

# Monte Carlo simulation of $\beta$ – $\gamma$ coincidence system using plastic scintillators in $4\pi$ geometry

M.S. Dias<sup>a,\*</sup>, H. Piuvezam-Filho<sup>a</sup>, A.M. Baccarelli<sup>b</sup>, M.N. Takeda<sup>c</sup>, M.F. Koskinas<sup>a</sup>

<sup>a</sup>*Instituto de Pesquisas Energéticas e Nucleares: IPEN-CNEN/SP, Av. Prof. Lineu Prestes 2242, 05508-000 São Paulo, SP, Brazil*

<sup>b</sup>*Departamento de Física-PUC/SP-Rua Marquês de Paranaguá 111, 01303-050 São Paulo, SP, Brazil*

<sup>c</sup>*Universidade Santo Amaro, UNISA-Rua Prof. Enéas da Siqueira Neto 340, 04829-300 São Paulo, SP, Brazil*

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## Abstract

A modified version of a Monte Carlo code called Esquema, developed at the Nuclear Metrology Laboratory in IPEN, São Paulo, Brazil, has been applied for simulating a  $4\pi\beta(\text{PS})$ – $\gamma$  coincidence system designed for primary radionuclide standardisation. This system consists of a plastic scintillator in  $4\pi$  geometry, for alpha or electron detection, coupled to a NaI(Tl) counter for gamma-ray detection. The response curves for monoenergetic electrons and photons have been calculated previously by Penelope code and applied as input data to code Esquema. The latter code simulates all the disintegration processes, from the precursor nucleus to the ground state of the daughter radionuclide. As a result, the curve between the observed disintegration rate as a function of the beta efficiency parameter can be simulated. A least-squares fit between the experimental activity values and the Monte Carlo calculation provided the actual radioactive source activity, without need of conventional extrapolation procedures. Application of this methodology to  $^{60}\text{Co}$  and  $^{133}\text{Ba}$  radioactive sources is presented and showed results in good agreement with a conventional proportional counter  $4\pi\beta(\text{PC})$ – $\gamma$  coincidence system.

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## 1. Introduction

The speed of new computers has made possible the simulation of several radiation detection systems by the Monte Carlo method, allowing better understanding and prediction of detection processes. For instance, the International Committee for Radionuclide Metrology (ICRM) Gamma-Ray Spectrometry Working Group has recently triggered an international comparison in order to calculate the HPGe detector efficiency curve by Monte Carlo, gathering different laboratories and applying several code packages [1].

As part of this effort for improving the results of calibration systems, the Nuclear Metrology Laboratory (Laboratório de Metrologia Nuclear, LMN) at the IPEN-

CNEN/SP has developed a Monte Carlo code, called ESQUEMA, and applied to a  $4\pi\beta(\text{PC})$ – $\gamma$  proportional counter coincidence system, designed for primary radionuclide standardisation [2]. This code simulates the behaviour of the extrapolation curve between the observed experimental activity and the  $4\pi$  detection efficiency parameter. In order to achieve this goal, all transitions in the decay scheme are followed, from the precursor nucleus to the ground state of the daughter radionuclide, and the deposited energy for all radiations in both detectors are determined. In this way, the code can reproduce the whole coincidence experiment accurately.

The present paper shows the application of this code to another  $4\pi\beta$ – $\gamma$  coincidence system, recently developed at the LMN for radionuclide standardisation. This system consists of a plastic scintillator in  $4\pi$  geometry, for alpha or electron detection, coupled to a NaI(Tl) counter for gamma-ray detection. It has provided the standardisation

\*Corresponding author. Fax: +55 011 38169188.

E-mail address: [msdias@ipen.br](mailto:msdias@ipen.br) (M.S. Dias).

of radionuclides which decay by different processes, namely:  $^{18}\text{F}$ ,  $^{60}\text{Co}$ ,  $^{133}\text{Ba}$  and  $^{241}\text{Am}$ , applying the extrapolation technique [3,4]. Two of these, namely  $^{60}\text{Co}$  and  $^{133}\text{Ba}$ , were chosen for the present simulation.

## 2. Coincidence equations

The general coincidence equations applied to this technique may be given by

$$\frac{N_{\beta}N_{\gamma}}{N_c} = N_0 \left\{ \frac{\sum_{i=1}^m a_i \left\{ \varepsilon_{\beta_i} + (1 - \varepsilon_{\beta_i}) \sum_{j=1}^n b_{ij} \frac{\varepsilon_{\text{CE}_{ij}} + (1 - \varepsilon_{\text{CE}_{ij}})\varepsilon_{(\text{X,A})_{ij}} + \varepsilon_{\beta_{\gamma_{ij}}}}{1 + \alpha_{ij}} \right\} \sum_{i=1}^m a_i \sum_{j=1}^n b_{ij} \varepsilon_{\gamma_{ij}} \frac{1}{1 + \alpha_{ij}}}{\sum_i a_i \left[ \varepsilon_{\beta_i} \sum_{j=1}^n b_{ij} \varepsilon_{\gamma_{ij}} \frac{1}{1 + \alpha_{ij}} + (1 - \varepsilon_{\beta_i}) \sum_{j=1}^n b_{ij} \varepsilon_{\text{C}_{ij}} \frac{1}{1 + \alpha_{ij}} \right]} \right\} \quad (1)$$

where  $N_{\beta}$ ,  $N_{\gamma}$  and  $N_c$  are the beta, gamma and coincidence counting rates, respectively;  $N_0$  is the disintegration rate;  $a_i$  and  $b_{ij}$  are the intensity per decay of the  $i$ th beta transition and relative intensity of the  $j$ th transition with respect to the  $i$ th transition;  $n$  is the number of daughter transitions following the  $i$ th beta transition;  $m$  the number of beta transitions;  $\varepsilon_{\beta_i}$  the beta efficiency associated to  $i$ th beta transition;  $\varepsilon_{\gamma_{ij}}$  and  $\varepsilon_{\beta_{\gamma_{ij}}}$  are the gamma detection efficiency and gamma efficiency of beta detector, respectively, associated to the  $ij$ th transition;  $\varepsilon_{\text{CE}_{ij}}$  and  $\varepsilon_{(\text{X,A})_{ij}}$  are the conversion electron detection efficiency and electron Auger or X-ray detection efficiency, respectively, associated to  $ij$ th transition, and  $\varepsilon_{\text{C}_{ij}}$  and  $\alpha_{ij}$  are the gamma–gamma coincidence detection efficiency and total internal conversion coefficient of the  $ij$ th transition.

A measure of the beta efficiency may be given by

$$\frac{N_c}{N_{\gamma}} = \frac{\sum_i a_i \left[ \varepsilon_{\beta_i} \sum_{j=1}^n b_{ij} \varepsilon_{\gamma_{ij}} \frac{1}{1 + \alpha_{ij}} + (1 - \varepsilon_{\beta_i}) \sum_{j=1}^n b_{ij} \varepsilon_{\text{C}_{ij}} \frac{1}{1 + \alpha_{ij}} \right]}{\sum_{i=1}^m a_i \sum_{j=1}^n b_{ij} \varepsilon_{\gamma_{ij}} \frac{1}{1 + \alpha_{ij}}} \quad (2)$$

In the conventional procedure, the observed activity  $N_{\beta}N_{\gamma}/N_c$  is plotted against the efficiency parameter  $(1 - N_c/N_{\gamma})/(N_c/N_{\gamma})$  and extrapolated to unity efficiency. The beta efficiency is changed electronically or by placing absorbers over and under the radioactive source. This procedure is valid if the points are sufficiently close to the origin and the curve is well behaved. The Monte Carlo procedure avoids this extrapolation procedure and the source activity is determined by least-squares fitting between experimental and calculated values, as described in the following sections.

## 3. Experimental procedure

The radioactive sources were prepared by dropping known aliquots of the solution on  $10 \mu\text{g cm}^{-2}$  Collodion films, previously coated with  $10 \mu\text{g cm}^{-2}$  gold layer. The latter coating was performed in order to measure the

sources in a conventional  $4\pi(\text{PC})\beta\text{--}\gamma$  coincidence system which makes use of a gas-flow proportional counter. This procedure allowed a comparison between the results from both systems. The film holder was made of a stainless-steel ring 0.1 mm thick, with 20 mm external diameter and a hole 10 mm in diameter. Additional details on the experimental set-up are given elsewhere [4]. Several measurements were performed changing the efficiency of the  $4\pi$  detector by placing absorbers over and under the radioactive sources.

As a result, an extrapolation curve between the observed activity and the efficiency parameter  $N_c/N_{\gamma}$  was obtained.

## 4. Monte Carlo procedure

### 4.1. Response curve determination

The deposited energy distribution for monoenergetic electrons and photons in  $4\pi$  plastic scintillator and for monoenergetic photons in NaI(Tl) detector has been calculated previously by means of code Penelope [5] for the geometry shown in Fig. 1.

The  $4\pi$  detector is a cylinder 9 mm high, 40 mm outside diameter and has a hole in the centre, 3 mm high and 20 mm in diameter. The plastic scintillator material has been made at the IPEN [6] and has similar characteristics as compared to NE102 [7]. It is surrounded by a plastic cap covered from inside with Teflon, in order to improve diffused light reflection, and coupled to a RCA 9850 phototube. The gamma-ray detector is a 50 mm  $\times$  50 mm

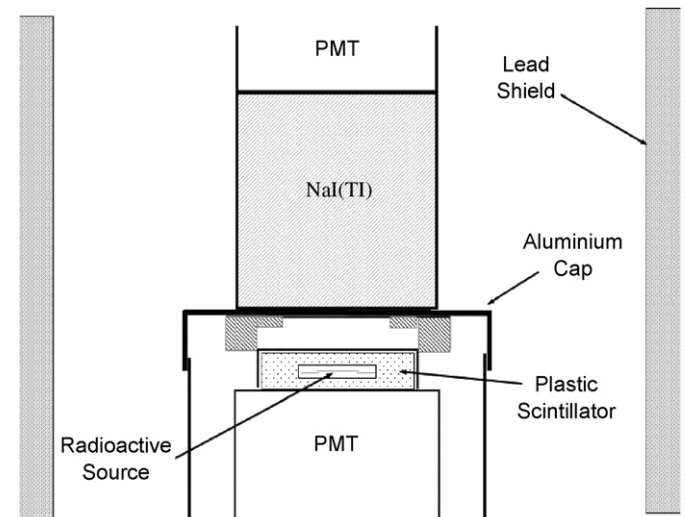


Fig. 1. Schematic diagram of the  $4\pi(\text{PS})\beta\text{--}\gamma$  coincidence system.

NaI(Tl) crystal positioned close to an aluminium cap which protects the plastic scintillator from external light. The detector system is surrounded by 50 mm lead shield.

#### 4.2. Activity determination

The second step in the Monte Carlo procedure was to run code Esquema for  $^{60}\text{Co}$  and  $^{133}\text{Ba}$  applying the response curves previously calculated by code Penelope. The necessary decay data was taken from the literature [8]. Code Esquema follows the decay scheme from the precursor nucleus to the ground state of the daughter nucleus, passing through all possible pathways. The simulation was performed changing the source absorber thickness in very small increments in order to get the simulated activity for the same efficiency values as obtained experimentally. The statistics in the calculation was kept better than 0.1% for all data points.

### 5. Data analysis

The source activity has been determined in two different ways. In the first, a linear least squares fit has been performed between  $N_{\beta}N_{\gamma}/N_c$  against the efficiency parameter  $(1-N_c/N_{\gamma})/(N_c/N_{\gamma})$ . The extrapolated value to unity efficiency ( $N_c/N_{\gamma} \rightarrow 1$ ) gives the activity value.

In the second approach, a least-squares fit between the experimental data and the Monte Carlo calculation has been performed, according to the following equation:

$$\chi^2 = (\vec{y}_{\text{exp}} - N_0 \vec{y}_{\text{MC}})^T V^{-1} (\vec{y}_{\text{exp}} - N_0 \vec{y}_{\text{MC}}) \quad (3)$$

where  $\vec{y}_{\text{exp}}$  is the experimental vector of  $N_{\beta}N_{\gamma}/N_c$ ;  $\vec{y}_{\text{MC}}$  the  $N_{\beta}N_{\gamma}/N_c$  vector calculated by Monte Carlo for unitary activity;  $N_0$  the specific activity of the radioactive solution;  $V$  the total covariance matrix, including both experimental and calculated uncertainties, and T stands for matrix transposition.

A series of simulated values were calculated for a wide range of beta efficiency parameter in small bin intervals. The  $\vec{y}_{\text{MC}}$  values used in Eq. (3) correspond to the same efficiency obtained experimentally.

### 6. Results and discussion

Fig. 2 shows the extrapolation curve obtained for  $^{60}\text{Co}$ . The closed dots are the experimental results and the open dots are the Monte Carlo calculation. It can be seen that the agreement is good in the whole plotted region. The numerical results for the activity are given in Table 1. Reasonable agreement is obtained with the linear fit and with the result taken from the conventional  $4\pi(\text{PC})\beta\text{-}\gamma$  system.

Fig. 3 shows the extrapolation curve for  $^{133}\text{Ba}$ .

The behaviour is well reproduced by the Monte Carlo simulation. The agreement with the experimental data points is good in the whole region. The numerical results

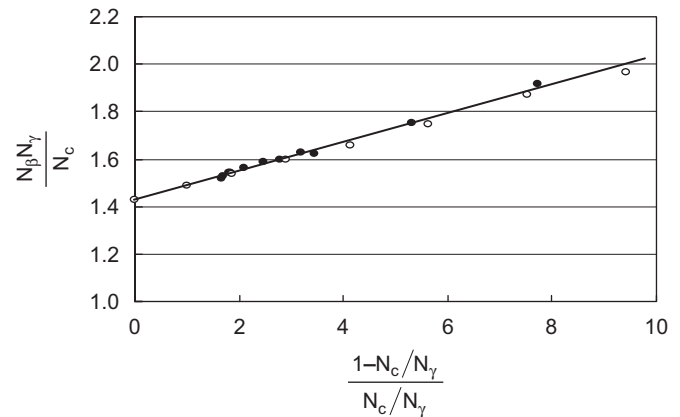


Fig. 2. Behaviour of  $N_{\beta}N_{\gamma}/N_c$  as a function of efficiency parameter  $(1-N_c/N_{\gamma})/(N_c/N_{\gamma})$  for  $^{60}\text{Co}$ . Closed circles correspond to experimental data and open circles to Monte Carlo calculation. The continuous curve corresponds to first degree polynomial fitting to the experimental points.

Table 1

Results of activity values of  $^{60}\text{Co}$  and  $^{133}\text{Ba}$ , for different system methodologies

	Condition	System	Activity ( $\text{kBq g}^{-1}$ )	Slope (%)
$^{60}\text{Co}$	Experimental	PC	$(142.72 \pm 0.29)$	
	Experimental	PS	$(142.42 \pm 0.54)$	$(4.32 \pm 0.13)$
	Monte Carlo	PS	$(143.22 \pm 0.33)$	
$^{133}\text{Ba}$	Experimental	PC	$(755.5 \pm 2.5)$	
	Experimental	PS	$(750.3 \pm 4.5)$	$(6.10 \pm 0.08)$
	Monte Carlo	PS	$(752.7 \pm 2.1)$	

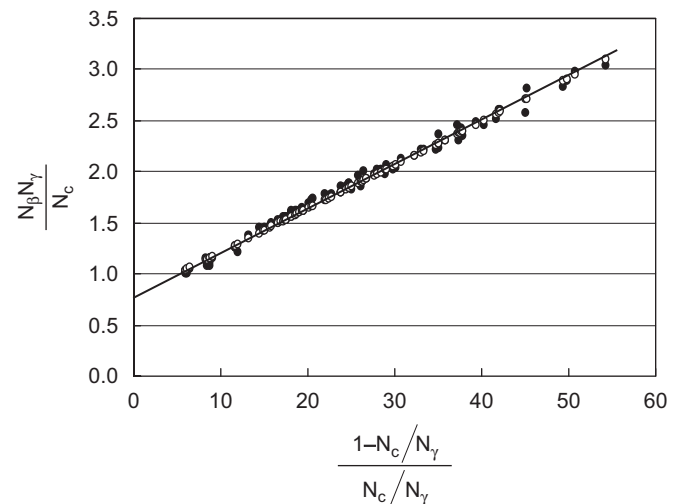


Fig. 3. Behaviour of  $N_{\beta}N_{\gamma}/N_c$  as a function of efficiency parameter  $(1-N_c/N_{\gamma})/(N_c/N_{\gamma})$  for  $^{133}\text{Ba}$ . Closed circles correspond to experimental data and open circles to Monte Carlo calculation. The continuous curve corresponds to first degree polynomial fitting to the experimental points.

for the activity are also given in Table 1. Good agreement is obtained between the Monte Carlo value and the result obtained by the conventional  $4\pi(\text{PC})\beta\text{-}\gamma$  system. As can be seen the Monte Carlo procedure showed good results and

presents the advantage of being based on a realistic behaviour of the extrapolation curve instead of the conventional approach, using polynomial fitting.

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