

# Analytical Formulation of an Yb-doped Tandem-Pumping Fiber Amplifier

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**Abstract**—An analytical formulation of a multi-kilowatt ytterbium-doped double-cladding tandem-pumped fiber amplifier was developed and is reported in this paper. The model consists of a system including the laser rate, pumping and seed power equations. The model was validated by describing a fiber amplifier behavior with a 5.986kW, 1018nm pumping and a signal seed of 75W, 1080nm, yielding a maximum power output of 5.448kW, with an efficiency of 91.1%. The thermal equations are solved for the temperature distribution along the fiber with a maximum temperature of 43.24°C, when the convective coefficient of 4000W · m<sup>-2</sup> · K<sup>-1</sup> is used.

**Index Terms**—high power fiber laser, ytterbium-doped fiber, tandem pump, rate equation

## I. INTRODUCTION

In recent years, significant advancements in fiber lasers technology have been occurring due to the fast development of high-performance rare-earth ion-doped fibers, like ytterbium-doped fibers (YDF), mainly caused by their advantages such as efficiency, beam quality and flexible operation. Yet the escalating of power has been limited by the pump source brightness, nonlinear effects, optical damage and by thermally induced modal instabilities. To solve these problems, pumping active fibers with high-brightness fiber lasers instead of laser diodes have been demonstrated to be an excellent technical solution to enhance the pump brightness [1]. This tandem pumping technique showed some useful characteristics like potential for mode-area scaling, easier heat management, mitigating photodarkening and a higher modal instability threshold.

In order to study YDF and develop a model that could approximate the behavior of these fibers when working as an amplifier in a tandem pumping configuration, this paper presents a solution for the rate equations in the steady-state regime. Taking into account the work of Peysokhan [2], this article solves the rate equations for an amplifier using the boundary conditions presented by Haag [3]. To validate the solutions for the power distribution along the fiber and for the Yb<sup>3+</sup> excited ions distribution, our results were compared to those obtained by Yan [4], and a very good agreement was obtained including the power distribution along the fiber.

To fulfill the goal of the present work, this paper starts in Section 2 formulating the theoretical model, the boundary conditions, and a solution for the rate equations for the laser.

In Section 3 the validation of the model is obtained by taking the experimental data, running the numerical code in MATLAB and comparing the results. In Section 4, a discussion is presented about the results and Section 5 concludes the work.

## II. THEORETICAL MODEL

According to Paschotta [5], the Yb<sup>3+</sup> ion spectroscopy is simpler compared to other rare earth ions. Thus, for all  $k$  wavenumbers, only the atomic sublevels  ${}^2F_{7/2}$  (laser transition inferior level) and  ${}^2F_{5/2}$  (metastable state) are important. In addition, the population thermalization within each sub-level is so fast that it can be considered instantaneous; therefore, it is only necessary to specify the populations  $N_1$  and  $N_2$  at the lowest and highest sublevels, respectively. Using the mathematical model proposed by Giles [6], defining the total power  $P_k(z)$  by the light intensity distribution  $\mathcal{I}_k(r, \varphi, z)$  as:

$$P_k(z) = \int_0^{2\pi} \int_0^\infty \mathcal{I}_k(r, \varphi, z) r dr d\varphi. \quad (1)$$

Paschotta states that the populations at the upper and lower levels of Yb<sup>3+</sup> ions behave in the same way as Er<sup>3+</sup> ions, according to the rate equations described by Giles in his work with erbium-doped fiber amplifier (EDFA) [6]. So the increasing in the total power of the light beam when passing through an active medium is a function of the cross-section  $\sigma_{21}$  and the degeneracies ( $g_1$ ) and ( $g_2$ ), and can be described as follows:

$$dP_k(z) = \sigma_{21} P_k(z) \left[ N_2 - \left( \frac{g_2 N_1}{g_1} \right) \right] dz \quad (2)$$

$$P_k^{out}(z) = P_k(z) \exp \left\{ \sigma_{21} \left[ N_2 - \left( \frac{g_2 N_1}{g_1} \right) \right] z \right\}. \quad (3)$$

Assuming that the amplifier is working in the steady state, the sum of the populations  $N_1$  and  $N_2$  is always constant, and the rate of change of the highest sublevel population  $dN_2/dt$  will be dependent on the phenomena of stimulated emission ( $e$ ), spontaneous emission and absorption ( $a$ ). The rate equations depend of the total dopant concentration  $N_t$ , the normalized light intensity  $i_k$ , the wave frequency  $\nu_k$ , the

$$\frac{N_2(z)}{N_t(z)} = \frac{\left(\frac{\Gamma_p}{h\nu_p A}\right) \sigma_p^a [P_p^+(z) + P_p^-(z)] + \left(\frac{\Gamma_s}{h\nu_s A}\right) \sigma_s^a [P_s^+(z)]}{\left(\frac{\Gamma_p}{h\nu_p A}\right) (\sigma_p^a + \sigma_p^e) [P_p^+(z) + P_p^-(z)] + \left(\frac{\Gamma_s}{h\nu_s A}\right) (\sigma_s^a + \sigma_s^e) [P_s^+(z)] + \frac{1}{\tau_2}} \quad (6)$$

$$\left(\left(\frac{dP_s^+}{dz}\right) + \alpha_s (P_s^+)\right) + \frac{\nu_s}{\nu_p} \left(\left(\frac{dP_p^+}{dz} - \frac{dP_p^-}{dz}\right) + \alpha_p (P_p^+ + P_p^-)\right) + h\nu_s A \frac{\overline{N_2}(z)}{\tau_2} = 0 \quad (13)$$

$$\int_0^z dP_s^+ + \int_0^z \frac{\nu_s}{\nu_p} ((\alpha_p - \alpha)(P_p^+ + P_p^-)) dz + \int_0^z h\nu_s A \frac{\overline{N_2}(z)}{\tau_2} dz + \int_0^z \alpha_s P_s^+ dz = 0 \quad (17)$$

radiative lifetime  $\tau_2$  and the Planck's constant  $h$ . They are described by Giles as follows:

$$N_t(r, \varphi, z) = N_1(r, \varphi, z) + N_2(r, \varphi, z) \quad (4)$$

$$\begin{aligned} \frac{dN_2}{dt} = & \sum_k \frac{P_k i_k \sigma_k^a}{h\nu_k} N_1(r, \varphi, z) \\ & - \sum_k \frac{P_k i_k \sigma_k^e}{h\nu_k} N_2(r, \varphi, z) - \frac{N_2(r, \varphi, z)}{\tau_2}. \end{aligned} \quad (5)$$

Expanding (5) and taking into account the incidence of two types of light power, the signal  $k = \{s\}$  and the pumping  $k = \{p\}$  beams. The signal laser  $P_s^+$  and the pumping laser  $P_p^\pm$  have different spectral properties each, and the pumping laser can travel in both directions, depending on the experiment configuration (single-pumping or dual-pumping). Assuming that the laser is working in continuous wave mode (CW), and after the end of the initial transient, the rate of change  $dN_2/dt$  is zero, and Eq. (6), as seen above, is obtained.

The propagation of the signal and pumping lasers along the fiber are described by Eqs. (7)-(9). These differential equations are a function of the filling factor  $\Gamma_{s,p}$  and the cross sections  $\sigma_{s,p}^{a,e}$ , representing the gain, and the term  $\alpha_{s,p}$  representing the power losses:

$$\frac{dP_s^+}{dz} = \Gamma_s [(\sigma_s^e + \sigma_s^a) N_2 - \sigma_s^a N_t] P_s^+ - \alpha_s P_s^+ \quad (7)$$

$$\frac{dP_p^+}{dz} = \Gamma_p [(\sigma_p^e + \sigma_p^a) N_2 - \sigma_p^a N_t] P_p^+ - \alpha_p P_p^+ \quad (8)$$

$$\frac{dP_p^-}{dz} = -\Gamma_p [(\sigma_p^e + \sigma_p^a) N_2 - \sigma_p^a N_t] P_p^- + \alpha_p P_p^- \quad (9)$$

#### A. Basic Rate Equations

In [3] Haag studied the influence of the amplified spontaneous emission in oscillator and amplifier systems. He established a model for the essential rate equations under a permanent regime, and he defined the following rate equations for the  $P_L^\pm$  laser intensity (coherent flux),

$$\frac{dP_L^\pm}{dz} = \pm P_L^\pm (g^* - \alpha^*) \quad (10)$$

where  $g^*$  denotes the saturable gain,  $\alpha^*$  is the consideration for the non-saturable linear losses.

#### B. Boundary conditions

Haag [3] defines the boundary conditions in a more generic form for  $P_L^\pm$  taking into consideration both the cases of oscillators and amplifiers. Then, to characterize the operation of the laser in the case of an amplifier system, the coherent flux coupled  $\Phi$  was defined to specify the input powers in the fibers.

$$P_L^+(0) = R_1 P_L^-(0) + \Phi \quad (11)$$

Therefore, according to [3], for a single pass laser amplifier, the boundary conditions are:  $R_1 = 0$ ,  $R_2 = 0$  and  $P_L(0) = \Phi$ , resulting in the following solution for the laser output.

$$P_L(L) = L_{out} \quad (12)$$

#### C. Model

According to [2], the proposed solution for a laser working in a high power regime, took into account that the pumping lasers have enough power to saturate the gain even if the signal laser power is strong enough to suppress spontaneous emission. Under these conditions, a high-power amplifier, in an ytterbium-doped optical fiber with tandem pumping, can be described by Eq. (13) above.

Peysokhan [2] still took into account that  $N_2(z) \ll N_t$  to find a solution for the pumping power:

$$\alpha = g_p + \alpha_p = \Gamma_s \sigma_p^a N_t + \alpha_p \quad (14)$$

$$P_p^+ = P_p^+(0) \exp(-\alpha z) \quad (15)$$

$$P_p^- = P_p^-(L) \exp(-\alpha(L-z)). \quad (16)$$

Replacing the pumping power in Eq. (13) leads to the integration shown in Eq. (17), as seen above.

To find the  $N_2(z)$  solution it is necessary to consider that the doping is found only in the core, and that the total gain  $G$  occurs due to the iteration of the signal laser propagating in the fiber core interacting with the  $N_2(z)$  population:

$$\begin{aligned} G = & \int_0^L (dP_s^+)/P_s^+ \\ = & \int_0^L (\Gamma_s [(\sigma_s^e + \sigma_s^a) N_2 - \sigma_s^a N_t] - \alpha_s) dz \\ = & \Gamma_s (\sigma_s^e + \sigma_s^a) \int_0^L N_2 dz - (\Gamma_s \sigma_s^a N_t + \alpha_s) L \end{aligned} \quad (18)$$

$$P_s^+(z) - P_s^+(0) - \frac{\nu_s}{\nu_p} \left(1 - \frac{\alpha_p}{\alpha}\right) \left(P_p^+(0) (1 - e^{-\alpha z}) + P_p^-(L) (e^{-\alpha(L-z)} - e^{-\alpha L})\right) + h\nu_s A \frac{\overline{N_2}(z)}{\tau_2} z + \delta = 0 \quad (21)$$

$$f(z) = \frac{\nu_s}{\nu_p} \left(1 - \frac{\alpha_p}{\alpha}\right) \left(P_p^+(0) (1 - e^{-\alpha z}) + P_p^-(L) (e^{-\alpha(L-z)} - e^{-\alpha L})\right) - h\nu_s A \frac{\overline{N_2}(z)}{\tau_2} z \quad (23)$$

$$\delta(z) \approx \alpha_s (P_s^+(0) + f(z)) z \quad (24)$$

$$\overline{N_2}(z) = \frac{1}{L} \int_0^L N_2 dz = \frac{G}{L} + \frac{\Gamma_s \sigma_s^a N_t + \alpha_s}{\Gamma_s (\sigma_s^e + \sigma_s^a)} \quad (19)$$

$$G = \frac{P_{out}}{\Phi}. \quad (20)$$

Also, applying the first order perturbation theory, with  $\delta$  being a small power variation, it is possible to get (21).

Therefore, a solution was obtained for the output power amplifier  $P_{out}$ , and thus the following model is proposed for a high-power ytterbium-doped fiber laser amplifier with tandem pumping:

$$P_{out} = P_s^+(z=L) = P_s^+(0) + f(z=L) - \delta \quad (22)$$

With  $f(z)$  and  $\delta(z)$  given by Eqs. (23) and (24), as seen above.

### III. RESULTS AND DISCUSSION

The parameters in Table I were used to run the computational code developed in MATLAB to solve the rate equations described previously. These data were obtained from Yan [4], although the filling factors  $\Gamma_s$  and  $\Gamma_p$  have not been provided. In addition to the factor  $N_t$ , the  $Yb^{3+}$  dopant concentration in the fiber, has been obtained from the work of Peysokhan [2] and Li [7]. All these parameters had to be adjusted in order to obtain compatible results with the experiment.

The filling factor used for the signal laser was 0.82, in both studies, and the pumping fill factor was determined by  $A_{core}/A_{clad}$  like in [8], and thus it was possible to obtain the axial distribution of the  $N_2$  population, Fig. 1 compares this excited  $Yb^{3+}$  axial distribution obtained from our model (blue) with the results from Yan's work [4] (red).

In order to obtain the output power of the amplified signal  $P_{s,MOD}^+(L)$ , shown in Fig. 2 in blue, it was necessary to obtain the dopant concentration along the fiber. For this, it started with the value of  $6.0 \times 10^{25} m^{-3}$  according to [2] and [7] models. So with  $N_t$  was possible to estimate an output power of  $5474W$ . Considering the decrease in the  $N_t$  concentration, it was possible to adjust the output power in the simulation. So, when a concentration of  $5.885 \times 10^{25} m^{-3}$  was employed, it was possible to obtain a good agreement with  $5448W$ , the same obtained by Yan  $P_{s,EXP}^+(L)$  (RED), when comparing Fig. 2 with reference [4].

When the evolution of the signal power along the axial length was compared to the experimental data, it was possible to evaluate the average error that stands around  $-0.23\%$ . Fig. 2 shows that the error at the beginning of the fiber has a very

TABLE I  
FIBER AND THERMAL PARAMETERS USED FOR SIMULATING [4] AND [2]

Symbol	Parameter	Value
YDF	$Yb^{3+}$ doped fiber	$30/250\mu m$
$A_{eff}$	Effective core area	$1.6997 \times 10^{-9} m^2$
$L$	Fiber length	$38m$
$NA$	Numerical Aperture	$0.06/0.46$
$P_p^+(z)$	Pumping power (PP)	$5986W$
$P_s^+(z)$	Seed power (SP)	$75W$
$\lambda_s$	(SP) wavelength	$1.080\mu m$
$\lambda_p$	(PP) wavelength	$1.018\mu m$
$\Gamma_s$	Filling factor (SP)	$0.82$
$\Gamma_p$	Filling factor (PP)	$0.0144$
$N_t$	$Yb^{3+}$ concentration	$5.885 \times 10^{25} m^{-3}$
$\tau_2$	Radiative lifetime	$1ms$
$\sigma_p^a$	(PP) absorption cross section	$74.6 \times 10^{-27} m^{-2}$
$\sigma_p^e$	(PP) emission cross section	$580.0 \times 10^{-27} m^{-2}$
$\sigma_s^a$	(SP) absorption cross section	$2.30 \times 10^{-27} m^{-2}$
$\sigma_s^e$	(SP) emission cross section	$282.0 \times 10^{-27} m^{-2}$
$\alpha_p$	Scattering loss coefficient (PP)	$2.0 \times 10^{-3} m^{-1}$
	Scattering loss coefficient (SP)	$2.0 \times 10^{-3} m^{-1}$
$\alpha_s$	Absorption coefficient	$0.45 dB m^{-1}$
$\alpha_\alpha @ 1.018\mu m$	Absorption coefficient	$0.45 dB m^{-1}$
$\kappa$	Heat conductive coefficient	$1.38 W \cdot m^{-1} \cdot K^{-1}$
$h_c$	Convective coefficient	$4000 W \cdot m^{-2} \cdot K^{-1}$

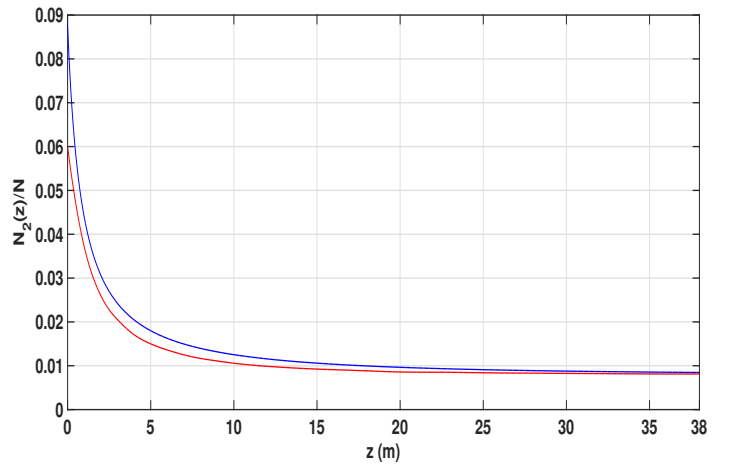


Fig. 1.  $N_{2,MOD}(z)/N_t$  (BLUE),  $N_{2,EXP}(z)/N_t$  (RED).

high rate, but decreasing in a very fast way. This difference can be explained by the fact that the initial model does not take into account the Raman gain as well as ASE.

Using the heat rate equations described by Li [7], it is possible to obtain the difference between the core and the

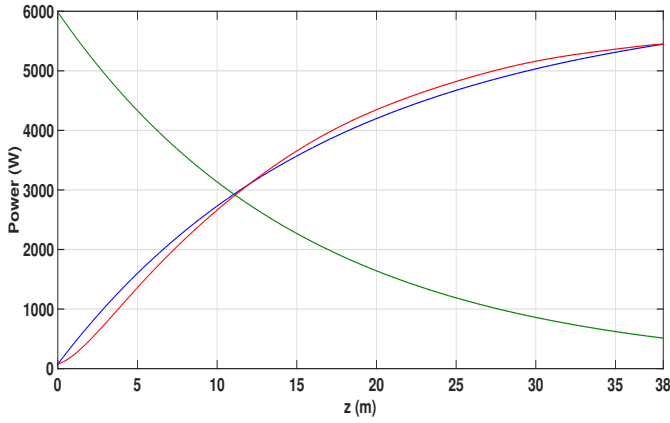


Fig. 2.  $P_{s,MOD}^+(z)$  (BLUE),  $P_{s,EXP}^+(z)$  (RED) and  $P_p^+(z)$  (GREEN)

clad external temperatures,  $\Delta T$ . To do that it is necessary to determine the heat distribution in the fiber core  $Q(z)$ :

$$Q(z) = \frac{\alpha(z) P_p(z)}{A_{eff}} (1 - \eta_q) \quad (25)$$

$$\Delta T = \frac{Q(z) a^2}{2\kappa} \left( \frac{1}{2} + \ln \left( \frac{b}{a} \right) \right). \quad (26)$$

The  $\alpha(z)$  is the sum of the scattering loss of the seed power  $\alpha_s$  and the absorption coefficient  $\alpha_a$ , where  $\eta_q$  is the quantum efficiency, with a theoretical value is given by  $\lambda_p/\lambda_s$ .

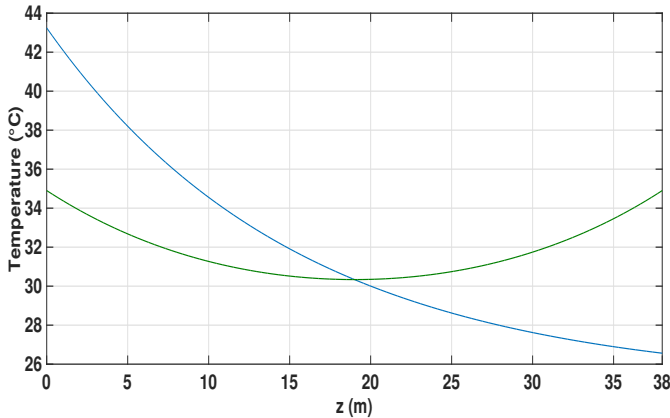


Fig. 3. Core's temperature distribution, Forward-pumped (BLUE) and Dual-pumped (GREEN),

Considering the high convective coefficient value  $h_c$  from Table I, initially the temperature value in the core is calculated taking into account that the surface temperature is close to  $25^\circ C$ , considering that the fiber must have a powerful cooling system, probably using liquid coolant. Thus it was possible to verify that the maximum temperature found in the simulation was  $43.24^\circ C$  as shown in Fig. 3, agreeing with the temperature distribution in [4]. Comparing the forward pumping system with the dual pumping scheme, it was found that in the later the maximum temperature is lower, around  $34.9^\circ C$ , and it is distributed along the whole fiber. Considering

only the temperature contribution, this laser design indicates the possibility of increasing the output power, since the fiber would support a high power pumping system, although it is important to take care with the emergence of non-linear effects.

#### IV. CONCLUSION

In this study, an analytical formulation was developed for a multi-kilowatt ytterbium-doped double-cladding tandem-pumped fiber amplifier, solving the signal and pumping power propagation and the variation of the upper-level population along the full axial length of the active fiber. This model describes the pumping power evolution in the forward and backward directions of the fiber for future simulations in a dual-pumped configuration. According to [2], these calculations are important because the rate equations require the use of numerical techniques involving iterative solutions of those coupled differential equations, making it necessary to use powerful computers or to consume lots of computational time. Another benefit of this approach is that it facilitates the optimization in the designing of fiber amplifiers because it makes easy to change parameters like length, dopant concentration, output power, efficiency, heat generated and temperature distribution. It was observed that the model results are in a good agreement with the work of Yan [4], with a maximum error under  $-5.4\%$ . Future works can be developed in the improvement of the model by taking into account non-linearities, such as stimulated Raman and Brillouin scattering (SRS and SBS) or ASE. The possibility of increasing the output power while keeping a low temperature was demonstrated in the dual-pumped simulations.

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