Use of neural network for the simulation of a gas centrifuge

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Abstract

The prediction of the separation of uranium isotopes using a gas centrifuge process by mathematical model is a hard task. The gas motion can be described by analytical or numerical solution of the system of equations, defined by the equation of continuity, the Navier-Stokes equation and the equation of energy. However, these calculations can not be performed for actual centrifuges.

Neural network is an alternative to model complex problems that show many difficulties to be solved by phenomenological model. A neural network is constituted by processing neurons arranged in layers, that process the information, and by flow channels that transfer the information between the neurons, named interconnects. A neural network, due to its parallel characteristics, has the ability to "learn" the non-linearity of a process presented to the network.

The authors propose the use of neural networks for the simulation and prevision of the separative and operational parameters of a gas centrifuge separating uranium isotopes. The last set of experimental data in the open literature, seems to be presented by Zippe (1960). The results from the uranium separation experiments are compiled and presented to the neural network in the learning and testing processes. The best structure is then chosen to minimize the total error. The parameters calculated by the neural network are then compared with the experimental data and with some theoretical results. The prediction using the neural network model shows good agreement with the experimental data.

1. Introduction

The prediction of the separation of the uranium isotopes by a gas centrifuge process employing mathematical models is quite difficult. The calculations require the simultaneous solution of the equations of gas motion (equation of continuity, the Navier-Stokes equation and equation of energy) and the diffusion equation. The diffusion equation may be solved independently, once the equations of gas motion have already been solved.

The separation analysis of the countercurrent centrifuge was first defined by Cohen [1] in the 40's through the solution of the diffusion equation using the method developed by Furry, Jones & Onsager [2] for the thermal diffusion column. This became a classical solution, named Cohen-Onsager equation. This model assumes a constant axial countercurrent flow and a radial averaged concentration. These simplifying hypotheses, which introduce errors when comparing the results with actual centrifuges, have gradually been improved. A one-dimensional axially varying countercurrent has been studied by Olander [3]. In the 70's and 80's many authors studied a bi-dimensional analysis of the internal flow and the separation performance [4, 5, 6, 7, 8]. Kai [8] reviewed the studies performed by Power Reactor and Nuclear Fuel Development Corp. (PNC) emphasizing the difficulties of predicting the separative performance of a gas centrifuge.

The solutions of these model-based equations, analytically or numerically, always require the use of approximations or linearization of the equations. The representation of the inside components, for example the scoops and rotating baffles, as a boundary condition is difficult. Consequently, none of the existing methods of calculation are valid for an actual centrifuge, although they are valuable for understanding the physical phenomena that take place in the gas centrifuge.

We propose here the use of neural networks for the simulation and prediction of the separative performance of a gas centrifuge. A neural network, due to its parallel characteristics, is able to "learn" the non-linearity of a process presented to the network in the training set.

2. Neural Network

Neural networks are one of the fastest growing areas of artificial intelligence in Chemical and Nuclear Engineering. The main applications are in fault diagnosis (Hoskins et.al.[9]) and in process control and modeling (Bhat & McAvoy [10], Su & McAvoy [11], Chan & Nascimento [12]). In nuclear technology, the use of neural networks began at the end of the 80's. They have been widely used in High Energy Physics (Denby [13]) and in Nuclear Power Plants (Uhrig [14], Eryurek et.al.[15]).

The application of neural networks in the simulation of chemical and nuclear processes, specifically in the isotope separation by the gas centrifuge, has great interest due to the non-linearity of the process. Migliavacca et.al. [16] employed neural network to optimize a centrifuge separation. The prediction of the maximum separation was tested in a new experiment and a difference of less then 1% was verified. This technique leads to numerical models, valid for actual centrifuges, avoiding the difficulties of the phenomenological model described above. The success of this kind of modeling depends strongly on the knowledge of the main variables affecting the process and the availability of a good data base with the necessary information over the desired domain.

There are two main structures for neural networks: (i) the multilayer feedforward network, shown in Figure 1, used for modeling steady state systems, and (ii) the recurrent network, which is better for dynamic modeling.

A neural network is constituted of processing neurons, represented by circles in Figure 1, and by flow channels that transfer the information between the neurons, named interconnects. The boxes represent neurons where the inputs for the network are stored. Each neuron calculates first the weighted sum of the input signals of the previous layer, and then generates an output through an activation function, usually defined by Eq.2.

$$S_v = \sum_{u=1}^{n} W_{u,v} x_u + W_{n+1,v} \qquad (1)$$

$$f(S_v) = \frac{1}{1 + e^{-Sv}} \qquad (2)$$

In process modeling, the most used neural network is the three layered network, consisting of an input layer, a hidden layer and an output layer. The input layer consists of ni+1 neurons, where ni is the number of input variables, and there is no processing in this layer. Besides the inputs, a bias is given to the network. The number of neurons in the hidden layer is defined by the user. According to Pollard [17] the final precision is only slightly sensitive to the number of neurons in the hidden layer after a minimum value. The output layer consists of a number of neurons equivalent to the number of outputs of the process.

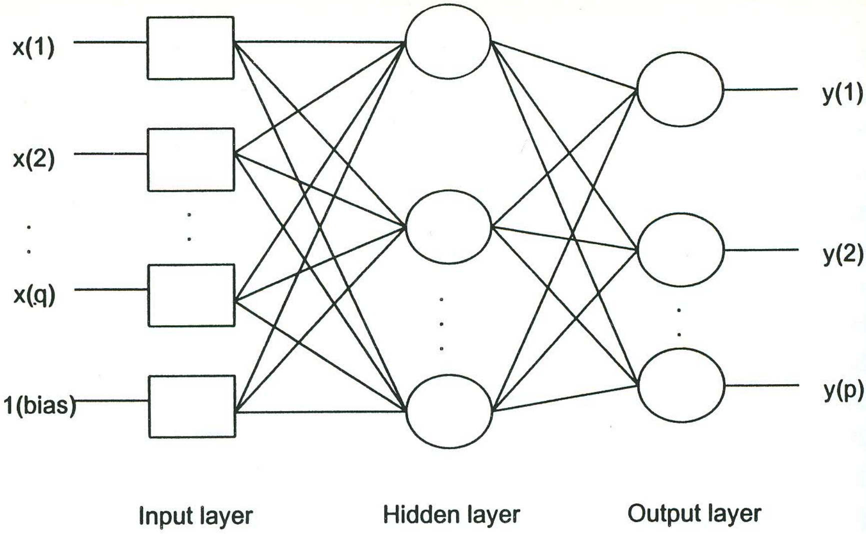


Figure 1: Three layer feedforward neural network

The system "learns" by changing the weights (Wu,v) in such a way as to minimize the sum of the squared differences (E) values expressed by

$$E = \sum_{m=1}^{r} \sum_{k=1}^{p} (y_k^{(m)} - O_k^{(m)})^2 \quad (3)$$

where y_k is the value presented to the neural network in the training set and O_k is the corresponding value obtained in the output layer, calculated by

$$O_k = f(S_k) \quad (4)$$

The backpropagation algorithm is the most used procedure for training three-layered feedforward neural networks (Rumelhart & McClelland [18]). This algorithm is a generalization of the steepest decent method.

3. Zippe's experiments

During the 2nd World War the process of gas centrifuge uranium enrichment achieved great interest among the most important countries. By that time, an intense theoretical and experimental research was developed. Jordan [19] reviews the development at that time.

The basis of the theory of the centrifugation process was established by Cohen et.al. in the United States of America and by Martin and Kuhn in Germany. By the same time, gas centrifuges were constructed in the USA supervised by Beams and in Germany supervised by Groth. In the former URSS, from 1946 to 1954, the uranium enrichment by the gas centrifuge has also been studied under the supervision of Steenbeck, Scheffel and Zippe. Later, in 1954 Zippe developed a short bowl ultra-centrifuge, still in the Soviet Union, based in a new and revolutionary construction concept. That centrifuge presented an excellent performance, about its mechanics and its separation, and became the precursor of the modern centrifuges. From 1958 to 1960 Zippe [20] reproduced in the University of Virginia (USA) his experiments, under the supervision of Beams.

That work could still be divulged, showing all the construction and operational details and the experimental results of the gas centrifuge in the uranium enrichment experiments. In the middle of 1960, the Atomic Energy American Commission prohibited the divulgence of any technical or operational information about the american centrifuges. A few months later, Germany and Holland took the same decision. So, after 1960 no technological information about the centrifuge process has been published in the open literature.

3.1. The uranium enrichment experiments (Zippe [20])

In this work, the authors use Zippe's experimental results to model the separative performance of the short bowl ultra-centrifuge using neural networks. Figure 2 presents a sketch of Zippe's centrifuge.

Zippe has performed three kinds of experiments in order to determine the separative and mechanical performance of the centrifuge in separating uranium isotopes. In the first type of experiment the separation was measured as function of the feed flow. The second type of experiment was performed in a very low feed flow rate, as function of the pressure in the center of the centrifuge rotor. The third type of experiments was very similar to the first one, but using more accurate experimental procedures.

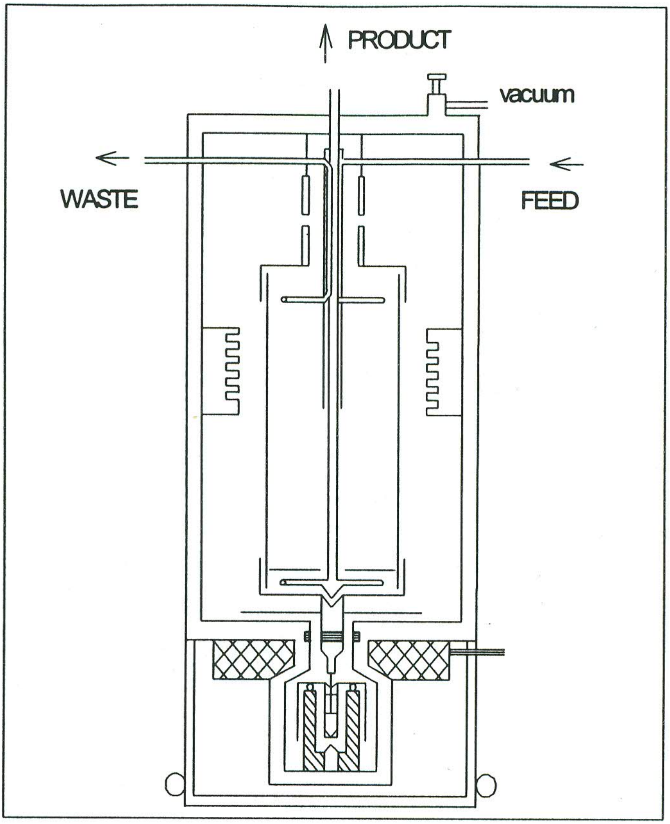


Figure 2: Sketch of Zippe's short bowl ultra-centrifuge

About 40 different arrangements of scoops and rotating baffles were used in tests of the first type of experiment by Zippe. Only 9 arrangements, the most important ones, are reported by Zippe [20]. Tests were also performed with different angle and location of the feed stream in the rotor. Using the experimental data of Zippe [20] we will model different centrifuge configurations. In this case we will search for an optimal centrifuge configuration. The data are reproduced and codified to be presented to the neural network in Table 1. This will be named CASE 1.

Table 1

Experimental data for the study of CASE 1

(extracted from Zippe [20])

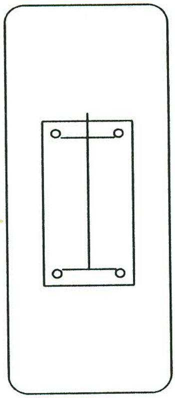
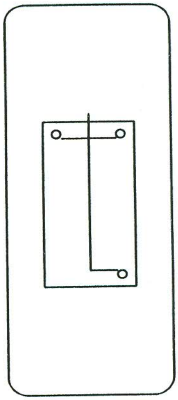
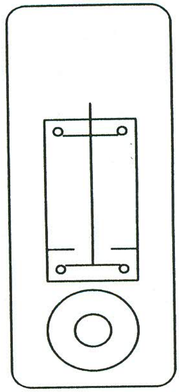
Syst. No.		10	5a	4
				
Top scoop		double - 2	double - 2	double - 2
Bottom scoop		double - 2	single - 1	double - 2
Top baffle	existence	no - 0	no - 0	no - 0
	NPH [(1)]	no - 0	no - 0	no - 0
	fins	no - 0	no - 0	no - 0
Bottom baffle	existence	no - 0	no - 0	yes - 1
	NPH [(1)]	no - 0	no - 0	12
	fins	no - 0	no - 0	no - 0
	spider	no - 0	no - 0	no - 0
EDCH [(2)]		0	0	0.2
RFL [(3)]		0.5	0.5	0.5
Feed rate[(4)] **(mg/s)**		-	4.6, 3.2	2.8
α		-	1.123, 1.132	1.15
δU x10^6 [(5)] **(g/s)**		-	7.73, 6.14	6.83
Power (W)		-	10.8, 7.9	14
Remarks		Practically no separation		Rotor became too hot

Table 1 (cont.)
Experimental data for the study of CASE 1
(extracted from Zippe [20])

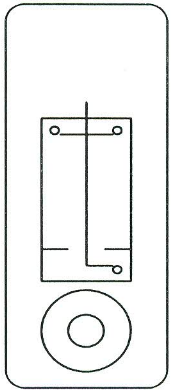
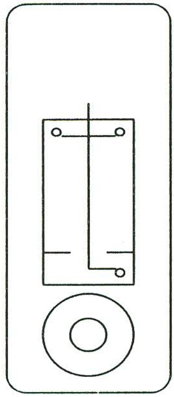
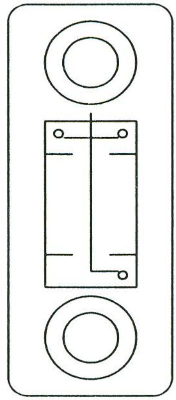
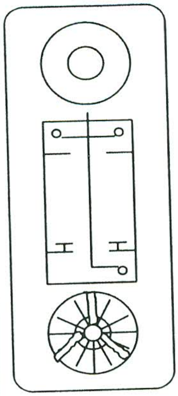
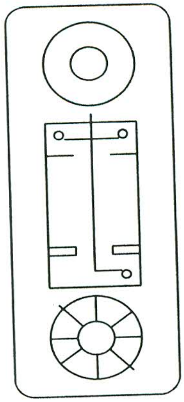
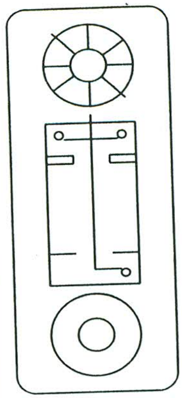
Syst. No.		5	6	13
				
Top scoop		double - 2	double - 2	double - 2
Bottom scoop		single - 1	single - 1	single - 1
Top baffle	existence	no - 0	no - 0	yes - 1
	NPH (1)	no - 0	no - 0	12
	fins	no - 0	no - 0	no - 0
Bottom baffle	existence	yes - 1	yes - 1	yes - 1
	NPH (1)	12	12	no - 0
	fins	no - 0	no - 0	no - 0
	spider	no - 0	no - 0	no - 0
EDCH (2)		0.2	0.2	0.5
RFL (3)		0.5	0.5	0.33
Feed rate(4) (mg/s)		3.81, 2.55	5.48, 0.5, 4.7, 7.4	6.18, 1.637
α		1.193, 1.216	1.155, 1.267, 1.157, 1.125	1.138, 1.228
δU x10^6 (5) (g/s)		14.79, 12.15	14.20, 3.48, 12.47, 12.82	12.89, 8.58
Power (W)		7.9, 6.25	8.9, 6.13, 9.1, 11.1	10.5, 7.7
Remarks				

Table 1 (cont.)
Experimental data for the study of CASE 1
(extracted from Zippe [20])

Syst. No.		29	30	31
				
Top scoop		double - 2	double - 2	double - 2
Bottom scoop		single - 1	single - 1	single - 1
Top baffle	existence	yes - 1	yes - 1	yes - 1
	NPH (1)	24	24	12
	fins	no - 0	no - 0	yes - 1
Bottom baffle	existence	yes - 1	yes - 1	yes - 1
	NPH (1)	0	12	24
	fins	yes - 1	yes - 1	no - 0
	spider	yes - 3	no - 0	no - 0
EDCH (2)		0.2	0.2	0.2
RFL (3)		0.5	0.5	0.5
Feed rate(4) (mg/s)		very low - 0.1	very low - 0.1	5.53, 3.8, 0.477
α		1.355	1.355	1.180, 1.202, 1.274
δU x10^6 (5) (g/s)		1.14	1.14	18.89, 16.03, 3.48
Power (W)		6.9	10.5	10, 6.6, 4.6
Remarks		no time to optimize δU		

where:

(1) NPH = No. of peripheral holes

(2) EDCH = Equivalent diameter of the center hole of the baffle or baffles, defined by

$$\frac{\text{diam. of center hole}}{\text{diam. of rotor}}$$

(3) RFL = Relative feed location, defined by the relation

$$\frac{\text{distance from the top end cap to the feed position}}{\text{height of the rotor}}$$

(4) when feed rate is indicated to be very small, we assumed the value 0.1 mg/s

(5) calculated by the expression

$$\delta U = F \cdot \vartheta \left(1 - \vartheta\right) \frac{\alpha - 1}{\alpha + 1} \cdot \ln(\alpha \qquad (5)$$

where: F is the feed rate,

α is the separation factor,

ϑ is the cut, defined by the relation between the product and the feed rate, assumed 0.5.

The System No. 6, as presented in Table 1, was more exhaustively tested. The results of these tests are reproduced in Table 2. In this case we will search for an optimal process condition of a given centrifuge configuration. These data will be used to model the separative performance of Zippe's centrifuge with arrangement No. 6. This will be called CASE 2.

Table 2

Experimental data for the study of CASE 2 - System No. 6

(extracted from Zippe [20])

Peripheral speed v (m/s)	Feed rate F (mg/s)	Separation factor α	Separative work δU (g/s)	Power (W)
350	2.20	1.048	0.6 E-6	4.30
350	2.50	1.062	1.4 E-6	4.10
350	3.10	1.100	3.7 E-6	4.70
350	3.70	1.126	6.7 E-6	5.80
350	3.90	1.126	7.8 E-6	6.50
350	4.30	1.141	9.7 E-6	7.35
350	4.90	1.139	11.4 E-6	7.60
350	5.30	1.134	12.1 E-6	8.30
350	5.70	1.130	11.5 E-6	8.70
350	6.10	1.130	12.2 E-6	9.20
350	6.50	1.108	12.0 E-6	10.50
350	4.72	1.157	12.7 E-6	9.10
350	5.40	1.155	13.9 E-6	8.90
350	7.39	1.125	12.7 E-6	11.10
245	0.80	1.080	0.5 E-6	7.10
245	1.20	1.088	0.9 E-6	7.80
245	1.50	1.088	1.1 E-6	8.20
245	1.80	1.088	1.1 E-6	9.00
245	2.10	1.078	1.5 E-6	9.40
245	2.30	1.080	1.5 E-6	9.50
245	2.50	1.078	1.7 E-6	10.30
245	2.90	1.074	1.7 E-6	10.80
245	3.30	1.065	1.6 E-6	11.60
303	1.50	1.053	1.5 E-6	5.30
303	3.20	1.094	3.2 E-6	7.20
303	4.10	1.115	5.1 E-6	8.20
303	5.10	1.970	5.5 E-6	9.20
303	5.30	1.094	5.5 E-6	9.70
303	6.70	1.085	5.3 E-6	10.90
303	6.80	1.076	4.5 E-6	11.50
350	1.50	1.043	0.5 E-6	4.80
350	2.80	1.104	3.9 E-6	5.80
350	4.10	1.128	8.5 E-6	7.20
350	5.20	1.144	12.0 E-6	7.80
350	5.30	1.134	11.4 E-6	7.80
350	5.80	1.136	12.5 E-6	8.90
350	6.40	1.132	11.5 E-6	9.40
350	6.50	1.126	12.5 E-6	9.80

4. Case 1

4.1. The experimental data

In CASE 1 experiments with different centrifuge arrangements will be treated. The data used are those presented in Table 1, excluding system No.10. These data are presented to the neural network in the learning process with the following input variables:

- bottom scoop (1=single, 2=double)
- top baffle existence (0=no, 1=yes)
- top baffle number of peripheral holes
- top baffle fins (0=no, 1=yes)
- bottom baffle existence (0=no, 1=yes)
- bottom baffle number of peripheral holes
- bottom baffle fins (0=no, 1=yes)
- bottom baffle spider (0=no, 3=yes, with 3 wings)
- equivalent diameter of the center hole of the baffle or baffles
- relative feed location
- feed rate (mg/s)

and the output variables:

- separation factor α
- power consumption (W)
- separative work δU (g/s)

4.2. Learning and testing whit the neural network

The experimental data were divided into two groups: a 'learning set' with 12 data sets and a 'test set' with 4 data sets. The learning set was used to train the neural network and the test set to check the neural network results. A three-layered feedforward neural network was used and trained with the backpropagation algorithm.

We tested seven different networks, with 4 to 10 neurons in the hidden layer. The number of presentations employed to train the neural network was up to 100 000. Table 3 shows the errors calculated in training of each network for the learning set and for the test set.

Table 3

Total errors calculated after the training of neural networks with different numbers of neurons in the hidden layer for CASE 1

NH	"LEARNING SET"		"TEST SET"	
	No. SETS	RMST	No. SETS	RMSTT
4	100 000	9.31×10^{-4}	36 400	1.18×10^{-1}
5	100 000	4.04×10^{-5}	6 600	9.13×10^{-2}
6	95 600	2.26×10^{-12}	31 300 - 100 000	7.87×10^{-2}
7	100 000	1.97×10^{-7}	92 000 - 100 000	7.14×10^{-2}
8	100 000	6.53×10^{-11}	44 000 - 100 000	5.53×10^{-2}
9	97 200	4.23×10^{-12}	54 600 - 100 000	4.36×10^{-2}
10	100 000	5.57×10^{-12}	38 900 - 100 000	5.33×10^{-2}

A network with 6 neurons in the hidden layer was chosen, although the test set show that the best agreement would be the network with 9 neurons. We decided to pick up the network with 6 neurons in sense to avoid overfitting (Pollard [17]). The networks with 6 and 9 neurons gave similar results. The sum of errors as function of the number of presentations is shown in Figure 3 for the network with 6 neurons.

4.3. Comparison of experimental versus calculated data

After the training of the neural network the weights are chosen by those that minimize the errors in the test set. The comparison of the calculated variables α, δU and power consumption against the experimental values are shown in Figures 4, 5 and 6, respectively. The agreement between the experimental and the calculated values for those variables is satisfactory. Unfortunately we did not have information about the experimental error, otherwise we could do a statistical evaluation of neural network prediction.

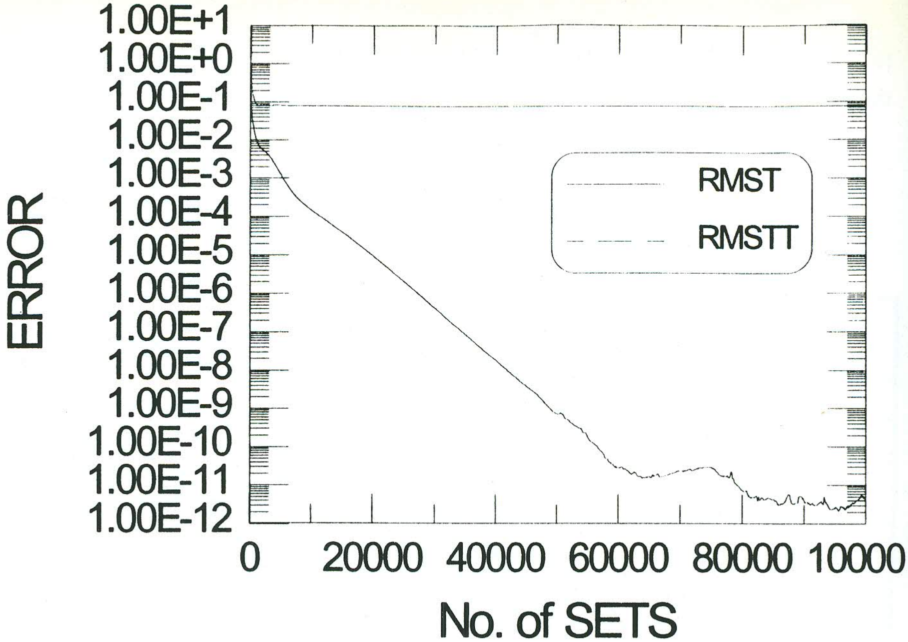


Figure 3: Global error in the training of a neural network with 6 neurons in the hidden layer for the CASE 1: learning set and test set

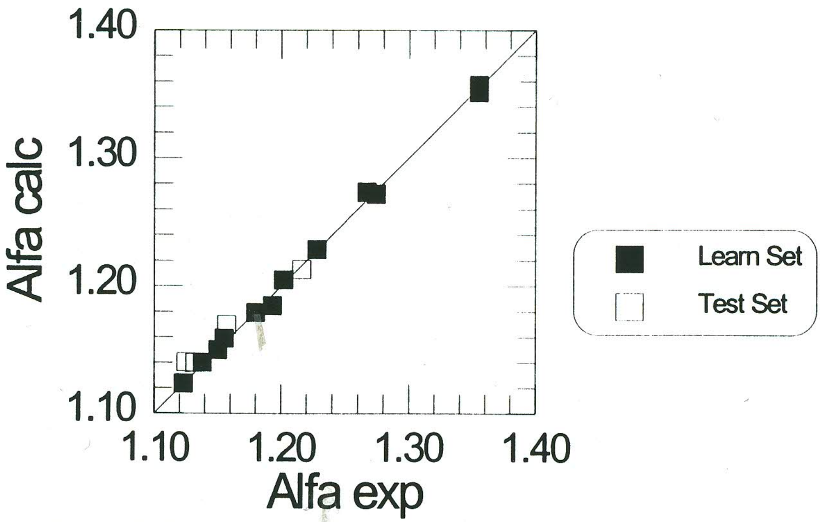


Figure 4: Comparison of the experimental and the neural network calculated values of the separation factor α

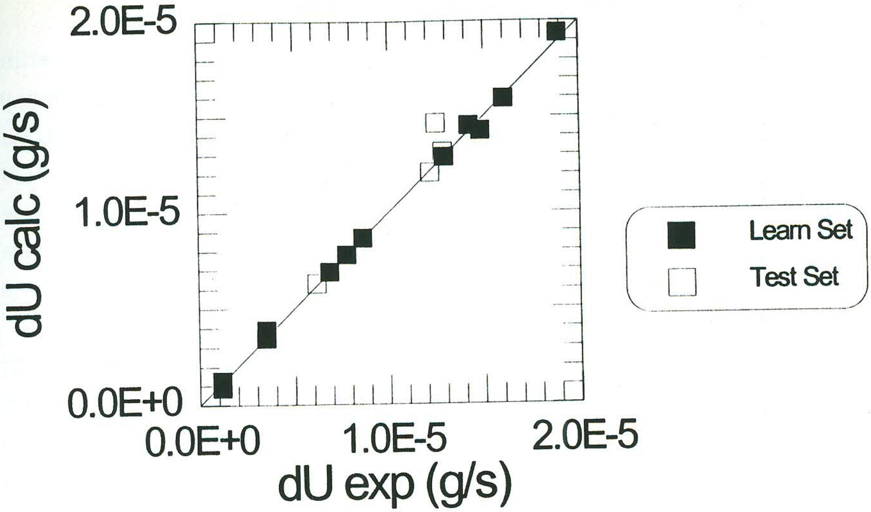


Figure 5: Comparison of the experimental and the neural network calculated values of the separative work δU

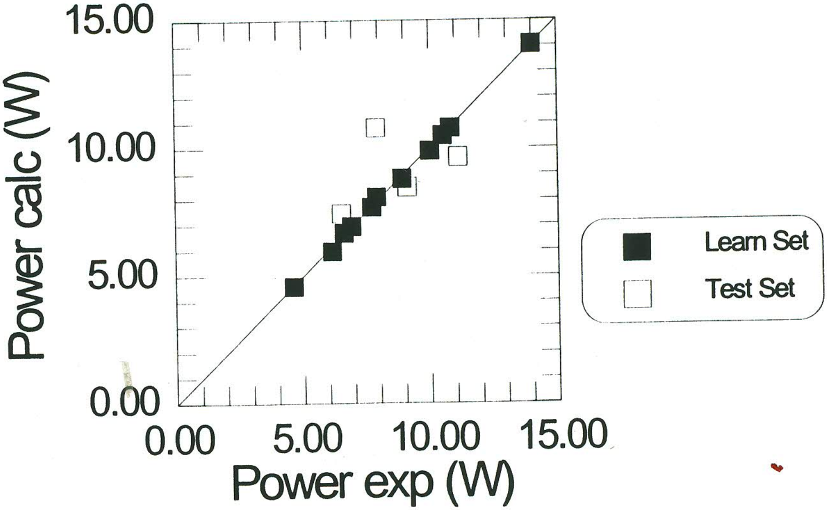
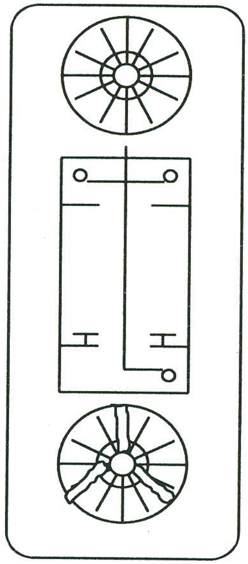


Figure 6: Comparison of the experimental and the neural network calculated values of the power consumption

Table 4

Optimal centrifuge

		
Top scoop		double - 2
Bottom scoop		single - 1
Top baffle	existence	yes - 1
	NPH	24
	fins	yes - 1
Bottom baffle	existence	yes - 1
	NPH	24
	fins	yes - 1
	spider	yes - 3
EDCH		0.2
RFL		0.3
Feed rate (mg/s)		9.5
α		1.199
δU x10^6 (g/s)		2.06
Power (W)		12.6

4.4. Optimization of the centrifuge

Neural network was trained to represent the centrifuge assembled with different scoop and baffles systems. Then we proceeded a scanning of the neural network solution to search the combination that would result in the maximum separative work δU. The performance of different centrifuges, using the scoops and baffles shown in Table 1, with feed rates from zero to 10 mg/s was calculated. The neural network calculated 36288 different solutions. The system pointed out as the best arrangement is shown in Table 4, this arrangement was different from those proposed by Zippe [20].

The separative performance of the optimal centrifuge can also be predicted using the neural network. The expected values for the separation factor and the separative work are plotted in Figures 7 and 8, and the power consumption in Figure 9.

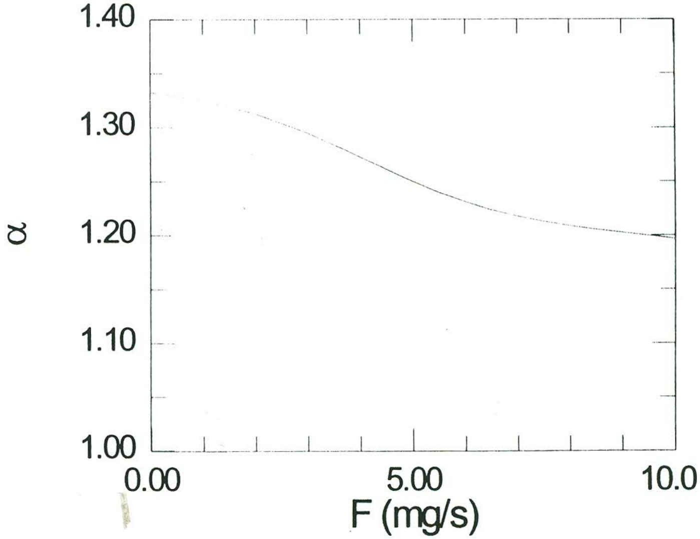


Figure 7: Prediction of the separation factor α as function of the feed rate for the optimal centrifuge

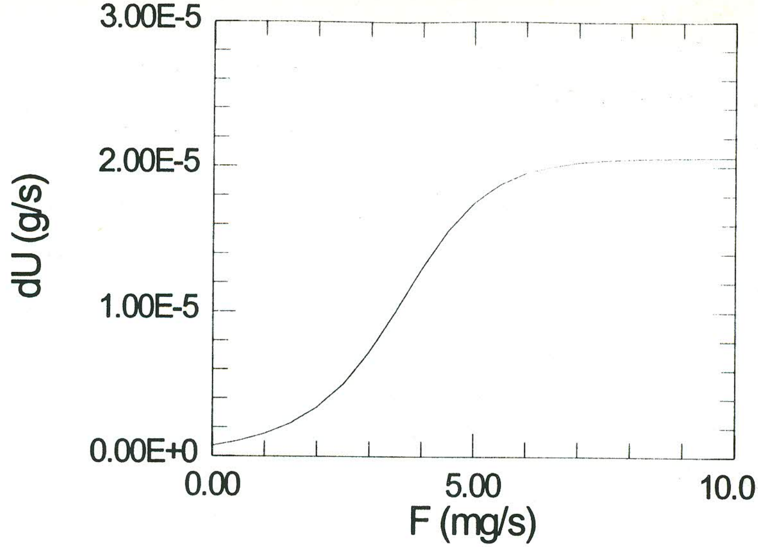


Figure 8: Prediction of the separative work δU as function of the feed rate for the optimal centrifuge

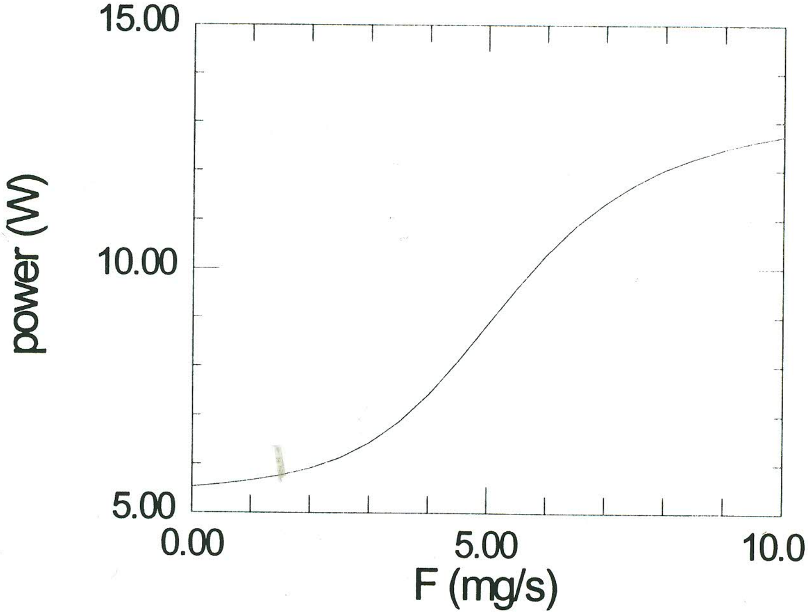


Figure 9: Prediction of the power consumption as function of the feed rate for the optimal centrifuge

5. Case 2

5.1. The experimental data

In CASE 2, the results from experiments realized with system No. 6, described in Table 1, will be treated. The data used are those presented in Table 2. For each velocity a bias was added representing zero feed rate with null separative power. The power consumption was extrapolated from the curves presented by Zippe [20]. These data are presented to the neural network in the learning process with the following input variables:

- feed rate (mg/s)
- peripheral velocity (m/s)

and the output variables:

- separation factor α
- power consumption (W)
- separative work δU (g/s)

5.2. Learning and testing with the neural network

The experimental data were divided into two groups: a 'learning set' with 21 data sets and a 'test set' with 21 data sets.

We tested nine different networks, with 4 to 12 neurons in the hidden layer. The number of presentations employed to train the neural network was up to 100 000. Table 5 shows the errors calculated in training of each network for the learning set and for the test set.

Although the network with 8 neurons in the hidden layer was the one that presented the smaller errors, the network that better described the process, to prevent over-fitting was the neural network with six neurons in the hidden layer. The sum of errors as a function of the number of presentations is shown in Figure 10 for the network with 6 neurons.

Table 5
Total errors calculated after the training of neural networks with different numbers of neurons in the hidden layer for CASE 2

NH	"LEARNING SET"		"TEST SET"	
	No. SETS	RMST	No. SETS	RMSTT
4	9 500	0.0937	12 300	0.2740
5	100 000	0.0570	46 400	0.1981
6	100 000	0.0452	100 000	0.1696
7	100 000	0.0466	69 300	0.1892
8	100 000	0.0304	100 000	0.1445
9	100 000	0.0372	61 000	0.1539
10	100 000	0.0328	67 000	0.1475
11	100 000	0.0302	94 900	0.1474
12	100 000	0.0314	83 600	0.1529

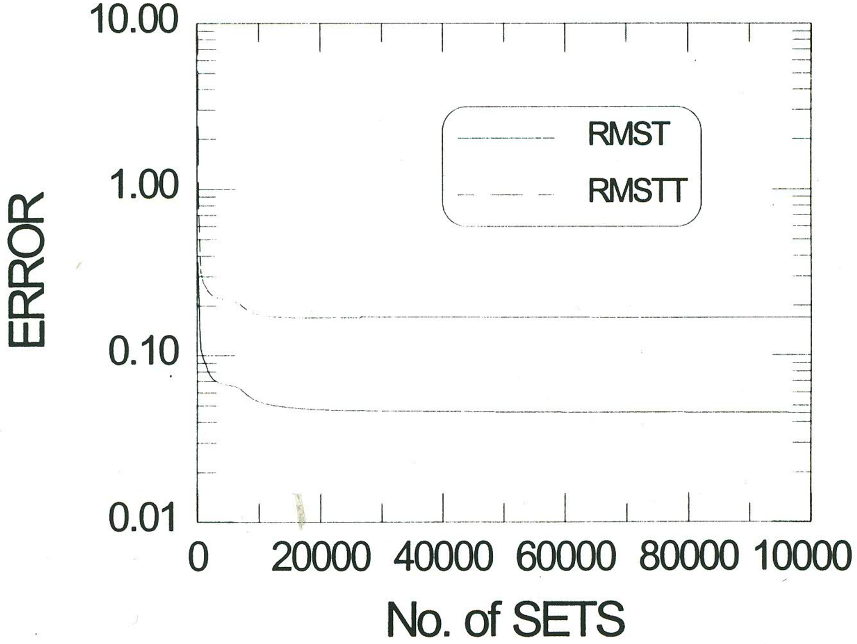


Figure 10: Global error in the training of a neural network with .6 neurons in the hidden layer for the CASE 2: learning set and test set

5.3. Comparison of experimental versus calculated data

The procedure to choose the weights was very similar to that used in CASE 1. The comparison of the calculated variables α, δU and power against the experimental values are shown in Figures 11, 12 and 13, respectively. The agreement between the experimental and the calculated values for those variables is satisfactory. Normally the training set are always fitted (neural networks as used in this work are called as universal approximators (Hornik [21]). The most important, however, is the prediction that the neural network showed with the test set.

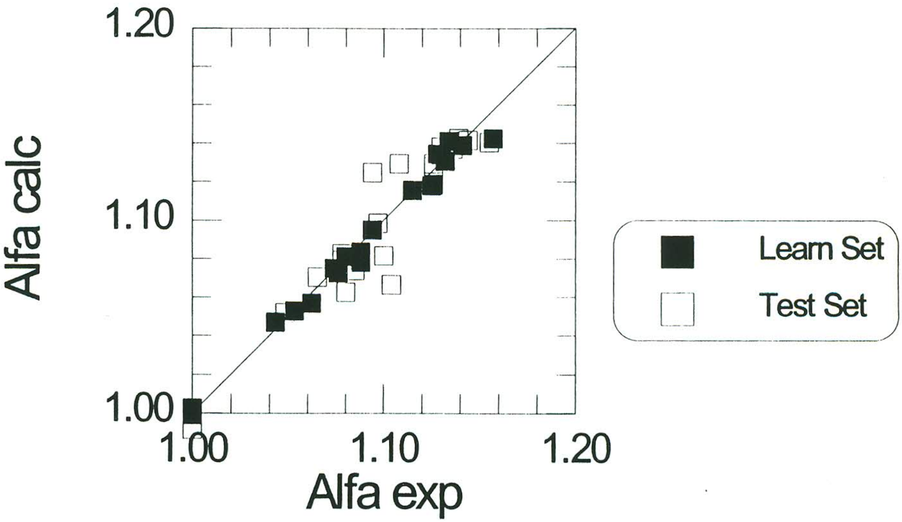


Figure 11: Comparison of the experimental and the neural network calculated values of the separation factor α

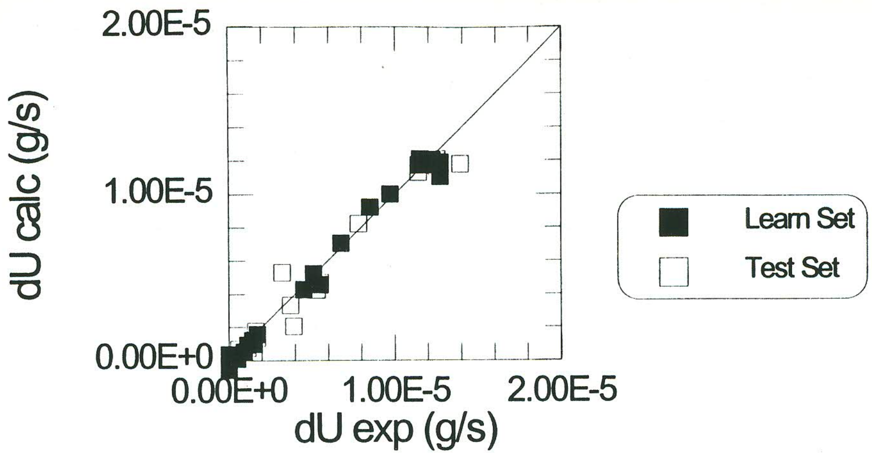


Figure 12: Comparison of the experimental and the neural network calculated values of the separative work δU

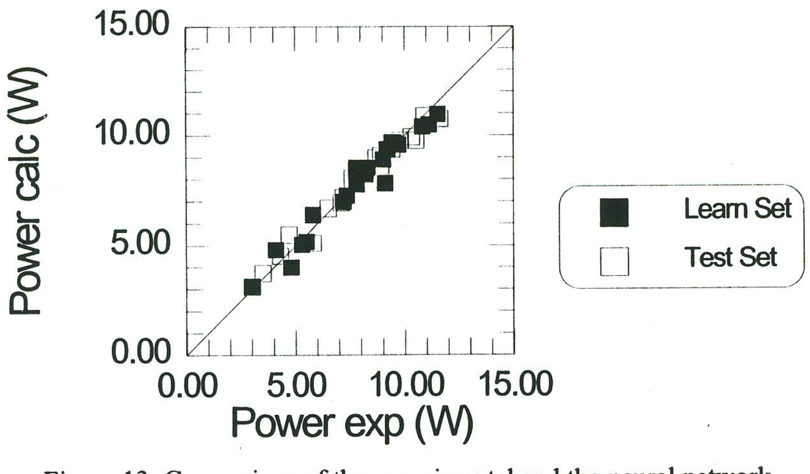


Figure 13: Comparison of the experimental and the neural network calculated values of the power consumption

5.4. Prediction of the separative performance of the centrifuge

Having the neural network trained to represent the centrifuge, the solution of the problem was mapped on a grid in the domain of the learning set data. Figure 14 represents the solution of the separative work δU as a function of the feed rate F for three peripheral velocities. In the same way, Figures 15 and 16 show the variation of the separation factor α and solution of the power consumption, respectively.

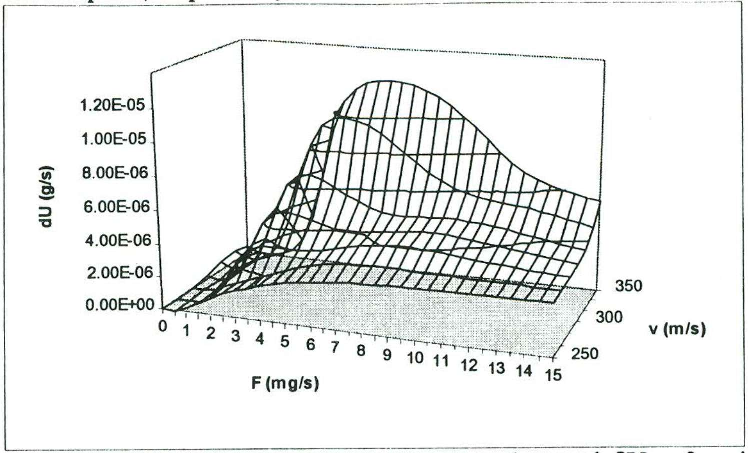


Figure 14: Response surface for the separative work δU as function of the peripheral velocity v and the feed rate F

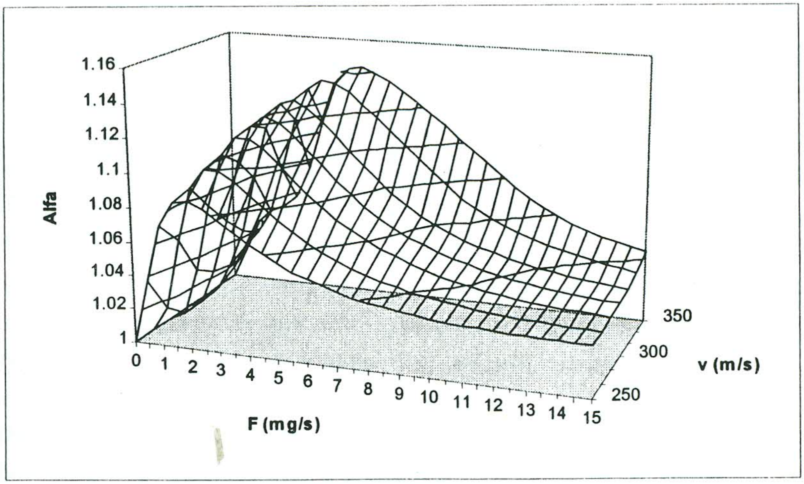


Figure 15: Response surface for the separation factor α as function of the peripheral velocity v and the feed rate F

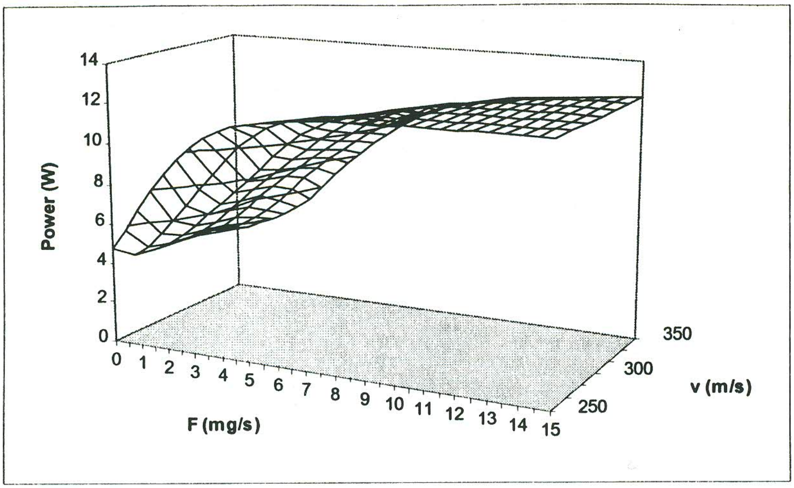


Figure 16: Response surface for the power consumption as function of the peripheral velocity v and the feed rate F

These surfaces show continuous and smooth responses, as expected. The separative work increases with the feed rate and then decreases, defining an optimum feed rate for each peripheral velocity. The separation factor also assumes a maximum value and then decreases with the feed rate. This initial increase with the feed rate is probably caused by the establishment of the internal countercurrent. The decrease observed is a characteristic of any countercurrent separation system, as it is disturbed from its equilibrium. These tendencies are highlighted with the increase of the peripheral velocity, as predicted by the theory. The power consumption increases with the feed rate, and the highest values are at low velocities.

Figure 17 shows the comparison of the experimental data for the peripheral velocity of 350 m/s with the phenomenological model presented by Kai [8] and with the neural network proposed in the present work. Kai [8] suggests that the temperature difference between the upper and the lower end plates ΔT is -10 °C at F=3 mg/s, 0°C at 4.5 mg/s and 10 °C at 6 mg/s, concluding that his model is in good agreement with the experimental results. This temperature difference was not measured. From our practical experience we do not measure such a strong variation in ΔT. Also analyzing Zippe[20] data for a different centrifuge configuration the difference was no higher than 3°C.

The neural network prediction, on the other hand, showed a very good agreement, although we should use this methodology very carefully because it is not based on basic phenomena.

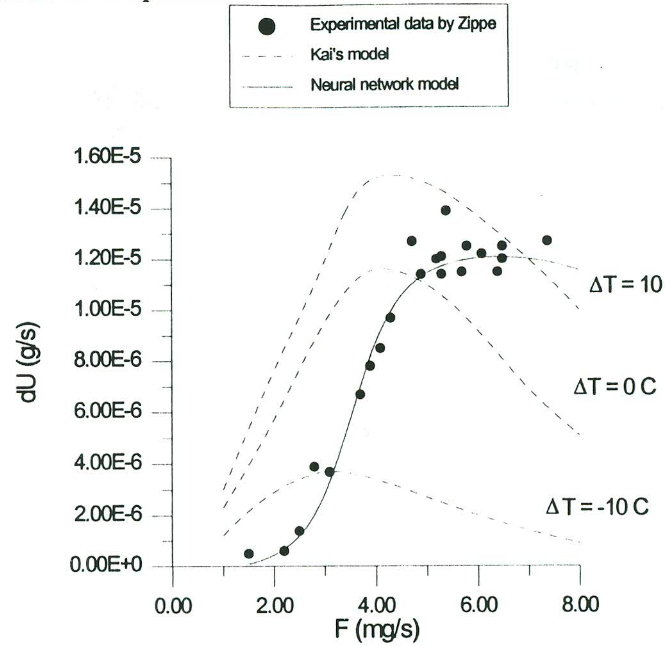


Figure 17: Comparison of theoretical results from Kai [8] with neural network model and with experimental ones by Zippe [20]

5. Conclusion

Modeling by neural networks has been shown an important tool to simulate the separation of uranium isotopes. Experimental data of good quality is fundamental for obtaining reasonable results. Migliavacca et. al. [16] have obtained a good representation of an actual centrifuge, with differences between the optimal predicted values and the experimental verification close to 1%.

In CASE 1, there were few data available, but even so a neural network could be trained, with good results comparing the experimental and the calculated data. An optimization was realized resulting in reasonable values of the output variables. It is not possible to verify if the arrangement found in the optimization process is realistic. That means, we can not ensure the results of this optimization without a confirming experiment. But the most important is that we verified that this procedure can be used for the optimization of actual centrifuges.

In CASE 2, the experimental data covered a good domain of the variables used as input of the model. The neural network trained could represent very well the data from the learned set and from the test set. This is a model to be used for the prediction of the separative performance of the centrifuge. Neural networks can model very accurately an actual centrifuge, in a desired domain of operational variables.

While the learning process of a neural network takes some time, the model that is obtained is a very fast running model. This aspect is an interesting characteristic for optimization and solution of the separation process in a cascade.

Notation

E - quadratic deviation

f - Sigma function, in Eq. (2)

F - feed flow rate (mg/s)

I - measured light intensity vector

n - number of input variables in the neural network model

NH - number of neurons in the hidden layer

No. SETS - number of presentations of the data to the network

O_j - output from neuron j

O_k - output from neuron k in the output layer

p - number of output variables in the neural network model

q - number of input variables in the network

r - number of input/output pairs in the learning set

RMST - learning set sum of errors

RMSTT - test set sum of errors

S_v - weighted sum of inputs to a neuron

$W_{u,v}$ - weight of variable i, in neuron j

y_k - experimental output variable k

Greek symbols

α - separation factor

δU - separative work (g/s)

ΔT - difference of temperature between the top and the bottom end cap

ϑ - cut

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