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EVALUATION OF THE COLLAPSE PRESSURES OF A RING-STIFFENED CYLINDRICAL SHELL UNDER EXTERNAL HYDROSTATIC PRESSURE USING CODE FORMULATIONS AND FEA

Miguel Mattar Neto and Carlos A. J. Miranda
Comissão Nacional de Energia Nuclear
São Paulo, Brazil

Helvécio C. K. da Silveira
Centro Tecnológico da Marinha
São Paulo, Brazil

ABSTRACT

In this paper the collapse of a ring-stiffened cylindrical shell under external hydrostatic pressure is evaluated using code formulations (ASME (1992), BSI (1990), and GL (1988)) and nonlinear elastic-plastic finite element analysis. Some conclusions and comments are addressed from the comparison between the results obtained from the used approaches.

INTRODUCTION

Cylinders under external pressure or equivalent conditions, such as vacuum, may fail by buckling if precautions to prevent it are not taken. One way of strengthening the cylinder is to use ring-stiffeners, either internally or externally. Even so, buckling can still take place in different modes.

The following principal failure modes can occur: local inter frame collapse of the shell between ring frames, and overall collapse of the shell-and-rings combination between rigid sections such as flat heads.

The local inter frame collapse of the shell between ring frames is an interaction between elastic-plastic buckling (called lobar buckling) and axisymmetric membrane shell yielding at midbay.

Lobar buckling is an elastic-plastic instability of the shell between ring frames and is characterized by inward and outward lobes, which may or may not develop around the entire periphery of the cylindrical shell. The failure may occur in one or more ring spaces. This mode of failure indicates that the rings have greater resistance to buckling than the shell between them.

Axisymmetric shell yielding is initiated at the extreme fibers at both the outer surface of the shell midway between ring frames and the inner surface of the shell at the stiffeners. The yield leads to elastic-plastic collapse that is characterized by an accordion type of pleat extending around the periphery of the cylinder. This failure may occur in one or more spaces between the rings.

Overall collapse of the shell-and-rings combination occurs between rigid sections such as flat heads or heavy stiffeners. It is associated with rings which are weak to resist either out-of-circularity bending leading to premature yielding or sideways tripping of the rings, when precipitates out-of-circularity bending. This failure mode is also characterized by inward and outward lobes, but the lobes are fewer (usually just two or three) than the number of lobes in lobar buckling.

Other than the physical and geometric properties of the ring-stiffened cylindrical shells, there are several factors that influence the failure modes and the respective collapse pressures. These factors must be considered in the design and are called "imperfections" related to the geometry of the structure, boundary conditions, materials and loads (including residual stresses). The presence of imperfections can cause the shell to reach yield and collapse at pressure lower than that one for a geometrically perfect ring-stiffened cylinder.

In this paper we investigate the collapse pressures of a ring-stiffened cylindrical shell under external hydrostatic pressure using Code formulations from ASME (1992), BSI (1990) and GL (1988) and nonlinear elastic-plastic finite element analysis (FEA). Some comments and conclusions are addressed from the comparison between the obtained results.

THE RING-STIFFENED CYLINDRICAL SHELL UNDER EVALUATION

The cylindrical shell with internal ring-stiffeners has the following characteristics:

shell mean radius	2,500 mm

shell thickness	24 mm
ring thickness	24 mm
ring height	223 mm
distance between ring-stiffeners	370.6 mm
distance between rigid sections	9,265 mm

The material of the shell and of the ring stiffeners is similar to SA 537 Class 1. Its main mechanical properties are: 205,000 MPa as the Young's modulus, 0.3 as the Poisson's ratio; and the stress-strain data of Table 1. From these data, the yielding stress is 337.30 MPa.

Table 1: Stress-strain data

Stress (MPa)	254.20	321.77	344.92	356.30	399.86
Strain (ustrain)	1.24	2.70	4.70	8.50	46.50

COLLAPSE PRESSURES FROM CODE FORMULATIONS

Ring-stiffened cylindrical shells subjected to external hydrostatic pressure are of considerable interest to the various navies of the world and some civilian design codes GL (1988), and BSI (1990) are, in fact, based to a large extent on the various military investigations.

According to GL (1988), that is a code for naval applications, the collapse pressures corresponding to local interframe failure of the shell between ring-stiffeners can be predicted using the stresses in the shell and in the stiffeners obtained from Pulos and Salerno (1960) and formulae shown in Reynolds (1960) and Lunchick (1961) for lobar buckling and axisymmetric shell yielding, respectively. Corrections for symmetric buckling can be done following the prescriptions of Pulos and Krenske (1965).

The local modes of failure are less sensitive to imperfections than the overall mode and the simultaneous occurrence of all failure modes has been argued by theoreticians (Franitza, 1989 and Kendrick, 1972) as being the only criterion to consider for optimum design. Thus, it is usual to design so that the collapse is precipitated by local inter ring failure of the shell. The stiffening is usually designed so that local interframe collapse leads to overall collapse if the pressure is maintained. In the overall mode, the collapse pressures are greatly reduced by out-of-circularities which create an eccentric load path for the compressive hoop force producing bending moments in the rings.

The collapse pressure corresponding to the overall mode of an imperfect ring-stiffened cylindrical shell between hard sections (such as flat heads) can be predicted using the formulae shown in Franitza (1988) and Mattar Neto et al. (1995). In the formulations the ring-stiffeners are assumed equally spaced and the imperfections expressed as sinusoidal shapes having a maximum amplitude of the out-of-circularity in the central ring and half wave and n waves in the axial and circumferential directions of the shell, respectively.

The collapse pressures obtained from the analytical formulae, corresponding to the three modes of failure, are shown in Table 2. The amplitude of the sinusoidal shape imperfection, as described above, used in the calculation of the collapse pressures of the overall failure modes is an out-of-circularity in the central ring of the structure of 0.3 % of the inner radius of the shell.

Table 2: Collapse pressures from GL (1988) formulations

Mode of Failure	Collapse pressures (MPa)
Lobar buckling	4.27
Axisymmetric shell yielding	4.28
Overall mode (n=2 waves)	5.05
Overall mode (n=3 waves)	4.35
Overall mode (n=4 waves)	4.40

It can be seen that the collapse pressures of all three modes have about the same value if we take the minimum value for the overall mode (n=3 waves in the circumferential direction of the shell).

From formulations of ASME (1992) and BSI (1990), using the tables and figures corresponding to SA 537 Class 1 in ASME Code and to a similar material in BSI, we have the maximum allowable pressures shown in Table 3. It is important to notice that both codes have design factors of three.

Table 3: Maximum allowable pressures from BSI (1990) and ASME (1992)

Code	Maximum Allowable Pressure (MPa)
ASME (1992)	1.05
BSI (1990)	1.00

COLLAPSE PRESSURES FROM NONLINEAR ELASTIC-PLASTIC FEA

A detailed finite element model of the structure was built. Taking into account the longitudinal symmetry of the structure and of the loads only one half of the structure is modeled as shown in Figure 1.

The shell and the ring-stiffeners were modeled using quadrilateral shell finite elements with 4 nodes and 6 degrees of freedom per node. In the shell discretization, there are two elements between the ring-stiffeners. Displacement boundary conditions adequate with the longitudinal symmetry were imposed in the plane of symmetry. To simulate the rigid sections, the radial displacements were restrained in the radial direction of the shell. The external pressure and the corresponding end-forces were applied in the shell.

Two reasons may justify the use of the longitudinal symmetry to reduce the model: the shape imperfections introduced in the analysis have longitudinal symmetry also, and in this type of structure under external pressure the hoop stress is the most significant stress. Thus, the model can capture the most important failure modes although it precludes other ones by the symmetry assumption.

The FEA was a nonlinear static analysis. The stress-strain data of Table 1 were input in a multilinear plasticity model. The elastic-plastic collapse pressure in each analysis was obtained from the nonconvergence of the finite element solution and from the asymptotic behavior of a selected displacement in a load-displacement plot.

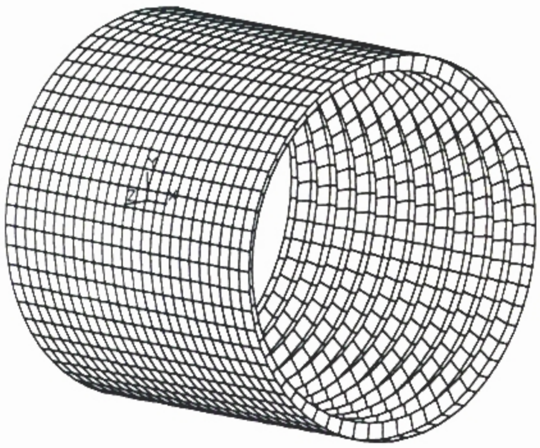


Figure 1: Finite element model of half of the ring-stiffened cylindrical shell

First, to evaluate the local interframe elastic-plastic collapse of the shell between rings, no shape imperfection was taken into account in the finite element model. Then, the same shape imperfections with the same amplitude used in the evaluation with the GL (1988) formulations were introduced in the finite element model with n= 2, 3 and 4 waves. As it has already said , the imperfections were expressed as sinusoidal shapes having a maximum amplitude of the out-of-circularity in the central ring and half wave and n waves in the axial and circumferential directions of the shell, respectively. The amplitude of the sinusoidal shape imperfection is an out-of-circularity in the central ring of the structure of 0.3 % of the inner radius of the shell.

Due to symmetry, from the finite element model, without shape imperfections, it was possible to evaluate only the local axisymmetric shell yielding. The corresponding elastic-plastic FEA collapse pressures obtained are shown in Table 3.

In Table 3, the collapse pressures of the shell with shape imperfections correspond to failure of the it in the central region. This can be seen in Figure 2 for n = 3 waves and the same occurs for n = 2 and 4 waves. If it is possible to maintain the applied hydrostatic pressure, the shell failure precipitates the overall collapse with the yielding of the rings. In Figure 2, it can be seen that the stresses in the central frames are near the yield stress of the material.

Table 3: Collapse pressures from FEA

Mode of Failure	Collapse pressures (MPa)
Axisymmetric shell yielding	4.29
n=2 waves	5.11
n=3 waves	4.75
n=4 waves	5.08

It was necessary to reinforce the shell near the flat head (rigid sections on extremes) in order to force that the failure occurs in the center of the shell. Figure 3 shows the displacement field, just before the collapse, for the structure with n = 2 waves, without and with a reinforcement of 25 % of the thickness of the shell that extends over a distance of two and a half ring spaces from the discontinuity. The reinforcement of the shell near discontinuities is a recommended design practice Franitza (1989) to avoid weak points in the structure.

CONCLUSIONS

For the shell under study, the elastic-plastic FEA collapse pressures are in good agreement with the collapse pressures obtained from GL (1988) formulation.

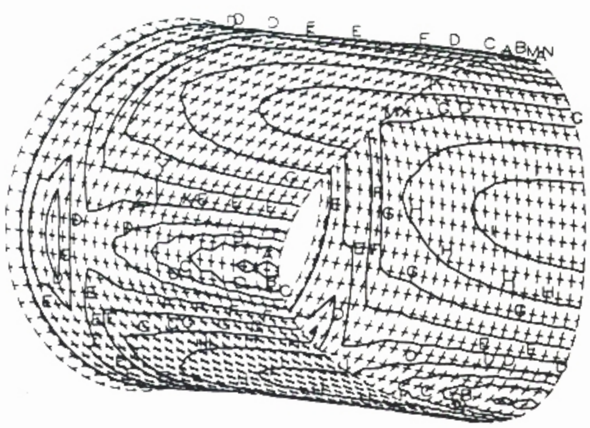
The maximum allowable pressures obtained from ASME (1992) and from BSI (1990) have almost the same values. They are 4 to 4.5 times smaller than the collapse pressures calculated with FEA and GL (1988). Most of the difference can be attributed to the safety factors of ASME (1992) and BSI (1990) and the amplitude of the out-of-circularity. In ASME (1992) and BSI (1990) the amplitudes are assumed as 0.45% and 0.5% of the inner radius of the shell, respectively. Remember that the out-of-circularity used in FEA and GL (1988) formulations is 0.3% of the inner radius of the shell.

Assuming an inversely proportional relationship between the out-of-circularity amplitude and the collapse pressures, we can estimate the corresponding values for 0.3% of the inner radius of the shell from ASME (1992) and BSI (1990) are 1.58 MPa and 1.67 MPa, respectively. It can be noticed that a reduction of the amplitude of the out-of-circularity is not acceptable in the ASME (1992) and BSI (1990) codes although both indicate indirectly the linear relation mentioned.

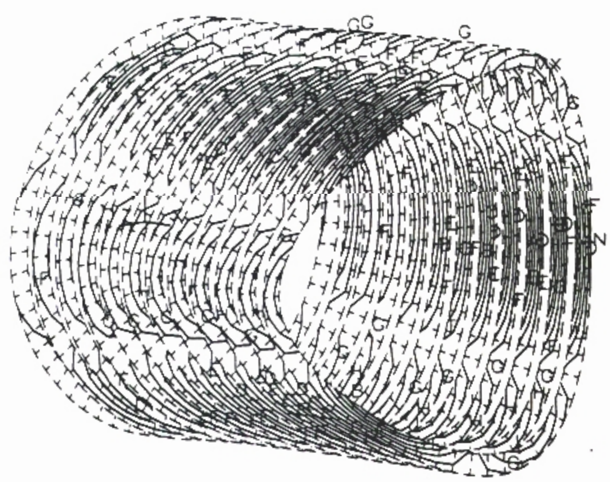
In this situation the collapse pressures obtained from FEA and GL (1988) formulations are 2.8 to 3 times greater than the maximum allowable pressures from ASME (1992) and BSI (1990) and these factors are very close to the codes safety factors (3.0).

It is important to mention that the large discontinuities near rigid sections are weak points in these types of structures. Reinforcement must be provided in these regions as indicated by Franitza (1989) and by the FEA results.

FEA and the more detailed formulations of GL (1988) may be unnecessary in many cases. However, in some situations of thin shells they may be useful by giving more information and physical insight of the structural behavior in the design phase and better recommendations in how to control the out-of-circularity amplitudes in the construction phase, with the consequent economic savings.

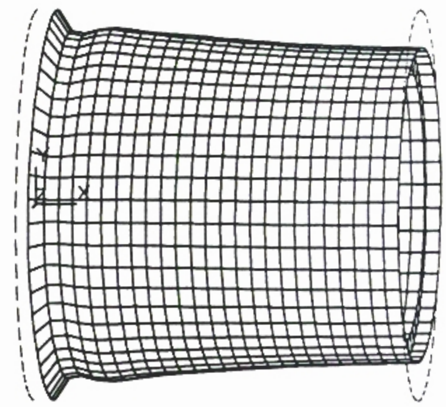


A=227 MPa to I = 357 MPa
(a) Shell

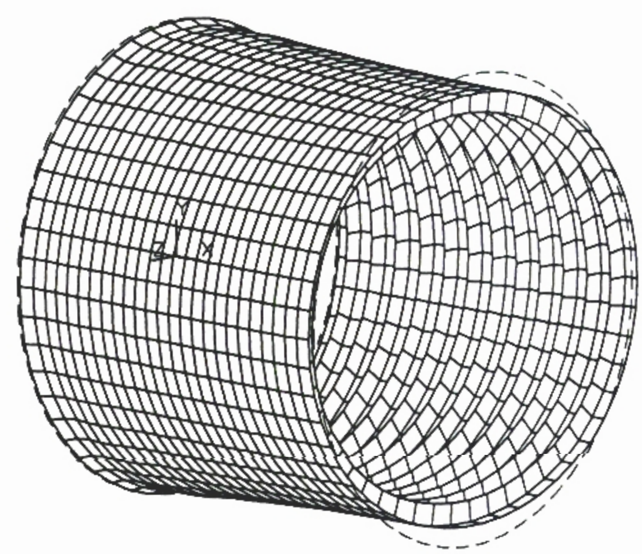


A = 114 MPa to I = 341 MPa
(b) Ring frames

Figure 2: Von Mises Stress Intensities (MPa)



(a) without reinforcement



(b) with reinforcement

Figure 3: Displacements of the shell (n = 2 waves)

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