Impedance Analysis of Tape-Helix Slow-Wave Structures

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ABSTRACT — **In this paper is reported an investigation about the interaction impedance behavior for slow-wave structures (SWS) in order to provide some guidelines to aid on traveling-wave tubes (TWT) and backward-wave oscillators (BWO) design. The analysis is made rigorous by removing the usual approximation of considering the radial propagation constant to be the same in the different structure regions. Furthermore, a tape-helix model instead of a simpler sheath-helix model has been used to consider the space-harmonic effects on the interaction impedance. Functional variations are given of interaction impedance with frequency and with the effective relative dielectric constant of support rods. Additionally, the dispersion equation is solved for the first backward-mode and its interaction impedance curve is shown.**

Index Terms **— Helix, interaction impedance, traveling-wave tube (TWT).**

I. INTRODUCTION

Among power microwave tubes, traveling-wave tubes (TWT) have an uncommon feature due to their gain and bandwidth that turn these devices the main amplifier devices in microwave communications systems, frequency agile wide-band electronic counter measure, and electronic counter-counter measure systems.

In this paper, a rigorous field analysis is developed for both the dispersion and the interaction impedance characteristics by an improvement of the existing analysis of an inhomogeneously-load helix by considering, first, different radial propagation constants in different structure regions and, second, the space-harmonic effects. The results of the propagation characteristics of SWS presented here are rather general and can used for any space-harmonic of interest. For instance, one can use them to predict and to control the potential backward-mode oscillation impedance for a TWT.

This paper is organized as follows: Section II describes the field theoretical analysis of a SWS. Section III shows and discusses the results of the theoretical evaluation of the interaction impedance for some SWS in order to obtain its interaction impedance plot. Section IV relates the conclusions.

II. THEORETICAL ANALYSIS

Solving the Maxwell's equations for a SWS under the propagation hypothesis $e^{j(\omega t-\beta z)}$, where ω is the angular frequency and $\beta=\beta(\omega)$ is the axial propagation constant, one can obtain a set of field component expressions that describes the electromagnetic field behavior on the SWS that is shown in Fig. 1, and according with Floquet theorem, the axial propagation constant β_n, to nth space-harmonic, is given by [1]

$$\beta_n a=\beta_0 a+n\frac{2\pi a}{p}=\beta_0 a+n\cot\psi\,, \tag{1}$$

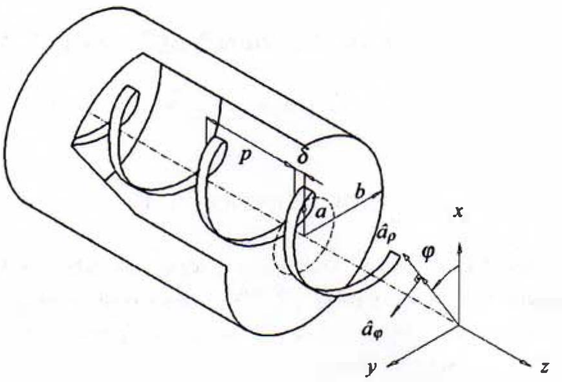

Fig. 1. Slow-wave structure analyzed in this paper.

where a is the helix radio. p is the helix pitch and ψ is the helix angle. The electric and magnetic field expressions in cylindrical coordinates (ρ,φ,z), in terms of space-harmonics for $(\rho\le a)$, are given by

$$E_{\rho 1n}(\rho,\varphi)=\left[\frac{j\beta_n a}{h_{1n}a}A_{1n}I'_n(h_{1n}\rho)-naZ_0\frac{k_0 a}{\rho(h_{1n}a)^2}C_{1n}I_n(h_1\rho)\right]e^{jn\varphi}$$

$$E_{\varphi 1n}(\rho,\varphi)=\left[-na\frac{\beta_n a}{\rho(h_{1n}a)^2}A_{1n}I_n(h_{1n}\rho)-jZ_0\frac{k_0 a}{h_{1n}a}C_{1n}I'_n(h_{1n}\rho)\right]e^{jn\varphi}$$

$$E_{z1n}(\rho,\varphi)=A_{1n}I_n(h_{1n}\rho)e^{jn\varphi}\text{, and}$$

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$$H_{\rho 1n}(\rho,\varphi)=\left[\frac{naZ_0(k_0a)}{\rho(h_{1n}a)^2}A_{1n}I_n(h_{1n}\rho)+\frac{j\beta_n a}{h_{1n}a}C_{1n}I_n'(h_{1n}\rho)\right]e^{jn\varphi},$$

$$H_{\varphi 1n}(\rho,\varphi)=\left[\frac{jZ_0(k_0a)}{h_{1n}a}A_{1n}I_n'(h_{1n}\rho)-\frac{na\beta_n a}{\rho(h_{1n}a)^2}C_{1n}I_n(h_{1n}\rho)\right]e^{jn\varphi},$$

$$H_{z1n}(\rho,\varphi)=C_{1n}I_n(h_{1n}\rho)e^{jn\varphi}\ , \tag{2}$$

where $I_n(h_{1n}\rho)$ represents the modified Bessel functions of order n of the first kind and the prime indicates its derivative with respect to the argument. Z_0 is the free-space impedance and $j=\sqrt{-1}$. In a similar way, the field expression in terms of space-harmonics for ($a \le \rho \le b$), where b is the circular waveguide radio, are given by

$$E_{\rho 2n}(\rho,\varphi)=\left[\frac{j\beta_n a}{h_{2n}a}A_{2n}G_{10}^n(\rho,b)-\frac{naZ_0k_0a}{\rho(h_{2n}a)^2}C_{2n}G_{01}^n(\rho,b)\right]e^{jn\varphi}$$

$$E_{\varphi 2n}(\rho,\varphi)=\left[-\frac{na\beta_n a}{\rho(h_{2n}a)^2}A_{2n}G_{00}^n(\rho,b)-\frac{jZ_0k_0a}{h_{2n}a}C_{2n}G_{11}^n(\rho,b)\right]e^{jn\varphi}$$

$$E_{z2n}(\rho,\varphi)=A_{2n}G_{00}^n(\rho,b)e^{jn\varphi}\ \text{,and}$$

$$H_{\rho 2n}(\rho,\varphi)=\left[n\frac{\varepsilon_r k_0aZ_0}{\rho(h_{2n}a)^2}A_{2n}G_{00}^n(\rho,b)+\frac{j\beta_n a}{h_{2n}a}C_{2n}G_{11}^n(\rho,b)\right]e^{jn\varphi}$$

$$H_{\varphi 2n}(\rho,\varphi)=\left[\frac{na\varepsilon_r Z_0(k_0a)}{\rho(h_{2n}a)^2}A_{2n}G_{00}^n(\rho,b)+\frac{j\beta_n a}{h_{2n}a}C_{2n}G_{11}^n(\rho,b)\right]e^{jn\varphi}$$

$$H_{z2n}(\rho,\varphi)=C_{2n}G_{01}^n(\rho,b)e^{jn\varphi}, \tag{3}$$

where k_0 is the free-space wave number and the $G_{ij}^n(\rho,b)$ functions are given in appendix. The radial propagation constants can be written as

$$h_{1n}^2=\beta_n^2-k_0^2 \quad \text{to } \rho \le a,$$
$$h_{2n}^2=\beta_n^2-\varepsilon_r k_0^2 \quad \text{to } a \le \rho \le b.$$

The dispersion equation and the field coefficients A_{1n}, C_{1n}, A_{2n}, and C_{2n}, also given in appendix, can be found using the Sensiper's boundary conditions to $\rho = a$, and can be expressed in terms of $\hat{J}_{\|n}$, the complex Fourier component of surface current density in the direction of the tape associated with the nth space-harmonic. The dispersion equation to the propagation wave problem is given by

$$\sum_{n=-\infty}^{\infty}\frac{1}{\Delta}\frac{sin(\beta_n\delta/2)}{(\beta_n\delta/2)}\frac{G_{00}^n(a,b)G_{11}^n(a,b)}{h_{2n}a}\left\{\left[1-\frac{n\beta_n a}{(h_{1n}a)^2}\cot\psi\right]\right.$$
$$\left[F_{1n}^{2,1}-\cot\psi\frac{n\beta_n a}{(h_{1n}a)^2}F_{1n}^{1,2}\right]-\cot^2\psi\frac{(k_0a)^2}{(h_{1n}a)^2}F_{2n}^{1,2}$$
$$+\cot\psi\frac{n\beta_n a}{(h_{2n}a)^2}\left[I_n^2(h_{1n}a)-F_{1n}^{2,1}\left[1-\frac{(h_{2n}a)^2}{(h_{1n}a)^2}\right]\right.$$
$$\left.\left.\left[1-\cot\psi\frac{n\beta_n a}{(h_{2n}a)^2}\right]\right]\right\}=0 \tag{4}$$

where δ is the helix width and ε_r is the effective relative dielectric constant. $\Delta, F_{1n}^{1,2}, F_{1n}^{2,1}$, and $F_{2n}^{2,1}$ are defined in appendix.

III. INTERACTION IMPEDANCE

The slow-wave interaction impedance on-axis of nth space-harmonic K_n is an important part of the design process for a TWT because this parameter is related to the gain and efficiency of the device. K_n, proportional to the strength of coupling between the electromagnetic wave and the electron beam, is defined by [2]

$$K_n=\frac{|E_{zn}(\rho=0)|^2}{2\beta_n^2P_T}, \tag{6}$$

where $E_{zn}(\rho=0)$ is the value of the nth space-harmonic of the electric field on axis z and P_T is the total power propagated in the structure, given by

$$P_T=\frac{1}{2}\mathrm{Re}\oint\vec{E}\times\vec{H}^*\cdot\hat{a}_z\rho d\rho d\varphi. \tag{7}$$

P_T can be split in two contributing parts. First, the propagated power in $\rho \le a$, P_1, and second, in $a \le \rho \le b$, P_2, so that one can write

$$P_T=P_1+P_2=\pi\frac{k_0a}{Z_0}\sum_{n=-\infty}^{\infty}(p_{1n}+p_{2n})$$

where p_{1n} and p_{2n} are given by

$$p_{1n}=\frac{\beta_n a}{(h_{1n}a)^2}\left\{\left[|A_{1n}|^2+Z_0^2|C_{1n}|^2\right]\int_0^a I_n'^2(h_{1n}\rho)\rho d\rho\right.$$
$$\left.+\frac{(na)^2}{(h_{1n}a)^2}\left[|A_{1n}|^2+Z_0^2|C_{1n}|^2\right]\int_0^a\frac{1}{\rho}I_n^2(h_{1n}\rho)d\rho\right\}$$
$$-j2na\frac{Z_0}{k_0a}\frac{(\beta_n a)^2}{(h_{1n}a)^3}\left(1+\frac{(k_0a)^2}{(\beta_n a)^2}\right)A_{1n}C_{1n}\int_0^a[I_n'(h_1\rho)I_n(h_1\rho)]\rho d\rho$$

$$p_{2n} = \frac{\beta_n a}{(h_{2n}a)^2} \int_a^b \left\{ \varepsilon_r |A_{2n}|^2 \left[G_{10}^n(\rho,b) \right]^2 + Z_0^2 |C_{2n}|^2 \left[G_{11}^n(\rho,b) \right]^2 \right\} \rho d\rho$$
$$+ (na)^2 \frac{\beta_n a}{(h_{2n}a)^4} \int_a^b \left\{ \varepsilon_r |A_{2n}|^2 \left[G_{00}^n(\rho,b) \right]^2 + Z_0^2 |C_{2n}|^2 \left[G_{01}^n(\rho,b) \right]^2 \right\} \frac{1}{\rho} d\rho$$
$$+ jna \frac{Z_0}{k_0 a} \frac{(\beta_n a)^2}{(h_{2n}a)^3} A_{2n} C_{2n} \left\{ \left(1 - \varepsilon_r \frac{(k_0 a)^2}{(\beta_n a)^2} \right) \int_a^b \left[G_{10}^n(\rho, b\; G_{01}^n(\rho,b) \right] d\rho \right.$$
$$\left. - \left(1 + \varepsilon_r \frac{(k_0 a)^2}{(\beta_n a)^2} \right) \int_a^b \left[G_{00}^n(\rho,b) G_{11}^m(\rho,b) \right] d\rho \right\}$$

III. RESULTS AND DISCUSSION

The first step in obtaining the interaction impedance for a SWS is to obtain its dispersion curve by solution of the transcendental equation (4). Fig. 2 shows a dispersion curve for two different effective relative dielectric constants ε_r (for region between helix and guide), illustrating the allowed and forbidden regions.

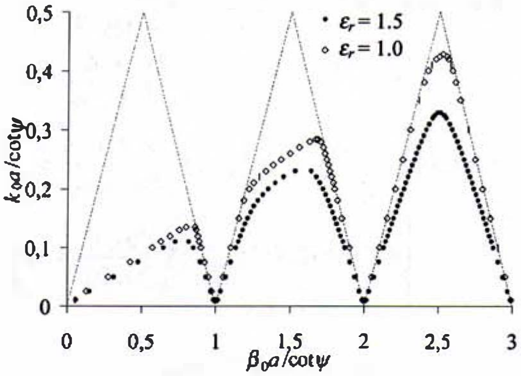

Fig. 2. Dispersion plot obtained for the fundamental propagation mode (n = 0) for two different ε_r, using the tape-model which considers the space-harmonic effects.

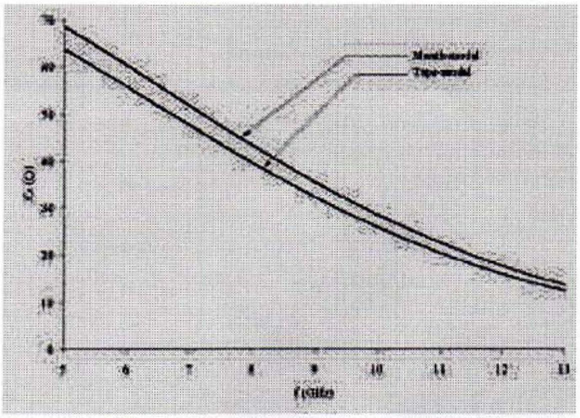

Fig. 3. Interaction impedance comparison betwen sheath and tape models (a = 1.09mm, r (= b/a) = 1.633, p = 1.247mm, ψ = 10.32°, δ = 0.76, and ε_r = 1.78).

Fig. 2 shows that the maximum frequency which allows a propagation mode is limited by ε_r. Fig. 3 shows a impedance interaction comparison between the sheath-model and the tape-model. The sheath-model impedance interaction is a particular case of the tape-model impedance interaction (n = 0). K_0 obtained using the tape-model is shown to be smaller if compared with the sheath-model. Fig. 4 shows the effect of the effective relative dielectric constant on the interaction impedance. The curve was plotted for the same parameters used in Fig. 3, varying ε_r. It shows that the interaction impedance is increased by a decreasing in ε_r.

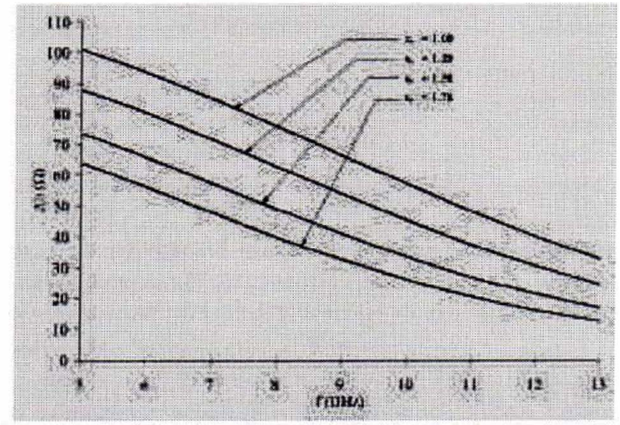

Fig. 4. Interaction impedance as function of frequency for 4 different effective relative dielectric constants of support rods.

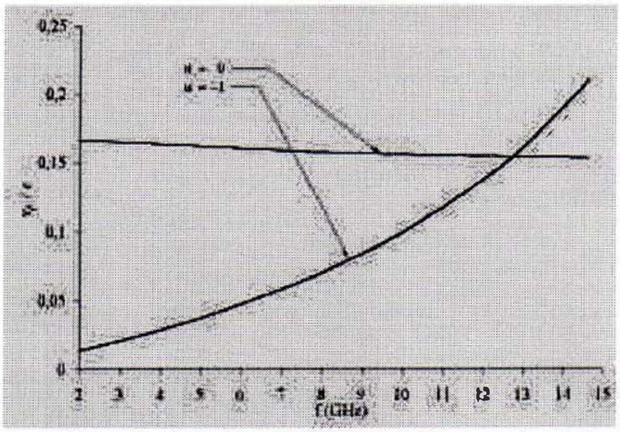

Fig. 5. Normalized phase velocity (v_p/c) as function of frequency f, plotted using the tape-helix model for both fundamental and backward-mode at ρ = 0.25a (a = 1.588mm, r = 1.634, p = 1.813mm, ψ = 10.3°, and ε_r =1.683).

Figure 5 shows both the fundamental (n = 0) and the first backward (n = −1) modes normalized phase velocities using the present tape-model. Plotting a curve for interaction impedance for fundamental and first-backward mode, one obtains the Fig. 6. One can observe that for a frequency about 14GHz, the SWS under analysis becomes an oscillatory structure.

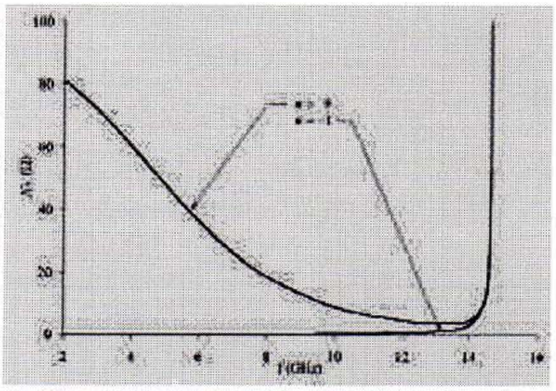

Fig. 6. The fundamental interaction impedance K_0 at the helix axis ($\rho = 0$) and the first backward-mode K_{-1} ($\rho = 0.25a$).

IV. CONCLUSION

In this paper, it was presented a tape-helix model in order to determine the interaction impedance of a SWS for both fundamental and first-backward mode. The model was validated using usual structure parameter reported in literature [3][4].

APPENDIX

The expressions to the spatial-harmonic coefficients of the fields A_{1n}, C_{1n}, A_{2n} e C_{2n} are given by:

$$A_{1n} = \frac{j}{\Delta}\frac{\hat{J}_{\parallel n}\sin\psi}{\omega\varepsilon_0 a}\frac{(k_0a)^2}{h_{2n}a}\frac{G_{00}^n(a,b)G_{11}^n(a,b)}{I_n(h_{1n}a)}$$
$$\left[F_{1n}^{2,1} - \cot\psi\frac{n\beta_n a}{(h_{1n}a)^2}F_{1n}^{1,2}\right],$$

$$C_{1n} = \frac{\hat{J}_{\parallel n}\sin\psi}{\Delta}\frac{G_{00}^n(a,b)G_{11}^n(a,b)}{I_n'(h_{1n}a)}\left\{\cot\psi\frac{(k_0a)^2}{(h_{2n}a)(h_{1n}a)}F_{2n}^{1,2}\right.$$
$$-\frac{n\beta_n a}{(h_{2n}a)^2}I_n(h_{1n}a)I_n'(h_{1n}a)\frac{G_{01}^n(a,b)}{G_{11}^n(a,b)}$$
$$\left.\left[1-\frac{(h_{2n}a)^2}{(h_{1n}a)^2}\right]\left[1-\cot\psi\frac{n\beta_n a}{(h_{2n}a)^2}\right]\right\},$$

$$A_{2n} = \frac{I_n(h_{1n}a)}{G_{00}^n(a,b)}A_{1n},$$

$$C_{2n} = \frac{I_n(h_{1n}a)}{G_{01}^n(a,b)}C_{1n} - \frac{\hat{J}_{\parallel n}\cos\psi}{G_{01}^n(a,b)},$$

where Δ is written as:

$$\Delta = \frac{G_{00}^n(a,b)G_{11}^n(a,b)(k_0a)^2}{h_{2n}a}\left\{\frac{F_{1n}^{2,1}F_{2n}^{1,2}}{I_n(h_{1n}a)I_n'(h_{1n}a)h_{1n}a}\right.$$
$$\left.+n^2\frac{(\beta_n a)^2}{(k_0a)^2}\frac{I_n^2(h_{1n}a)}{(h_{2n}a)^3}\frac{G_{01}^n(a,b)}{G_{11}^n(a,b)}\left[1-\frac{(h_{2n}a)^2}{(h_{1n}a)^2}\right]^2\right\},$$

and the Fourier coefficient $\hat{J}_{\parallel n}$ is given by

$$\hat{J}_{\parallel n} = J_0\left(\frac{\delta}{p}\right)\frac{\sin(\beta_n\delta/2)}{(\beta_n\delta/2)}.$$

The $G_{00}^n(\rho,b)$ functions are defined in terms of modified Bessel functions as

$$G_{00}^n(\rho,b) = I_n(h_{2n}\rho)K_n(h_{2n}b) - I_n(h_{2n}b)K_n(h_{2n}\rho),$$
$$G_{01}^n(\rho,b) = I_n(h_{2n}\rho)K_n'(h_{2n}b) - I_n'(h_{2n}b)K_n(h_{2n}\rho),$$
$$G_{10}^n(\rho,b) = I_n'(h_{2n}\rho)K_n(h_{2n}b) - I_n(h_{2n}b)K_n'(h_{2n}\rho),$$
$$G_{11}^n(\rho,b) = I_n'(h_{2n}\rho)K_n'(h_{2n}b) - I_n'(h_{2n}b)K_n'(h_{2n}\rho).$$

The $F_{jn}^{p,q}$ functions are defined in terms of modified Bessel functions of first I_n and second K_n kinds, and in terms of $G_{ij}^n(\rho,b)$ functions as:

$$F_{1n}^{2,1} = I_n^2(h_{1n}a) - \frac{h_{2n}a}{h_{1n}a}I_n(h_{1n}a)I_n'(h_{1n}a)\frac{G_{01}^n(a,b)}{G_{11}^n(a,b)},$$

$$F_{2n}^{1,2} = I_n'^2(h_{1n}a) - \varepsilon_r\frac{h_{1n}a}{h_{2n}a}I_n(h_{1n}a)I_n'(h_{1n}a)\frac{G_{10}^n(a,b)}{G_{00}^n(a,b)},$$

$$F_{1n}^{1,2} = I_n^2(h_{1n}a) - \frac{h_{1n}a}{h_{2n}a}I_n(h_{1n}a)I_n'(h_{1n}a)\frac{G_{01}^n(a,b)}{G_{11}^n(a,b)}$$

$$F_{2n}^{2,1} = I_n'^2(h_{1n}a) - \varepsilon_r\frac{h_{2n}a}{h_{1n}a}I_n(h_{1n}a)I_n'(h_{1n}a)\frac{G_{10}^n(a,b)}{G_{00}^n(a,b)}.$$

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