	0.18697		6.0085D-07	0.12137	0.27644	1.2714D+00
6	1.2666BD-07	0.13773		1.3447D-07	0.09553	0.23163	2.0523D+00
7	3.4557D-08	0.19707		5.9425D-08	0.38908	0.47330	1.2372D-01
8	9.9329D-09	0.24349		2.0743D-08	0.24692	0.28005	3.0717D-01
9	6.2361D-09	0.68128		5.8422D-09	0.33160	0.33411	1.7033D-01
10	1.0023D-09	0.49815		1.4623D-09	0.37279	0.41650	1.3476D-01

DETECTOR PARTICLE DISTRIBUTION IN PERCENTAGE BINS RELATIVE TO INCREASING VALUE OF CONTRIBUTIONS

DETECTOR	0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100.0
1	247165	0	0	0	0	19974	21712	2183	2061	890	158
2	209621	0	43915	30314	2570	3398	1046	275	19	4	
3	237775	28522	16993	5227	441	801	42	63	8	0	
4	244152	36525	6560	1379	459	129	49	23	1	1	
5	273149	13344	1903	593	149	37	13	7	2	0	
6	267957	18414	23116	721	125	84	6	4	1	1	
7	286616	2900	446	27	17	5	3	0	0	0	
8	289571	691	53	13	0	3	1	1	0	0	
9	289756	841	32	13	4	2	0	1	0	0	
10	290482	465	38	10	3	6	1	0	0	0	

NUMBER OF COLLISIONS OF TYPE NCOLL  
SOURCE SPLIT(D) PSLN GAMGEN RBNLCM ALBEDO  
100000 0 0 0 189879 0 99126 55  
55 99945 0 0 0 0 0 0

TIME REQUIRED FOR THE PRECEDING BATCH WAS 17 MINUTES, 49 SECONDS.

## \*\*\*SMART BATCH 4

RANDOM-C19P64C8B3BP

SOURCE DATA  
 WAVE 1.00 1.00 -0.00E6 -0.0027 0.0006 0.0 XAVE 0.0 0.0 AGAVE  
 1.00E+05 0.00E+05 0.00E+05 0.00E+05 0.00E+05 0.00E+05 0.00E+05 0.00E+05 0.00E+05

PARTICLES PER CM\*\*2 PER SECOND

DETECTOR	PARTICLE	RESPONSES(DETECTOR)		NEUTRON FLUX		PSD	CY(SD)	FOM
		PSD	ACCUMULATED	PSD	RESPONSE			
1	RESPONSE	0.02955	3.0386D-04	0.01211	0.00032	9.5714D+01		
2	3.9975D-04	0.03807	4.5115D-05	0.04281	0.22480	7.6597D+00		
3	4.3695D-05	0.06284	9.1721D-06	0.07245	0.28023	2.6741D+00		
4	7.9846D-06	0.20961	2.3686D-06	0.10678	0.29830	1.2310D+00		
5	2.5077D-06	0.42952	6.7847D-07	0.16311	0.37187	5.1487D-01		
6	9.0974D-07	0.13330	1.3785D-07	0.07851	0.18837	2.2771D+00		
7	1.4B04D-07	0.17820	5.6320D-08	0.31913	0.46651	1.4594D-01		
8	4.7006D-08	0.15543	1.8103D-08	0.21329	0.27711	3.0853D-01		
9	1.0194D-08	0.13361	5.0230D-09	0.28976	0.33295	1.6718D-01		
10	2.5654D-09	0.15417	1.2754D-09	0.32129	0.41462	1.3598D-01		

DETECTOR PARTICLE DISTRIBUTION IN PERCENTAGE BINS RELATIVE TO INCREASING VALUE OF CONTRIBUTIONS

56	DETECTOR	0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100.0
		329363	0	0	0	26758	29111	2845	2969	1045	139	10
1	279176	0	58615	40614	3386	4215	1424	580	112	10	10	10
2	316751	31761	26433	7785	2501	789	274	116	21	21	21	21
3	329392	45013	8839	1833	610	184	66	19	2	1	1	1
4	366931	15938	2285	605	62	44	6	6	1	0	0	0
5	359586	22629	2929	751	174	110	16	9	7	7	7	7
6	382177	3856	522	131	45	4	5	3	0	0	0	0
7	385734	1235	174	21	8	2	2	1	0	0	0	0
8	385546	1862	156	35	11	4	2	1	0	0	0	0
9	387066	B78	94	9	0	2	1	0	0	0	0	0
10												

NUMBER OF COLLISIONS OF TYPE NCOLL  
 SOURCE SPLIT(D) FISHN GAMGEN REACCOLL ALBEDO BDRYX ESCAPE  
 100000 0 0 0 189773 0 99196 59 99341E-CUT TIMEKILL R R KILL R R SURV GAMLIST  
 0 0 0 0 0 0 0 0 0

TIME REQUIRED FOR THE PRECEDING BATCH WAS 17 MINUTES, 50 SECONDS.

\*\*\*STANDARD BATCH

5

RANDOM-OCLEMIRUCA4AF

SOURCE DATA  
WAVE 1.00 -0.0009 0.0035 0.0044  
1.000E+05

PARTICLES PER CM\*\*2 PER SECOND

DETECTOR	BATCH RESPONSE	RESPONSES [NEUTRATOR] FSID	NEUTRON FLUX ACCUMULATED RESPONSE	FSID	CV( SD )	FOM
1	3.8594D-04	0.02529	3.8427D-04	0.01093	0.07949	9.3948D+01
2	4.7310D-05	0.09346	4.5554D-05	0.03908	0.19146	7.3487D+00
3	8.3560D-06	0.06706	9.0089D-06	0.06031	0.26855	3.0857D+00
4	2.6277D-06	0.23210	2.4205D-06	0.09761	0.25087	1.1779D+00
5	5.4350D-07	0.11686	6.5116D-07	0.13893	0.36455	5.8148D-01
6	1.2134D-07	0.08476	1.3456D-07	0.06614	0.17845	2.5652D+00
7	3.4158D-08	0.22320	5.1680D-08	0.27186	0.46134	1.5185D-01
8	1.0061D-08	0.43582	1.6496D-08	0.19468	0.25836	2.9613D-01
9	3.1820D-09	0.38605	4.6548D-09	0.25565	0.31928	1.7171D-01
10	7.7008D-10	0.29357	1.1743D-09	0.28179	0.40696	1.4133D-01

PARTICLE DISTRIBUTION IN PERCENTAGE BINS RELATIVE TO INCREASING VALUE OF CONTRIBUTIONS

DETECTOR	0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100.0
1	411246	0	0	0	0	33573	36822	3569	3713	1290	171
2	348547	0	73254	51120	4290	5595	1424	763	105	9	
3	395456	39924	33235	9785	3145	893	421	140	35	4	
4	415652	54026	9262	2890	201	254	77	23	2	2	
5	455252	22537	3636	613	208	65	20	12	2	1	
6	446245	30955	3906	1200	193	129	44	26	13	1	
7	477683	4892	575	213	74	19	7	3	0	0	
8	482198	1254	184	45	18	1	3	1	1	0	
9	401923	2282	268	42	12	6	2	0	1	0	
10	483851	1036	138	34	14	5	0	1	0	0	

NUMBER OF COLLISIONS OF TYPE NCOLD  
SOURCE SPLIT(D)  
100000 0 0 0 0 0 0 0 0 0 0 0

TIME REQUIRED FOR THE PROCEEDING BATCH WAS 17 MINUTES, 51 SECONDS.

\*\*\*START BATCH 6

SOURCE DATA  
WAVE 1.00 0.0020 0.0012 0.0014 0.0 0.0

PARTICLES PER CM\*\*2 PER SECOND

RAMPDM-3C2E98RCAB

DETECTOR	BATCH RESPONSE	RESPONSES(DETECTOR)				NEUTRON FLUX FSD	FSD	CV( SD )	FOM
		FSI	ACUMULATED RESPONSE	XAVE	ZAVE				
1	4.0937D-04	0.03452	3.8846D 04	0.01086	0.06968	7.9325D+01			
2	4.1039D-05	0.03509	4.4601D-05	0.03354	0.18672	8.3148D+00			
3	8.6497D-06	0.05580	8.9490D-06	0.05138	0.26036	3.5432D+00			
4	1.9491D-06	0.11055	2.3419D-06	0.08546	0.24309	1.2810D+00			
5	1.2141D-06	0.36437	7.4498D-07	0.14155	0.25251	4.6696D-01			
6	5.4674D-07	0.51612	2.0326D-07	0.23425	0.45784	1.7050D-01			
7	9.3560D-08	0.28522	5.6667D-08	0.21351	0.40497	2.0523D-01			
8	1.4453D-08	0.16652	1.6156D-08	0.16750	0.25272	3.3345D-01			
9	3.6859D-09	0.18078	4.4933D-09	0.22208	0.31534	1.8969D-01			
10	9.1095D-10	0.19769	1.1304D-09	0.24539	0.40221	1.5537D-01			

DETECTOR PART(CUR DISTRIBUTION IN PERCENTAGE BINS RELATIVE TO INCREASING VALUE OF CONTRIBUTIONS

DETECTOR	0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100.0
	493545	0	0	0	0	40204	44033	4260	4706	1309	207
1	410609	0	87715	61190	5141	63336	2076	865	192	15	
2	474818	47695	39771	11181	4351	1078	500	166	44	6	
3	493592	67664	13228	2644	1046	269	115	50	1	3	
4	255655	19531	2746	670	112	52	15	4	3	0	
5	562098	13980	1976	343	112	17	11	3	1	0	
6	573180	58712	940	76	42	13	5	3	1	0	
7	577035	3424	222	71	18	10	3	2	1	0	
8	578294	2760	304	78	19	10	4	1	1	0	
9	580614	1257	161	46	22	7	3	1	0	0	
10											

NUMBER OF COLLISIONS OF TYPE MCOLL  
SOURCE SPLIT(D) FISHN GAMGEN REALCOLL ALBEDO EDRYX ESCAPE  
100000 0 0 0 188636 0 98973 47  
TIME REQUIRED FOR THE PRECEDING BATCH WAS 17 MINUTES, .47 SECONDS.

E CUT TIMEKILL R R KILL R SURV GAMOST  
99953 0 0 0 0 0 0

## \*\*\*START BATCH 7

RANDOM-C9F1 F9A47EAR

## SOURCE DATA

WAVE

1.000

0.99916

0.30003

0.00004

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

## PARTICLES PER CH\*\*2 PER SECOND

WAVE

0.30003

0.00004

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

0.0

## RESPONSE(BATCH)

RESPONSE

3.9492D-04

0.03325

0.07962

0.04958

0.05357

0.28177

0.42497

0.02650

0.15088

0.23298

0.14657

0.23364

0.02338D-08

1.5456D-08

4.2886D-09

1.0466D-09

0.22784

0.0

4.6873

71537

13059

5130

3237

2316

1079

244

98

196

91

20

14

15

11

14

7

1

4

1

1

0

0

RESPONSE(DETECTOR)  
ACUMULATED  
RESPONSE  
3.9492D-04  
9.4637D-05  
8.9094D-06  
2.2522D-06  
7.3337D-07  
1.8912D-07  
5.6238D-08  
0.19222  
0.15202  
0.20270  
0.040234  
0.24644  
0.31403  
0.39989

NEUTRON FLUX  
FSI)  
FSI)  
CV( SD )  
7.3247D+01  
8.3766D+00  
3.9992D+00  
1.3744D+00  
4.8540D-01  
1.6812D-01  
2.1698D-01  
3.4691D-01  
1.9513D-01  
1.5443D-01

FOM  
7.3247D+01  
8.3766D+00  
3.9992D+00  
1.3744D+00  
4.8540D-01  
1.6812D-01  
2.1698D-01  
3.4691D-01  
1.9513D-01  
1.5443D-01

PERCENTAGE BINS RELATIVE TO INCREASING VALUE OF CONTRIBUTIONS

59  
0.0 10.0 20.0 30.0 40.0 50.0 60.0 70.0 80.0 90.0 100.0

NUMBER OF COLLISIONS OF TYPE NCOLL  
SOURCE SPLIT(D) FISHN GAMEN REACCOLL ALBEDO DORYX ESCAPE  
100000 0 0 189061 0 93413 40  
TIME CUT TIMERILL R R KILL R R SURV GAMLOSS  
99952 0 0 0 0 0 0

TIME REQUIRED FOR THE PRECEDING BATCH WAS 17 MINUTES, 50 SECONDS.

\*\*\*START BATCH 0

RANDOM-B0E4BD2C4A17

SOURCE DATA

WAVE 1.00

WAVE -0.0021

WAVE 0.0024

WAVE 0.0

WAVE 0.0

WAVE 0.0

WAVE 0.0

WAVE 0.0

WAVE 0.0

PARTICLES PER CM<sup>-2</sup> PER SECOND

DETECTOR	BATCH RESPONSE	RESPONSES(DETECTOR)		NEUTRON FLUX		CV( SD )	POT
		PSD	ACCUMULATED RESPONSE	PSD	ACCUMULATED		
1	3.9315D-04	0.03379	3.8985D-04	0.01009	0.06970	6.8916D+01	
2	4.9785D-05	0.10155	4.5280D-05	0.03011	0.15079	7.7312D+00	
3	7.9145D-06	0.05531	8.7851D-06	0.04022	0.24815	4.2349D+00	
4	1.8952D-06	0.09951	2.2080D-06	0.06901	0.23591	1.4721D+00	
5	5.4595D-07	0.07778	7.0994D-07	0.11654	0.23347	5.1622D-01	
6	2.2764D-07	0.33281	1.9394D-07	0.19262	0.41951	1.1889D-01	
7	4.3966D-08	0.23969	5.4521D-08	0.17455	0.39471	2.3013D-01	
8	1.3263D-08	0.25663	1.584D-08	0.13827	0.23668	3.6670D-01	
9	3.8514D-09	0.35037	4.1814D-09	0.18384	0.29964	2.0745D-01	
10	B.2293D-10	0.27035	1.0186D-09	0.20665	0.39295	1.6419D-01	

DETECTOR

DETECTOR	PARTICLE DISTRIBUTION IN PERCENTAGE BINS RELATIVE TO INCREASING VALUE OF CONTRIBUTIONS									
	0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0
1	658062.	0	0	0	53486	58798	5697	6304	1747	274
2	557855.	0	117020	81862	6880	8414	2793	1207	226	20
3	632777	56397	60570	13962	5503	2483	650	187	93	11
4	640882	103604	10966	5551	1628	696	215	36	24	4
5	737559	28696	3853	1259	126	109	9	13	3	0
6	741932	27199	2654	433	143	52	22	3	2	0
7	764194	7823	1085	240	107	9	11	7	1	0
8	768311	5580	255	147	33	21	2	1	2	0
9	769421	5169	574	88	44	10	8	1	1	0
10	772134	3464	395	126	21	16	1	3	0	0

NUMBER OF COLLISIONS OF TYPE NCOLL

SOURCE SPLIT(D) FISIN GAMEN RREALOM ALREDX BDRX ESCAPE 99466 54 E-CUT TIMEKILL R R KILL R SURV GAMOST

100000 0 0 0 189675 0 99466 54 99466 0 0 0 0

TIME REQUIRED FOR THE PRECEDING BATCH WAS 17 MINUTES, .53 SECONDS.

\*\*\*START BATCH 9

RANDOM-F82DDE70E5FF

SOURCE DATA  
 XAVE 0.000E+05 1.00 -0.0014 -0.0059 G.0008  
 PARTICLES PER CM\*2 PER SECOND

DETECTOR	BATCH RESPONSE	RESPONSE(FSD)	NOMINON FSD	FSD	CV(%)	FOM
	ACCUMULATED	RESPONSE	ACCUMULATED	FSD		
1	4.0154D-04	0.03399	3.9115D-04	0.00974	0.06437	6.5685D+01
2	5.4195D-05	0.12517	4.6291D-05	0.03158	0.14705	6.2514D+00
3	8.4099D-06	0.09020	6.7434D-06	0.03719	0.23244	4.5057D+00
4	1.7494D-06	0.01466	2.1570D-06	0.06312	1.5646D+00	
5	7.2271D-07	0.20546	7.1136D-07	0.10596	0.22280	5.5515D-01
6	1.4988D-07	0.13557	1.8891D-07	0.17616	0.41761	2.0084D-01
7	3.8305D-08	0.12517	5.2720D-08	0.16077	0.39315	2.4113D-01
8	1.0050D-08	0.83420	1.7947D-08	0.23151	0.40162	1.1628D-01
9	1.8052D-09	0.85224	5.7226D-09	0.32169	0.43233	6.0227D-02
10	1.9662D-09	0.336664	1.1239D-09	0.19646	0.31167	1.6146D-01

DETECTOR

PARTICLE DISTRIBUTION IN PERCENTAGE BINS RELATIVE TO INCREASING VALUE OF CONTRIBUTIONS

	0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100.0
1	740468	0	0	0	60023	66136	6371	7417	7448	7448	227
2	627553	0	149852	73934	7733	10578	2670	788	193	193	10
3	711802	63459	60160	15710	6169	2788	955	209	104	104	13
4	704967	132823	21391	6219	1831	606	321	39	25	25	4
5	629083	32297	4735	1007	147	125	15	10	1	1	2
6	634804	30599	2972	463	194	53	28	2	3	3	0
7	659764	8801	1162	203	95	31	17	10	1	1	0
8	867854	2824	193	85	32	2	4	1	1	1	0
9	870112	1914	118	33	12	1	2	0	0	0	0
10	870879	1982	271	59	32	0	5	0	0	0	0

NUMBER OF COLLISIONS OF TYPE NCOLL  
 SOURCE SPLIT(D) FISHN GAMEN REACOLL ALBEDO RDRX ESCAPE  
 100000 0 0 0 169195 0 99061 51 R-CUT TIMEKILL R R KILL R R SURV GAMLOSS  
 99949 0 0 0 0 0 0 0

TIME REQUIRED FOR THE PRECEDING BATCH WAS 17 MINUTES, 49 SECONDS.

\*\*\*START BATCH 10

RANDOM=ACF'112D08EDB

SOURCE DATA  
WAVE  
1.000E+05 1.00 -0.0019 0.0022 0.0032 0.0 0.0 0.0 0.0

PARTICLES PER CM\*\*2 PER SECOND

DETECTOR	BATCH RESPONSE	RESPONSES(DETECTOR)		NEUTRON FLUX ACCUMULATED RESPONSE	PSD	CV(5D)	FOM
		FSI	PSD				
1	3.8024D-04	0.02493	3.9006D-04	0.00912	0.06178	6.7442D+01	
2	4.2527D-05	0.03825	4.5916D-05	0.02979	0.14028	6.3208D+00	
3	1.2067D-05	0.10180	9.0757D-06	0.05148	0.30909	2.1171D+00	
4	1.7908D-06	0.08807	2.1204D-06	0.05826	0.22978	1.6529D+00	
5	6.4603D-07	0.16061	7.0481D-07	0.09736	0.21780	5.9187D-01	
6	1.3432D-07	0.11139	1.8347D-07	0.16347	0.41652	2.0995D-01	
7	3.3903D-08	0.24804	5.6838D-08	0.15096	0.38844	2.4621D-01	
8	1.2554D-08	0.29538	1.740BD-08	0.21587	0.39772	1.2040D-01	
9	1.3514D-09	0.40431	5.5855D-09	0.29830	0.42753	6.3056D-02	
10	1.1817D-10	8.17964	1.2333D-09	0.21594	0.26580	1.2032D-01	

DETECTOR PARTICLE DISTRIBUTION IN PERCENTAGE BINS RELATIVE TO INCREASING VALUE OF CONTRIBUTIONS

DETECTOR	0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100.0
1	822591	0	0	0	66875	73435	7097	8228	1069	333	
2	696863	0	146584	102522	6582	11123	3570	870	201	18	
3	790633	80272	66264	19624	6252	1790	840	281	73	9	
4	782995	147859	23780	6419	2527	667	162	30	38	4	
5	922034	35882	47794	1585	161	140	10	19	1	2	
6	927492	33989	3311	456	270	43	48	1	4	0	
7	955274	9730	1164	469	101	39	19	8	3	1	
8	964274	3122	424	95	34	4	4	1	1	0	
9	966562	2324	125	40	15	1	1	0	0	0	
10	967635	2078	295	65	27	8	4	2	0	0	

NUMBER OF COLLISIONS OF TYPE NCOLL  
SOURCE SPLIT(D) FISHN GAMGEN REALCOLL ATRD00 BDYXX ESCAPE  
100000 0 0 0 189336 0 98637 56 9944

TIME REQUIRED FOR THE PRECEDING BATCH WAS 17 MINUTES, 46 SECONDS.

## A.2 Sample Problem 2

\*\*\*START BATCH 1

NAMINFM=40000,LFPA71A

SOURCE DATA  
YOU ARE USING THE DEFAULT VERSION OF SOURCE WHICH SHOT'S WAVE TO MDF AND PROVIDES AN ENERGY IC.  
WAVE WAVE WAVE XAVE YAVE ZAVE AGAVE  
9.6E7E+04 10.23 0.0007 -0.0001 -0.0027 0.0 0.0 0.0 0.0

4 PT R\*\*2 FLUENCE

DETECTOR	BATCH	RESPONSE(FD)	NEUTRON FLUX ACCUMULATED	PSD	CV(SD)	FOM
1	1.9356D+00	0.01002	1.9156D+00	0.01002	0.06790	1.5650D+03
2	2.1357D+00	0.01358	2.1357D+00	0.01358	0.09257	8.5118D+02
3	1.8077D+00	0.01382	1.8077D+00	0.01382	0.06938	8.2183D+02
4	6.6802D-01	0.01746	6.6802D-01	0.01746	0.07040	5.1510D+02
5	4.3622D-01	0.01763	4.3622D-01	0.01763	0.04574	5.0528D+02
6	1.7872D-01	0.02763	1.7872D-01	0.02763	0.08732	2.0563D+02
7	4.2268D-02	0.04378	4.2268D-02	0.04378	0.12017	8.1923D+01

DETECTOR	0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100.0
1	54507	21578	0	6594	5412	3180	2963	0	676	298	
2	53277	17817	6512	5591	3485	3430	0	599	399	334	
3	52490	13827	5429	4839	2998	2862	0	525	351	283	
4	38218	11400	0	2219	1946	1115	948	0	212	128	
5	33332	5916	3919	1514	819	991	785	0	146	74	
6	23365	3780	3779	0	638	304	414	189	31	35	
7	12721	698	1170	966	0	118	85	71	26	9	

NUMBER OF COLLISIONS OF TYPE NCOLL  
SOURCE SPLIT(D) FISHN GAMGEN REALCOLL ALBEDO BDRYX ESCAPE E-CUT TIMEKILL R KILL R SURV GAMLOSS  
100000 1960 0 0 1917108 0 1084495 0 90000 0 11960 632 0

TIME REQUIRED FOR THE PRECEDING BATCH WAS 6 MINUTES, 22 SECONDS.

\*\*\*START BATCH 1

RANMOD-195E7A42575A

## SOURCE DATA

WAVE 1AVE

	WAVE	1AVE	WAVE	1AVE	WAVE	1AVE	WAVE	1AVE
1	9.657E+04	10.023	-0.0007	0.0045	0.0007	0.0	0.0	0.0

4 P) R\*\*2 FLUXENCE

DETECTOR	BATCH RESPONSE	RESPONSE(DETECTOR)	NEUTRON FLUX	FSD	CW(ED)	PROM
	PSD	ACCUMULATED RESPONSE	PSD	CW(ED)		
1	1.9364D+00	0.00965	1.9323D+00	0.00509	0.03785	1.5200D+03
2	2.0632D+00	0.01234	2.0821D+00	0.00669	0.07246	8.8054D+02
3	1.7536D+00	0.01129	1.7815D+00	0.00678	0.05849	6.5936D+02
4	6.5794D-01	0.01681	6.5927D-01	0.00886	0.04068	5.0220D+02
5	4.3723D-01	0.01896	4.3334D-01	0.00960	0.03323	4.2843D+02
6	1.7664D-01	0.02762	1.7415D-01	0.01414	0.08946	1.9727D+03
7	4.1097D-02	0.03931	4.0930D-02	0.02236	0.09756	7.8876D+01

DETECTOR PARTICLE DISTRIBUTION IN PERCENTAGE BINS RELATIVE TO INCREASING VALUE OF CONTRIBUTIONS

	0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100.0
1	21B391	93707	0	26647	21541	12806	11736	0	2675	1185	
2	212557	71346	26538	14053	19413	16649	1170	0	2393	1562	1192
3	209494	55161	22094	19438	11965	11508	0	2140	1409	1044	
4	152410	45703	0	8688	7537	4354	3793	0	904	479	
5	123567	32732	15823	6192	3144	3996	2974	0	542	271	
6	92951	15200	15051	0	2432	1170	1453	967	133	128	
7	50402	3695	4646	2052	1627	503	352	240	87	46	

NUMBER OF COMMISSIONS OF TYPE NCOLL  
 SOURCE SPLIT(D) FILEN GAMMA RESCOLL ALISTRO UDRYX ESCPL  
 100000 1764 0 0 1910548 0 1082420 89976 0 117RR 877 0  
 GAMOST

TIME REQUIRED FOR THE PRECEDING BATCH WAS 6 MINUTES, 20 SECONDS.

\*\*\*START BATCH B

RANDOM-E18DB/3979A2

SOURCE DATA

ZAVE

WTANG

0.0006

WAVE

0.0023 -0.0006

XAVE

0.0 -0.0008

YAVE

0.0 0.0

4 PT R\*\*2 FINNCE.

DETECTOR

BATCH

RESPONSE

RESPONSE(DEFLECTION)

NEUTRON FLUX

FSD

FSI

ACUMULATED

RESPONSE

0.01336

1.9363D+00

1

1.4682D+00

0.01050

2.0506D+00

0.01507

1.7800D+00

0.01868

6.5762D-01

0.02124

4.3619D-01

0.02359

1.6936D-01

0.04340

3.9773D-02

0.0704D-02

0.02215

0.02215

0.25553

DETECTION

0.0

10.0

20.0

30.0

40.0

50.0

60.0

70.0

80.0

90.0

100.0

PARTICLE DISTRIBUTION IN PERCENTAGE BINS RELATIVE TO INCREASING VALUE OF CONTRIBUTIONS

1	437702	186731	53153	43326	25502	23435	0	5417	2349
2	425354	142806	52742	27905	39041	33274	0	1796	3099
3	420041	110016	43804	34619	16221	28678	0	4138	2754
4	305183	91010	31791	17699	15086	3620	7538	0	1801
5	247532	65714	31791	12246	6246	7958	5098	0	1078
6	186051	30806	29902	0	4812	2417	3782	1554	281
7	106953	7476	9302	4167	3358	1138	438	629	240

NUMBER OF COLLISIONS OF TYPE NCULL  
SOURCE SPLIT(D) FISH GAMGEN REALCOLL

100000 1791 0 1901602 0 1082298 0 89847 0 11944 0 849 0

TIME REQUIRED FOR THE PRECEDING BATCH WAS 6 MINUTES, 20 SECONDS.

\*\*START BATCH 9  
 SOURCE DATA  
 WTAVE 9.68E-04 10.23  
 YAVE 0.0044 0.0034  
 WAVE 0.0008 0.0  
 XAVE 0.0 0.0  
 ZAVE 0.0 0.0  
 AGEAVE 0.0 0.0  
 1 P1 R\*\*2 FIENNE

4AND0M=34.3E3UGM5RGA

DETECTOR	BATCH	RESPONSE(FSII)	RESPONSE(DETECTOR)		NEUTRON FLUX		PSD	CV(%)	POW
			FSII	ACCUMLATED	RESPONSE	FSII			
1	1.9292D+00	0.01005	1.9355D+00	0.00369	0.00557	1.2876D+03			
2	2.1031D+00	0.01235	2.0716D+00	0.00443	0.06004	6.9132D+02			
3	1.7628D+00	0.01144	1.7786D+00	0.00446	0.04096	6.7957D+02			
4	6.6494D-01	0.02153	6.5794D-01	0.00614	0.04018	4.6374D+02			
5	4.3820D-01	0.02111	4.3267D-01	0.00653	0.02528	4.1115D+02			
6	1.7186D-01	0.02725	1.7476D-01	0.00931	0.05359	2.0186D+02			
7	3.9749D-02	0.05783	4.0598D-02	0.02072	0.23381	4.0779D+01			

DETECTOR PARTICLE DISTRIBUTION IN PERCENTAGE BINS RELATIVE TO INCREASING VALUE OF CONTRIBUTIONS

	0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100.0
1	492473	210126	0	59699	48725	286772	26386	0	6104	2635	
2	478452	160782	59350	31469	43961	17351	0	5374	3505	2649	
3	472611	123689	49149	13934	26885	25905	0	4673	3124	2403	
4	343184	102559	0	19949	16905	9682	8490	0	2044	1038	
5	278473	73966	35804	13712	7017	8913	6642	0	1227	595	
6	209353	34641	33618	0	5427	2715	3676	1746	318	295	
7	113578	8439	10468	4626	3778	1172	485	690	270	91	

NUMBER OF COLLISIONS OF TYPE NCOLL  
 SOURCE SPLIT(D) FISHN GAMGEN REACCOLL ALBEDO EDORYX ESCAPE  
 100000 1699 0 0 1907716 0 1081844 0 89972 0 11727 811 GAMCOST  
 0

TIME REQUIRED FOR THE PRECEDING BATCH WAS 6 MINUTES, 20 SECONDS.

• START BATCH 10

RANDOM=6A8A5FC1202

SOURCE DATA

WAVE

WAVE

WAVE

WAVE

WAVE

9.687E+04 10.23 -0.0008 -0.0001 0.0029 0.0 0.0

AGRAVE

0.0

PI R\*\*2 FLUENCE

DETECTOR	BATCH RESPONSE	RESPONSIVENESS FSI	ACUMULATED RESPONSE	NEUTRON FLUX	FSD	CY(5D)	YOM
1	1.9403D+00	0.01053	1.9360D+00	0.00348	0.04639	1.3998D+03	
2	2.1167D+00	0.01896	2.0762D+00	0.00442	0.07557	8.0479D+02	
3	1.7697D+00	0.01374	1.7777D+00	0.00424	0.03745	8.7482D+02	
4	6.6807D-01	0.01736	6.5892D-01	0.00580	0.03690	4.6913D+02	
5	4.3582D-01	0.02043	4.3298D-01	0.00622	0.02446	4.0742D+02	
6	1.7285D-01	0.04121	1.7457D-01	0.00933	0.08119	1.8099D+02	
7	3.7749D-02	0.03733	4.0313D-02	0.01910	0.22601	4.3177D+01	

DETECTOR PARTICLE DISTRIBUTION IN PERCENTAGE BINS RELATIVE TO INCREASING VALUE OF CONTRIBUTIONS

	0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100.0
1	547397	233200	0	66455	54167	31151	29356	0	6761	28532	
2	531797	178649	65945	34897	48774	41508	0	5968	3934	2967	
3	525358	137393	54507	48902	22747	35862	0	5154	3475	2652	
4	381996	113951	0	22222	18850	10745	9453	0	2284	1154	
5	309433	82274	39828	15236	7791	9984	7363	0	1363	652	
6	232767	38494	37346	0	6042	2991	4070	1982	331	318	
7	126159	9453	9125	7702	4160	1308	535	753	298	100	

NUMBER OF COLLISIONS OF TYPE NCOLL,  
SOURCE SPIN ID FISHN GAMGN RADLCOL ALBEDO BDRX ESCAPE  
100000 1765 0 1914825 0 1084431 0 89874 0 11891 0 11 0

TIME REQUIRED FOR THE PRECEDING BATCH WAS 6 MINUTES, 21 SECONDS.

### **A.3 Sample Problem 3**

#### **A.3.1 Without Source Step Biasing**

\*\*\*START BATCH 1  
RANCOM-11579BDSUB97

SOURCE DATA

YOU ARE USING THE DEFAULT VERSION OF GTMID WHICH ASSUMES GEOMETRY AND XSECT MEDIA ARE IDENTICAL.

\*TAVE TAVE YAVE YAVE XAVE YAVE ZAVE AGAVE  
1.000E+04 1.00 0.0050 -0.0036 0.4944 2.192E-01 1.607E-01 9.996E-05 0.0

PARTICLES PER CM\*\*2 PER SECOND

DETECTOR	BATCH RESPONSE	RESPONSE(DETECTOR)	NEUTRON FLUX	VSD	CV(SD)	POW
1	3.3598D-09	0.16022	3.3598D-09	0.16022	0.09256	2.2377D+01

DETECTOR PARTICLE DISTRIBUTION IN PERCENTAGE BINS RELATIVE TO INCREASING VALUE OF CONTRIBUTIONS

0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100.0
1	9281	23	0	0	0	5	0	5	0	0

NUMBER OF COLLISIONS OF TYPE NCOLL

SOURCE SPLIT(D) FISHN GAGEN REA001 ALR00 ESCAPE E-CUT TIMEKILL R R KILL R SURV GAMLST  
14600 0 0 0 112485 0 12841 6122 3878 0 0 0 0 0

TIME REQUIRED FOR THE PRECEDING BATCH WAS 1 MINUTE, 44 SECONDS.

\*\*\*START BATCH 4

SOURCE DATA  
XAVE 1.00  
YAVE 0.0005  
ZAVE 0.5050  
WAVE -5.0968E-01  
XAVE 5.3528E-01  
YAVE 9.996E-05  
ZAVE 0.0

PARTICLES PER CM\*\*2 PER SECOND

DETECTOR	BATCH RESPONSE	RESPONSE(FD)	NEUTRON FLUX	FSD	Cv(SD)	FOM
1	2.9534D-09	0.16513	3.5668D-09	0.09836	0.12074	1.4634D+01

DETECTOR PARTICLE DISTRIBUTION IN PERCENTAGE BINS RELATIVE TO INCREASING VALUE OF CONTRIBUTIONS

0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100.0
1	37191	103	0	0	0	0	21	0	2	3

NUMBER OF COLLISIONS OF TYPE MCCOLL  
SOURCE SPLIT(D) FISHM GAMGEN REACTION,M ALREDO BDRYX ESCAPE E-CUT TIMEKILL R R KILL R R SURY GAMMOS  
10000 0 0 114911 0 12950 6109 3691 0 0 0 0

TIME REQUIRED FOR THIS PRECEDING BATCH WAS 1 MINUTE, 46 SECONDS.

\*\*\*START BATCH 8 RANDOM=BE6E7B79CACF

SOURCE DATA  
WAVE 1.00 0.0012 0.0040 0.4985 2.509E-01 1.348E+00 9.996E-05 AGRAVE  
1.000E+04

PARTICLES PER CM\*\*2 PER SECOND

DETECTOR	BATCH RESPONSE	RESPONSE(DETECTOR) FSID	NEUTRON FLUX ACCUMULATED RESPONSE	FSID	CY(SD)	FCM
1	3.0808D-09	0.16312	3.3484D-09	0.066688	0.07893	1.5792D+01

PARTICLE DISTRIBUTION IN PERCENTAGE BINS RELATIVE TO INCREASING VALUE OF CONTRIBUTIONS

0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100.0
1	74.328	21.1	0	0	0	0	1.5	24	5	5

NUMBER OF COMMISSIONS OF TYPE NCOLL  
SOURCE SPLIT(D) FISHN GAMCN REACCOL, ALHEDC BDRYX ESCAPE E-CUT TIMEKILL R R KILL R SURV GAMLOSS  
16000 0 0 1174433 0 0 2917 6103 3897 0 0 0 0 0

TIME REQUIRED FOR THE PRECEDING BATCH WAS 1 MINUTE, 45 SECONDS.

\*\*\* START BATCH ? RANDOM-B2C9DDEBAP

SOURCE DATA	WAVE	UAVE	VAVE	WAVE	UAVE	VAVE	WAVE	UAVE	VAVE
1.000R+04	1.00	-0.0063	-0.0054	0.4925	4.811E-01	6.450E-01	9.996E-05	0.0	0.0

PARTICLES PER CM\*\*2 PER SECOND

DETECTOR	BATCH RESPONSE(FILTERATOR)	NEUTRON MIX	FSD	CY( SD )	FOM	
1	3.3047D-69	0.23897	3.3436D 09	0.06507	0.0B243	1.4B27D+01

DETECTOR PARTICLE DISTRIBUTION IN PERCENTAGE BINS RELATIVE TO INCREASING VALUE OF CONTRIBUTIONS

	0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100.0
1	89616	226	0	0	0	0	0	48	0	6	6

NUMBER OF COLLISIONS OF TYPE NCOLL  
 SOURCE SPLIT(D)  
 100000 0 0 0 0 0 0 0 0 0 0 0

GAMM

ESCAPE E-CUT TIMEKILL R R KILL R SURV GAMMOS

1.3916 6084 12620 0 0 0 0 0 0 0 0 0

\*\*\*START BATCH 10 RANDOM-3A17ICNBY7

SOURCE DATA  
XAVE 1.00 -0.0021 0.0031 XAVE 0.4962 -5.126E-01 1.146E+00 9.996E-05 AGEAVE  
1.000E+04

PARTICLES PER CM\*\*2 PER SECOND

DETECTOR	BATCH RESPONSE 2.9991D-09	RESPONSE PSD 0.17266	DETECTOR ACUMULATED RESPONSE 3.3091D 09	NEUTRON FLUX PSD 0.06120	CY(ED) 0.07737	FROM 1.5070D+01
DETECTOR	PARTICLE DISTRIBUTION IN PERCENTAGE BINS RELATIVE TO INCREASING VALUE OF CONTRIBUTIONS					
	0.0	10.0	20.0	30.0	40.0	50.0
1	92914	246	0	0	0	0
					56	0
					6	6

NUMBER OF COLLISIONS OF TYPE NCOLL,  
SOURCE SPLIT(0) FISIN GAMGEN UNALCOLL ALBERTO BORRYX ESCAPE  
16000 0 3 0 115350 0 13177 6062 E-CUT TIMEKILL R R KILL R R SURV CAMLOSS  
0 0 0 0 0 0 0 0 0 0 0 0

TIME REQUIRED FOR THE PRECEDING BATCH WAS 1 MINUTE, 47 SECONDS.

### **A.3.2 With Source Step Biasing**

\*\*\*START BATCH 1

RANDOM=13579HHDYDR97

SOURCE DATA

YOU ARE USING THE DYNAPART VERSION OF GTMED WHICH ASSUMES GEOMETRY AND XSECT MEDIA ARE IDENTICAL.  
WTAVE TAVE VAVE XAVE XAVE XAVE AGEAVE  
1.051E+04 1.00 0.0008 0.4940 -1.962E+00 -1.798E+00 1.00E-04 0.0

PARTICLES PER CM\*\*2 PER SECOND

DETECTOR	BATCH RESPONSE	RESPONSE (DETECTOR) PSD	NEUTRON FLUX ACCUMULATED RESPONSE	FSD	CV( SD )	PO4
1	3.6444D-09	0.14482	3.6444D-09	0.14482	0.42367	2.2735D+01

DETECTOR PARTICLE DISTRIBUTION IN PERCENTAGE RNS RELATIVE TO INCREASING VALUE OF CONTRIBUTIONS

0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100.0
1	8896	0	0	0	0	636	212	32	1	0

NUMBER OF COLLISIONS OF TYPE NCOLL SOURCE SPLIT(0) FISHN GAGEN REACTLL ALFEDO BDYX ESCAPE  
10000 0 0 119497 0 25875 4895 E-CMT TIMEKILL R R KILL R R SURV GAMLOSS  
0 0 0 0 0 0 0 0 0 0 0

TIME REQUIRED FOR THE PRECEDING BATCH WAS 2 MINUTES, 5 SECONDS.

\*\*\*START BATCH 4

RANDOM-CRAH9B10R42B

SOURCE DATA

WAVE	IAVE	UAVE	VAVE	WAVE	XAVE	ZAVE	AVERAGE
9.821E+03	1.00	0.0097	-0.0100	0.5000	1.636E+00	1.488E+00	0.001E-04
PARTICLES PER CM**2 PER SECOND							

RESPONSES(DETECTOR)

DETECTOR	SEARCH RESPONSE	PSD	ACCELERATOR RESPONSE	NEUTRON FLUX	FSD	CY( SD )	FOM
1	2.9534D-09	0.02186	3.1261D 09	0.04334	0.40199	6.2840D+01	

DETECTOR

PARTICLE DISTRIBUTION IN PERCENTAGE BINS RELATIVE TO INCREASING VALUE OF CONTRIBUTIONS

0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100.0
1	35475	0	0	0	0	1167	1278	890	255	30

NUMBER OF COLLISIONS OF TYPE NCOLL  
SOURCE SPLIT(D) PISHN GAMGEN REAGCOLL ALBEDO BDRX ESCAPE F-CUT TIMEKILL R R KILL R R SURV GAMLOSS  
10000 0 0 0 141886 0 26247 4888 51112 0 0 0 0 0

TIME REQUIRED FOR THE PRECEDING BATCH WAS 2 MINUTES, 7 SECONDS.

\*\*\*START BATCH 6

RANICM- /A4ENDEADA318

SOURCE DATA

	XAVE	YAVE	ZAVE	XAVE	YAVE	ZAVE	XAVE	YAVE	ZAVE	XAVE	YAVE	ZAVE
WAVE	1.00	0.0196	0.0123	0.4962	-1.036E+00	5.241E+00	1.001E+04	1.001E+04	0.0	0.0	0.0	0.0
9.718E+03												

PARTICLES PER CM\*\*2 PER SECOND

DETECTOR	BATCH RESPONSE	RESPONSE PSD	NEUTRON FLUX ACCUMULATED RESPONSE	PSD	CY( SD )	FOM
1	3.4362D-09	0.06156	3.2357D-09	0.02471	0.30983	9.5769D+01

DETECTOR PARTICLE DISTRIBUTION IN PERCENTAGE BINS RELATIVE TO INCREASING VALUE OF CONTRIBUTIONS

	0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100.0
1	70879	0	0	0	0	2374	2538	1836	580	40	
NUMBER OF COLLISIONS OF TYPE NCOLL											
SOURCE SPLIT(D)	FISHN	GAMGEN	REALCOLL	ALBFDO	BIDRYX	ESCAPE	E CUT	TIMEKILL	R KILL	R SURV	GAMLOSS
10000	0	0	143372	0	26649	4737	5263	0	0	0	0

TIME REQUIRED FOR THE PRECEDING BATCH WAS 2 MINUTES, 9 SECONDS.

\*\*\*START BATCH 9 RANDOM-RAA14P712B1P

SOURCE DATA  
WAVE 1.064E+04 1.00 0.0058 0.0070 0.5098 9.696E-01 1.867E+00 1.001E-04 AGAVE  
PARTICLES PER CM\*\*2 PER SECOND

DETECTOR	BATCH RESPONSES(FSD)	RESPONSES(DETECTOR)	NEUTRON FLUX	FSD	CV(SD)	FOM
1	2.9282D-09	0.02840	3.1153D-09	0.02233	0.30345	1.0446D+02

DETECTOR PARTICLE DISTRIBUTION IN PERCENTAGE BINS RELATIVE TO INCREASING VALUE OF CONTRIBUTIONS

0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100.0
1	79719	0	0	0	0	2650	2863	2080	612	72

NUMBER OF COLLISIONS OF TYPE NCOLL  
SOURCE SPLIT(D) FSUR GAMEN REALCOLL ALBEDO UDRYX ESCAPE R-CUT TIMEKILL R R KILL R SURV GAMMOS  
100000 0 0 140324 0 26042 4881 5119 0 0 0 0

79 TIME REQUIRED FOR THE PRECEDING BATCH WAS 2 MINUTES, 6 SECONDS.

\*\*\*START BATCH 10

SOURCE DATA  
WAVE 1.00 0.0109 -0.0012 0.4969 -2.11E+00 3.172E+00 1.00E-04  
9.892E+03

PARTICLES PER CM\*\*2 PER SECOND

DETECTOR	BATCH	RESPONSE(DETECTOR)	NEUTRON FLUX	FSD	CY( SD )	FOM
1	4.7195D-09	0.24259	ACCUMULATED RESPONSE	3.2757D-09	0.03984	0.24139

DETECTOR PARTICLE DISTRIBUTION IN PERCENTAGE BINS RELATIVE TO INCREASING VALUE OF CONTRIBUTIONS

0.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100.0
1	88620	0	0	0	0	2940	5474	457	287	4

NUMBER OF COLLISIONS OF TYPE NCOLL  
SOURCE SPLIT(D) FISIN GAMGEN REACOL ALBEDO BDRX ESCAPE E-CUT TIMERILL R R KILL R R SURY GAMMOS  
10000 0 0 0 141903 0 26173 4641 5157 0 0 0 0

## Appendix B

# Another Approach to the Monte Carlo Point-Estimator Concept

The usual formulation of the point-detector estimator is given by:

$$CON = W \times P_s \times P(\Omega \rightarrow \Omega') \frac{e^{-\Sigma_{tr}}}{r^2}, \quad (B.1)$$

where  $W$  is the particle current weight,  $P_s$  the non-absorption probability,  $P(\Omega \rightarrow \Omega')$  the probability per steradian of scattering into the detector direction,  $e^{-\Sigma_{tr}}$  the probability of hitting the detector, and  $r$  the distance between the collision point and the detector position.

The approach proposed in this work is based on the fact that instead of using  $P(\Omega \rightarrow \Omega')$  on a steradian basis,  $P(\Omega \rightarrow \Omega')$  is interpreted in terms of a probability that the particle hits a sphere of unit cross section area placed in the detector position as shown in Figure B.1-a. This eliminates the need for the  $1/r^2$  factor and the contribution becomes

$$CON = W \times P_s \times P'(\Omega \rightarrow \Omega') e^{-\Sigma_{tr}}. \quad (B.2)$$

This is accomplished by calculating the solid angle formed by the sphere of unit cross section area at the detector position and the collision point, i.e

$$SA = \frac{1}{4\pi} \int_0^{2\pi} d\varphi \int_0^\pi \sin \theta d\theta = \frac{1}{2}(1 - \cos \theta) \quad (B.3)$$

Table B.1: Comparison of the Calculated Results.

Detector	SXE	SPD	FSD	NPD	FSD
1	3.8269E-04	3.6421E-04	0.0071	3.9522E-04	0.0164
2	4.4637E-05	3.9779E-05	0.0228	4.3999E-05	0.0512
3	8.9468E-06	8.1489E-06	0.0363	8.7902E-06	0.0754
4	2.2423E-06	2.1442E-06	0.0822	2.1719E-06	0.0865
5	6.2343E-07	7.0446E-07	0.1363	7.1119E-07	0.1392
6	1.8914E-07	1.9343E-07	0.2276	1.9432E-07	0.2301
7	5.9068E-08	5.5311E-08	0.2102	5.5492E-08	0.2118
8	1.9177E-08	1.4919E-08	0.1699	1.4935E-08	0.1699
9	6.2713E-09	4.2579E-09	0.2194	4.2640E-09	0.2195
10	2.0408E-09	1.0450E-09	0.2492	1.0464E-09	0.2490

and the new probability  $P'(\Omega \rightarrow \Omega')$  is given by

$$P'(\Omega \rightarrow \Omega') = 4\pi P(\Omega \rightarrow \Omega') \frac{1}{2}(1 - \cos \theta). \quad (\text{B.4})$$

The factor  $4\pi$  eliminates the per steradian basis of  $P(\Omega \rightarrow \Omega')$  and the angle  $\theta$  is shown in Figure B.1-b.

It can be seen from Figure B.1-b that whenever  $r$  is less than  $r'$ ,  $\theta$  assumes the value  $\pi/2$  so that the emergent particle will be normally incident to a unit area in the plane that contains the detector and the collision point and the contribution becomes simply  $W/2$ .

Table B.1 shows the results of the calculations for Sample Problem 1 but with the first three groups being analyzed. The results for the standard point detector estimator (SPD) were obtained using Equation B.1 with a unboundness correction scheme which uses an average contribution when particles collide within a distance of 0.5642 cm off the detector position. The calculations for Detectors 1 and 2 show a very good agreement for results with acceptable standard deviations. However, the accuracy of the new formalism (NPD) is better when compared with the results obtained with a surface crossing estimator (SXE). The results for the outer detectors, although with unacceptable standard deviations ( $FSD > 0.5$ ), are presented to illustrate the excellent agreement between the two point detector estimators.

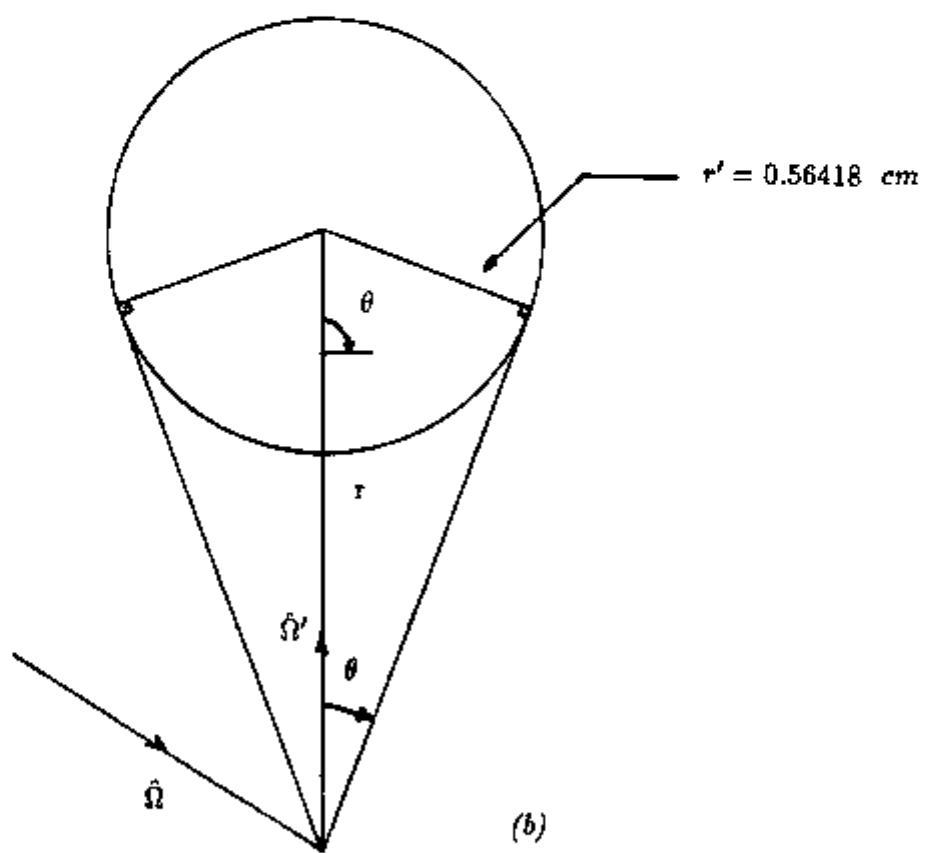
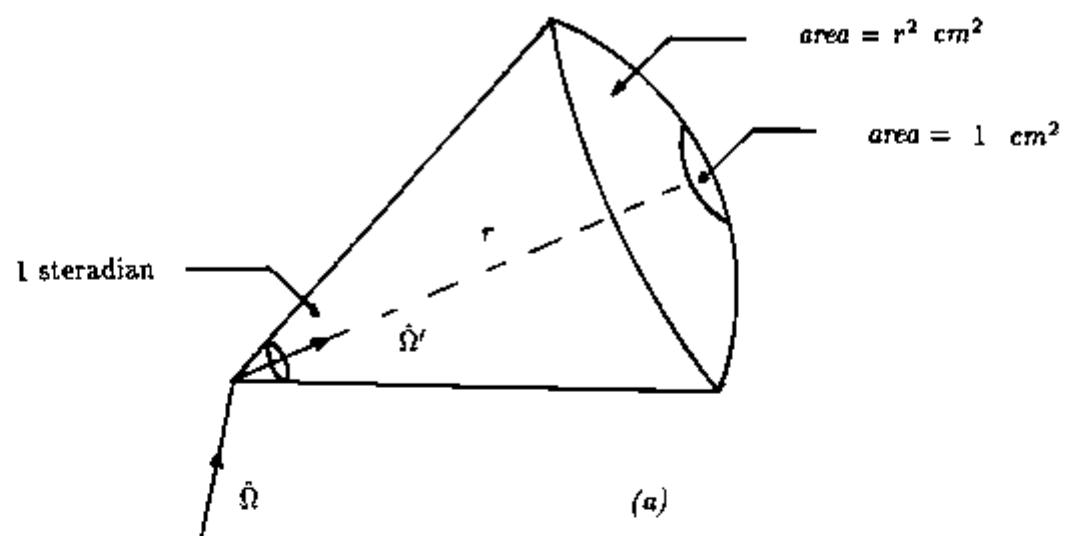


Figure B.1: Schemes of the New Point Detector Estimator.

## Appendix C

# Modified and Added Subroutines

The subroutines listed in this appendix are meant to be used as a guideline for the reader if he wants to implement the same modifications in his or her version of the MORSE code.

```
C**** WILSON MAY 11, 1989 ****          MOR 1150
C * * THIS IS THE MAIN ROUTINE * * * * * * * * * * * * * * * * *
C * *
C * * THE FOLLOWING CARD DETERMINES THE SIZE ALLOWED FOR BLANK COMMON * * * * *
C * * (REGION SIZE NEEDED IS ABOUT 150K + 4*(SIZE OF BLANK COMMON IN WORDS) ) *
C * * NOTE - THE ORDER OF COMMONS IN THIS ROUTINE IS IMPORTANT AND MUST CORRE-
C * * Spond TO THE ORDER USED IN DUMP ROUTINES SUCH AS HELP, XSCHLP, AND USRHLB
C * *
C * * LABELLED COMMONS FOR WALK ROUTINES * * * * * * * * * * * * * *
REAL * 8 SS
COMMON MC( 50000)
COMMON /APOLLO/ AGSTRT,DDF,DEADWT(26),ITOUT,ITIM
COMMON /FISBNK/ MFISTPP
COMMON /NUTRON/ NAME
C * *
C * * LABELLED COMMONS FOR CROSS-SECTION ROUTINES * * * * * * * * * *
COMMON /LOC SIG/ ISCCCG
COMMON /MEARS/ MM
COMMON /MOMENT/ MMOM
COMMON /QAL/ Q
COMMON /RESULT/ POINT
C * *
C * * LABELLED COMMONS FOR GEOMETRY INTERFACE ROUTINES * * * * * * * *
COMMON /NORMAL/ UNORM
C * *
C * * LABELLED COMMONS FOR USER ROUTINES * * * * * * * * * * * * * *
COMMON /PDET/ ND
COMMON /USER/ AGST
C * *
C * * COMMON /DUMMY/ WILL NOT BE FOUND ELSEWHERE IN THE PROGRAM * * *
COMMON /DUMMY/ DUM
COMMON /SSS/ SS(1000)
C * *
DATA JUNK/Z46484848/
```

```

NLFT = (LOC(DUM) - LOC(WC(1)))/4
DO 10 I=1,NLFT
10 WC(I) = JUNK
ITOUT = 6
ITIN = 6
NLFT = (LOC(AGSTRT) - LOC(WC(1)))/4
CALL MORSE(NLFT)
STOP
END
SUBROUTINE RELCOL                               RELCO 10
C * * THIS VERSION IS FOR USE WITH MORSE-CGA * * * * *
C                                         RELCO 20
C THIS VERSION IS FOR POINT DETECTORS LOCATED AT (XD,YD,ZD)   RELCO 30
C                                         RELCO 40
COMMON /USER/ AGSTAT,WTSTRT,XSTRT,YSTRT,ZSTRT,DFF,EBOTW,EBOTG,      RELCO 50
1 TCUT,IO,II,IADJM,WGPQT1,WGPQT2,WGPQT3,WGPQTG,WGPQTH,WITS,WLAST,    RELCO 51
2 NLFT,WMPG,WMTG,WSTRT
COMMON /PDET/ HD,HE,HE,WT,WA,WRESP,WEI,WEEND,WEEND,WEER,WEIR,WEIE,RELCO 60
1 NAME,WTEND,WEEND,WEEND,LOCRSP,LOCID,LOCIB,LOCDD,LOCCT,LOCUD,      RELCO 61
2 LOCSD,LOCQE,LOCQT,LOCQTE,LOCQAE,IMAX,EFIRST,EGTOP                RELCO 62
COMMON /NUTRON/ NAME,NAMEX,IG,IGO,WMED,WEDOLD,WREG,U,V,W,WOLD,VOLD,RELCO 70
1 ,WOLD,X,Y,Z,XOLD,YOLD,ZOLD,WATE,OLDWT,WTBC,BLZHT,BLZDN,AGE,OLDAGE,RELCO 71
COMMON BL(1)                                     RELCO 80
DIMENSION NL(1)                                 RELCO 90
EQUIVALENCE (BL(1),NL(1))                      RELCO 100
DATA WEST /1/. FWEST /1./                      RELCO 110

C     WEST + FWEST ARE THE NO. OF ESTIMATES TO BE MADE TO EACH DETECTOR   RELC 130
C * * * ISTAT MUST BE EQUAL TO 1.                                         * * * *
C * * * WEI MUST BE AT LEAST 1.                                         * * * *
C * * * WEEND MUST BE AT LEAST 1.                                         * * * *
DO 30 I=1,HD
IA=LOCID+I
XE = BL(IA)
YE = BL(IA+HD)
ZE = BL(IA+2*HD)                                RELC 160
                                         RELC 170
                                         RELC 180
                                         RELC 180
                                         RELC 200
C * * PTPT CALCULATES DISTANCE TO DETECTOR USING GLOBAL COORD. * *
CALL PTPT(XE,YE,ZE,A,B,C,THETA,BL,BL)          RELC 210
C     A = XE - I      - CALCULATED IN PTPT * *
C     B = YE - Y      - CALCULATED IN PTPT * *
C     C = ZE - Z      - CALCULATED IN PTPT * *
SD2=A*A+B*B+C*C
DS=SQRT (SD2)                                    RELC 240
                                         RELC 250
C * * * COS DEPENDS ON THE ANGLE OF INTEREST
COS=C/DS                                         RELC 270
C     THETA = (A*WOLD + B*VOLD + C*WOLD)/DS          RELC 280
IGOLD = IGO
IGQ = WGPQT3
IF (IGO.LE.WGPQT1) IGQ=WGPQT1
IA = LOCSP + WRESP*WMTG + 1
CALL PTHETA(WMED,IGOLD,IGQ,THETA,BL(IA),WMTG)    RELC 310
NES = 0                                         RELC 320
                                         RELC 330
                                         RELC 340

```

```

PSUM = 0.                               RELC 350
IA = IA - 1                           RELC 360
DO 5 IL=IGOLD,IGQ                   RELC 370
5   PSUM = PSUM + ABS (BL(IA+IL))    RELC 380
10  R = FLTRWF(0) * PSUM            RELC 390
DO 15 IL=IGOLD,IGQ                  RELC 400
IF (R-ABS (BL(IA+IL))) 20,20,15    RELC 410
15  R = R - ABS (BL(IA+IL))        RELC 420
IL = IGQ                            RELC 430
20  MARK=1                           RELC 440
AGED = AGE + DS/BL(NMTG+IL)         RELC 450
MEDIUM=NMED                         RELC 470
CALL EUCLID(MARK,X,Y,Z,XE,YE,ZE,DS,IL,ARG,O,MEDIUM,BLZNT,WREG)
COM = 0.0                             RELC 480
IF (ARG.LT.-1.E+64) GO TO 24        RELC 490
C*****BEWARE THIS VERSION WILL NOT WORK IF ENERGY BIASING IS USED
C*****WILSON 11/14/68 ****
C***** CORRECTED FOR THE UNBOUNDED CONTRIBUTION
C   COM = WATE*EXP (ARG)*SIGN (PSUM,BL(IA+IL))/FNEST      RELC 510
R1=0.564189584
ST= R1/DS
IF(ST.GT.1.0) GO TO 7
C   CALL NSIGTA(IG,NMED,TSIG,P)
C   DS1 = ARG-R1*TSIG
COM = WATE*EXP (ARG)*SIGN (PSUM,BL(IA+IL))/FNEST      RELC 510
CT=SQRT(SD2-R1*R1)/DS
CDM=COM+4.*3.1415962*(1-CT)/2.
GO TO 24
7   COM = WATE*0.5
C*****WILSON 11/14/88 ****
24  CALL FLUXST (I,IL,COM,AGED,COS,O)      RELC 530
25  NES = NES + 1                         RELC 540
INN=LOCXD+6*ND+I                        RELC 550
NL(INN)=NL(INN)+1                       RELC 560
IF (NES-NEST) 10,30,30                  RELC 570
30  CONTINUE
RETURN
END
C*****+
C                               +
C                               +
C                               +
C   SUBROUTINES OF INTEREST          +
C                               +
C   -PBANK      NOT MODIFIED
C   -FLUXST     MODIFIED
C   -GETINT    NOT MODIFIED
C   -GSTORE    MODIFIED
C   -INPUT1     "
C   -MORSE      "
C   -MSOUR     "

```

```

C      WBATCH      "
C      WPART      ADDED
C      WRWPRI     ADDED
C      - WRUN      MODIFIED
C      OUTPT      MODIFIED
C      SCORIN      "
C      STRTCH      "
C      STPART      ADDED
C      VAR2       MODIFIED
C      VAR3       MODIFIED
C      VAR4       ADDED
C
C      *          BDRY 380
C      *          BDRY 380
C      *          BDRY 380
C      *          BDRY 380
C
C*****SUBROUTINE FLUXST(I,IGE,FLUX,AGE,CDS,SWITCH)*****FLUX 10
C
C      REAL * 8 SS, SCORE,FLUX
C      INTEGER*4 SWITCH
C      * * SWITCH = 1 -- STORE IN ALL RELEVANT ARRAYS          FLUX 20
C      * * SWITCH = 0 -- STORE IN ALL RELEVANT ARRAYS EXCEPT UD          FLUX 30
C      * * SWITCH = -1 -- STORE IN ARRAY UD ONLY          FLUX 40
C
C      COMMON /USER/ AGSTR1,WTSTR1,I1TR1,Y1TR1,Z1TR1,DFF,E1OTW,E1OTG,          FLUX 60
C      1 TCUT,10,11,IADJM,NGPQT1,NGPQT2,NGPQT3,NGPQTG,NGPQTH,WITS,WLAST,          FLUX 61
C      2 WLEFT,WNGP,WNTG,WSTR1
C      COMMON /PDET/ ND,NE,SE,ST,BA,MRESP,WEI,WEIND,WEEND,WDNR,WTNR,WTNE,FLUX 70
C      1 NAME,WTNDWR,WTNEND,NAMEEND,LOCRSP,LOCID,LOCIB,LOCOC,LOCT,LOCUD,          FLUX 71
C      2 LOCSD,LOCQE,LOCQT,LOCQTE,LOCQAE,LMAX,EFIRST,EGTOP
C      COMMON BC(1)
C      COMMON /SSS/ SS(1)
C      DIMENSION BC(1)
C      EQUIVALENCE (BC(1),BC(1))
C      DATA K/1/
C      IF(IGE.GT.WNTG) GO TO 40
C      IF (SWITCH) 170,10,10
C 10  IF (WE) 60,60,20
C 20  J=BC(LOCIB+3*WE+IGE)
C      IF(J.LE.0) RETURN
C 50  IB = LOCQE + (I-1)*WE + J
C      SS(IB) = SS(IB) + FLUX
C 60  IF (NT) 90,90,70
C 70  IA = LOCT + (I-1)*NT + 1
C      DO 80 K=1,NT
C      IF (AGE-BC(IA)) 90,90,80
C 80  IA = IA + 1
C
C      K = NT
C      IA=IA-1
C      BC(IA)=AGE
C      IDT = LOCT + (ND+I)*NT
C      BC(IDT) = BC(IA) - BC(IA-1)
C 90  IF (WA) 120,120,100
C 100 IA = LOCCD + 1
C
C      FLUX 270
C      FLUX 290
C      FLUX 300
C      FLUX 310
C      FLUX 320

```

```

DO 110 L=1,WA          FLUX 330
  IF (COS-BC(IA)) 120,120,110   FLUX 340
110 IA = IA + 1         FLUX 360
40 CALL HELP(4EFXST,1,1,-1,1)   FLUX 180
                                CALL ERROR
120 IF (WE) 170,170,130   FLUX 190
130 IF (WT) 150,150,140   FLUX 370
140 ID = LOCQTE + (I-1)*WTNS + (J-1)*WT + X   FLUX 380
      SS(ID) = SS(ID) + FLUX
150 IF (WA) 170,170,160   FLUX 390
160 IE = LOCQAE + (I-1)*WANE + (J-1)*WA + L   FLUX 400
      SS(IE) = SS(IE) + FLUX
170 IS = (I-1)*WRESP + 1   FLUX 410
      IU = LOCUD + IS        FLUX 420
      IS = LOCSD + IS        FLUX 430
***** WILSON MAY 11, 1989 *****
      IW = LOCSD + (I-1)*WRESP + 3*MDNR + 1   FLUX 440
      IC = LOCQT + (I-1)*WTRA + (K-1)*WRESP + 1   FLUX 450
      IB = LDCRSP + IGE - WMTG                 FLUX 460
      DO 220 IR = 1,WRESP
      IB = IB + WMTG                         FLUX 470
      SCORE = FLUX*BC(IB)
      IF (SWITCH) 180,200,180   FLUX 480
180 SS(IU) = SS(IU) + SCORE
      IU = IU + 1                         FLUX 490
190 IF (SWITCH) 220,200,200   FLUX 500
200 SS(IS) = SS(IS) + SCORE
      SS(IW) = SS(IW) + SCORE
      IS = IS + 1                         FLUX 510
      IW = IW + 1                         FLUX 520
***** WILSON MAY 11, 1989 *****
      IF (WT) 220,220,210   FLUX 530
210 SS(IC) = SS(IC) + SCORE
      IC = IC + 1                         FLUX 540
220 CONTINUE
      RETURN
      END
      SUBROUTINE GSTORE(WSG,IGG)
C * * THIS VERSION OF GSTORE IS FOR MORSE-CGA * * * *
C   THIS ROUTINE CHECKS TO SEE IF THERE IS ROOM IN THE BANK AND IF SO GSTOR *
C       STORES DATA FOR THE GENERATED GAMMA                           GSTOR *
C   IT ASSUMES THAT THE GAMMA IS EMITTED UNIFORMLY IN DIRECTION           GSTOR *
COMMON /NUTRON/ NAME,NAMEX,IG,IGO,MMED,MEDOLD,MRREG,U,V,W,WOLD,VOLDGSTOR 20
1 ,WOLD,X,Y,Z,XOLD,YOLD,ZOLD,WATE,OLDWT,WTBC,BLZNT,BLZOW,AGE,OLDAGEGSTOR 21
COMMON /APOLLO/ AGSTRT,DDF,DEADWT(5),ETA,ETATH,ETAUSD,VIMP,VIMP, GSTOR 30
1 WIMP,WTSTRT,XSTRT,YSTRT,ZSTRT,TCUT,XTRA(10).                      GSTOR 31
2 IO,II,MEDIA,IADJM,ISBIAS,ISOUR,ITERS,ITIME,ITSTA,LOCWTS,LOCFWL,GSTOR 32
3 LOCEPR,LOCMSL,LOCFSI,MAXGP,MAXTM,MEDALB,MGPREG,MXREG,WAFL.          GSTOR 33
4 NDEAD(5),NEWNM,NGEOM,NGPQT1,NGPQT2,NGPQT3,NGPQTG,NGPQTH,BITS,      GSTOR 34
5 NKCALC,NKILL,NLAST,NNEM,NNGP,NNOST,WMTG,NOLEAK,NORMF,NPAST.          GSTOR 35
6 EPSCL(13),NQUIT,NSIGL,NSOUR,NSPLT,NSTRT,NXTRA(10)                  GSTOR 36

```

```

***** WILSON MAY 11, 1989 **** NOR 1150
      COMMON /MS/ MS          GSTOR 40
***** WILSON MAY 11, 1989 **** NOR 1150
      COMMON BC(1)           GSTOR 40
      DIMENSION NC(1)         GSTOR 50
      EQUIVALENCE (BC(1),NC(1)) GSTOR 60
      IF (NMOST-MS) 10,10,15   GSTOR 70
      10 IF (NPSCL(13) .EQ. 0) WRITE (IO,1010)   GSTOR 80
      1010 FORMAT ('WARNING * * * NO ROOM IN BANK FOR SECONDARIES * * *') GSTOR 90
      NPSCL(13) = NPSCL(13) + 1   GSTO 100
      RETURN                   GSTO 110
      15 MS = MS + 1           GSTO 120
      NEWNM = NEWNM + 1         GSTO 130
      HAS = NAME               GSTO 140
      WTS = WATE               GSTO 150
      IGS = IG                 GSTO 160
      NAME = NEWNM              GSTO 170
      WATE = W8G                GSTO 180
      IG = IGG                 GSTO 190
      US=U                     GSTO 200
      VS=V                     GSTO 210
      WS=W                     GSTO 220
      CALL GTISO(U,V,W)        GSTO 230
      CALL STORMT(MS,0)         GSTO 240
C     CALL BANKR(4) FOR GAMMA GENERATION ANALYSIS    * * *
      CALL BANKR(4)             GSTO 245
      NAME = HAS                GSTO 260
      WATE = WTS                GSTO 260
      IG = IGS                 GSTO 270
      U=US                     GSTO 280
      V=VS                     GSTO 290
      W=WS                     GSTO 300
      ISCT = LUCNSC + 6*BMTG*MIREG + (NREG-1)*BMTG + IG   GSTO 310
      NC(ISCT) = NC(ISCT) + 1   GSTO 320
      ISCT = ISCT + IGG - IG   GSTO 330
      NC(ISCT) = NC(ISCT) + 1   GSTO 340
      ISCT = ISCT + BMTG*MIREG GSTO 360
      BC(ISCT) = BC(ISCT) + W8G GSTO 360
      ISCT = ISCT + IG - IGG   GSTO 370
      BC(ISCT) = BC(ISCT) + WATE GSTO 380
      NPSCL(4) = NPSCL(4) + 1   GSTO 390
      RETURN                   GSTO 410
      END                      GSTO 420
      SUBROUTINE INPUT1
C * * THIS VERSION OF INPUT1 IS FOR MORSE-CGA * * *
C     THIS ROUTINE READS THE RANDOM WALK DATA AND CALLS ROUTINES TO READ * * *
C     SOURCE DATA AND GEOMETRY DATA.      INITIALIZES SOME VARIABLES * * *

```

```

***** WILSON MAY 11, 1989 **** NOR 1150
      1041 FORMAT (2I,18,9I5,F5.0,2I5)

```

```

1040 FORMAT (10I6,F6.0,2I5)
C**** WILSON MAY 11, 1989 **** MOR 1160
      WRITE(10,1050)MSTRT,MHOST,MITS,MQUIT,MGPQTW,MGPQTG,MNGP,MNTG,MCOLTPU 860
      1P,IADJM,AXTIM,MEDIA,MEDALS IMPU
1050 FORMAT('0',3I,'MSTRT',3I,'MHOST',4I,'MITS',3I,'MQUIT',2I,
      1'MGPQTW',2I,'MGPQTG',4I,'MNGP',4I,'MNTG',2I,'MCOLTP',3I,'IADJM' IMPU
      2,2I,'MAXTIM',3I,'MEDIA',2I,'MEDALS',/1I,10I8,F8.2,2I8) IMPU

      SUBROUTINE MORSE(MLEFT) MORSE 10
C THIS IS THE EXECUTIVE ROUTINE FOR THE RANDOM WALK PROCESS * * * MORSE *
C IT CONTROLS THE SUCCESSION OF EVENTS WHICH COMprise THE MORSE *
C * * * MONTE CARLO PROCESS * * * * * MORSE *
C **** THIS VERSION OF SUBROUTINE MORSE KILLS JOBS ON IO REQUESTS***** MORSE ****
C *** THIS VERSION OF MORSE IS FOR MORSE-CGA * * * *
      REAL*8 RANDOM, SS MORSE 15
      COMMON /APOLLO/ AGSTRT,DDF,DEADWT(6),ETA,ESTATE,ETAUSD,UINP,VIMP, MORSE 20
      1 WIMP,WTSTRT,XSTRAT,YSTRAT,ZSTRAT,TCUT,XTRA(10), MORSE 21
      2 IO,I1,MEDIA,IADJM,ISBIAS,ISOUR,ITERS,ITIME,ITSTR,LOCWTS,LOCFWL,MORSE 22
      3 LOCEPR,LOCMSC,LOCFSH,MAXGP,MAXTIM,MEDALS,MGPREG,MXREG,MALB, MORSE 23
      4 MDEAD(6),NEWHM,NEGEOM,MGPQT1,MGPQT2,MGPQT3,MGPQTG,MGPQTW,MITS, MORSE 24
      5 MKCALC,MKILL,MLAST,WMEM,WNGP,MHOST,MNTG,BOLEAK,WRMF,WPAST, MORSE 25
      6 MPSCL(13),MQUIT,MSIGL,MSOUR,MSPLT,MSTRAT,MTRA(10) MORSE 26
      COMMON /PDET/ MD,EHE,ME,MT,NA,MRESP,NEK,NEIND,NEND,NENDR,NTRR,NTME,
      1 NAE,NTEDER,NTEND,NAEND,LOCRESP,LOCAD,LOCIB,LOCJO,LOCCT,LOCUD,
      2 LOCSD,LOCQE,LOCQT,LOCQTE,LOCQAB,LMAX,EFIRST,EGTOP
      COMMON /HUTRON/ NAME,NAMEX,IG,IGO,IMED,MEOLD,MREG,U,V,W,VOLD,VOLDMORSE 30
      1 ,VOLD,X,Y,Z,ICLD,VOLD,ZOLD,WATE,OLDWT,WTRC,BLZET,BLZON,AGE,OLDAGEMORSE 31
      COMMON /FISBK/ MFISTP,MFISBN,MFISH,FTUTL,FWATE,WATEF MORSE 40
C**** WILSON MAY 11, 1989 **** MOR 1160
      COMMON /HS/ MS GSTOR 40
      COMMON /SSS/ SS(1) GSTOR 40
      COMMON /OUTB/ SWATE,XAVE,YAVE,ZAVE,EAVE,UAVE,VAVE,WAVE,AGEAVE GSTOR 40
C**** WILSON MAY 11, 1989 **** MOR 1160
      DIMENSION WTS(1) MORSE 60
      COMMON WTS(1) MORSE 60
      EQUIVALENCE(WTS(1),NTS(1)) MORSE 70
      CALL MSGR(IQ) MORS
C BEGIN NEW PROBLEM MORSE 80
10  MLAST=MLEFT MORSE 80
      CALL TIMER(-2,XTRA) MORS 100
      NXT = ICLOCK(0) MORS 110
      CALL INPUT MORS 120
C      READS CARDS A THRU D - CALLS SCORIN FOR CARDS E IF ISOUR .LE. ZERO 140
C      READS CARDS F THRU Q - CALLS JDMIN, XSEC AND SCORIN MORS 150
      CALL IOLEFT(NIO)
C ** A DUMMY IOLEFT IS PROVIDED FOR SITES NOT HAVING THIS CAPABILITY*
C      NIO IS TOTAL NO. IO'S LEFT AFTER INPUT ROUTINES FINISH
      WRITE(10,1001) NIO
1001 FORMAT(1I,'NUMBER OF IO REQUESTS LEFT AFTER INPUT IS ',I10)
      NIQB=NIO
      NTIO=0

```

```

ISIG=1
WTSTW=WTILL+WSPLT
MGPREG = MHTG*MIREG
MXTRA(8) = MGPREG
MCOMB = MGPQTH*MGPQTG
IA = LOCFWL + 1
IU = LOCFWL + MIREG
DO 15 I=IA,IU
IB = I + MIREG
15 WTS(IB) = WTS(I)
IAW = LOCWTS + 1
IUW = LOCWTS + 3*MGPREG
DO 20 I=IAW,IUW
IB = I + 12*MGPREG
20 WTS(IB) = WTS(I)
IMITS = MITS
IRUNS = MQUIT
INDEX=0
CALL TIMER(INDEX,XTRA)
WRITE (IO,1010) (XTRA(I),I=1,INDEX)

1010 FORMAT (29HOTIME REQUIRED FOR INPUT WAS ,10A4)           MORS 330
C BEGIN NEW RUN
25 MITS = IMITS
CALL BANKR(-1)                                                 MORS 320
DO 30 I=IA,IU                                                 MORS 350
IB = I + MIREG                                               MORS 360
30 WTS(I) = WTS(IB)
DO 35 I=IAW,IUW                                              MORS 370
IB = I + 12*MGPREG                                           MORS 380
35 WTS(I) = WTS(IB)
ITERS=MITS
ITSTR=0
***** WILSON APRIL 19, 1989 *****
ITIMEI=ICLOCK(0)
C BEGIN NEW BATCH
40 MMEM=NSTART
SWATE=0.
XAVE=0.
YAVE=0.
ZAVE=0.
UAVE=0.
VAVE=0.
WAVE=0.
AGEAVE=0.
EAVE=0.
***** WILSON APRIL 19, 1989 *****
IF (ITSTR) 45,50,45
45 MMEM = MFISH
50 CALL BANKR(-2)
C      CALLS STBTCH
MBATCH=MITS-ITERS+1                                         MORS 470
                                                               MORS 480
                                                               MORS 490
                                                               * * *
                                                               MORS 500

```

```

        CALL RMDOUT(RANDOM)                                MORS 601
        WRITE (IO,1015) WBATCH,RANDOM                     MORS 602
1015 FORMAT (15B1***START BATCH ,I4,25X,7RANDOM=,Z12/12H0SOURCE DATA) MORS 603
60  CALL MSOUR                                         MORS 610
C BEGIN NEW HISTORY                                     MORS 610
    MS = 1                                              MORS 660
    MMEM = MMEM - 1                                     MORS 650
61  CALL GETWT(MS,1)                                    MORS 660
    MS = MS - 1                                         MORS 660
    HALB = 0                                            MORS 570
    WGPQT = WGPQT1                                      MORS 580
65  IF (WATE) 70,165,70                                 MORS 590
70  IF(IG-WGPQT)>0,90,75                               MORS 600
75  IF(IG-WGPQT2)>0,160,80                            MORS 610
80  IF(IG-WGPQT3)>0,85,160                            MORS 620
85  WGPQT=WGPQT3                                      MORS 630
90  IGO=IG                                           MORS 640
    UOLD=U                                         MORS 650
    VOLD=V                                         MORS 660
    WOLD=W                                         MORS 670
    OLDWT=WATE                                     MORS 680
    XOLD=X                                         MORS 690
    YOLD=Y                                         MORS 700
    ZOLD=Z                                         MORS 710
    BLZON=BLZNT                                     MORS 720
    MEDOLD=MED                                       MORS 730
    DLDAGE=AGE                                      MORS 740
    IF(WTSTW.GT.0) CALL TESTW                         MORS 750
    IF(WATE)100,95,100                             MORS 760

95  MDEAD(1)=MDEAD(1)+1                               MORS 770
    DEADWT(1)=DEADWT(1)+OLDWT                        MORS 780
C R KILL                                             MORS 750
    GO TO 165                                         MORS 800
100 CALL WTCOL                                         MORS 810
    IF (WREG-MXREG) 102,102,101                      MORS
101 WRITE (IO,1016) WREG,MXREG                       MORS
1016 FORMAT (6H0NREG=,I5,8H, MXREG=,I5,8I8, MXREG ON CARD I MUST BE GEMORS
1 TO THE NUMBER OF REGIONS DESCRIBED IN GEOMETRY INPUT) MORS
    CALL EXIT                                         MORS
102 IF (TCUT.LE.0..OR.AGE.LT.TCUT) GO TO 110          MORS
    MDEAD(4)=MDEAD(4) +1                           MORS 840
    DEADWT(4) =DEADWT(4)+OLDWT                      MORS 850
C AGE KILL                                           MORS 820
    MPSCL(10) = MPSCL(10) + 1                        MORS 870
C CALL BANKR(10) FOR TIME-KILL ANALYSIS             MORS 840
    GO TO 165                                         MORS 890
110 IF(WATE)120,115,120                            MORS 900
115 MDEAD(2)=MDEAD(2)+1                           MORS 910
    DEADWT(2)=DEADWT(2)+WTBC                        MORS 920
C ESCAPE                                              MORS 890
    GO TO 165                                         MORS 940

```

```

120 IF(WALE)130,130,126                               MORS 950
125 ISCT = LOCMSC + 2*WGPREG + (WREG-1)*WMTG + IG      MORS 960
    WTS(ISCT) = WTS(ISCT) + 1                           MORS 970
    ISCT = ISCT + WGPREG                                MORS 980
    WTS(ISCT) = WTS(ISCT) + WATE                         MORS 990
    CALL ALBDO(IG,U,V,W,WATE,WMED,WREG)                  MUR 1000
    WPSCL(6) = WPSCL(6) + 1                             MUR 1010
    CALL BANKR(6)                                       MUR 1020
    GO TO 65                                         MUR 1030
130 CALL GTMED(WMED,IMED)                            MDR 1040
    IF (MFISTP) 140,140,135                           MUR 1050
135 IF (WTS(LOCFSH+(IMED-1)*WMTG+IG)) 140,140,136   MORS
136 CALL FPROB                                     MORS
140 IF (WCNFB) 155,155,145                           MDR 1070
145 IF(WTS(LOCFSH+(2*MEDIA+IMED-1)*WMTG+IG))155,155,150 MDR 1080
150 CALL GPROB                                     MDR 1090
C
155 ISCT = LOCMSC + (WREG-1)*WMTG + IG              MOR 1110
    WTS(ISCT) = WTS(ISCT) + 1                         MOR 1120
    ISCT = ISCT + WGPREG                                MOR 1130
    WTS(ISCT) = WTS(ISCT) + WATE                        MOR 1140
    ISCT = LOCMSC + 8*WGPREG + IMED                   MOR 1150
    WTS(ISCT) = WTS(ISCT) + 1                         MOR 1160
    CALL COLISH( IG,U,V,W,WATE,WMED,WREG)             MOR 1170
C
    WPSCL(5) = WPSCL(5) + 1                           MOR 1180
    CALL BANKR(5)                                       MDR 1200
C     CALLS RELCOL                                 MOR 1090
    GO TO 65                                         MOR 1220
160 RDEAD(3)=RDEAD(3)+1                            MDR 1230
    DEADWT(3)=DEADWT(3)+WATE                         MDR 1240
    WPSCL(9) = WPSCL(9) + 1                           MOR 1250
C     CALL BANKR(9)  FOR E-CUT ANALYSIS            MOR 1140
C     ENERGY CUTOFF                                MOR 1150
***** WILSON APRIL 18,1989 *****
165 IF (WS.GT.0) GO TO 61                           MDR 1280
    CALL MPART                                      MDR 1280
    CALL STPART                                     MDR 1280

    IF (WMEM) 170,170,60                           MDR 1280
***** WILSON APRIL 18,1989 *****
C     END OF HISTORY                            MOR 1170
170 CALL BANKR(-3)                                    MOR 1300
C     CALLS WBATCH                                 MOR 1190
***** WILSON APRIL 18,1989 *****
    IF (ITERS) 185,185,55                           MORS 520
55  CALL OUTPT(1)                                    MORS 530
    CALL WRPRI(ITIMEI,WBATCH)                      MUR 1410
***** WILSON APRIL 18,1989 *****
    CALL OUTPT(2)                                    MDR 1320
    IF(ICLOCK(0)-MIT-MAXIM) 181,181,175           MDR 1330
181 CALL IOLEFT(WIO)                                MUR

```

```

C      NIO IS NO. IO'S LEFT AFTER CURRENT BATCH COMPLETED          MOR
      NIOP=NIOB-NIO          MOR
C      NIOB IS NO. IO'S IN BATCH JUST COMPLETED          MOR
      IF(NIOB.GT.NTIO) NTIO=NIOB          MOR
      IF(NIO.GT.NTIO) GO TO 180          MOR
      ISIG=2          MOR
175   NITS = NITS - ITERS + 1          MOR 1340
      ITERS = 0          MOR 1350
      NQUIT = NQUIT - IRUNS          MOR 1360
      IRUNS = -NQUIT          MOR 1370
      IF(ISIG.EQ.1) WRITE(IO,1020) IRUNS,NITS,NITS
1020 FORMAT(1H0/39HORUN TERMINATED BY EXECUTION TIME LIMIT          MOR 1390
      1                  /18,SH RUNS OF,13,SH BATCHES,16H AND 1 RUN MOR 1391
      20F,13,19H BATCHES COMPLETED./)          MOR 1392
      IF(ISIG.EQ.2) WRITE(IO,1050) IRUNS,NITS,NITS
1050 FORMAT(1H0/39HORUN TERMINATED BY LIMIT ON IO          MOR
      1                  /18,SH RUNS OF,13,SH BATCHES,16H AND 1 RUN MOR
      20F,13,19H BATCHES COMPLETED./)
180   ITERS = ITERS - 1          MOR 1400
      IF(ITTERS)195,195,180          MOR 1410
185   IF(MSOUR)40,40,180          MOR 1420
190   ITSTR=1          MOR 1430
C     END OF BATCH          MOR 1320
      GO TO 40          MOR 1450
195   CALL BANKR(-4)          MOR 1460
C     CALLS WRUN          MOR 1350
      NQUIT=NQUIT-1          MOR 1480
      INDX = -1          MOR 1490
      CALL TIMER(INDX,XTRA)          MOR 1500
      WRITE (IO,1030) NITS,(XTRA(I),I=1,INDX)          MOR 1510
1030 FORMAT (32HOTIME REQUIRED FOR THE PRECEDING,14,13H BATCHES WAS ,10MOR 1520
      1A4)
      CALL TIMER(-2,XTRA)
C     END OF NITS BATCHES          MOR 1370
      IF (NQUIT) 200,200,25          MOR 1550
200   CALL OUTPT(3)          MOR 1560
C     END OF RUN          MOR 1400
      FTIME = ICLOCK(0) - MXT          MOR 1580
      FTIME=FTIME/8000.          MOR 1590
      WRITE(IO,1040) FTIME          MOR 1600
1040 FORMAT (37HOTOTAL CPU TIME FOR THIS PROBLEM WAS ,F6.2,9H MINUTES.)MOR 1610
      GO TO 10          MOR 1620
      END          MOR 1630

      SUBROUTINE MSOUR          MSOUR00100
C
C     THIS IS THE EXECUTIVE ROUTINE FOR THE GENERATION AND STORAGE OF      MSOUR *
C     SOURCE PARAMETERS AT THE START OF EACH BATCH      *      MSOUR *
C
C *** THIS VERSION OF MSOUR IS DESIGNED FOR THE COMBGEOM PKG WITH MARS      MSOUR *
      REAL*8 IDUM,DIST,UDUM
      COMMON /FISBNK/ MFISTP,EFISB,EFISE,FTOTL,FVATE,WATER      MSOUR00200

```

```

COMMON /UTRDB/ NAME,NAMEX,IG,IGO,INMED,MEDOLD,MREG,U,V,W,UCLD,VLDSNS000300
1 ,WOLD,X,Y,Z,XOLD,YOLD,ZOLD,WATE,OLDWT,WTBC,BLZNT,BLZOM,AGE,OLDAGENS000400
COMMON /APOLLO/ AGSTRT,DDF,DEADWT(5),ETA,ETATE,ETASD,UIWP,VINP,
1 VINP,WTSTRT,ISTRT,YSTRT,ZSTRT,TCUT,XTRA(10), MS000600
2 IO,11,MEDIA,IADJM,ISBIAS,ISOUR,ITERS,ITIME,ITSTR,LOCWTS,LOCFWL,MS000700
3 LOCEPR,LOCFSN,MAIGP,MAITIM,MEDALS,MGPREG,MREG,MALS, MS000800
4 NDEAD(5),NEWIN,NGEOM,MGPQT1,MGPQT2,MGPQT3,MGPQTG,MGPQTE,MITS, MS000900
5 NRCALC,NRILL,NLAST,NMEM,MGP,HOST,MNTG,NOLEAK,NORMF,NPAST, MS001000
6 NPSCl(13),NQUIT,NSIGL,NSOUR,NSPLT,NSTRT,NITRA(10) MS001100
COMMON/GOMLOC/ KMA,KFPD,KLCR,KPSD,KIOR,KRIZ,KACZ,KMIZ,KMCZ, MS001300
1 KKR1,KKR2,KNSR,KVOL,MADD,LDATA,LTMA,LFFD,NUMR,IRTRU,NUMB,NIR MS001400
2 ,KBLZ,KBCZ
COMMON /ORG1/ DIST ,MARK,INMED,MBLZ MS00
COMMON/ARAB/NSY,NLEV,NAR,YQ,IAW,IAY,IP,MI1(3) MS00
COMMON/REPEAT/ JP(20) MS00
COMMON /MGDMV/ MUS,MUZ,LL,IPRET,IFLOW,IECT,MLD,IGX MS00
COMMON /OUTB/ SWATE,XAVE,YAVE,ZAVE,EAVE,UAVE,VAVE,WAVE,AGEAVE GSTUR 40
COMMON NSTOR(1) MS001500
DIMENSIDE IDUM(3),UDUM(3) MS00
INTEGER BLZNT MS001600
VOLD=0. MS001700
VOLD=0. MS001800
WOLD=0. MS001900
ETATE=0. MS002000
XOLD=0. MS002100
YOLD=0. MS002200
ZOLD=0. MS002300
BLZOK=0. MS002400
OLDWT=WTSTRT MS002500
ETA=0. MS002600
IGO=0 MS002700
MEDOLD=0. MS002800
OLDAGE=0. MS002900
WATE=WTSTRT MS003700
I=ISTRT MS003800
T=YSTRT MS003900
Z=ZSTRT MS004000
AGE=AGSTRT MS004100
NAMEX=1 MS004200
CALL SOURCE(IG,UIWP,VINP,WIMP,I,Y,Z,WATE,INMED,AGE,ISOUR,ITSTR,MGP MS004300
1QT3,DDF,ISBIAS,MNTG) MS004400
C *** DEFINE SOURCE ANGLE BEFORE CALLING CALI - LOOKZ ** MS004400
IF(ABS(UIWP)+ABS(VINP)+ABS(WIMP)) 35,35,30 MS005400
30 U1=UIWP MS005500
V1=VINP MS005600
W1=WIMP MS005700
GO TO 40 MS005800
35 CALL GTISO(U1,V1,W1) MS006900
C SELECT ISOTROPIC DIRECTION COSINES * * * MS00700
40 U=U1 MS006000
V=V1 MS006100

```

```

W=W1                               MS006200
UDUM(1)=U                           MS004500
UDUM(2)=V                           MS004600

UDUM(3)=W                           MS004700
KDUM(1)=X                           MS004500
KDUM(2)=Y                           MS004600
KDUM(3)=Z                           MS004700
MIX = 6*MLEV+4                     MS00
IF(MLEV.LE.0) GO TO 110            MS00
DO 100 I=1,MIX                   MS00
IX1 = JP(12)+I-1                  MS00
100 NSTOR(IX1) = 0                MS00
110 CONTINUE                       MS00
JP4 = JP(4)                         MS00
JP5 = JP(5)                         MS00
JP7 = JP(7)                         MS00
JP8 = JP(8)                         MS00
JPO = JP(10)                        MS00
LL = 0                             MS00
JL = 0                             MS00
JLU = 0                            MS00
C   SUBROUTINE CALI WILL CALL LOOKZ
C   THIS CALL TO LOOKZ CORRESPONDS TO THE COMBGEOM VERSION OF LOOKZ
CALL CALI(JL,JLU,KDUM,UDUM,JP,NSTOR(JP4),NSTOR(JP5),NSTOR(JP7),
1 NSTOR(JP8),NSTOR(JPO),NSTOR(KPPD),NSTOR(KMA ),NSTOR(KLCR),
. NSTOR,NSTOR)                      MS004800
BLZMT=MBLZ                          MS004900
MREG=NSTOR(KRIZ+NMEDG-1)           MS005100
NMED=NSTOR(KMIZ+NMEDG-1)           MS005200
NAME=1                               MS005300
IF(ISOUR)50,50,45                  MS006300
45  IG=ISOUR                         MS006400
50  CALL STORT(1,0)                  MS006500
C   PLACE SOURCE PARAMETERS IN PARTICLE BANK *          *
MPSCL(1) = MPSCL(1) + 1             MS006600
C ** PARTICLE POSITION MUST BE IN LOCAL COORDINATES WHEN SDATA CALLED * MS00
C ** GETWT SELECTS X,Y,Z FROM BANK AND CONVERTS TO LOCAL COORD
CALL GETWT(1,1)                     MS00
SWATE=SWATE+WATE                  OUTP 330
XAVE=XAVE+WATE*X                  OUTP 340
YAVE=YAVE+WATE*Y                  OUTP 350
ZAVE=ZAVE+WATE*Z                  OUTP 360
EAVE=EAVE+WATE*IG                 OUTP 370
UAVE=UAVE+WATE*U                  OUTP 380
VAVE=VAVE+WATE*V                  OUTP 390
WAVE=WAVE+WATE*W                  OUTP 400
AGEAVE=AGEAVE+WATE*AGE            OUTP 410
CALL BANKR(1)                      MS006700
NEWNM=1                            MS006900
RETURN                            MS007300
END                                MS007400

```

```

SUBROUTINE UNPART                                MBATC 10
REAL * 8 F, BC

C THIS ROUTINE IS CALLED AT THE END OF EACH HISTORY TO DO THE SUMS   MBATC *
C NEEDED FOR ESTIMATED MEANS AND FOR CALC. OF BATCH STATISTICS   MBATC *
COMMON /PDET/ ND,NEE,NE,NT,NA,MRESP,NEX,NEND,NDER,NTMR,NTME,MBATC 20
1 NAME,NTDER,NTEND,NAME,LOCNSP,LOCSD,LOCQD,LOCIB,LOCQD,LOCUT,LOCUD,   MBATC 21
2 LOCSD,LOCQE,LOCQT,LOCQTE,LOCQAE,LMAX,EFIRST,EGTOP               MBATC 22
C COMMON BC(1)                                         MBATC 30
COMMON /PRE/ W(189),NFRQ(10,188)                  MBATC 30
COMMON /SSS/ BC(1)                                     MBATC 30
DIMENSION IW(9),JW(21)
DATA IW/1,2,3,4,5,6,7,8,9/,JW/-19,-18,-17,-16,-15,-14,-13,
*-12,-11,-10,-9,-8,-7,-6,-5,-4,-3,-2,-1,0,1/
DATA IFLA/0/

IF(IFLA.GT.0) GO TO 301
IFLA=1
K=0
DO 303 J=1,21
DO 302 I=1,9
K=K+1
W(K)=IW(I)*10.**JW(J)
302 CONTINUE
303 CONTINUE
301 DO 70 I=1,ND                                MBATC 40
IU = LOCUD+(I-1)*MRESP                         MBATC 60
IS = LOCSD +(I-1)*MRESP                         MBATC 60
IF = LOCQE +(I-1)*NE                            MBATC 70
IT = LOCQT +(I-1)*NTMR                          MBATC 80
IE = LOCQTE +(I-1)*NTME                          MBATC 90
IA = LOCQAE +(I-1)*NAME                          MBATC 100
IF (NE) 15,15,5                                  MBATC 110
5      DO 10 J=1,NE
ISUB = IF + J
F = BC(ISUB)                                     MBATC 120
ISUB = ISUB + NEND                             MBATC 130
BC(ISUB) = BC(ISUB) + F                         MBATC 140
IF (DABS(F) .LT. 1.0E-35) F = 0.0              MBATC 150
ISUB = ISUB + NEND                             MBATC 160
10     BC(ISUB) = BC(ISUB) + F*F                MBATC 170
15     DO 30 M=1,MRESP                           MBATC 180
MBATC 190
MBATC 200
MBATC 300
C *****THIS PART FOR THE UNCOLL*****
ISUB = IU + M                                    MBATC 280
F = BC(ISUB)                                     MBATC 290
ISUB = ISUB + NDMR                            MBATC 300
BC(ISUB) = BC(ISUB) + F                         MBATC 310
IF (DABS(F) .LT. 1.0E-35) F = 0.0              MBATC 320
ISUB = ISUB + NDMR                            MBATC 330
BC(ISUB) = BC(ISUB) + F*F                      MBATC 340
C *****THIS PART FOR THE BATCH *****
ISUB = IS + M                                    MBATC 300
ISUB = ISUB + 3*NDMR                           MBATC 260
MBATC 300

```

```

F = BC(ISUB) MBAT 290
ISUB = ISUB + MDER MBAT 300
BC(ISUB) = BC(ISUB) + F MBAT 310
IF (DABS(F) .LT. 1.0E-35) F = 0.0 MBAT 320
ISUB = ISUB + MDER MBAT 330
BC(ISUB) = BC(ISUB) + F*F MBAT 340
C *****THIS PART FOR THE ACCUMULATIVE *****
ISUB = ISUB + MDER MBAT 300
BC(ISUB) = BC(ISUB) + F MBAT 310
C WRITE(50,650) I,F MBAT 300
650 FORMAT(1H ,I2,3X,1PE11.2)
C*****FOR FREQUENCY CALCULATIONS*****
IF(F.GT.0.) GO TO 300 MBAT 300
MFREQ(I,1)=MFREQ(I,1)+1 MBAT 300
GO TO 310 MBAT 310
300 IF(F.LT.90.) GO TO 322 MBAT 320
MFREQ(I,188)=MFREQ(I,188)+1 MBAT 330
GO TO 310 MBAT 340
322 DO 323 L=2,167 MBAT 350
TEST=W(L)-F MBAT 360
IF(TEST.GT.0.) GO TO 333 MBAT 370
323 CONTINUE MBAT 380
333 MFREQ(I,L)=MFREQ(I,L)+1 MBAT 390
C*****FOR FREQUENCY CALCULATIONS*****
310 IF (DABS(F) .LT. 1.0E-35) F = 0.0 MBAT 400
ISUB = ISUB + MDER MBAT 410
BC(ISUB) = BC(ISUB) + F*F MBAT 420
ISUB = IS + M MBAT 430
ISUB = ISUB + 10*MDER MBAT 440
BC(ISUB) = BC(ISUB) + F*F*F MBAT 450
ISUB = ISUB + MDER MBAT 460
BC(ISUB) = BC(ISUB) + F*F*F*F MBAT 470
IF (WT) 30,30,20 MBAT 480
20 DO 25 K=1,WT MBAT 490
ISUB = IT + (K-1)*MRESP + M MBAT 500
F = BC(ISUB) MBAT 510
ISUB = ISUB + MTMDER MBAT 520
BC(ISUB) = BC(ISUB) + F MBAT 530
IF (DABS(F) .LT. 1.0E-35) F = 0.0 MBAT 540
ISUB = ISUB + MTMDMR MBAT 550
25 BC(ISUB) = BC(ISUB) + F*F MBAT 560
30 CONTINUE MBAT 570
IF (WE) 70,70,35 MBAT 580
35 DO 65 J=1,WE MBAT 590
ITE = IE + (J-1)*WT MBAT 600
IAE = IA + (J-1)*WA MBAT 610
IF (WT) 50,50,40 MBAT 620
40 DO 45 K=1,WT MBAT 630
ISUB = ITE + K MBAT 640
F = BC(ISUB) MBAT 650
ISUB = ISUB + MTMEND MBAT 660

```

```

BC(ISUB) = BC(ISUB) + F          MBAT 540
IF (DABS(F) .LT. 1.0E-35) F = 0.0 MBAT 550
ISUB = ISUB + MTEEND            MBAT 560
45 BC(ISUB) = BC(ISUB) + F+F    MBAT 570
60 IF (NA) 66,65,65             MBAT 580
65 DO 60 L=1,NA                MBAT 590
     ISUB = IAE + L              MBAT 600
     F = BC(ISUB)               MBAT 610
     ISUB = ISUB + MAEEND      MBAT 620
     BC(ISUB) = BC(ISUB) + F    MBAT 630
     IF (DABS(F) .LT. 1.0E-35) F = 0.0 MBAT 640
     ISUB = ISUB + MAEEND      MBAT 650
60 BC(ISUB) = BC(ISUB) + F+F    MBAT 660
65 CONTINUE                      MBAT 670
70 CONTINUE                      MBAT 680
     RETURN                      MBAT 690
     END                         MBAT 700

SUBROUTINE MBATCH(MSDRC)          MBATC 10
C THIS ROUTINE IS CALLED AT THE END OF EACH BATCH TO DO THE SUMS MBATC *
C NEEDED FOR ESTIMATED MEANS AND FOR CALC. OF BATCH STATISTICS MBATC *
REAL * 8 F ,BC
COMMON /PDET/ MD,NNE,NE,NT,NA,WRESP,BEI,WEEND,WEND,NDNR,WTNR,NTNE,MBATC 20
1 WANE,WTEND,WTEND,WAEND,LOCRSP,LOCXD,LOCIB,LOCCO,LOCT,LOCUD, MBATC 21
2 LOCSD,LOCQE,LOCQT,LOCQTE,LOCQAE,LMAX,EFIRST,EGTOP           MBATC 22
COMMON /SSS/ BC(1)               MBATC 30
DO 70 I=1,MD                  MBATC 40
     IS = LOCSD + (I-1)*WRESP   MBATC 60
     DO 30 M=1,WRESP            MBATC 200
     ISUB = IS + M              MBATC 280
     F = BC(ISUB)               MBATC 290
     ISUB = ISUB + NDNR         MBATC 300
     BC(ISUB) = BC(ISUB) + F    MBATC 310
     IF (DABS(F) .LT. 1.0E-36) F = 0.0 MBATC 320

     ISUB = ISUB + NDNR         MBATC 330
     BC(ISUB) = BC(ISUB) + F+F/MSDRC MBATC 340
30 CONTINUE                      MBATC 440
70 CONTINUE                      MBATC 680
     RETURN                      MBATC 690
     END                         MBATC 700

SUBROUTINE WRMPAI(ITIMEI,MBATCH)   MRUN  10
C THIS ROUTINE IS CALLED AT THE END OF EACH BATCH MRUN  +
C IT NORMALIZES AND OUTPUTS CALCULATED QUANTITIES ALONG WITH MRUN  +
C FRACTIONAL STANDARD DEVIATIONS          MRUN  +
REAL * 8 FM1,FM2,FSAVE1,FSAVE2,E,ANUMB,FOM,FTIME0
COMMON /USER/ AGSTRT,WTSTR,ISTRT,YSTR,2STR,DFF,EBOTN,EBOTG, MRUN  20
1 TCUT,IO,I1,IADJM,WGPQT1,WGPQT2,WGPQT3,WGPQTG,WGPQTN,MITS,MLAST, MRUN  21
2 NLEFT,WGP,WTG,YSTRT             MRUN  22
COMMON /PDET/ MD,NNE,NE,NT,NA,WRESP,WE,WEEND,WEND,NDNR,WTNR,NTNE,MRUN  30
1 WANE,WTEND,WTEND,WAEND,LOCRSP,LOCXD,LOCIB,LOCCO,LOCT,LOCUD, MRUN  31

```

```

2 LOCSD,LOCQE,LOCQT,LOCQTE,LOCQAE,LMAX,EFIRST,EGTOP           MRUN  32
COMMON /SSS/ E(1)
COMMON /PRE/ W(189),WFREQ(10,188)
COMMON A(1)
DIMENSION NUMB(1),IHOLL(10),ANUMB(1),P(10),PP(10),NSUM(20,10)   MRUN  50
EQUIVALENCE (A(1),NUMB(1)), (IHOLL(1),STR)
EQUIVALENCE (E(1),ANUMB(1))
DATA IH1/1H1/, IH2/1H2/, IHO/1H0/                                MRUN  60
DATA P/0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,0.999/                 MRUN  70
DATA PP/10.,20.,30.,40.,50.,60.,70.,80.,90.,100./                MRUN  70
NPART1= NSTART
IFL = 1
IF(NITS.EQ.NBATCH) IFL = -1
NPART2= NBATCH*NSTART
NPART = NBATS*NSTART
DO 350 K=1,188
WRITE(60,670)W(K),(WFREQ(I,K),I=1,ND)
670 FORMAT(1H ,1PE9.2,10(1X,I6))
350 CONTINUE
FM1= 1.0/NPART1
FM2= 1.0/NPART2
IF (IADJM) 10,10,5
5 FM1= FM1*DFF
FM2= FM2*DFF
10 IX = LOCSD + 4*NDMR + 1
IX2 = IX + NDMR
CALL VAR2(E(IX),E(IX2),NRESP,ND,1,NSTART)
IX1 = LOCSD + 6*NDMR
IX3 = IX1 + NDMR
IX = IX3 + NDMR
IX2 = IX + NDMR
DO 300 I=1,NDMR
E(IX+I) = E(IX1+I)
E(IX2+I) = E(IX3+I)
300 CONTINUE
IX = LOCSD + 8*NDMR + 1
IX2 = IX + NDMR
IX3 = IX2 + NDMR
IX4 = IX3 + NDMR
IX5 = IX4 + NDMR
IX6 = IX5 + NDMR
CALL VAR4(E(IX),E(IX2),E(IX3),E(IX4),E(IX5),E(IX6),
*NRESP,ND,1,NPART2,IFL)
C * * THE STATEMENTS IN THE FOLLOWING LOOP NORMALIZE ALL ARRAYS TO BE OUTPUT
FSAVE1= FM1
FSAVE2= FM2
DO 120 I=1,ND
FM1= FSAVE1*A(LOCSD+5*ND+I)
FM2= FSAVE2*A(LOCSD+5*ND+I)
IA = LOCSD + (I-1)*NRESP + 4*NDMR
IB = LOCSD + (I-1)*NRESP + 6*NDMR

```

```

DO 76 M=1,MRESP          MRUN 470
E(IA+M) = E(IA+M)*FW1    MRUN 490
E(IB+M) = E(IB+M)*FW2    MRUN 490
76 CONTINUE                MRUN 490
120 CONTINUE                MRUN 490
     IHP = IBO                MRUN 890
C * * OUTPUT SBD, SBD2, SAD AND SAD2 ARRAYS IN THE FOLLOWING LOOP    MRUN 870
DO 125 M=1,MRESP          MRUN 910
     IHOL = MLAST + (M-1)*20 + 1           MRUN 920
     IHOL2 = IHOL + 19           MRUN 930
     WRITE (IO,1020) IHP,(A(I),I=IHOL,IHOL2)    MRUN 940
1020 FORMAT (A1,20I4)        MRUN 950
     IHP = IBS                MRUN 960
     IST = LOCQE - 9           MRUN 970
     WRITE (IO,1030) (ANUMB(I),I=IST,LOCQE)    MRUN 980
1030 FORMAT (1H0,32X,20HRESPONSES(DETECTOR),10A8,/,5X,8HDETECTOR,9X,   MRUN 990
  1 6HBATCH ,12X,3HFSD,10X,11HACCUMULATED,10I,3EFSD,11I,6HCV(SD),   MRUN 991
  2 11I,3HFOM,/,,
  3 21I,BHRESPONSE,26I,8HRESPONSE)           MRUN 992
C1030 FORMAT (1H0,32X,20HRESPONSES(DETECTOR),10A8,/,5X,8HDETECTOR,7X,   MRUN 990
C  1 6HBATCH ,10X,3HFSD, 8I,11HACCUMULATED, 8X,3EFSD, 9I,7HCV(SDI),   MRUN 991
C  2 8I,6HCV(SD),9I,3HFOM,/,,
C  3 19I,BHRESPONSE,24X,8HRESPONSE)           MRUN 992
     WRITE(40,222) MPART2
222 FORMAT (1H ,/,14X,' NUMBER OF PARTICLES ',18,/)
     WRITE(30,232)          MPART2
232 FORMAT (1H ,/,20X,18,/)
     DO 125 I=1,ND          MRU 1000
     IA = LDCSD + 4*MDDR + (I-1)*MRESP + M    MRU 1030
     IB = IA + MDDR          MRU 1040
     IC = LDCSD + 8*MDDR + (I-1)*MRESP + M    MRU 1030
     ID = IC + MDDR          MRU 1040
     IL = LDCSD + 12*MDDR + (I-1)*MRESP + M   MRU 1030
     IF = IL + MDDR          MRU 1040
     ITIMEQ = ICLOCK(0)-ITIMEI      MRU 1040
     FTIMEQ = ITIMEU/6000.0       MRU 1040
     FOM = 1./(E(ID)**2*FTIMEQ)    MRU 1040
C125 WRITE (IO,1040) I,E(IA),E(IB),E(IC),E(ID),E(IL),FOM    MRU 1050
     WRITE (30,1051) I,E(IA),E(IB),E(IC),E(ID),E(IF),FOM    MRU 1050
     WRITE (40,1041) I,E(IA),E(IB),E(IC),E(ID),E(IF),FOM    MRU 1050
1051 FORMAT (2I,13,'&',1PE11.4,'&',OPPF7.4,'&',
           *1PE11.4,'&',OPPF7.4,'&',OPPF7.4,'&',
           *1PE11.4)           MRU 1060
     1041 FORMAT (I3,1PE13.4,OPPF10.4,1PE13.4,OPPF10.4,OPPF10.4,   MRU 1060
           *1PE13.4)
     125 WRITE (IO,1040) I,E(IA),E(IB),E(IC),E(ID),E(IF),FOM    MRU 1050
     1040 FORMAT (I10,1PE20.4,OPPF15.5,1PE19.4,OPPF15.5,OPPF16.5,1PE18.4)  MRU 1060
C1040 FORMAT (I10,1PE18.4,OPPF13.5,1PE17.4,OPPF13.5,OPPF13.5,   MRU 1060
C  *OPPF13.5,1PE16.4)           MRU 1060
     DO 414 I=1,ND
     NPA=0

```

```

SUM=0.0
TSUM=0.0

DO 408 K=2,188
DEL=(V(K)+W(K+1))/2.
TSUM=TSUM+MFREQ(I,K)*DEL
408 CONTINUE
J=1
DO 412 K=2,188
WPA=WPA+MFREQ(I,K)
DEL=(W(K)+W(K+1))/2.
SUM=SUM+DEL*MFREQ(I,K)
TEST=SUM/TSUM
IF(TEST.LT.P(J)) GO TO 412
406 MSUM(I,J)=WPA
J=J+1
WPA=0
IF(J.GT.10) GO TO 414
IF(TEST.GT.P(J)) GO TO 406
412 CONTINUE
414 CONTINUE
WRITE(10,634)
634 FORMAT(1H //,5X,'DETECTOR',10X,' PARTICLE DISTRIBUTION IN',
*' PERCENTAGE BINS RELATIVE TO INCREASING VALUE OF',
*' CONTRIBUTIONS',/)
PR=0.0
WRITE(10,636) PR,(PP(J),J=1,10)
636 FORMAT(1H ,10X,11(5X,F5.1)/)
DO 422 I=1,ND
WRITE(10,644) I,(MSUM(I,J),J=1,10)
644 FORMAT(1H ,1B,4I,10(3X,I7))
422 CONTINUE
RETURN
END

SUBROUTINE NRUN(NBATS,NRUNS)                               NRUN  10
C THIS ROUTINE IS CALLED AT THE END OF EACH RUN           NRUN  **
C IT NORMALIZES AND OUTPUTS CALCULATED QUANTITIES ALONG WITH NRUN  **
C FRACTIONAL STANDARD DEVIATIONS                         NRUN  **
REAL * 8 FM, FSAVE,E,ANUMB
COMMON /USER/ AGSTRT,WTSTRT,ISTRRT,YSTRT,ZSTRT,DFF,EBOTM,EBOTG,   NRUN  20
1 TCUT,1Q,1I,1ADJM,MGPQT1,MGPQT2,MGPQT3,MGPQTG,MGPQTM,MITS,MLAST,  NRUN  21
2 MLEFT,MNGP,MNTG,MSTRRT                                NRUN  22
COMMON /PDET/ ND,NE,NE,NT,NA,RESP,PER,WEED,WEED,MDMR,MTMR,MTME,NRUN  30
1 NAE,NTMDR,NTMED,NAMEHD,LOCRESP,LOCKD,LOCIB,LOCDO,LOCUD,    NRUN  31
2 LOCSD,LOCQE,LOCQT,LOCQTE,LOCQAE,LMAX,EFIRST,EGTOP      NRUN  32
COMMON /SSS/ E(1)                                         NRUN  40
COMMON A(1)                                              NRUN  50
DIMENSION NUMB(1),ANUMB(1),IEOLL(10)                     NRUN  60
EQUIVALENCE (A(1),NUMB(1)), (IEOLL(1),STR)
EQUIVALENCE (E(1),ANUMB(1))                                NRUN  60
DATA IH1/1H1/, IH2/1H2/, IHO/1H0/                          NRUN  70

```

```

      MPART = NBATS*MSTRT          NRUN  80
      FM = 1.0/NPART               NRUN  90
      IF (IADJM) 10,10,5           NRUN 100
  5   FM = FM*DFF                NRUN 110
      WRITE (IO,1000)              NRUN 120
  1000 FORMAT (82H2THIS IS AN ADJOINT PROBLEM - ADJOINT ENERGY DEPENDENT
     1FLUENCE IS NOT DIFFERENTIAL) NRUN 130
  10   IX = LOCUD + MDNR + 1      NRUN 131
      IX2 = IX + MDNR             NRUN 140
      CALL VAR2(E(IX),E(IX2),NRESP,MD,1,MPART) NRUN 150
  *****WILSON SEPT/26/89*****
      IX = LOCSD + 2*MDNR + 1      NRUN 160
      IZ = LOCSD + 6*MDNR + 1      NRUN 160
      IY = IZ + MDNR               NRUN 160
      CALL FTEST(E(IX),E(IY),E(IZ),NRESP,MD,NBATS,MPART) NRUN 160
  *****WILSON SEPT/26/89*****
      IX = LOCSD + MDNR + 1      NRUN 170
      IX2 = IX + MDNR             NRUN 180
      CALL VAR2(E(IX),E(IX2),NRESP,MD,NBATS,MPART) NRUN 190
      IF (NE) 20,20,15             NRUN 190
  15   IX = LOCQE + NEND + 1      NRUN 200
      IX2 = IX + NEND             NRUN 210
      CALL VAR2(E(IX),E(IX2),NE,MD,1,MPART) NRUN 220
  20   IF (NT) 35,35,25           NRUN 230
  25   IX = LOCQT + NTMDNR + 1    NRUN 240
      IX2 = IX + NTMDNR           NRUN 250
      CALL VAR3(E(IX),E(IX2),NRESP,NT,MD,1,MPART) NRUN 260
      IF (NE) 45,45,30             NRUN 270
  30   IX = LOCQTE + NTNEND + 1   NRUN 280
      IX2 = IX + NTNEND           NRUN 290
      CALL VAR3(E(IX),E(IX2),NT,NE,MD,1,MPART) NRUN 300
  35   IF (NA) 45,45,40           NRUN 310
  40   IX = LOCQAE + NAREND + 1   NRUN 320
      IX2 = IX + NAREND           NRUN 330
      CALL VAR3(E(IX),E(IX2),NA,NE,MD,1,MPART) NRUN 340
  45   IF = LOCIB + 2*NE          NRUN 350
C * * THE STATEMENTS IN THE FOLLOWING LOOP NORMALIZE ALL ARRAYS TO BE OUTPUT
      FSAVE = FM                  NRUN 360
      DO 120 I=1,MD                NRUN 390
      FM = FSAVE*I(LOCXD+6*MD+I)   NRUN 400
      IA = LOCUD + (I-1)*NRESP+MDNR NRUN 410
      IB = LOCSD + (I-1)*NRESP + 6*MDNR NRUN 420
      IC = LOCQE + (I-1)*NE + NEND  NRUN 430
      IF (NE) 60,60,50             NRUN 440
  50   DO 55 J=1,NE                NRUN 450
  55   E(IC+J) = E(IC+J)/I(IF+J)*FM NRUN 460
  60   DO 75 M=1,NRESP             NRUN 470
      E(IA+M) = E(IA+M)*FM        NRUN 480
      E(IB+M) = E(IB+M)*FM        NRUN 490
      IF (NT) 75,75,65             NRUN 500
  65   IE = LOCQT + (I-1)*NTNR + NTMDNR NRUN 510

```

```

IDT = LOCT + ND*NT + (I-1)*NT      MRUN 520
DO 70 K=1,NT                         MRUN 530
  IQE = IE + (K-1)*NRESP + N        MRUN 540
  IDT = IDT + 1                     MRUN 550
70   E(IQE) = E(IQE)*FM/A(IDT)      MRUN 560
75   CONTINUE                         MRUN 570
    IF (NE) 120,120,80                MRUN 580
80   IQTE = LOCQTE + (I-1)*NTNE + NTEND     MRUN 590
  IQAE = LOCQAE + (I-1)*NAWE + NAWEND     MRUN 600
  DO 115 J=1,NE                      MRUN 610
    IF (NT) 95,95,85                MRUN 620
85   IQT = IQTE + (J-1)*NT          MRUN 630
  IDT = LOCT + ND*NT + (I-1)*NT      MRUN 640
  DO 90 K=1,NT                      MRUN 650
  IQ = IQT + K                      MRUN 660
  IDT = IDT + 1                    MRUN 670
90   E(IQ) = E(IQ)/A(IDT)/A(IF+J)*FM    MRUN 680
95   IF (NA) 115,115,100              MRUN 690
100  IQA = IQAE + (J-1)*NA          MRUN 700
C   THE DA ARRAY IS BEING STORED IN THE FIRST NA CELLS IN QAE
  IDA = LOCQAE + 1                  MRUN 690
                                         MRUN 720

  IC = LOCQI + 1                  MRUN 730
  E(IDA) = (A(IC) + 1.0)*6.2832    MRUN 740
  DO 105 L=2,NA                  MRUN 750
  IDA = IDA + 1                  MRUN 760
  IC = IC + 1                    MRUN 770
105  E(IDA) = (A(IC) - A(IC-1))*6.2832    MRUN 780
  IDA = LOCQAE                  MRUN 790
  DO 110 L=1,NA                  MRUN 800
  IQ = IQA + L                  MRUN 810
  IDA = IDA + 1                  MRUN 820
110  E(IQ) = E(IQ)/A(IDA)/A(IF+J)*FM    MRUN 830
115  CONTINUE                      MRUN 840
120  CONTINUE                      MRUN 850
    CALL DATE (IBOLL,I)
    WRITE (IO,1010) (IEOLL(J),J=1,I)
1010 FORMAT (22B1THIS CASE WAS RUN ON ,10I4)    MRUN 860
    IHP = IH0                      MRUN 870
C   * * OUTPUT SUD, SUD2, SSD AND SSD2 ARRAYS IN THE FOLLOWING LOOP
  DO 125 M=1,NRESP                MRUN 870
    IMOL = MLAST + (M-1)*20 + 1    MRUN 890
    IMOL2 = IMOL + 19             MRUN 910
    WRITE (IO,1020) IHP,(A(I),I=IMOL,IMOL2)    MRUN 920
1020 FORMAT (A1,20I4)               MRUN 930
    IHP = IB2                      MRUN 940
    IST = LOCQE - 9                MRUN 950
    WRITE (IO,1030) (ANUMB(I),I=IST,LOCQE)    MRUN 960
1030 FORMAT (1HO,32I,20HRESPONSES(DETECTOR) ,10A8,/,5X,8HDETECTOR,9X,1
  6HUNCOLL,12X,3HFSD,13X,6HTOTAL,13X,3HFSD,12X,3HFSD,10X,
  3 7BF VALUE,/,21X,8HRESPONSE,26X,8HRESPONSE,10X,5HBATCH)    MRUN 970
    DO 125 I=1,ND                  MRUN 980
                                         MRUN 990
                                         MRUN 991
                                         MRUN 992
                                         MRU 1000

```

IA = LOCUD + MDRR + (I-1)*BRESP + M	MRU 1010
IB = IA + MDRR	MRU 1020
IC = LOCSD + MDRR + (I-1)*BRESP + M	MRU 1030
ID = IC + MDRR	MRU 1040
IC = LOCSD + 6*MDRR + (I-1)*KRESP + M	MRU 1030
IF = IC + MDRR	MRU 1040
IE = LOCSD + 9*MDRR + (I-1)*KRESP + M	MRU 1030
125 WRITE (IO,1040) I,E(IA),E(IB),E(IC),E(ID),E(IF),E(IF)	MRU 1050
1040 FORMAT (I10,1PE20.4,OPF15.5,1PE19.4,OPF15.5,OPF15.5,	MRU 1060
+OPF15.5)	
IF (ME) 210,210,130	MRU 1070
130 NDM = (ND-1)/10 + 1	MRU 1080
NEM = (ME-1)/17 + 1	MRU 1090
IE1 = IE1	MRU 1100
C * * OUTPUT SQE AND SQE2 IN THE FOLLOWING LOOP	MRU 1180
DO 205 IND = 1,NDM	MRU 1120
ID1 = (IND-1)*10 + 1	MRU 1130
IF (IND-NDM) 135,140,140	MRU 1140
135 ID2 = ID1 + 9	MRU 1150
GO TO 145	MRU 1160
140 ID2 = ND	MRU 1170
145 DO 205 INE=1,NEM	MRU 1180
IE1 = (INE-1)*17 + 1	MRU 1190
IF (INE-NEM) 150,155,155	MRU 1200
150 IE2 = IE1 + 16	MRU 1210
GO TO 160	MRU 1220
155 IE2 = ME	MRU 1230
160 IF (INE-1) 165,165,170	MRU 1240
165 ETOP = EFIRST	MRU 1260
GO TO 175	MRU 1260
170 ETOP = A(LOCIB+ME+IE1-1)	MRU 1270
175 IST = LDCQT - 9	MRU 1280
WRITE (IO,1050) IMP,(AFUMB(I),I=IST,LDCQT),(I,I=ID1,ID2)	MRU 1290
1050 FORMAT (A1,27I,24HFLUENCE(ENERGY,DETECTOR),1I,10A8,/,16H0)	DETECTMRU 1300
10R NO.,18,9I10)	MRU 1301
IF (ME-7) 180,180,186	MRU 1310
180 IEP = IE2	MRU 1320
185 WRITE (IO,1060) ETOP	MRU 1330
1060 FORMAT (9H ENERGYIES,/,1I,1PE11.3)	MRU 1340
DO 205 IE = IE1,IE2	MRU 1350
IF (NHE) 190,200,190	MRU 1360
190 IF (IE-NHE-1) 200,195,200	MRU 1370
195 WRITE (IO,1060) EGTOP	MRU 1380
200 IEP = LOCIB + ME + IE	MRU 1390
ID11 = LOCQE + (ID1-1)*ME + NEND + IE	MRU 1400
ID12 = ID11 + (ID2-ID1)*ME	MRU 1410
WRITE (IO,1070) (E(I),I=ID11,ID12,ME)	MRU 1420
1070 FORMAT (17X,1P10E10.3)	MRU 1430
ID11 = ID11 + NEND	MRU 1440
ID12 = ID12 + NEND	MRU 1450
WRITE (IO,1080) (E(I),I=ID11,ID12,ME)	MRU 1460

```

1080 FORMAT (17X,10(F9.3,1X))          MRU 1470
205  WRITE (IO,1090) A(IPR)           MRU 1480
1090 FORMAT (1X,1PE11.3)               MRU 1490
210  IF (NT) 375,375,215             MRU 1500
215  ITRM = (NRESP-1)/10 + 1        MRU 1510
     ITM = (NT-1)/17 + 1            MRU 1520
     IHP = IH1                      MRU 1530
C * * OUTPUT SQT AND SQT2 IN THE FOLLOWING LOOP      MRU 1580
DO 275 I=1,ND                                     MRU 1550
DO 275 IWR=1,MRM                     MRU 1560
     IR1 = (IWR-1)*10 + 1            MRU 1570
     IF (IWR-MRM) 220,225,225       MRU 1580
220  IR2 = IR1 + 9                         MRU 1590
     GO TO 230                       MRU 1600
225  IR2 = NRESP                        MRU 1610
230  DO 275 INT=i,WTM                  MRU 1620
     IST = LOCQTE - 9                MRU 1630
     WRITE (IO,1100) IHP,I,(ANUMB(IPR),IPR=IST,LOCQTE)   MRU 1640
1100 FORMAT (A1,11HDETECTOR NO,I3,5X,32HRESPONSE(RESPONSE,TIME,DETECTOR
     1),1X,10AB)                      MRU 1650
     IF (NT-8) 235,235,240           MRU 1660
235  IHP = IH2                         MRU 1670
240  IT1 = (INT-1)*17 + 1            MRU 1680
     IF (INT-WTM) 245,250,250       MRU 1690
245  IT2 = IT1 + 16                  MRU 1700
     GO TO 255                       MRU 1710
250  IT2 = NT                         MRU 1720
255  IF (INT-1) 260,260,265         MRU 1730
260  AGS = A(LOCID + 4*ND + I)      MRU 1740
     GO TO 270                       MRU 1750
265  ISUB = LOCT + (I-1)*NT + IT1 - 1    MRU 1760
     AGS = A(ISUB)                   MRU 1770
270  WRITE (IO,1110) (IR,IR=IR1,IR2)     MRU 1780
1110 FORMAT (4X,9H RESPONSE,10I10)      MRU 1790
     WRITE (IO,1120) AGS              MRU 1800
1120 FORMAT (7H TIMES /1PE12.3)        MRU 1810
DO 275 IT=IT1,IT2                      MRU 1820
     IMDT = (I-1)*NT + IT + LOCT    MRU 1830
     ITR1 = LOCQT + NTMDMR + (I-1)*NTMR + (IT-1)*NRESP + IR1   MRU 1840
     ITR2 = ITR1 + IR2 - IR1        MRU 1850
     WRITE (IO,1070)      (E(IP),IP=ITR1,ITR2)   MRU 1860
     ITR1 = ITR1 + NTMDMR          MRU 1870
     ITR2 = ITR2 + NTMDMR          MRU 1880
     WRITE (IO,1080) (E(IP),IP=ITR1,ITR2)   MRU 1890
275  WRITE (IO,1090) A(IMDT)           MRU 1900
     IF (NE) 475,475,280           MRU 1910
280  NEM = (NE-1)/10 + 1            MRU 1920
     NTM = (NT-1)/17 + 1            MRU 1930
     IHP = IH1                      MRU 1940
C * * OUTPUT SQTE AND SQTE2 IN THE FOLLOWING LOOP      MRU 1970
DO 370 I=1,ND                      MRU 1980

```

```

DO 370 IHE=1,NHE
IE1 = (IHE-1)*10 + 1
IF (IHE-NHE) 285,290,290
285 IE2 = IE1 + 9
GO TO 295
290 IE2 = HE
295 IF (IHE-1) 300,300,306
300 ETOP = EFIRST
GO TO 310
305 ETOP = A(LOCIB + HE + IE1 - 1)
IF (IE1.EQ.NHE+1) ETOP=EGTOP
310 I1E = LOCIB + HE + IE1
DO 370 INT=1,NTM
I2 = LOCIB + HE + IE2 - 1
IST = LOCQAE - 9
WRITE (IO,1130) IHP,I,(AMUMB(IPR),IPR=IST,LOCQAE)
1130 FORMAT (A1,11HDETECTOR NO,13,8X,29HFLUENCE(TIME,ENERGY,DETECTOR),
1 IX,10A8)
IF (NT-8) 315,315,320
315 IHP = IH2
320 IT1 = (INT-1)*17 + 1
IF (INT-NTM) 325,330,330
325 IT2 = IT1 + 16
GO TO 335
330 IT2 = NT
335 IX = LOCIB + HE + NHE
ESAV = A(IX)
IF (NHE .LT. IE1 .OR. NHE .GE. IE2) GO TO 340
A(IX) = ECTOP
340 IF (I1E .LE. I2) GO TO 345
WRITE (IO,1140) ETOP
GO TO 350
345 WRITE (IO,1140) ETOP,(A(IP),IP=I1E,I2)
1140 FORMAT (1B0,3X,8HENERGIES,5X,1P10E10,3)
350 A(IX) = ESAY
I2 = I2 + 1
IF (INT-1) 355,355,360
355 AGS = A(LOCID + 4*ND + I)
GO TO 365
360 ISUB = LOCT + (I-1)*BT + IT1 - 1
AGS = A(ISUB)
365 WRITE (IO,1150) (A(IP),IP=I1E,I2)
1150 FORMAT (2I,5HTIMES,10I,1P10E10,3)
WRITE (IO,1160) AGS
1160 FORMAT (1PE12.3)
DO 370 IT=IT1,IT2
INTT = (I-1)*BT + IT + LOCT
ITE1 = (I-1)*BTNE + (IE1-1)*BT + IT + LOCQTE + BTWEND
ITE2 = ITE1 + (IE2-IE1)*BT
WRITE (IO,1070)      (E(IP),IP=ITE1,ITE2,BT)
ITE1 = ITE1 + BTWEND

```

```

ITE2 = ITE2 + NTEEND
WRITE (IO,1080) (E(IP),IP=ITE1,ITE2,NT) BRU 2470
370 WRITE (IO,1090) A(INDT) BRU 2480
375 IF (NA) 475,476,380 BRU 2490
380 NEM = (NE-1)/10 + 1 BRU 2500
     NAM = (NA-1)/17 +1 BRU 2510
     IHP = IH1 BRU 2520
C *   * OUTPUT SQAE AND SQAE2 IN THE FOLLOWING LOOP BRU 2530
DO 470 I=1,ND BRU 2540
DO 470 IWE=1,NEM BRU 2550
IE1 = (IME-1)*10 + 1 BRU 2560
IF (IWE-NEM) 385,390,390 BRU 2570
385 IE2 = IE1 + 9 BRU 2580
GO TO 385 BRU 2590
390 IE2 = NE BRU 2600
395 IF (IME-1) 400,400,405 BRU 2610
400 ETOP = EFIRST BRU 2620
     GO TO 410 BRU 2630
405 ETOP = A(LOCIB + NE + IE1 - 1) BRU 2640
410 I1E = LOCIB + NE + IE1 BRU 2650
     DO 470 IMA=1,NAM BRU 2660
     I2 = LOCIB + NE + IE2 - 1 BRU 2670
     IST = LMAX - 9 BRU 2680
     WRITE (IO,1170) IIP,I,(ANUMB(IPR),IPR=IST,LMAX) BRU 2690
1170 FORMAT (A1,11BDETECTOR NO,I9,6X,31HFLUXENCE(COSINE,ENERGY,DETECTOR)) BRU 2700
     1,1I,10A8) BRU 2711
     IF (NA-8) 415,415,420 BRU 2720
415 IHP = IH2 BRU 2730
420 IX = LOCIB + NE + NME BRU 2740
     ESAV = A(IX) BRU 2750
     IF (NME .LT. IE1 .OR. NME .GE. IE2) GO TO 425 BRU 2760
     A(IX) = EGTOP BRU 2770
425 IF (I1E .LE. I2) GO TO 430 BRU 2780
     WRITE (IO,1140) ETOP BRU 2790
     GO TO 435 BRU 2800
430 WRITE (IO,1140) ETOP,(A(IP),IP=I1E,I2) BRU 2810
435 A(IX) = ESAV BRU 2820
     I2 = I2 + 1 BRU 2830
     IA1 = (IMA-1)*17 + 1 BRU 2840
     IF (IMA-1) 440,440,445 BRU 2850
440 FM1 = -1.0 BRU 2860
     GO TO 450 BRU 2870
445 FM1 = A(LOCOC + IA1 - 1) BRU 2880
450 WRITE (IO,1180) (A(IP),IP=I1E,I2) BRU 2890
1180 FORMAT (5X,7HCOSINES,5X,1P10E10.3) BRU 2900
     WRITE (IO,1180) FM1 BRU 2910
1180 FORMAT (2X,F12.6)
     IF (IMA-NAM) 455,460,460 BRU 2930
455 IA2 = IA1 + 16 BRU 2940
     GO TO 465 BRU 2950
460 IA2 = NA BRU 2960

```

```

465 DO 470 IA=IA1,IA2                                ERU 2970
      ICO = LDCC0 + IA
      IAE1 = (I-1)*NAME + (IE1-1)*NA + IA + LOCQAE + NANEND   ERU 2980
      IAE2 = IAE1 + (IE2-IE1)*NA                           ERU 2990
      WRITE (IO,1070)          (A(IP),IP=IAE1,IAE2,NA)
      IAE1 = IAE1 + NANEND                                 ERU 3000
                                         ERU 3020
      IAE2 = IAE2 + NANEND                               ERU 3030
      WRITE (IO,1080) (A(IP),IP=IAE1,IAE2,NA)
470  WRITE (IO,1180) A(ICO)                           ERU 3050
475  IF (NEIND) 490,490,480                         ERU * *
C       OUTPUT EXTRA ARRAYS OF LENGTH ND * * * * *
480  WRITE (IO,1200)                                 ERU 3060
1200 FORMAT (26H1EXTRA ARRAYS OF LENGTH ND/)
      DO 485 I=1,NEXND                                ERU 3070
      STR = 0.0                                         ERU 3080
      CALL INSERT(STR,1,4,4HEXTR)                      ERU 3090
      CALL INTBCD(I,DUM,L)                            ERU 3100
      LP = 5 - L                                       ERU 3120
      CALL INSERT(STR,LP,L,DUM)                        ERU 3130
      IST = LOCAD + (5+I)*ND + 1                     ERU 3140
485  CALL HELPER(A(IST),1,ND,STR,IO)                ERU 3150
490  CALL ENDRUN                                     ERU 3160
      RETURN                                         ERU 3170
      END                                             ERU 3180

C*
      SUBROUTINE OUTPT(KEY)                           OUTPT 10
C ** THIS VERSION OF OUTPT IS FOR MORSE-CGA * * *
C THIS ROUTINE CONTROLS CALCULATION AND OUTPUT OF AVERAGE VALUES OF OUTPT *
C SOURCE PARAMETERS (KEY=1), THE COLLISION COUNTERS AT THE END OF OUTPT *
C EACH BATCH (KEY=2), AND OUTPUTS THE COUNTERS FOR NUMBER OF               OUTPT *
C SCATTERINGS, ETC., AT END OF RUN (KEY=3)                                OUTPT *
COMMON /NUTRON/ NAME,NAMEX,IG,IGO,NMED,MEDOLD,MREG,U,V,W,WOLD,VOLD,VOLOUTPT 30
1 ,WOLD,X,Y,Z,XOLD,YOLD,ZOLD,WATE,OLDWT,WTBC,BL2NT,BL2OW,AGE,OLDAGEOUTPT 31
COMMON /APOLLO/ AGSTRT,DDF,DEADWT(6),ETA,ETATH,ETAUSD,UIMP,VIMP,    OUTPT 40
1 WIMP,WTSTRT,XSTRT,YSTRT,ZSTRT,TCUT,XTRA(10),                          OUTPT 41
2 IO,I1,MEDIA,IADJM,ISBIAS,ISOUR,ITERS,ITIME,ITSTR,LOCWTS,LOCFWL,OUTPT 42
3 LOCEPR,LOCNSC,LOCPSN,MAXGP,MAXTIN,MEDALS,MGPREG,MREG,HALB,        OUTPT 43
4 NDEAD(6),NEWMM,NGEOM,NGPQT1,NGPQT2,NGPQT3,NGPQTG,NGPQTM,NITS,    OUTPT 44
5 NCALC,NKILL,NLAST,NMEM,NMGP,NHOST,NMTG,NOLEAK,NORMF,NPAST,        OUTPT 45
6 MPSCL(13),NQUIT,NSIGL,NSOUR,NSPLT,NSTRT,XTRA(10)                   OUTPT 46
COMMON /FISBNK/ MFISTP,WFISBN,WFISH,FTOTL,FWATE,WATEF                 OUTPT 50
COMMON /OUTB/ SWATE,XAVE,YAVE,ZAVE,EAVE,UAVE,VAVE,WAVE,AGEAVE        GSTOR 40
COMMON WTS(1)                                              OUTPT 60
DIMENSION NC(1)                                              OUTPT 70
EQUIVALENCE (WTS(1),NC(1))                                     OUTPT 80
REAL*8 RANDOM                                              OUTPT
DATA FKX/0.0/, FKSUM/0.0/, VARK/0.0/, FMKFW/0.0/, FKFW/0.0/      OUTPT 90
DATA NITSX/0/
GO TO (1111,2222,3333), KEY                                OUTPT 100
1111 NBATCH=NITS-ITERS+1                                    OUTPT 110
                                         OUTPT 120

```

```

      IF (NBATCH-1) 20,15,20          OUTP 130
15   ITCUT = 1                      OUTP 140
      FKFW =0.                        OUTP 150
      FKFW =0.                        OUTP 160
20   FKKT = BMEM                   OUTP 170
      FKFW = FKFW + FKKT            OUTP 180
      IF(SWATE.EQ.0.) GO TO 30       OUTP 190
      XAVE=IAVE/SWATE              OUTP 200
      YAVE=YAVE/SWATE              OUTP 210
      ZAVE=ZAVE/SWATE              OUTP 220
      EAVE=EAVE/SWATE              OUTP 230
      UAVE=UAVE/SWATE              OUTP 240
      VAVE=VAVE/SWATE              OUTP 250
      WAVE=WAVE/SWATE              OUTP 260
      AGEAVE=AGEAVE/SWATE          OUTP 270
30   WRITE (10,1010)                 OUTP

1010 FORMAT ('    WTAVE      IAVE      UAVE      VAVE      WAVE      XAVE' OUTP
             1    YAVE      ZAVE      AGEAVE')           OUTP
      WRITE (10,1011)SWATE,EAVE,UAVE,VAVE,WAVE,XAVE,YAVE,ZAVE,AGEAVE OUTP
1011 FORMAT(1H ,1PE10.3,2I,0PF7.2,2I,3(F7.4,1I),1I,3(1PE10.3,1X),1X, OUTP
             1E10.3)                  OUTP
      RETURN                         OUTP 540
2222 WRITE(10,1030)                 OUTP 550
1030 FORMAT(/36HNUMBER OF COLLISIONS OF TYPE NCOLL/3I,6HSOURCE,1I, OUTP 560
             1  6HSPLIT(D),4I,5HFISHM,3I,6HGAMGEH,1I,8HREALCOLL ,3I,6HALBEDO,4I,OUTP 561
             2 5HBDRYX,3I,6HESCAPE,4I,5HE-CUT,1I,8HTIMERKILL ,1I,8HR R KILL,1I, OUTP 562
             3 8HR R SURV, 2I,7HGAMLDST)           OUTP 563
      WRITE (10,1040) (NPSCL(I),I=1, 13)        OUTP 570
1040 FORMAT(13I9)                   OUTP 580
      LENGTH = 0                      OUTP 590
      CALL TIMER(LENGTH,XTRA)         OUTP 600
      WRITE (10,1050) (XTRA(I),I=1,LENGTH)     OUTP 610
1050 FORMAT (43HOTIME REQUIRED FOR THE PRECEDING BATCH WAS ,10A4) OUTP 620
      IF (ITCUT) 50,50,40            OUTP 630
40   IF (NPSCL(10)) 50,50,45          OUTP 640
45   ITCUT = 0                      OUTP 650
50   DO 55 I=1,13                  OUTP 660
55   NPSCL(I)=0                  OUTP 670
      IF (NORMF) 65,65,60            OUTP 680
60   FKFW = FKFW + FKKT*FTOTL/SWATE OUTP 690
      IF (WFISH) 65,65,65            OUTP 700
65   ESTK = FKFW/FKFW*WFISH/NSTRT OUTP 710
      IF (ESTK) 65,65,70            OUTP 720
70   DO 75 I=1,NIREG               OUTP 730
75   WTS(LOCFWL+I) = WTS(LOCFWL+I)*ESTK OUTP 740
      WTCHNG = FWATE/WFISH/SWATE*FKKT OUTP 750
      NG1 = LOCWTS+1                OUTP 760
      NG2 = LOCWTS+3*NGPREG         OUTP 770
      DO 80 I=NG1,NG2               OUTP 780
80   WTS(I) = WTS(I)*WTCHNG        OUTP 790
85   IF (NKCALC) 105,105,90          OUTP 800

```

```

90   FKT = FTOTL/SWATE                                OUTP 810
      IF (WHATCH .LT. WKCALC .OR. ITCUT .EQ. 0) GO TO 95    OUTP 820
      FKSUM = FKSUM + FWKT*FKT                            OUTP 830
      VARK = VARK + FWKT*FKT*FKT                          OUTP 840
      FKK = FKK + FWKT                                    OUTP 850
      BITSK = BITSK + 1                                  OUTP 860
      GO TO 100                                         OUTP 870
95   WRITE(10,1060)                                     OUTP 880
1060 FORMAT(63E --- K FOR THIS BATCH WILL NOT BE USED IN AVERAGE K CALCOUTP 890
     1ULATION )                                       OUTP 891
100   WRITE (10,1070) FKT      ,FTOTL,FWATE,BFISH          OUTP 900
1070 FORMAT(1H0/3H K=,F9.5,15I,6HFTOTL=,E13.5,5I,6HFWATE=,E13.5,
     15I,6HBFISH=,15)                                     OUTP 910
106   RETURN                                         OUTP 920
3333 WRITE (10,1080) (NDEAD(I),DEADWT(I),I=1,4)        OUTP 930
1080 FORMAT(16H1NEUTRON DEATHS,20X,6HNUMBER,16X,6HWEIGET/
     127H KILLED BY RUSSIAN ROULETTE,8X,16, 9X,E13.5/
     26H ESCAPED,27I,16, 9X,E13.5/                           OUTP 941
     322H REACHED ENERGY CUTOFF,13I,16, 9X,E13.5/20H REACHED TIME CUTOFFOUTP 943
     4 15I,16, 9X,E13.5)                                     OUTP 944
     IF(MEDIA)125,125,117                                 OUTP 945
117   WRITE(10,1090)                                     OUTP 950
1090 FORMAT(1H0/22HNUMBER OF SCATTERINGS)                OUTP 960
C
      WRITE(10,1100)                                     OUTP 980
1100 FORMAT(7H0MEDIUM 13I,6HNUMBER)                      OUTP 990
      M123=0                                           OUTP 995
      DO 115 MMED=1,MEDIA                            OUT 1000
      M1 = LOCMSC + 8*MMTG*MIREG + MMED               OUT 1010
      M123=M123+MC(M1)                               OUT
115   WRITE(10,1110)MMED,WTS(M1)                       OUT 1020
1110 FORMAT (15,10X,17)                                OUT 1030
      WRITE (10,1112) M123                            OUT
1112 FORMAT (' TOTAL', 9X,17)                          OUT
120   WRITE (10,1120)                                     OUT 1050
1120 FORMAT ('1REAL SCATTERING COUNTERS')              OUT 1060
      M1 = LOCMSC + 1                                OUT 1070
      CALL OUTPT2(WTS(M1),MC(M1),MMTG,MIREG,10)       OUT 1080
125   IF (MEDALB-7777) 130,135,130                  OUT 1090
130   WRITE (10,1130)                                     OUT 1100
1130 FORMAT ('1ALBEDO SCATTERING COUNTERS')           OUT 1110
      M1 = LOCMSC + 2*MMTG*MIREG + 1                 OUT 1120
      CALL OUTPT2(WTS(M1),MC(M1),MMTG,MIREG,10)       OUT 1130
135   IF (MFISTP) 145,145,140                         OUT 1140
140   WRITE (10,1140)                                     OUT 1150
1140 FORMAT ('1FISSION PRODUCTION COUNTERS')          OUT 1160
      M1 = LOCMSC + 4*MMTG*MIREG + 1                 OUT 1170
      CALL OUTPT2(WTS(M1),MC(M1),MMTG,MIREG,10)       OUT 1180
145   IF (MGPQTM*MGPQTG) 150,155,150                  OUT 1190
150   WRITE (10,1150)                                     OUT 1200
1150 FORMAT ('1SECONDARY PRODUCTION COUNTERS (BOTH THE GROUPS CAUSING POUT 1210

```

```

1RDUCTION AND RESULTING FROM PRODUCTION') OUT 1211
    M1 = LOCHSC + 6*MNTG*MREG + 1 OUT 1220
    CALL OUTPT2(WTS(M1),MC(M1),MNTG,MREG,IO) OUT 1230
155 IF (MSPLT) 165,165,160 OUT 1240
160 WRITE(IO,1160) OUT 1250
1160 FORMAT(21H1NUMBER OF SPLITTINGS) OUT 1260
    M1 = LOCWTS+4*MGPREG+1 OUT 1270
    CALL OUTPT2(WTS(M1),MC(M1),MAXGP,MREG,IO) OUT 1280
    WRITE (IO,1170) OUT 1290
1170 FORMAT(47H1NUMBER OF SPLITTINGS PREVENTED BY LACK OF ROOM) OUT 1300
    M1 = M1 + 2*MGPREG OUT 1310
    CALL OUTPT2(WTS(M1),MC(M1),MAXGP,MREG,IO) OUT 1320
165 IF(MKILL)176,175,170 OUT 1330
170 WRITE(IO,1180) OUT 1340
1180 FORMAT(33H1NUMBER OF RUSSIAN ROULETTE KILLS) OUT 1350
    M1 = LOCWTS + 8*MGPREG + 1 OUT 1360
    CALL OUTPT2(WTS(M1),MC(M1),MAXGP,MREG,IO) OUT 1370
    WRITE(IO,1190) OUT 1380
1190 FORMAT(97H1NUMBER OF RUSSIAN ROULETTE SURVIVALS) OUT 1390
    M1 = M1 + 2*MGPREG OUT 1400
    CALL OUTPT2(WTS(M1),MC(M1),MAXGP,MREG,IO) OUT 1410
175 IF(MKCALC)185,185,180 OUT 1420
180 VARK = VARK/FWK OUT 1430
    FKSUM = FKSUM/FWK OUT 1440
    VARK = SQRT ((VARK-FKSUM**2)/(NITSK-1)) OUT 1450
    WRITE(IO,1200) FKSUM,VARK,NITSK OUT 1460
1200 FORMAT(11H1AVERAGE K=,E15.4,10X,14HSTANDARD DEV.=,E15.4,5X,3HEFOR.IOUT 1470
14,8H BATCHES) OUT 1471
185 FWK= 0. OUT 1480
    FKSUN=0. OUT 1490
    VARK=0. OUT 1500
    NITSK=0 OUT 1510
    CALL RNDOUT(RANDOM) OUT

    WRITE (IO,1210) RANDOM OUT
1210 FORMAT('0    ** NEXT RANDOM NUMBER IS   ',Z12) OUT
    RETURN OUT 1520
    END OUT 1530

    SUBROUTINE SCORIN SCORI 10
C * * * *
C THIS ROUTINE READS INPUT DATA FOR THE ANALYSIS MODULE * * * * * * * *
C * * * *
REAL * 8 SS, LNK1
COMMON /USER/ AGSTRT,WTSTRT,ISTRRT,YSTRRT,ZSTRRT,DFF,EBOTW,EROTG, SCORI 20
1 TCUT,IO,I1,IADJM,MGPQT1,MGPQT2,MGPQT3,MGPQTG,MGPQTM,NITS,BLAST, SCORI 21
2 NLEFT,WMGP,WMTG,MSTRAT SCORI 22
COMMON /PDET/ ND,MME,ME,MT,WA,BRESP,WEI,WEEND,WEND,WDWR,WTWR,WTWE,SCORI 30
1 WANE,WTEND,WTNEND,WANEND,LOCRSP,LOCID,LOCIB,LOCJO,LOCT,LOCUD, SCORI 31
2 LOCSD,LOCQE,LOCQT,LOCQTE,LOCQAE,LMAX,EFIRST,EGTOP SCORI 32
COMMON BLNK(1) SCORI 40
COMMON /SSS/ SS(1)

```

DIMENSION LNK(1),LNK1(1),IBOL(20),IBF(6),IBA(6),IBP(6)	SCORI 50
EQUIVALENCE(BLNK(1),LNK(1)),(SS(1),LNK1(1))	SCORI 60
DATA IEDLR,IBF,IBA/1H ,4BPRIM,4HARY ,4HEVER,4HGY B,4HIMS ,1H ,	SCORI 70
1 4HSECO,4HNDAR,4HY EN,4HERGY,4H BIR,1BS/,JUP/3HUPP/,JLD/3HLOW/	SCORI 71
 C**** WILSON JUNE 2, 1989 ****	MOR 1150
C LOCUD = LOCT + 2*MD*IT	SCOR 680
LOCUD = 0	SCOR 680
C**** WILSON JUNE 2, 1989 ****	MOR 1150
LOCSD = LOCUD + 3*MDMR	SCOR 690
C**** WILSON APRIL 19, 1989 ****	MOR 1150
C LOCQE = LOCSD + 3*MDMR + 20	SCOR 700
LOCQE = LOCSD + 14*MDMR + 20	SCOR 700
C**** WILSON APRIL 19, 1989 ****	MOR 1150
 RETURN	SCO 3330
END	SCO 3340
 SUBROUTINE STBTCH (MBAT)	STBTC 10
C THIS ROUTINE IS CALLED AT THE START OF EACH BATCH	STBTC *
C THIS ROUTINE ZEROES THE ARRAYS USED TO ACCUMULATE ESTIMATED	STBTC *
C QUANTITIES DURING A BATCH .	STBTC *
C AT THE START OF THE FIRST BATCH, THE ARRAYS WHICH ACCUMULATE	STBTC *
C ESTIMATES AND SQUARED ESTIMATES ARE ALSO ZEROED . * *	STBTC *
REAL * 8 BL	
COMMON /PDET/ MD,NNE,NZ,NT,NA,NRESP,NEX,NEMD,NEND,NDYR,NTHR,NTNE,STBTC 30	
1 NANE,NTNDMR,NTNEND,NAPEnd,LOCNSP,LOCID,LOCIB,LOCQO,LOCT,LOCUD,	STBTC 31
2 LOCSD,LOCQE,LOCQT,LOCQTE,LOCQAE,LMAX,EFIRST,EGTOP	STBTC 32
COMMON /SSS/ BL(1)	STBTC 40
IF (MBAT) 10,20,150	STBTC 50
10 CALL ERROR	STBTC 60
20 IA = LOCUD + 1	STBTC 70
C**** WILSON APRIL 18, 1989 ****	MOR 1150
C IB = IA + 6*MDMR - 1	STBTC 80
IB = IA +17*MDMR - 1	STBTC 80
C**** WILSON APRIL 18, 1989 ****	MOR 1150
DO 30 I=IA,IB	STBTC 90
30 BL(I) = 0.0	STBT 100
IF (NE) 60,60,40	STBT 110
40 IA = LOCQE + 1	STBT 120
IB = IA + 5*NEND - 1	STBT 130
DO 50 I=IA,IB	STBT 140
50 BL(I) = 0.0	STBT 150
60 IF (NT) 110,110,70	STBT 160
 70 IA = LOCQT + 1	STBT 170
IB = IA + 3*NTNDMR - 1	STBT 180
DO 80 I=IA,IB	STBT 190
80 BL(I) = 0.0	STBT 200
IF (NE) 140,140,90	STBT 210
90 IA = LOCQTE + 1	STBT 220

```

        IB = IA + 3*NTHEND - 1                      STBT 230
        DO 100 I=IA,IB                               STBT 240
100   BL(I) = 0.0                                STBT 260
110   IF (IA*NE) 140,140,120                      STBT 260
120   IA = LOCQAE + 1                            STBT 270
        IB = IA + 3*NHANEND - 1                  STBT 280
        DO 130 I=IA,IB                               STBT 290
130   BL(I) = 0.0                                STBT 300
140   RETURN                                     STBT 310
150   IA = LOCSD                                 STBT 330
        IJ = LOCSD + 4*EDMR                      STBT 330
        IV = LOCSD + 5*EDMR                      STBT 330
        DO 290 I=1,ND                               STBT 380
        DO 210 M=1,NRESP                           STBT 430
        IA = IA + 1                                STBT 450
        IJ = IJ + 1                                STBT 450
        IV = IV + 1                                STBT 450
        BL(IA) = 0.0                                STBT 470
        BL(IJ) = 0.0                                STBT 470
        BL(IV) = 0.0                                STBT 470
210   CONTINUE                                    STBT 480
290   CONTINUE                                    STBT 480
        RETURN                                     STBT 480
        END                                         STBT 480

        SUBROUTINE STPART                          STBTC 10
C   THIS ROUTINE IS CALLED AT THE START OF EACH SOURCE PARTICLE    STBTC *
C   THIS ROUTINE ZEROES THE ARRAYS USED TO ACCUMULATE ESTIMATED    STBTC *
C   QUANTITIES DURING A BATCH .                                     STBTC *
REAL * 8 BL
COMMON /PDET/ ND,NE,NT,IA,NRESP,NEX,NEEND,NEND,EDMR,NTMR,NTME,STBTC 30
1  NANE,NTHDMR,NTHEND,NHANEND,LOCRSP,LOCAD,LOCIB,LOCQO,LOCUD,    STBTC 31
2  LOCSD,LOCQE,LOCQT,LOCQTE,LOCQAE,LMAX,EFIRST,EGTOP          STBTC 32
COMMON /SSS/ BL(1)                                         STBTC 40
        IA = LOCUD                                STBT 320
        IJ = LOCSD + 3*EDMR                      STBT 330
        IC = LOCQE                                STBT 340
        ID = LOCQT                                STBT 350
        IE = LOCQTE                               STBT 360
        IF = LOCQAE                               STBT 370
        DO 290 I=1,ND                               STBT 380
        IF (NE) 180,180,160                         STBT 390
160   DO 170 J=1,NE                                STBT 400
        IC = IC + 1                                STBT 410
170   BL(IC) = 0.0                                STBT 420
180   DO 210 M=1,NRESP                           STBT 430
        IA = IA + 1                                STBT 440
        IJ = IJ + 1                                STBT 450
        BL(IA) = 0.0                                STBT 460
        BL(IJ) = 0.0                                STBT 470
        IF (NT) 210,210,190                         STBT 480
190   DO 200 K=1,NT                                STBT 490

```

```

        ID = ID + 1                      STBT 500
200  BL(ID) = 0.0                   STBT 510
210  CONTINUE                      STBT 520

        IF (IE) 290,290,220              STBT 530
220  DO 280 J=1,IE                STBT 540
        IF (ET) 250,250,230              STBT 550
230  DO 240 K=1,ET                STBT 560
        IE = IE + 1                  STBT 570
240  BL(IE) = 0.0                 STBT 580
250  IF (EA) 280,280,260              STBT 590
260  DO 270 L=1,EA                STBT 600
        IF = IF + 1                  STBT 610
270  BL(IF) = 0.0                 STBT 620
280  CONTINUE                      STBT 630
290  CONTINUE                      STBT 640
        RETURN                        STBT 650
        END                           STBT 660

        SUBROUTINE TESTW               TESTW 10
C * * THIS VERSION OF TESTW IS FOR MORSE-CGA * * *
C      THIS ROUTINE TESTS WHETHER RUSSIAN ROULETTE OR SPLITTING OPTIONS TESTW **
C      ARE IN EFFECT AND THEN PERFORMS THE SPLITTING OR ROULETTE      TESTW **
C      SPLITTING IS PERFORMED UNTIL THE WEIGHT FALLS BELOW WTHIR      TESTW **
COMMON /NUTRON/ NAME,NAMEIX,IG,IGO,BMED,MEDOLD,WREG,U,V,W,UOLD,VOLDTESTW 20
1 ,WOLD,I,Y,Z,XOLD,YOLD,ZOLD,WATE,OLDWT,WTBC,BLZYT,BLZON,AGE,OLDAGETESTW 21
COMMON /APOLLO/ AGSTRT,EDP,DEADWT(6),ETA,ETATE,ETAUSD,UIWP,VINP, TESTW 30
1 VIWP,WTSTRT,ISTRRT,YSTRRT,ZSTRRT,TCUT,ITRA(10),                TESTW 31
2 IO,I1,MEDIA,IADJM,ISBIAS,ISOUR,ITERS,ITIME,ITSTR,LOCWTS,LOCFWL,TESTW 32
3 LOCEPR,LOCFSN,MAXGP,MAXTIM,MEDALS,MGPREG,MXREG,HALB,           TESTW 33
4 NDEAD(5),NEWHM,EGEOM,NGPQT1,NGPQT2,NGPQT3,NGPQTG,NGPQTM,RITS,   TESTW 34
5 NKCALC,NKILL,ELAST,NMEM,VMGP,BMST,BMTG,BOLEAK,BORMF,BPAST,     TESTW 35
6 NPSC(13),NSQUIT,NSIGL,NSOUR,NSPLT,NSTRT,NXTRA(10)            TESTW 36
COMMON /NS/ NS
COMMON NWTS(1)
DIMENSION WTS(1)
EQUIVALENCE (NWTS(1),WTS(1))
IF (IG-MAXGP) 10,10,65
10 IF (NKILL+NSPLT) 65,65,15
15 NWT = (WREG-1)*MAXGP+IG+LOCWTS
WTHIR = WTS(NWT)
NWT = NWT+MGPREG
WTLOR = WTS(NWT)
NWT = NWT+MGPREG
WTAVR = WTS(NWT)
IF (NKILL) 45,45,20
20 IF(WTLOR-ABS (WATE))40,40,25
25 IF(FLTRMF(MARG)+WTAVR-ABS (WATE))35,35,30
30 WATE=0.
NWT = NWT + 6*MGPREG
NWTS(NWT)=NWTS(NWT)+1
NWT = NWT + MGPREG

```

```

        WTS(NWT) = WTS(NWT) + OLDWT          TEST 220
C WEIGHT KILLED ENTERING COLLISION IS SCORED
        MPSCL(11) = MPSCL(11) + 1            TEST 240
C CALL BANKR(11) FOR R R KILL ANALYSIS
C R R KILL
        RETURN                                TEST 270
35   W1=WATE                                TEST 280
        WATE=SIGN (WTAVR,W1)                TEST 290
        NWT = NWT + 8*MGPREG                TEST 300
        NWTS(NWT)=NWTS(NWT)+1              TEST 310
        NWT = NWT + MGPREG                 TEST 320
        WTS(NWT) = WTS(NWT) + W1           TEST 330
C WEIGHT ENTERING COLLISION (BUT BELOW WTLOW) AND SURVIVING IS SCORED TEST *
        MPSCL(12) = MPSCL(12) + 1            TEST 350
C CALL BANKR(12) FOR R R SURV ANALYSIS
C R R SURVIVAL
        OLDWT=WATE                            TEST 380
        RETURN                                TEST 390
40   IF (NSPLT) 65,65,45                      TEST 400
45   IF(WTHEIR-ABS (WATE)) 50, 65, 65       TEST 410
50   IF(NHOST-NS) 60,60,55                  TEST 420
55   NS=NS+1                                 TEST 430
        NEWNM=NEWNM+1                         TEST 440
        WATE=WATE*.5                          TEST 450
        NAME1=NAME                            TEST 460
        NAME=NEWNM                            TEST 470
        CALL STORNT(NS,1)                     TEST 480
        MPSCL(2) = MPSCL(2) + 1              TEST 490
C CALL BANKR(2) FOR SPLIT DAUGHTER ANALYSIS
C SPLITD
        NAME=NAME1                            TEST 520
        OLDWT=WATE                            TEST 530
        NWS = NWT+2*MGPREG                  TEST 540
        NWTS(NWS) = NWTS(NWS)+1             TEST 550
        NWS = NWS + MGPREG                 TEST 560
        WTS(NWS) = WTS(NWS) + WATE         TEST 570
C WEIGHT AFTER SPLITTING IS SCORED
        GO TO 45                            TEST *
40   NWS = NWT + 4*MGPREG
        NWTS(NWS) = NWTS(NWS)+1
        NWS = NWS + MGPREG
        WTS(NWS) = WTS(NWS) + WATE
C WEIGHT WHICH COULD HAVE SPLIT IS SCORED
65   RETURN                                TEST 650
        END                                  TEST 680

        SUBROUTINE FTTEST(SX, SY,SZ,M1,M2,NBAT,NPAR)      VAR2 10
C NBAT IS THE NO. OF INDEPENDENT BATCHES      VAR2 20
C NPAR IS THE TOTAL NUMBER OF PARTICLES PROCESSED    VAR2 30
C IT IS ASSUMED THAT THE SUMSQ ARRAY HAS ACCUMULATED THE NUMBER OF PARTICLES
C TIMES THE SQUARE OF THE BATCH AVERAGE (THIS IS OBTAINED BY DIVIDING      60

```

```

C      THE SQUARED BATCH SUM BY THE NUMBER OF PARTICLES STARTING THE BATCH)   60
REAL * 8 SX, SY, SZ,AN, AI, AJ,SSB,SSW,SST
DIMENSION SX(M1,M2), SY(M1,M2), SZ(M1,M2)                                     VAR2 70
AN = NPAR
AI = NBAT
AJ = AN/AI
DO 28 I=1,M1
DO 29 J=1,M2
      SSB = AJ*( SX(I,J)/AJ-SZ(I,J)**2/AJ/AN)
      SST = SY(I,J) - SZ(I,J)**2/AN
      SSW = SST - SSB
      SY(I,J) = (SSB/(AI-1.))/(SSW/(AI*(AJ-1.)))
C      SY(I,J) = DSQRT(SSB/(AI-1.)/AN)/(SZ(I,J)/AN)
C      SY(I,J) = DSQRT(SSW/(AI+(AJ-1.))/AN)/(SZ(I,J)/AN)
C      SY(I,J) = DSQRT(SST/(AN-1.)/AN)/(SZ(I,J)/AN)
29 CONTINUE
      RETURN                                         VAR2 240
      END                                         VAR2 250

SUBROUTINE VAR2(SX,SX2,M1,M2,NBAT,NPAR)                                         VAR2 10
C      NBAT IS THE NO. OF INDEPENDENT BATCHES                                         VAR2 20
C      NPAR IS THE TOTAL NUMBER OF PARTICLES PROCESSED                           VAR2 30
C      IT IS ASSUMED THAT THE SUMSQ ARRAY HAS ACCUMULATED THE NUMBER OF PARTICLES
C      TIMES THE SQUARE OF THE BATCH AVERAGE (THIS IS OBTAINED BY DIVIDING      50
C      THE SQUARED BATCH SUM BY THE NUMBER OF PARTICLES STARTING THE BATCH)   60
REAL * 8 SX, SX2, AN, DUM, BN, CN
DIMENSION SX(M1,M2),SX2(M1,M2)                                                 VAR2 70
IF (NBAT-1) 5,5,15                                            VAR2 80
***** WILSON APRIL 20,1989 *****
5   AN = NPAR
DO 29 I=1,M1
DO 29 J=1,M2
      IF (SX(I,J)) 24,19,24
19  SX2(I,J) = 0.0
      GO TO 29
C24 WRITE(6,651) SX(I,J),SX2(I,J),AN

C651 FORMAT(1H ,3(2X,1PE10.3))
24  DUM      = SX2(I,J)/AN - (SX(I,J)/AN)**2
      SX2(I,J) = DSQRT(DABS(DUM)/AN)/(SX(I,J)/AN)                            VAR2 210
C      WRITE(6,651) SX(I,J),SX2(I,J),DUM
29 CONTINUE
***** WILSON APRIL 20,1989 *****
      RETURN                                         VAR2 120
15  AN = NPAR
BN = NBAT
CN = DSQRT(BN - 1.)
DO 30 I=1,M1
DO 30 J=1,M2
      IF (SX(I,J)) 25,20,25
20  SX2(I,J) = 0.0
      GO TO 30

```

```

25 DUM      = SX2(I,J)*AM - SX(I,J)**2                         VAR2 210
      SX2(I,J) = DSQRT(DABS(DUM)) / (SX(I,J)*CM)                 VAR2 220
30 CONTINUE
      RETURN
      END

      SUBROUTINE VAR3(SX,SX2,M1,M2,M3,NBAT,NPART)
C   NBAT IS THE NO. OF INDEPENDENT BATCHES                      VAR3 10
C   NPART IS THE TOTAL NUMBER OF PARTICLES PROCESSED           VAR3 20
C   IT IS ASSUMED THAT THE SUMSQ ARRAY HAS ACCUMULATED THE NUMBER OF PARTICLES
C   TIMES THE SQUARE OF THE BATCH AVERAGE (THIS IS OBTAINED BY DIVIDING    50
C   THE SQUARED BATCH SUM BY THE NUMBER OF PARTICLES STARTING THE BATCH)   60
      REAL * 8 SX, SX2, DUM
      DIMENSION SX(M1,M2,M3), SX2(M1,M2,M3)                         VAR3 70
      CALL ERRSET( 208,300,-1,1,1,208)
      IF (NBAT-1) 5,5,15
5   DO 10 I=1,M1
      DO 10 J=1,M2
      DO 10 K=1,M3
      IF (SX2(I,J,K)) 24,19,24
19  SX2(I,J,K) = 0.0
      GO TO 10
24  DUM      = SX2(I,J,K)/AM - (SX(I,J,K)/AM)**2                         VAR2 210
      SX2(I,J,K) = DSQRT(DABS(DUM)/AM) / (SX(I,J,K)/AM)                 VAR2 220
10  CONTINUE
      RETURN
15  DO 30 I=1,M1
      DO 30 J=1,M2
      DO 30 K=1,M3
      IF (SX(I,J,K)) 25,20,25
20  SX2(I,J,K) = 0.0
      GO TO 30
25  SX2(I,J,K) = (SX2(I,J,K)/NPART-(SX(I,J,K)/NPART)**2)/(NBAT-1.)  VAR3 200
      IF (SX2(I,J,K).LT.1.E-70) GO TO 30
      SX2(I,J,K) = DSQRT(DABS(SX2(I,J,K)))/SX(I,J,K)*NPART             VAR3 210
30  CONTINUE
      CALL ERRSET( 208, 10,-1,1,1,208)
      RETURN
      END

      SUBROUTINE VAR4(SX,SX2,SX3,SX4,SX5,SX6,
*M1,M2,NBAT,NPART,IFL)
C   NBAT IS THE NO. OF INDEPENDENT BATCHES                      VAR2 10
C   NPART IS THE TOTAL NUMBER OF PARTICLES PROCESSED           VAR2 20
C   IT IS ASSUMED THAT THE SUMSQ ARRAY HAS ACCUMULATED THE NUMBER OF PARTICLES
C   TIMES THE SQUARE OF THE BATCH AVERAGE (THIS IS OBTAINED BY DIVIDING    50
C   THE SQUARED BATCH SUM BY THE NUMBER OF PARTICLES STARTING THE BATCH)   60
      REAL * 8 AM, X1, X2, X3, X4, DUM2, DUM4, DUM42
      REAL * 8 SX, SX2, SX3, SX4, SX5, SX6
      REAL * 8 BETA, BETAJ, S2, S4, AJ, V2S2, V2SL
      DIMENSION SX(M1,M2),SX2(M1,M2),SX3(M1,M2),SX4(M1,M2),SX5(M1,M2)  VAR2 70
      *,SX6(M1,M2)

```

```

***** WILSON APRIL 20, 1989 *****
      AN = MPART
      AJ = MPART/51.
      DO 29 I=1,M1
      DO 29 J=1,M2
      IF (SX(I,J)) 24,19,24
19    SX2(I,J) = 0.0
      SX5(I,J) = 0.0
      GO TO 29
24    X1      = (1./AN)*SX(I,J)
      X2      = (1./AN)*SX2(I,J)
      X3      = (1./AN)*SX3(I,J)
      X4      = (1./AN)*SX4(I,J)
      S2      = X2 - X1**2
      S4      = X4-4.*X1*X3+8.*X1**2*X2-4.*X1**4-X2**2
      S4      = X4-4.*X1*X3+6.*X1**2*X2-3.*X1**4
      BETA    = S4/S2**2
      BETAJ   = BETA/AJ + 3.*(AJ-1.)/AJ
      V2S2   = 1./51.* (BETAJ - 48./50.)
C     IF(IFL .GT.0) GO TO 51
      V2SL   = (1./AN)+(BETA-(AN-3.)/(AN-1))
      SX5 (I,J) = DSQRT(DABS(V2S2/4.))
C51    SX2 (I,J) = DSQRT(DABS( S2)/AN)/(SX(I,J)/AN)
      SX2 (I,J) = DSQRT(DABS( S2)/AN)/(SX(I,J)/AN)
      SX6 (I,J) = DSQRT(DABS(V2SL/4.))
29    CONTINUE
      RETURN
      END

```

## Appendix D

# Derivation of Some Equations of Chapter 2

The derivations in this appendix are found in Hansen, Hurwitz, and Madow [7].

### D.1 Derivation of Equation 2.38

Equation 2.38 in Section 2.4 is given by

$$V_{S^2}^2 = \frac{1}{N} \left( \beta - \frac{N-3}{N-1} \right), \quad (\text{D.1})$$

where

$$\beta = \frac{\mu_4}{(\sigma^2)^2}, \quad (\text{D.2})$$

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2, \quad (\text{D.3})$$

and

$$\mu_4 = \frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^4. \quad (\text{D.4})$$

*Proof:*

The relative variance of the variance is given by

$$V_{S^2}^2 = \frac{\sigma_{S^2}^2}{(\sigma^2)^2}. \quad (\text{D.5})$$

By definition

$$V_{S^2}^2 = \frac{E(S^2 - ES^2)^2}{(ES^2)^2} = \frac{E(S^2 - \sigma^2)^2}{(\sigma^2)^2} = \frac{ES^4 - \sigma^4}{\sigma^4}. \quad (\text{D.6})$$

To evaluate  $ES^4$  the following transformation is used:

$$z_i - \mu = z_i \quad (D.7)$$

$$\bar{z} - \mu = \bar{z}, \quad (D.8)$$

which gives

$$ES^4 = E \left[ \frac{\sum_{i=1}^N (z_i - \bar{z})^2}{N-1} \right]^2, \quad (D.9)$$

$$ES^4 = \frac{1}{(N-1)^2} E \left[ \left( \sum_{i=1}^N z_i^2 \right)^2 - 2N\bar{z}^2 \sum_{i=1}^N z_i^2 + N^2\bar{z}^4 \right], \quad (D.10)$$

where

$$\left( \sum_{i=1}^N z_i^2 \right)^2 = \sum_{i=1}^N z_i^4 + \sum_{i \neq j}^N z_i^2 z_j^2, \quad (D.11)$$

$$\bar{z}^2 \sum_{i=1}^N z_i^2 = \frac{1}{N^2} \left[ \sum_{i=1}^N z_i^4 + 2 \sum_{i \neq j}^N z_i^2 z_j + \sum_{i \neq j}^N z_i^2 z_j^2 + \sum_{i \neq j \neq k}^N z_i^2 z_j z_k \right], \quad (D.12)$$

and

$$\bar{z}^4 = \frac{1}{N^4} \left[ \sum_{i=1}^N z_i^4 + 4 \sum_{i \neq j}^N z_i^2 z_j + 3 \sum_{i \neq j}^N z_i^2 z_j^2 + 6 \sum_{i \neq j \neq k}^N z_i^2 z_j z_k + \sum_{i \neq j \neq k \neq m}^N z_i^2 z_j z_k z_m \right]. \quad (D.13)$$

For sampling with replacement, independent samples, and using the fact that  $Ez_j = 0$ , when  $i \neq j \neq k \neq m$  it follows that

$$Ez_i^2 z_j^2 = Ez_i^2 Ez_j^2, \quad (D.14)$$

$$Ez_i^2 z_j = Ez_i^2 Ez_j = 0, \quad (D.15)$$

$$Ez_i^2 z_j z_k = Ez_i^2 Ez_j Ez_k = 0, \quad (D.16)$$

and

$$Ez_i^2 z_j z_k z_m = Ez_i^2 Ez_j Ez_k Ez_m = 0. \quad (D.17)$$

Therefore

$$E \left( \sum_{i=1}^N z_i^2 \right)^2 = N\mu_4 + N(N-1)\sigma^4, \quad (D.18)$$

$$E\bar{z}^2 \sum_{i=1}^N z_i^2 = \frac{1}{N^2} [N\mu_4 + N(N-1)\sigma^4], \quad (\text{D.19})$$

and

$$E\bar{z}^4 = \frac{1}{N^4} [N\mu_4 + 3N(N-1)\sigma^4]. \quad (\text{D.20})$$

Substituting equations D.18, D.19, and D.20 in Equation D.6, subtracting and dividing by  $\sigma^4$

$$V_{S^2}^2 = \frac{1}{N} \left( \beta - \frac{N-3}{N-1} \right). \quad (\text{D.21})$$

## D.2 Derivation of Equations 2.46 and 2.47

For  $I$  random groups of  $J$  elements

$$V_{S^2}^2 = \frac{1}{I} \left( \beta_J - \frac{I-3}{I-1} \right), \quad (\text{D.22})$$

where

$$\beta_J = \frac{\beta}{J} + 3 \frac{J-1}{J}. \quad (\text{D.23})$$

*Proof:*

In Chapter 2 it was shown that

$$S_b^2 = \frac{J}{I-1} \sum_{i=1}^I (\bar{z}_i - \bar{z})^2. \quad (\text{D.24})$$

The relative variance of  $S_b^2$  is also given by

$$V_{S_b^2}^2 = \frac{\sigma_{S_b^2}^2}{(\sigma^2)^2}. \quad (\text{D.25})$$

Similar to Equation D.6

$$V_{S_b^2}^2 = \frac{ES_b^4 - \sigma^4}{\sigma^4}. \quad (\text{D.26})$$

Using the following transformation:

$$\bar{z}_i - \mu = \bar{z}_i \quad (\text{D.27})$$

$$\bar{z} - \mu = \bar{z}, \quad (\text{D.28})$$

Equation D.24 can be written as

$$S_b^2 = \frac{J}{I-1} \sum_{i=1}^I (\bar{z}_i - \bar{z})^2, \quad (D.29)$$

which gives

$$ES_b^4 = \frac{J^2}{(I-1)^2} E \left[ \sum_i^I (\bar{z}_i - \bar{z})^2 \right]^2, \quad (D.30)$$

or

$$ES_b^4 = \frac{J^2}{(I-1)^2} E \left[ \left( \sum_i^I \bar{z}_i^2 \right)^2 - 2I\bar{z}^2 \sum_i^I \bar{z}_i^2 + I^2 \bar{z}^4 \right]. \quad (D.31)$$

The first term of Equation D.31 is given by

$$E \left( \sum_i^I \bar{z}_i^2 \right)^2 = E \sum_i^I \bar{z}_i^4 + E \sum_{i \neq h}^I \bar{z}_i^2 \bar{z}_h^2, \quad (D.32)$$

$$E \left( \sum_i^I \bar{z}_i^2 \right)^2 = E \sum_i^I \left( \frac{\sum_j^J z_{ij}}{J} \right)^4 + E \sum_{i \neq h}^I \left( \frac{\sum_j^J z_{ij}}{J} \right)^2 \left( \frac{\sum_j^J z_{ih}}{J} \right)^2, \quad (D.33)$$

which gives

$$E \left( \sum_i^I \bar{z}_i^2 \right)^2 = I \left[ \frac{\mu_4}{J^3} + \frac{3(J-1)\sigma^4}{J^3} \right] + I(I-1) \frac{\sigma^4}{J^2}. \quad (D.34)$$

The second term in Equation D.31 is given by

$$E \left( 2I\bar{z}^2 \sum_i^I \bar{z}_i^2 \right) = 2IE \left[ \left( \frac{\sum_i^I \bar{z}_i}{I} \right)^2 \sum_i^I \bar{z}_i^2 \right] = \frac{2}{I} E \left( \sum_i^I \bar{z}_i^2 \right), \quad (D.35)$$

which becomes

$$E \left( 2I\bar{z}^2 \sum_i^I \bar{z}_i^2 \right) = 2 \left[ \frac{\mu_4}{J^3} + \frac{3(J-1)\sigma^4}{J^3} \right] + 2(I-1) \frac{\sigma^4}{J^2}. \quad (D.36)$$

The third term in Equation D.31 becomes

$$EI^2 \bar{z}^4 = \frac{1}{J^2} E \left( \sum_i^I \bar{z}_i \right)^4 = \frac{1}{J^2} E \left( \sum_i^I \bar{z}_i^4 + 3 \sum_{i \neq h}^I \bar{z}_i^2 \bar{z}_h^2 \right), \quad (D.37)$$

which gives

$$EI^2 \bar{z}^4 = \frac{1}{J^2} \left\{ I \left[ \frac{\mu_4}{J^3} + \frac{3(J-1)\sigma^4}{J^3} \right] + 3I(I-1) \frac{\sigma^4}{J^2} \right\}. \quad (D.38)$$

Substituting equations D.34, D.36, and D.38 in Equation D.31  $Es_b^4$  will be given by

$$Es_b^4 = \frac{\mu_4}{JJ} + \frac{3(J-1)}{IJ}\sigma^4 + \frac{I^2 - 2I + 3}{(I-1)J}\sigma^4. \quad (\text{D.39})$$

Substituting Equation D.39 in Equation D.26

$$V_{S_b^2}^2 = \frac{1}{I} \left[ \frac{1}{J} \frac{\mu_4}{\sigma^4} + \frac{3(J-1)}{J} - \frac{I-3}{I-1} \right], \quad (\text{D.40})$$

$$V_{S_b^2}^2 = \frac{1}{I} \left[ \beta_J + \frac{3(J-1)}{J} - \frac{I-3}{I-1} \right], \quad (\text{D.41})$$

or finally

$$V_{S_b^2}^2 = \frac{1}{I} \left( \beta_J - \frac{I-3}{I-1} \right). \quad (\text{D.42})$$

### D.3 Derivation of Equation 2.48

In terms of the relative variance Equation 2.48 in Section 2.4 can be written as

$$V_S^2 = \frac{V_{S^2}^2}{4}. \quad (\text{D.43})$$

*Proof:*

Since

$$E(S - \sigma)^2 = ES^2 - 2\sigma ES + \sigma^2 = 2\sigma(\sigma - ES), \quad (\text{D.44})$$

then

$$V_S^2 = 2 \left( \frac{\sigma - ES}{\sigma} \right). \quad (\text{D.45})$$

Using the expansion

$$\frac{S - \sigma}{\sigma} = \frac{(S^2 - \sigma^2)}{2\sigma^2} - \frac{(S^2 - \sigma^2)^2}{8\sigma^4} + \frac{(S^2 - \sigma^2)^3}{16\sigma^6} - \dots \quad (\text{D.46})$$

it follows that

$$\frac{\sigma - ES}{\sigma} = \frac{1}{8\sigma^4} E(S^2 - \sigma^2)^2 - \frac{1}{16\sigma^6} E(S^2 - \sigma^2)^3 + \dots \quad (\text{D.47})$$

Since

$$\frac{V_S^2}{2} = \frac{\sigma - ES}{\sigma}, \quad (\text{D.48})$$

Equation D.47 becomes

$$\frac{V_S^2}{2} = \frac{V_{S^2}^2}{8} - \frac{1}{16\sigma^6} E(S^2 - \sigma^2)^3 + \dots \quad (\text{D.49})$$

Finally for sufficiently large  $N$

$$V_S^2 = \frac{V_{S^2}^2}{4}. \quad (\text{D.50})$$

## VITA

Wilson José Vieira was born in Anápolis, Goiás, Brazil on November 7, 1955. In 1979 he received his B.S. degree in Civil Engineering from the Universidade Federal de Uberlândia and in 1982 he received his M.S. degree in Nuclear Engineering from the Universidade de São Paulo. He started to work in 1982 for the Instituto de Pesquisas Energéticas e Nucleares in São Paulo in radiation shielding calculations. In 1986 he started his work toward the Ph.D. degree in Nuclear Engineering at the University of Tennessee, Knoxville.