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EBWR CORE SHIELDED BY WATER, IRON AND CONCRETE (*)

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SUMÁRIO

Neste estudo o reator EBWR foi considerado operando a 100 Mw e com uma blindagem de 28,6 cm de água, 10 cm de ferro e 300 cm de concreto. Uma esfera de igual volume ao do caroço, de 73 cm de raio, foi escolhida como a melhor geometria.

Os calculos dos fluxos rápido e térmico foram baseados na teoria de dois grupos, e executados em máquina de calcular de mesa e em um computador digital IBM-704.

O método utilizado é exposto e os resultados obtidos são apresentados sob forma de tabelas e gráficos.

SUMMARY

In this study the operation of the EDWR at 100 Mw when shielded by 28.6 cm of water, 10 cm of iron and 300 cm of concrete was considered. The equal-volume sphere core, with a radius of 73 cm was chosen for the study as indicated by calculations for the best core geometry.

The fast and thermal neutron flux calculations were done by two-group analysis. Calculations for the latter were made both by hand and by computer.

The procedures followed and the calculations made in connection with the study are explained, and the results are tabulated and plotted.

RÉSUMÉ

Dans cette étude on a pris en considération l'opéra

tion du EBWR à la puissance de 100 MW et avec une protection par 28,6 cm d'eau, 10 cm de fer et 300 cm de béton. Un coeur d'égale volume, en sphère, avec un rayon de 73 cm, comme indiqué par les calculs de la meilleure géométrie, a été choisi pour la réalisation de cette étude.

Les calculs du flux neutrons rapides et thermiques ont été effectués par la théorie de deux groupes, autant à main, comme aussi à l'aide d'un ordinateur IBM-704.

Les procédés suivis et les calculs faits, pour cette étude, sont expliqués, et les résultats résumés par des tableaux et des graphiques.

CALCULATION OF THE NEUTRON FLUX DISTRIBUTION IN THE
EBWR CORE SHIELDED BY WATER, IRON AND CONCRETE (*)

by

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INTRODUCTION

In this study the operation of the EBWR at 130 MW when shielded by 28.6 cm of water, 10 cm of iron and 300 cm of concrete was considered. The equal-volume sphere core, with a radius of 73 cm was chosen for the study as indicated by calculations for the best core geometry.

The fast and thermal neutron flux calculations were done by two-group analysis. Calculations for the latter were made both by hand and by computer.

The procedures followed and the calculations made in connection with the study are explained, and the results are tabulated and plotted.

REMOVAL CROSS-SECTIONS

The fuel assemblies are made up of 144 elements, 3.75 by 3.75 inches in cross-section and with an active height of 4 ft. Approximately 20% of the assemblies are spiked (ANL 5781 Addendum). The calculations for the removal cross-sections for

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the various components of the fuel elements, made in accordance with

AERE-R3216 - "Methods of Calculation for Use in the Design of Shields for Power Reactors"

NAA-SR-2380 - "Application of Fast Neutron Removal Theory to the Calculation of Thermal Neutron Flux Distributions in Reactor Shields"

Price, B. T., C. C. Horton and K. T. Spinney, "Radiation Shielding", International Series of Monographs on Nuclear Energy, Pergamon Press gave the following values:

$$\begin{aligned}\sigma_{\lambda}(\text{H}_2\text{O}) &= 2.99 && \text{barns} \\ \sigma_{\lambda}(\text{U}) &= 3.6 \\ \sigma_{\lambda}(\text{Zr}) &= 1.9694 \\ \sigma_{\lambda}(\text{Nb}) &= 2.0063 \\ \sigma_{\lambda}(\text{Ca}) &= 1.9719 \\ \sigma_{\lambda}(\text{Fe}) &= 1.8614\end{aligned}$$

Removal cross-sections for the fuel elements were then determined, using the data indicated in Table 1, which yielded values as follows:

Thin Elements

$$\begin{aligned}\text{Enriched} & \quad \Sigma_{\lambda} = 0.111 \text{ cm}^{-1} \\ \text{Natural} & \quad \Sigma_{\lambda} = 0.111\end{aligned}$$

Thick Elements

$$\begin{aligned}\text{Enriched} & \quad \Sigma_{\lambda} = 0.115 \\ \text{Natural} & \quad \Sigma_{\lambda} = 0.115\end{aligned}$$

Spikes

$$\Sigma_{\lambda} = 0.094$$

The core was taken as divided into four regions, each

TABLE -1

VOLUME FRACTIONS AND ATOMIC DENSITIES IN 12.75" x 12.75" x 32.385 cm (32.385 cm x 32.385 cm) CELL AT ROOM TEMPERATURE

FUEL ELEMENT TYPE MATERIAL	THIN ELEMENTS				THICK ELEMENTS				SPIKES	
	ENRICHED		NATURAL		ENRICHED		NATURAL		VOLUME FRACTION	ATOMIC DENSITY ($\times 10^{24}$)
	VOLUME FRACTION	ATOMIC DENSITY ($\times 10^{24}$)	VOLUME FRACTION	ATOMIC DENSITY ($\times 10^{24}$)	VOLUME FRACTION	ATOMIC DENSITY ($\times 10^{24}$)	VOLUME FRACTION	ATOMIC DENSITY ($\times 10^{24}$)		
U^{235}	.00244	.0001156	.001276	.0000604	.00336	.0001592	.001739	.0000823	.0036	.0001702
U^{238}	.1598	.00756	.1610	.007615	.2230	.01055	.22474	.01063	.000262	.000124
U	.1623	.007675	.1623	.007675	.2264	.01071	.2264	.01071	.003862	.0001826
H_2O	.668	.02239	.668	.02239	.588	.01970	.588	.01970	.582	.01950
Zr	.134	.005650	.134	.005650	.148	.006245	.148	.006245	.250	.01057
Nb	.00578	.0003151	.00578	.0003151	.00813	.0004431	.00813	.0004431		
Ca									.0365	.0008511
O in Spike Meat										.00757
Fe	.0302	.00256	.0302	.00256	.0302	.00256	.0302	.00256	.0302	.00256

NOTES: The rod follower has a volume fraction of .0302; in the follower $N(U^{238}) = .0001737 \times 10^{24}$, $N(Fe) = .0848 \times 10^{24}$.
 The control rod has the same volume fraction as the follower; in the rod $N(Fe) = .07605 \times 10^{24}$, $N(B) = .00875 \times 10^{24}$.
 U^{235} in the rod follower is included with U^{238} in the cell.

4.

a spherical shell with a thickness of

Region 1	- 34.46 cm (radius of inner sphere)
2	- 13.20
3	- 8.90
4	- 16.44

Fig. 3 shows the core loading arrangement with the fuel elements distributed as follows:

Region	No. of Elements
1	36 thin elements, enriched
2	28 spikes
3	16 thick elements, enriched 20 thin elements, enriched
4	40 thick elements, enriched 4 spikes 4 thick elements, natural

The results of the calculations for the macroscopic removal cross-sections for each core region are listed below.

TABLE 2

MACROSCOPIC REMOVAL CROSS-SECTION	
Region	$\Sigma_r (\text{cm}^{-1})$
1	0.111
2	0.094
3	0.113
4	0.113

FAST NEUTRON FLUX AT THE EDGE OF THE CORE

The fast neutron flux at the edge of the core was found by comparing the results derived from three methods:

METHOD 1

1. The flux at the first interface was determined, using a cross-section Σ_{s1} and the formula

$$\phi(0) = \frac{Q_1}{2\Sigma_{s1}} \left[1 - \frac{1}{2\Sigma_{s1}R_{s1}} \left(1 - e^{-2\Sigma_{s1}R_{s1}} \right) \right] \text{ n/cm}^2 \cdot \text{sec}$$

2. Using the same formula and a cross-section Σ_{s2} for the entire region from the core center to the second interface, a corresponding value of flux at the latter point was calculated.
3. Similarly, a flux calculation was made again for the first interface, this time using a cross-section Σ_{s2} .
4. The effect of the inner sphere, when treated as having a cross-section of Σ_{s2} on the flux at the second interface was calculated.
5. The value found from Step 4 was subtracted from the value obtained in Step 2 to give the correct value of flux at the second interface.
6. For the other succeeding interfaces, a similar procedure was followed, whereby the effect on the flux at the outer face of a shell by the sphere within the shell is taken into account.
7. The flux at the edge of the core was finally determined by summing up all the contributions of partial fluxes at the interfaces, considering the corresponding attenuations.

METHOD 2

1. The flux at the first interface was determined using the formula

6.

$$\phi_1(r) = \frac{Q_1}{2\bar{\Sigma}_{s1}} \left[1 - \frac{1}{2\bar{\Sigma}_{s1} R_{s1}} \left(1 - e^{-2\bar{\Sigma}_{s1} R_{s1}} \right) \right] \quad \text{M/cm}^2 \times \text{sec.}$$

2. The contribution of the flux found from Step 1 to the flux at the edge of the core, considering attenuation, was calculated using the formula

$$\phi(R_{s4}) = \phi_1(0) \frac{R_{s1}}{R_{s4}} E_1 \left[\bar{\Sigma}_{s2} (R_{s2} - R_{s1}) + \bar{\Sigma}_{s3} (R_{s3} - R_{s2}) + \bar{\Sigma}_{s4} (R_{s4} - R_{s3}) \right] \quad \text{M/cm}^2 \times \text{sec.}$$

3. The second region (spherical shell) was taken as a finite slab source and the flux at the second interface calculated from the formula

$$\phi_2(r) = \frac{Q_2}{2\bar{\Sigma}_{s2}} \left\{ 1 - E_1 \left[\bar{\Sigma}_{s2} (R_{s2} - R_{s1}) \right] \right\} \quad \text{M/cm}^2 \times \text{sec.}$$

4. The contribution from this region (second) to the flux at the edge of the core was determined using the formula

$$\phi(R_{s4}) = \phi_2(0) \frac{R_{s2}}{R_{s4}} E_1 \left[\bar{\Sigma}_{s3} (R_{s3} - R_{s2}) + \bar{\Sigma}_{s4} (R_{s4} - R_{s3}) \right] \quad \text{M/cm}^2 \times \text{sec.}$$

5. A similar procedure was followed for the other regions, and the flux at the edge of the core was determined by adding all the contributions of the partial fluxes from each region.

METHOD 3

1. The core was treated as a homogeneous sphere with an average removal cross-section $\bar{\Sigma}_{R2} = 0.110 \text{ cm}^{-1}$ and an average source strength $\bar{Q} = 7.5 \times 10^{12} \text{ n/cm}^2 \times \text{sec.}$

2. Then the flux at the edge of the core was determined from the formula

$$\phi(R_{s4}) = \frac{\bar{Q}}{2\bar{\Sigma}_R} \left[1 - \frac{1}{\bar{\Sigma}_R} \left(1 - e^{-2\bar{\Sigma}_R R_s} \right) \right] \quad \text{M/cm}^2 \times \text{sec.}$$

Power and source strengths for the various core regions were calculated from the curves of Figs. 1 and 2, which give values as follows:

TABLE 3

POWER AND SOURCE STRENGTHS FOR THE VARIOUS CORE REGIONS

Region	Power P_i (watts)	Source Strength Q_i ($n/cm^3 \times sec$)
1	0.281×10^8	1.27×10^{13}
2	0.403	1.11
3	0.205	0.52
4	0.111	0.10

The results of the calculations based on the three methods are tabulated below.

TABLE 4

FAST NEUTRON FLUX AT THE EDGE OF THE CORE FROM EACH CORE REGION

Region	Method 1	Method 2	Method 3 (homogeneous core)
	(ϕ in $n/cm^2 \times sec$)		
1	7.76×10^{10}	7.76×10^{10}	
2	4.74×10^{11}	5.35×10^{11}	
3	8.32×10^{11}	9.42×10^{11}	
4	3.90×10^{12}	4.20×10^{12}	
Total	5.28×10^{12}	5.75×10^{12}	3.20×10^{13}

FAST NEUTRON FLUX DISTRIBUTION IN THE RADIAL SHIELD

The flux based on a homogenous core was chosen for these calculations, with the flux distribution throughout the shield being determined from the formula

8.

$$\phi(r) = \frac{Q}{2\epsilon_s} \cdot \frac{R_s}{R_s + a} \left\{ A_1 E_1 \left[\sigma_1 a_1 + \sigma_2 a \right] + A_2 E_2 \left[\sigma_2 a_1 + \sigma_1 a \right] \right\} \text{ n/cm}^2 \cdot \text{sec}$$

and the following data:

$$\begin{aligned} Q &= 7.5 \times 10^{12} \text{ fast neutrons/cm}^3 \times \text{sec} \\ \Sigma_s &= 0.110 \text{ cm}^{-1} \\ R_s &= 73 \text{ cm} \\ A_1 &= 1 \\ A_2 &= 0.121 \\ \sigma_1 &= 0.129 \\ \sigma_2 &= 0.091 \end{aligned} \quad \left. \vphantom{\begin{aligned} Q \\ \Sigma_s \\ R_s \\ A_1 \\ A_2 \\ \sigma_1 \\ \sigma_2 \end{aligned}} \right\} \text{ NAA-SR-2380}$$

The resulting flux at the boundary of each shield is given in Table 5, and the distribution is plotted in Fig. 4.

SHIELDING CONFIGURATION

Dimensions:

1. Diameter of pressure vessel
84 inches
2. Inner diameter of reflector
80 inches
3. Outer diameter of core
57.4 inches
4. Thickness of H_2O reflector
28.6 cm
5. Thickness of Fe
10 cm
6. Thickness of concrete
300 cm

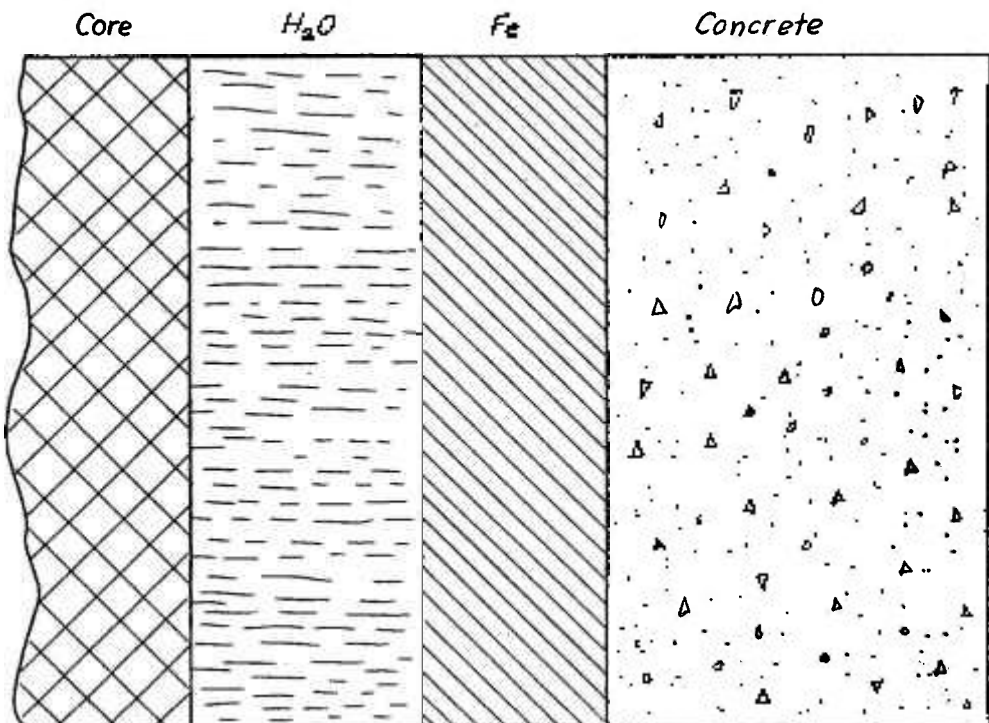
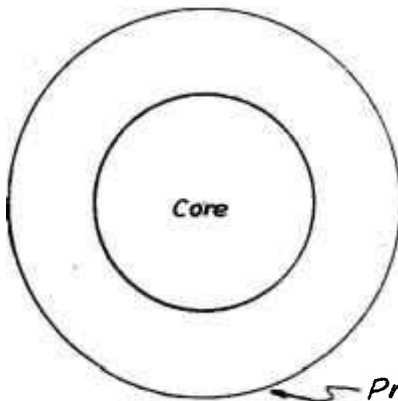


TABLE 5

FAST FLUX AT THE BOUNDARY OF EACH SHIELD

Material	Thickness (cm)	Removal Cross-section (cm ⁻¹)	Flux ϕ (n/cm ² x sec)
Core	73	0.110	
H ₂ O	28.6	-	1.98×10^{11}
Fe	10	0.168	2.42×10^{10}
Concrete	300	0.112	2.70×10^{-6}

THERMAL NEUTRON FLUX AT THE EDGE OF THE CORE

By use of the formula

$$\phi_s = \phi^*(0) \frac{\phi_s}{\phi_s^*} \quad n/cm^2 \times sec$$

and an average thermal neutron flux value of

$$\bar{\phi}_s = 3.0 \times 10^{13} \quad n/cm^2 \times sec \quad ANL-5101 \quad addendum$$

and likewise values of

$$\begin{aligned} \phi_s^* &\approx 9.5 \quad (\text{arbitrary units}) \\ \phi_s &\approx 18.555 \quad (\text{arbitrary units}) \end{aligned}$$

from the reactor core calculations (refer to Figs. 1 and 2), the thermal neutron flux at the edge of the core was calculated to be

$$\phi_s(0) = 1.54 \times 10^{13} \quad n/cm^2 \times sec$$

THERMAL NEUTRON FLUX DISTRIBUTION IN THE SHIELD

Table 6 lists the constants used in the calculation by hand of the thermal neutron distribution.

TABLE 6

TABLE OF CONSTANTS FOR THERMAL NEUTRON CALCULATION

Material	K	D	(slope)
H ₂ O	0.22	0.246	0.1780
Fe	0.64	0.345	0.21
Concrete	0.379	0.418	0.123

The thermal flux distribution calculation by hand was made using the formula

$$\phi_i^s(x_i) = A_i e^{k_i x_i} + B_i e^{-k_i x_i} + \frac{\sigma_i \phi_i^f(0) e^{-\sigma_i x_i}}{D(k_i^2 - \sigma_i^2)} \quad \text{m/cm}^2 \times \text{sec} \quad \text{ANL-6000}$$

Taking $C_i = \frac{\sigma_i \phi_i^f(0)}{D(k_i^2 - \sigma_i^2)}$ then

$$\phi_i^s(x_i) = A_i e^{k_i x_i} + B_i e^{-k_i x_i} + C_i e^{-\sigma_i x_i} \quad \text{m/cm}^2 \times \text{sec}$$

Boundary conditions: Each region was taken as infinite; then

$$\phi_{si}(0) = A_i + B_i + C_i$$

$$A_i = 0$$

The B_i and C_i values obtained by the above calculations are tabulated below.

TABLE 7

VALUES OF B_i , C_i AND THE FLUX AT THE EDGE OF EACH SHIELD MATERIAL

Material	B_i	C_i	ϕ (n/cm ² x sec)
H ₂ O	-1.395×10^{15}	1.41×10^{15}	6.09×10^{12}
Fe	45.76×10^{12}	3.30×10^{11}	4.94×10^{10}
Concrete	-5.80×10^9	5.52×10^{10}	5.08×10^{-6}

The distribution curve is plotted in Fig. 4.

12.

Calculations for thermal neutron flux, taking:
Region 1 finite and region 2 infinite

$$1. \begin{cases} \phi_{s1}(0) = A_1 + B_1 + C_1 \end{cases}$$

$$2. \begin{cases} \phi_{s1}(x_1) = A_1 e^{k_1 x_1} + B_1 e^{-k_1 x_1} + C_1 e^{-\sigma_1 x_1} \\ \phi_{s2}(0) = A_2 + B_2 + C_2 \end{cases}$$

$$\therefore A_1 e^{k_1 x_1} + B_1 e^{-k_1 x_1} - A_2 - B_2 = C_2 - C_1 e^{-\sigma_1 x_1}$$

$$3. \begin{cases} J_1(x_1) = -D_1 A_1 k_1 e^{k_1 x_1} + D_1 B_1 k_1 e^{-k_1 x_1} + D_1 C_1 \sigma_1 e^{-\sigma_1 x_1} \\ J_2(0) = -D_2 A_2 k_2 + D_2 B_2 k_2 + D_2 C_2 \sigma_2 \end{cases}$$

$$\therefore -D_1 A_1 k_1 e^{k_1 x_1} + D_1 B_1 k_1 e^{-k_1 x_1} - D_2 A_2 k_2 - D_2 B_2 k_2 = D_2 C_2 \sigma_2 - D_1 C_1 \sigma_1 e^{-\sigma_1 x_1}$$

$$4. \begin{cases} \phi(\infty) = 0 \implies A_2 = 0 \end{cases}$$

Then we have the equations:

$$\begin{aligned} A_1 + B_1 &= \phi_{s1}(0) - C_1 \\ A_1 e^{k_1 x_1} + B_1 e^{-k_1 x_1} - B_2 &= C_2 - C_1 e^{-\sigma_1 x_1} \\ -A_1 D_1 k_1 e^{k_1 x_1} + B_1 D_1 k_1 e^{-k_1 x_1} - B_2 D_2 k_2 &= D_2 C_2 \sigma_2 - D_1 C_1 \sigma_1 e^{-\sigma_1 x_1} \end{aligned}$$

Then we have the determinant:

$$\Delta = \begin{vmatrix} A_1 & B_1 & B_2 \\ 1 & 1 & 0 \\ e^{k_1 x_1} & e^{-k_1 x_1} & -1 \\ -D_1 k_1 e^{k_1 x_1} & D_1 k_1 e^{-k_1 x_1} & -D_2 k_2 \end{vmatrix} \begin{vmatrix} \phi_{s1}(0) - C_1 \\ C_2 - C_1 e^{-\sigma_1 x_1} \\ D_2 C_2 \sigma_2 - D_1 C_1 \sigma_1 e^{-\sigma_1 x_1} \end{vmatrix}$$

The values obtained for A_i and B_i from the determinants are in Table 8.

TABLE 8

VALUES OF A_i , B_i AND THE FLUX AT THE EDGE OF EACH SHIELD MATERIAL

Material	A_i	B_i	ϕ (n/cm ² x sec)
H ₂ O	$- 7.25 \times 10^9$	$- 1.397 \times 10^{15}$	2.29×10^{12}
Fe	$+ 1.08 \times 10^6$	$+ 2.32 \times 10^{12}$	4.52×10^{10}
Concrete	-	$- 9.24 \times 10^9$	5.08×10^{-6}

The thermal flux distribution curve for this latter case (taking two regions together and assuming the first one finite and the second one infinite) is also plotted in Fig. 4.

Neutron Flux Calculation By Computer

The calculations for the thermal neutron flux were done by computer, using IBM-704, Code RE-34. Calculations were made for slab geometry, taking the slope of fast neutron distribution. They were also made for sphere geometry, taking both the slope of the fast neutron flux and the removal cross-section. (Refer to the attached programs.)

The results are plotted in Fig. 4.

S = Spike
m = Thin enriched
k = Thick enriched
h = Thick natural

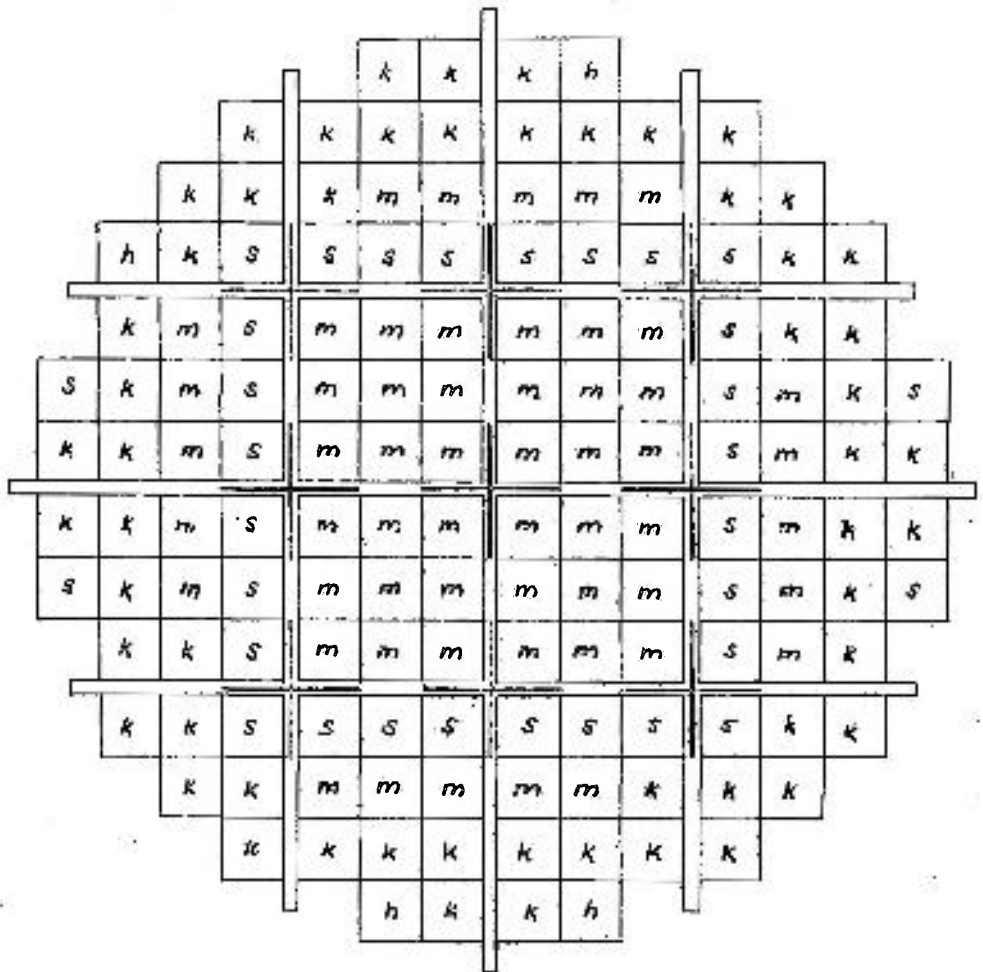


Fig. 3 CORE LOADING

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