

Full length article

Uncertainty minimization in electronic stopping cross-section measurements using the backscattering method

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ARTICLE INFO

Keywords:

Stopping cross-section
Backscattering method
Uncertainty optimization
Systematic uncertainty

ABSTRACT

Accurate determination of electronic stopping cross-sections is critical for ion beam analysis and related applications. While transmission methods are well established, backscattering approaches remain less explored from a metrological perspective, often lacking a systematic treatment of uncertainties. In this work, we present a quantitative framework to optimize experimental geometry in backscattering-based stopping measurements, explicitly accounting for both statistical and systematic errors. Applying the method to helium ions in gold thin films, we identify angular conditions that balance precision and accuracy, achieving average total uncertainties of 4.7% over a wide energy range. The results, benchmarked against SRIM, DPASS, and ICRU-49, demonstrate that our approach improves the reliability of RBS-derived stopping data and strengthens their use for reference purposes and model validation.

1. Introduction

The accurate determination of electronic stopping cross-sections is essential for a broad range of applications in nuclear and materials science, including ion beam analysis (IBA), ion implantation, radiation damage studies, and detector development. The stopping cross-section represents a fundamental quantity in the description of energy loss processes, and its precise knowledge is required both for modeling and for experimental quantification [1,2]. Although the theoretical understanding of the physical processes involved in energy loss phenomena is advancing [3–8], experimental work focusing on traceable measurements has become crucial to benchmark the models [9]. Accurate experimental data also help to improve semiempirical approaches to produce tables that are used in simulations targeting a wide range of applications.

Historically, several experimental approaches have been developed to measure stopping cross-sections, the most established being the transmission method and Rutherford backscattering spectrometry (RBS)-based method [10,11]. While the transmission method is often regarded as more direct, it is not exempt from systematic uncertainties, including foil non-uniformity, energy calibration issues, and beam stability [9,12]. Moreover, the perception that it offers intrinsically

higher accuracy has contributed to an imbalance in the literature regarding the metrological scrutiny of alternative methods.

In particular, despite its wide use in IBA, the backscattering method remains relatively underexplored from a metrological standpoint. Although early works by Bauer [10] and Mertens [12] have laid important foundations for the method, and some efforts have been made to evaluate the accuracy of RBS-derived stopping data [13–15], recent literature still lacks systematic frameworks to quantify and minimize both statistical and systematic uncertainties in these measurements. As a consequence, experimental stopping cross-section data obtained via RBS often omit a complete uncertainty budget, which compromises their traceability and utility as reference data [2].

There are different methodologies and strategies to experimentally determine the stopping cross-section in backscattering experiments. Some are based on relative measurements to tables of other better-known elements [16,17], some are constrained by a fitting formula [15, 18], and others require thickness markers [19].

This work aims to address this gap by providing a detailed optimization strategy for stopping cross-section measurements using the backscattering method. We present a methodology that identifies optimal geometrical configurations to minimize total uncertainty, incorporating both random and systematic components. Our approach builds

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<https://doi.org/10.1016/j.nimb.2026.166114>

Received 24 October 2025; Received in revised form 30 March 2026; Accepted 1 April 2026

Available online 18 April 2026

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upon earlier formulations [10,13], but extends them with explicit error propagation, correlation analysis, and benchmarking against reference models such as SRIM [20], DPASS [21] and ICRU [22] tables. The methodology is applied to helium ions backscattered from thin gold films, for which precise reference data are available.

By offering a reproducible and uncertainty-aware approach to stopping cross-section determination via backscattering, we contribute to improving the reliability of this technique and enable its broader adoption in precision applications.

2. Methods

2.1. The backscattering method

There are several approaches to measuring the stopping cross-section using backscattering data [10]. In this work, we adopt the method based on determining the peak width in thin-film measurements, as described in [23]. Fig. 1 shows a typical spectrum of a pure metallic film deposited on a light substrate. The peak is formed by all scattered ions from the surface down to the film–substrate interface. Its width (ΔE) is the difference between the energy of ions scattered by the film atoms at the surface ($K \cdot E_0$, with K being the kinematic factor and E_0 the incidence beam energy) and the exit energy of ions re-emerging after being scattered by film atoms at the interface (E_{out}). Assuming homogeneous stopping in the test film of thickness t along both the entry and exit paths, the energy difference can be written as:

$$\Delta E = \frac{K \cdot t}{\cos(\theta_{in})} [\epsilon]_{in} + \frac{t}{\cos(\theta_{out})} [\epsilon]_{out} \quad (1)$$

with:

$$\epsilon_j = \frac{1}{N} \frac{dE}{dx} \Big|_j \quad (2)$$

being the stopping cross-section in the j path, either the entry (index in) or the exit (index out) of the film. θ_{in} and θ_{out} represent the angles between the ion's entry and exit paths and the surface normal, respectively. N is the material atomic density.

The fundamental principle of this methodology is that by measuring the peak width in two different geometrical configurations (i.e. two pairs of values of θ_{in} and θ_{out}) for the same incident energy, one can obtain a system of equations. The solutions for $[\epsilon]_{in}$ and $[\epsilon]_{out}$ yield the stopping cross-section at two different mean energies. In matrix formalism, this can be expressed as:

$$\begin{pmatrix} \Delta E^A/t \\ \Delta E^B/t \end{pmatrix} = \begin{pmatrix} \frac{K}{\cos(\theta_{in}^A)} & \frac{1}{\cos(\theta_{out}^A)} \\ \frac{K}{\cos(\theta_{in}^B)} & \frac{1}{\cos(\theta_{out}^B)} \end{pmatrix} \begin{pmatrix} [\epsilon]_{in} \\ [\epsilon]_{out} \end{pmatrix} \quad (3)$$

where indexes A and B denote the two different measurements. To better constrain the problem, we impose the conditions $\theta_{in}^A = \theta_{out}^B = \Theta_1$ and $\theta_{in}^B = \theta_{out}^A = \Theta_2$. This assumption is necessary to reduce the number of variables, thereby enhancing result visualization and simplifying the experimental procedure.

Two things are important to note: the first is that if $\Theta_1 = \Theta_2$ the equation system of Eq. (3) becomes undetermined; and the second is that the factor K implicitly depends on the scattering angle, which remains fixed throughout the analysis. As a consequence, the experimental procedure involves tilting the sample in such a way that, in the first measurement, the incidence angle is Θ_1 and the exit angle is Θ_2 , while in the second measurement, the angles are reversed, with the incidence angle set to Θ_2 and the exit angle to Θ_1 .

There are different approximations to determine the mean energy at the entry and exit paths of the film [23]. Here, we adopt the following approximations:

$$\begin{aligned} \bar{E}_{in} &= E_0 - \frac{1}{4} \Delta \bar{E} \\ \bar{E}_{out} &= K \cdot E_0 - \left(\frac{2 \cdot K + 1}{4} \right) \Delta \bar{E} \end{aligned} \quad (4)$$

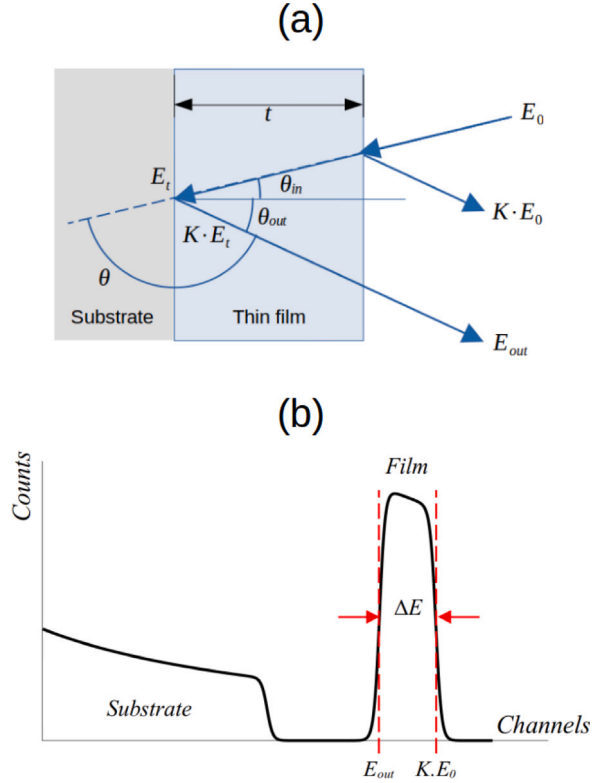


Fig. 1. Illustrative spectrum for the backscattering method of stopping cross-section determination.

with $\Delta \bar{E} = (\Delta E^A + \Delta E^B)/2$. This is an alternative definition based on the symmetrical mean energy approximation proposed in [23], which is more convenient for an experimental approach.

Both the assumption of homogeneous energy loss and the approximations used to calculate the mean energy along the ion's entry and exit paths in the film introduce systematic errors. These assumptions do not account for the non-linearity of the stopping cross-section curve. The impact of these approximations on the final result will be evaluated in Section 2.3.

2.2. Accounting for random uncertainties

The random uncertainty in this work arises primarily from the counting statistics of the backscattering spectra, which determine the precision with which the edges of the thin-film peak can be located. In practice, this translates into an uncertainty associated with the discretization of the spectrum into channels, where a uniform probability distribution is assumed within each channel when evaluating the peak width (ΔE).

From Eq. (3), we can propagate the uncertainties in the determination of ΔE_j to the values of $[\epsilon_j]$ using the definition of a covariance matrix [24]:

$$V_{\bar{A}} = (X^T V^{-1} X)^{-1} \quad (5)$$

with:

$$X = \begin{pmatrix} \frac{K}{\cos(\Theta_1)} & \frac{1}{\cos(\Theta_2)} \\ \frac{K}{\cos(\Theta_2)} & \frac{1}{\cos(\Theta_1)} \end{pmatrix} \quad (6)$$

$$V = \begin{pmatrix} \sigma_A^2 & 0 \\ 0 & \sigma_B^2 \end{pmatrix} \quad (7)$$

Let σ_A and σ_B be the uncertainties associated with the measurements of $\Delta E^A/t$ and $\Delta E^B/t$, respectively. In this work, we assumed $\sigma_A = \sigma_B = \sigma$, and its estimation is discussed in Section 2.2 in which we provide a conservative estimate of the precision with which the peak width can be determined from histogram data in the high-statistics regime.

The adjusted values of the stopping cross-section for a pair of measurements are then given by Eq. (8).

$$\begin{pmatrix} [\epsilon]_{in} \\ [\epsilon]_{out} \end{pmatrix} = (X^T V^{-1} X)^{-1} \cdot X^T \cdot V^{-1} \cdot \begin{pmatrix} \Delta E^A/t \\ \Delta E^B/t \end{pmatrix} \quad (8)$$

Note that arranging the matrix X in this format implies alternating Θ_1 and Θ_2 as angles of the entry and exit paths in two consecutive measurements at the same incidence beam energy.

Eq. (5) results in a matrix whose diagonal contains the uncertainties of our system:

$$\sigma_{[\epsilon]_{in}} = \frac{\sigma}{K} \cdot \frac{\sqrt{\sec^2(\Theta_1) + \sec^2(\Theta_2)}}{|\sec^2(\Theta_1) - \sec^2(\Theta_2)|} \quad (9)$$

$$\sigma_{[\epsilon]_{out}} = \sigma \cdot \frac{\sqrt{\sec^2(\Theta_1) + \sec^2(\Theta_2)}}{|\sec^2(\Theta_1) - \sec^2(\Theta_2)|} \quad (10)$$

and the outer diagonal results in the covariance between the $[\epsilon]_{in}$ and $[\epsilon]_{out}$ values, which is used to calculate the correlation factor r as:

$$r = \frac{\text{cov}([\epsilon]_{in}, [\epsilon]_{out})}{\sigma_{[\epsilon]_{in}} \cdot \sigma_{[\epsilon]_{out}}} = -\frac{2 \sec(\Theta_1) \sec(\Theta_2)}{\sec^2(\Theta_1) + \sec^2(\Theta_2)} \quad (11)$$

Note that the correlation factor is always negative, regardless of the combination of Θ_1 and Θ_2 .

2.2.1. Random uncertainties in $\Delta E/t$ determination

To determine the thin film peak width at half of its maximum, it is necessary to identify the channels in which the half height occurs on the trailing and falling edges. Being conservative, we can assume a uniform probability distribution within one channel, leading to $\sigma_{\text{edge}} = 1/\sqrt{12}$ (standard deviation of a uniform distribution of size 1). With ΔE calculated as the difference in channels between the trailing and falling edges (ΔC) times the gain factor G .

The uncertainty of ΔC can be calculated as:

$$\sigma_{\Delta C} = \sqrt{\left(\frac{1}{\sqrt{12}}\right)^2 + \left(\frac{1}{\sqrt{12}}\right)^2} = \frac{1}{\sqrt{6}} \quad (12)$$

Eq. (12) quantifies only the contribution of histogram discretization to the uncertainty on ΔC . It assumes that the film peak is sufficiently intense and well above the background, so that the uncertainty on the edge position is dominated by the finite channel width rather than by counting statistics.

Here, it is necessary to include a disclaimer that underlies this assumption. According to Scott [25], the parameter that quantifies how close histogrammed data ($\hat{f}(x)$) is to the true probability distribution ($f(x)$) that generates it is the Mean Integrated Squared Error (MISE) given by:

$$MISE(\hat{f}(x)) \approx \frac{1}{n \cdot h} + \frac{h^2}{12} \int_{-\infty}^{\infty} f'(x) \cdot dx \quad (13)$$

where n is the number of counts and h is the bin width. Usually, $f(x)$ is not known; however, in backscattering experiments, the simplest structure in a spectrum is a Gaussian shape. This leads to:

$$MISE(\hat{f}(x)) \approx \frac{1}{n \cdot h} + \frac{h^2}{48 \cdot \sigma^3 \cdot \sqrt{\pi}} \quad (14)$$

with σ being the standard deviation of the Gaussian shape. Minimizing the MISE leads to:

$$h \approx 3.5 \cdot \frac{\sigma}{n^{1/3}} \quad (15)$$

It is useful to rewrite this expression in terms of detector resolution (full-width at half maximum) $\Gamma = 2 \cdot \sqrt{2 \cdot \ln(2)} \cdot \sigma$.

$$h \approx 1.5 \cdot \frac{\Gamma}{n^{1/3}} \quad (16)$$

In practice, the bin width (gain) is not arbitrary: for a given detector resolution Γ and number of counts n in the peak, the bin size should be chosen close to the optimal value (Eq. (16)). Using a much smaller bin width than this optimum (i.e., oversampling well below the intrinsic detector resolution) increases the MISE and does not improve the precision on ΔE due to the increasing variance of the binned data. On the other hand, a much larger bin width also increases the MISE by introducing bias (a bias variance trade-off problem).

For practical purposes, under conditions of counts near the lower-quantification limit ($n \approx 100$), it reduces to $h \approx \Gamma/2$. Commercially available detectors have a typical resolution close to 10 keV, resulting in an optimal bin width of approximately 5 keV. In the case of higher counts in the peak, the bin width can be lower; however, this value can be considered optimal for lower counts and is a conservative value to adopt. In our experiment, the gain was set to 4.84(8) keV.

Therefore, thought experiments in which the number of channels is increased at a fixed detector resolution and statistics, or where counting statistics is increased while the bin width is kept far from the optimal value, correspond to deliberately non-optimal histogramming conditions. Our uncertainty model is not intended to describe such cases but rather realistic RBS measurements in which the bin width is chosen according to good practice, i.e., comparable to $\Gamma/2$.

Finally, accounting for uncertainties related to calibration, peak width determination, and film thickness determination, the $\Delta E/t$ uncertainty results in:

$$\sigma = \left(\frac{\Delta E}{t}\right) \sqrt{\left(\frac{\sigma_G}{G}\right)^2 + \left(\frac{G}{\Delta E \sqrt{6}}\right)^2 + \left(\frac{\sigma_t}{t}\right)^2} \quad (17)$$

2.3. Accounting for systematic error

In this work, the systematic error refers to the deviation introduced by the approximations used in the analytical method, such as assuming a constant stopping cross-section along the ion path and estimating the mean energy using simplified expressions. These deviations are quantified by comparing the analytical results with reference values obtained from the numerical integration of the SRIM stopping cross-section data.

To estimate the systematic uncertainties, we tested Eqs. (4) and (8) against the SRIM curve. The peak width, ΔE , has been computed directly using the integral (as in Eq. (18)) form and SRIM data, while the methodology described in Section 2.1 is applied to determine $[\epsilon]$ using Eq. (8). These values are then compared to the SRIM values themselves interpolated at the mean energies given by Eq. (4). The differences between the interpolated and the output of the matrix calculation are considered as systematic errors. This accounts for systematic errors inserted either by the surface approximation or by the calculation of the mean energy in the entry and exit paths.

$$\begin{aligned} E_1 &= E_0 - \int_0^{t/\cos(\theta_{in})} \left[\frac{1}{N} \frac{dE}{dx}(E(x), x) \right]_{E(0)=E_0} dx \\ E_{out} &= K \cdot E_1 - \int_0^{t/\cos(\theta_{out})} \left[\frac{1}{N} \frac{dE}{dx}(E(x), x) \right]_{E(0)=K \cdot E_1} dx \\ \Delta E &= K \cdot E_0 - E_{out} \end{aligned} \quad (18)$$

This approach is justified because SRIM data provide a reliable approximation of the stopping values in light of the current knowledge status of this process. A new measurement aims to improve this knowledge status by reducing the uncertainties. Since our objective is to minimize uncertainties, this method helps identify geometries where our assumptions remain valid.

$$s([\epsilon]_j) = [\epsilon]_j - \text{SRIM}(\bar{E}_j) \quad (19)$$

with $s([\epsilon]_j)$ being the systematic error and $\text{SRIM}(\bar{E}_j)$ being the SRIM interpolation at the energy $\bar{E}_j$. j is an index that denotes *in* or *out*.

2.4. Other uncertainty sources

In addition to the statistical and systematic uncertainties explicitly addressed in this work, several other effects contribute to the overall uncertainty in stopping cross-section measurements. These include uncertainties related to the beam energy calibration, scattering angle, and tilt angle.

Energy-loss straggling also contributes to the broadening of the backscattering peak. Although not explicitly included in our analytical propagation, its effect largely follows the same trend as the systematic deviations discussed in Section 2.3, i.e., it increases with the effective path length of ions inside the film. Thus, minimizing the systematic error in practice also minimizes the impact of straggling.

Most of the uncertainties mentioned above (particularly those related to beam energy and geometrical alignment) are type-B uncertainty components. Their explicit propagation requires a Monte Carlo approach in which these parameters are varied according to their known distributions. For example, Eq. (8) can be used in this Monte Carlo evaluation to assess the influence of the incidence and exit angle uncertainties on the final result. This will be discussed later in Section 3.

In our experimental experience and in the RBS literature, these type-B geometrical uncertainties (e.g., beam-energy calibration, scattering angle, tilt angle) are typically at least one order of magnitude smaller than the uncertainties arising from the determination of the peak width ΔE and film thickness t [9]. As a result, even a full Monte Carlo propagation of these effects would not modify the dominant uncertainty trends nor the geometry-optimization strategy proposed here.

2.5. Finding the best geometry

With the definitions of random and systematic uncertainties for the determination of $[\epsilon]$, the total uncertainty can be defined as:

$$\sigma_{total} = \sqrt{s([\epsilon]_j)^2 + \sigma_{[\epsilon]_j}^2} \quad (20)$$

Note that the common factor in the statistical uncertainty of $[\epsilon]_{in}$ and $[\epsilon]_{out}$, as in Eq. (9) and (10), can be written as the function $f(\theta_1, \theta_2)$:

$$f(\theta_1, \theta_2) = \frac{\sqrt{\sec^2(\theta_1) + \sec^2(\theta_2)}}{|\sec^2(\theta_1) - \sec^2(\theta_2)|} \quad (21)$$

which diverges when $\theta_1 \approx \theta_2$. In particular, when $\theta_1 = \theta_2$, the system in Eq. (3) becomes undetermined. $f(\theta_1, \theta_2)$ reaches a minimum at $\theta_1 = 0$ and $\theta_2 = \pi/2$ or $\theta_1 = \pi/2$ and $\theta_2 = 0$. However, as θ_1 or θ_2 approaches $\pi/2$, the length of the corresponding ion's path at entry or exit increases, leading to greater systematic errors.

Therefore, an optimal choice of θ_1 and θ_2 must balance reducing random uncertainties while also minimizing systematic errors. Since systematic uncertainty cannot be evaluated analytically, this procedure must be performed numerically.

3. Results and discussion

To illustrate the application of the method, we calculated both the random and systematic uncertainties, along with the correlation factor, to optimize an experiment aimed at measuring the electronic stopping cross-section of gold for helium ions. The adopted film thickness in these calculations was 45 nm ($\approx 266 \times 10^{15}$ at./cm²).

Observing Eqs. (3) and (18), both the random and systematic uncertainties depend on the film thickness t . In this context, the validity of the method relies on representative measurements of $\Delta E/t$. This imposes an upper limit on the incident beam energy, as it becomes constrained by the achievable depth resolution—i.e., when ΔE approaches the detector resolution, the depth information is no longer reliable. Assuming a detector resolution of 12 keV (FWHM), depth resolution

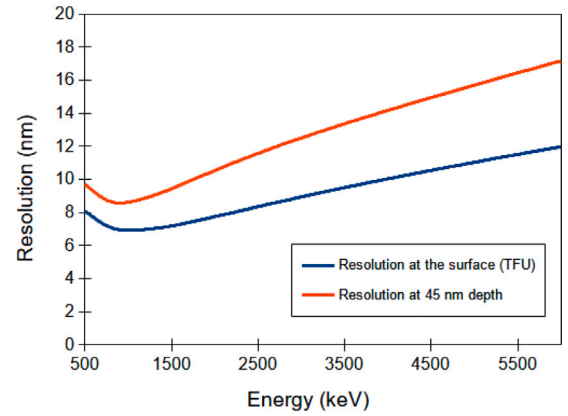


Fig. 2. Depth-resolution calculated using ResolNRA demonstrating the 45 nm thick film is suitable for stopping cross-section measurements in the energy range of this study.

calculations using ResolNRA [26] indicate that a 45 nm film is above this limitation within our energy range of 500–5100 keV, see Fig. 2.

At the highest beam energy considered (5.1 MeV), the measured width of the Au signal is about 50 keV for both geometrical configurations. The intrinsic detector resolution (FWHM) is significantly smaller than the observed peak width and therefore does not dominate the signal broadening, in agreement with ResolNRA calculations. Additional broadening may arise from energy-loss straggling, but the experimental geometries adopted in this work were chosen to minimize the associated systematic contribution.

The results for each contribution to the total uncertainty are presented in the panel of Fig. 3.

As mentioned above, the two types of uncertainty exhibit opposite behavior, and the correlation factor presents only negative values (overestimating in the stopping cross-section during the ion's entry into the film leads to underestimation during its exit, and vice versa). The correlation factor weakens if $\theta_1 > 60^\circ$ and $\theta_2 < 20^\circ$. Uncorrelated measurements are only achievable at specific angle combinations that lead to high systematic errors and are therefore not recommended.

The combination of random and systematic uncertainties yields the total uncertainties, as shown in Fig. 4. We also calculated the final uncertainties for incident energies of 500 keV and 5000 keV, with the results presented in Figs. 5 and 6, respectively.

From these results, it is clear that, to keep the total uncertainty low across the overall energy range, θ_1 should be kept between 50° and 80° , and θ_2 should be below 30° . Furthermore, it can be observed that total uncertainties below 1% are not feasible in this experiment. If higher accuracy is needed, alternative methods should be explored [9].

Finally, we performed experiments with incident energies ranging from 500 keV to 5100 keV. Typical spectral data are shown in Fig. 8. A thin film of gold deposited by magnetron sputtering on top of a silicon substrate was used. The nominal thickness was 45 nm, with the experimentally determined areal atomic density at $243.9(2.9) \times 10^{15}$ at./cm² by proton scattering ($79.5(7)$ $\mu\text{g}/\text{cm}^2$ or $41.2(3)$ nm). The low energy loss of 2.6 MeV protons (below five keV) in this film allowed for density determination using a thin film approximation (independent of energy loss models). No carbon or oxygen contamination was observed in proton scattering experiments at their respective resonant energies. No inhomogeneities were detected in sequenced measurements.

The film characterization was performed using two detectors positioned at scattering angles of 120° and 170° , respectively. The areal densities obtained from the net peak counts of the two detectors are denoted as x_1 and x_2 , with corresponding standard uncertainties σ_1 and σ_2 . These values were found to be consistent within the experimental uncertainties. Since both measurements share common uncertainty contributions, most notably the determination of the accumulated charge,

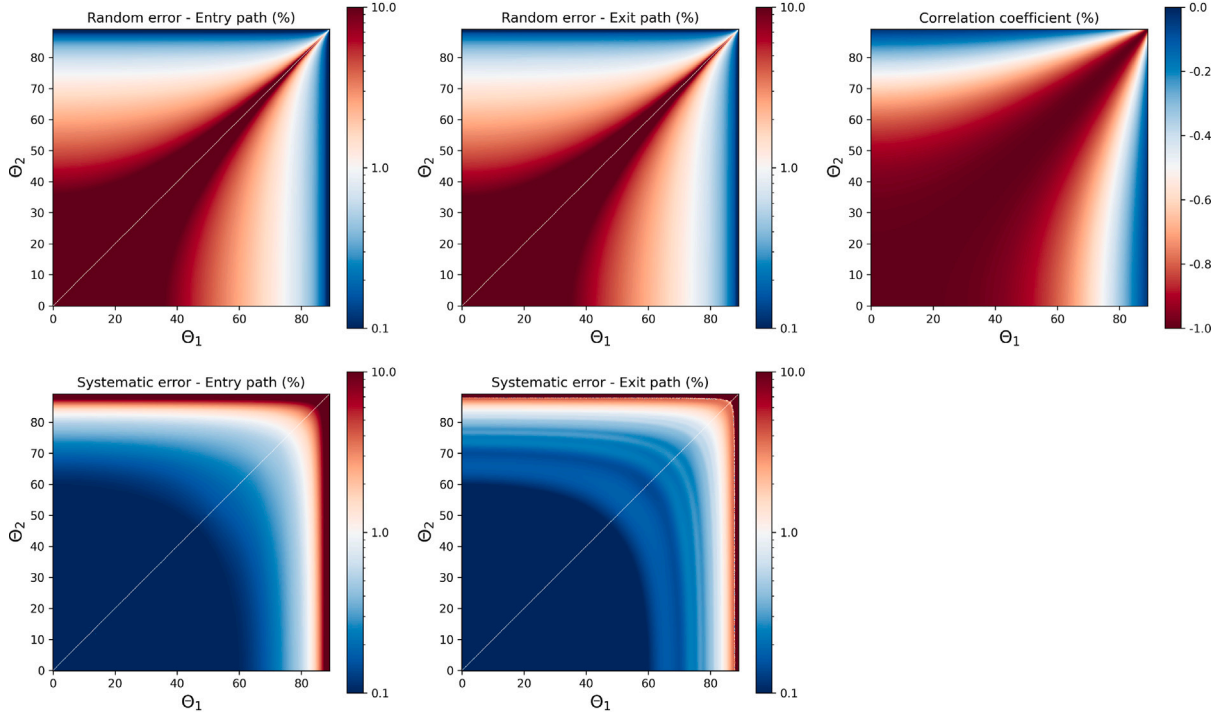


Fig. 3. Panel showing different contributions to the final uncertainty calculated assuming the measurement of helium stopping cross-section in a 3×10^{17} at./cm² (45 nm) thick gold film. The incident alpha beam energy is 1000 keV and the detector was placed at 120° scattering angle. Top line shows the relative random uncertainty calculated for the way-in (left) and way-out (center) the film, and the correlation factor of these both values (right). The bottom line shows systematic uncertainties for the way-in (left) and way-out (center) the film. Color scale in uncertainty plots are in logarithm scale and saturated at 10% for improved visualization. The color scale in the correlation factor map in linear and restricted between -1 and 0 since it is always negative.

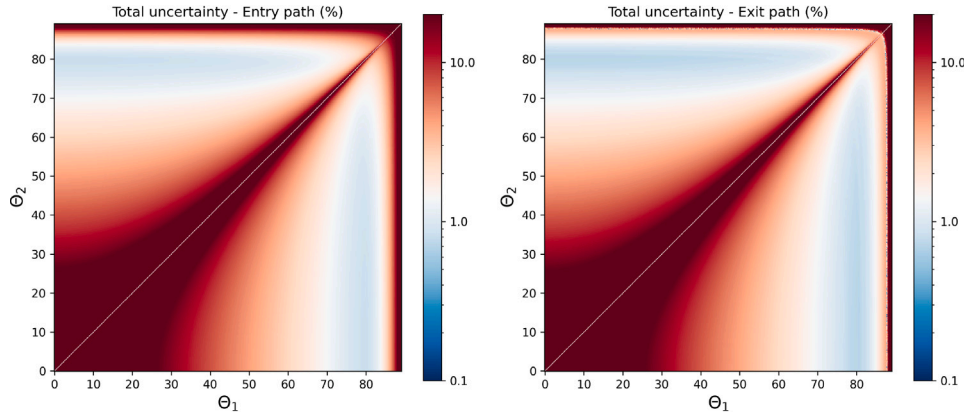


Fig. 4. Heat maps showing calculated total uncertainty assuming the measurement of helium stopping cross-section in a 3×10^{17} at./cm² (45 nm) thick gold film. The incident energy is 1000 keV and detector placed at 120° scattering angle. Total uncertainty in the way-in (left) and way-out (right). Color scale in uncertainty plots are in logarithm scale and saturated at 10% for improved visualization.

they are partially correlated. The final Au areal density was therefore obtained using a covariance-based weighted average with Eq. (22), in which common-mode and detector-specific uncertainty contributions were treated separately. The covariance between the two measurements was estimated as $\text{cov}_{12} = x_1 x_2 r_Q^2$, where r_Q is the relative uncertainty associated with the integrated charge parameter (the dominant shared contribution). Using this treatment, the combined relative uncertainty obtained from Eq. (23) is 1.2%. The corresponding uncertainty budget is presented in Table 1. Finally, the averaged areal density was used as input in a SIMNRA [27] simulation as an independent cross-check (not as a fit), showing good agreement, as illustrated in

Fig. 7.

$$\bar{x} = \frac{(\sigma_2^2 - \text{cov}_{12}) x_1 + (\sigma_1^2 - \text{cov}_{12}) x_2}{\sigma_1^2 + \sigma_2^2 - 2 \text{cov}_{12}} \quad (22)$$

$$\sigma^2(\bar{x}) = \frac{\sigma_1^2 \sigma_2^2 - \text{cov}_{12}^2}{\sigma_1^2 + \sigma_2^2 - 2 \text{cov}_{12}}. \quad (23)$$

The uncertainty associated with the charge–solid-angle product ($Q \cdot \Omega$) in Table 1 was determined through a dedicated calibration procedure based on repeated measurements under well-defined Rutherford scattering conditions. The validation experiment was performed using an amorphous silicon sample irradiated with a 1.5 MeV ⁴He beam, a

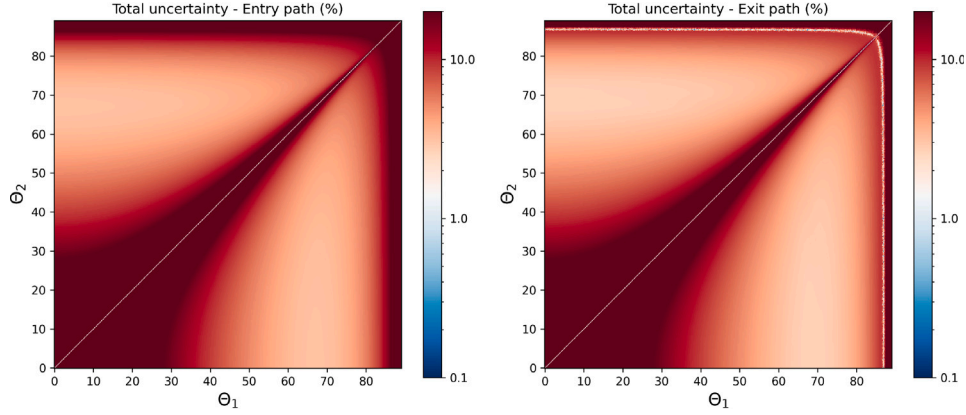


Fig. 5. Heat maps showing calculated total uncertainty assuming the measurement of helium stopping cross-section in a 3×10^{17} at./cm² (45 nm) thick gold film. The incident energy is 500 keV and detector placed at 120° scattering angle. Total uncertainty in the way-in (left) and way-out (right). Color scale in uncertainty plots are in logarithm scale and saturated at 10% for improved visualization.

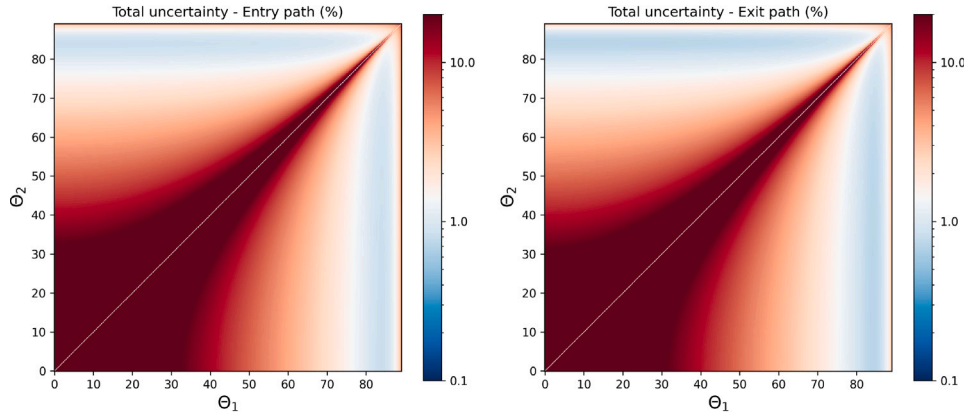


Fig. 6. Heat maps showing calculated total uncertainty assuming the measurement of helium stopping cross-section in a 3×10^{17} at./cm² (45 nm) thick gold film. The incident energy is 5000 keV and detector placed at 120° scattering angle. Total uncertainty in the way-in (left) and way-out (right). Color scale in uncertainty plots are in logarithm scale and saturated at 10% for improved visualization.

Table 1

Uncertainty budget for the gold areal density determination by proton scattering. See text for details.

Parameter	Value	Abs. Uncert.	Value	Abs. Uncert.
Energy (keV) [9]	2596.7	2.5	2596.7	2.5
Scattering angle (°)	120.0	0.3	170.0	0.3
	Value	Rel. Uncert.	Value	Rel. Uncert.
Counts	60819	0.41%	9516	1.0%
Part*sr (msr)	1.16E+11	1.0%	3.30E+10	1.0%
Cross-section (mb) [28]	2118.29	0.6%/Scatt. 0.23%/En. 0.16%/Screen. 0.65%/Total	1209.83	0.11%/Scatt. 0.23%/En. 0.16%/Screen. 0.26%/Total
Au areal dens. (10 ¹⁵ at./cm ²)	247	1.3%	239	1.5%
Common-mode covariance	$\text{cov}_{12} = 247 \times 239 \times (0.01)^2 = 5.90$			
Average Au areal density			243.9	1.2%

condition for which the stopping power accuracy is estimated to be approximately 0.8% [28]. For each detector, a series of spectra was acquired at fixed beam energy using multiple values of integrated charge, and the yield parameter $Y = Q \cdot \Omega$ was extracted for each acquisition. The random contribution to the uncertainty was obtained from the standard deviation of repeated measurements at each charge value, thereby directly accounting for the stability and reproducibility of the charge integration system. Systematic contributions were incorporated

by propagating the uncertainties associated with the stopping power (0.8%) and the scattering cross-section (0.7% at 170° and 0.3% at 120°, accounting for energy uncertainty of 1.5-keV, angular uncertainty of 0.3° and screening [28]), which were explicitly included in the uncertainty expression for Y . Additional contributions arising from the linear fit parameters and from the minimum measurable integrated charge of the current integrator were also taken into account in the systematic uncertainty. The resulting combined relative uncertainty of

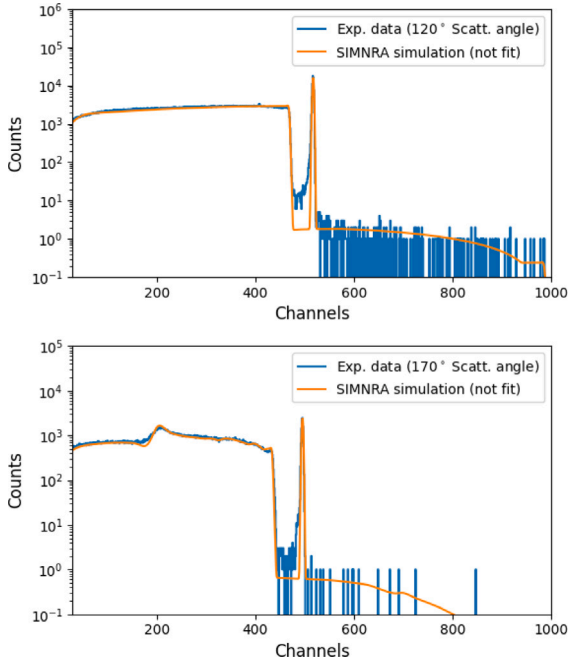


Fig. 7. Experimental data for the proton scattering experiment for target characterization. Gold areal density was determined using net peak area and SIMNRA simulation was used as a cross-check.

the $Q \cdot \Omega$ parameter is approximately 1% at $Q = 10\text{-}\mu\text{C}$ integrated charge, as in this work.

The energy reference of our accelerator was calibrated against the evaluated elastic-scattering cross-sections provided by SigmaCalc [29, 30], following a dedicated analysis as described in Ref. [9].

The set of angles was chosen as $\theta_1 = 60^\circ$ and $\theta_2 = 0^\circ$, following the constraints of our experimental setup. Within the uncertainty framework defined in Sections 2.2 and 2.3, this choice results in a maximum total uncertainty of approximately 3% for measurements performed at an incident energy of 1000 keV (which is close to the maximum of the stopping cross-section curve). This value is subsequently updated to include additional uncertainty contributions, and therefore increases accordingly. The correlation factor associated with this angular configuration is approximately 0.7.

To quantify the impact of geometrical uncertainties under realistic alignment conditions, we performed a Monte Carlo propagation in which independent Gaussian perturbations with 0.3° standard deviation were applied to each relevant angle in both measurements. The resulting mean total uncertainty changed from 4.63% to 4.64%, indicating a negligible contribution at this level and confirming that the uncertainty budget is dominated by peak-width and thickness terms in the present experiment. A sensitivity study was performed by propagating angular uncertainties of 0.3° , 0.5° and 1.0° via Monte Carlo. The resulting mean uncertainties changed from 4.63% to 4.64%, 4.65% and 4.69%, respectively, confirming the negligible impact of geometrical misalignment within this range.

Fig. 9 shows the experimental data obtained using the method described in 2.1 with a combination of angles chosen according to our approach. The results are compared to SRIM [20], DPASS [21], and ICRU-49 [22] data. The black points in the plot represent additional experimental data from the IAEA database [31]. In the upper plot, we observe the overall agreement of our data with the semi-empirical models, while in the bottom plot, we display the relative residuals with SRIM as the reference. We provide densely spaced experimental data to better define the statistical limits of the method. At lower energies, it is evident that all experimental data fall within 4% of the residuals

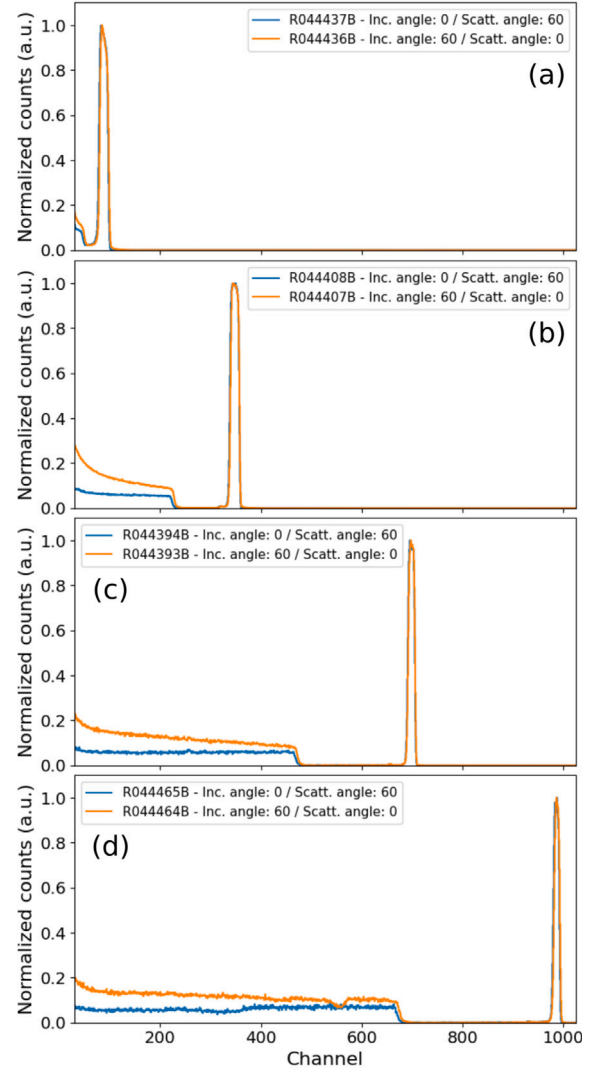


Fig. 8. Representative experimental data used for the determination of the He-Au stopping cross-section. Each panel displays two reciprocal geometries: beam incidence at 0° with detection at 60° (blue), and incidence at 60° with detection at 0° (orange). Angles measured relative to the sample surface normal. The beam energies are (a) 593 keV, (b) 1901 keV, (c) 3695 keV, and (d) 5199 keV.

amplitude, a value that is very close to the final uncertainty calculated using the method proposed here.

In the residuals plot in Fig. 9, our experimental data were labeled accordingly to determine whether they were obtained using the entry path (green) or the exit path (red). We observe great consistency in the data determined by both paths.

The comparison with the models shows good overall agreement within the experimental uncertainty bars for both SRIM [20] and ICRU-49 [22], with the latter providing a better description of the fine structure observed in the residuals. In contrast, DPASS [21] exhibits the largest deviations over the investigated energy range, reaching discrepancies of up to 17% with respect to the experimental data.

Interestingly, the magnitude of the resulting experimental uncertainties is comparable to the spread observed among independent measurements reported in the IAEA stopping power database [31]. For helium ions above 1 MeV, the available experimental data show a spread of 5.25%, which is consistent with the uncertainty budget obtained in the present work.

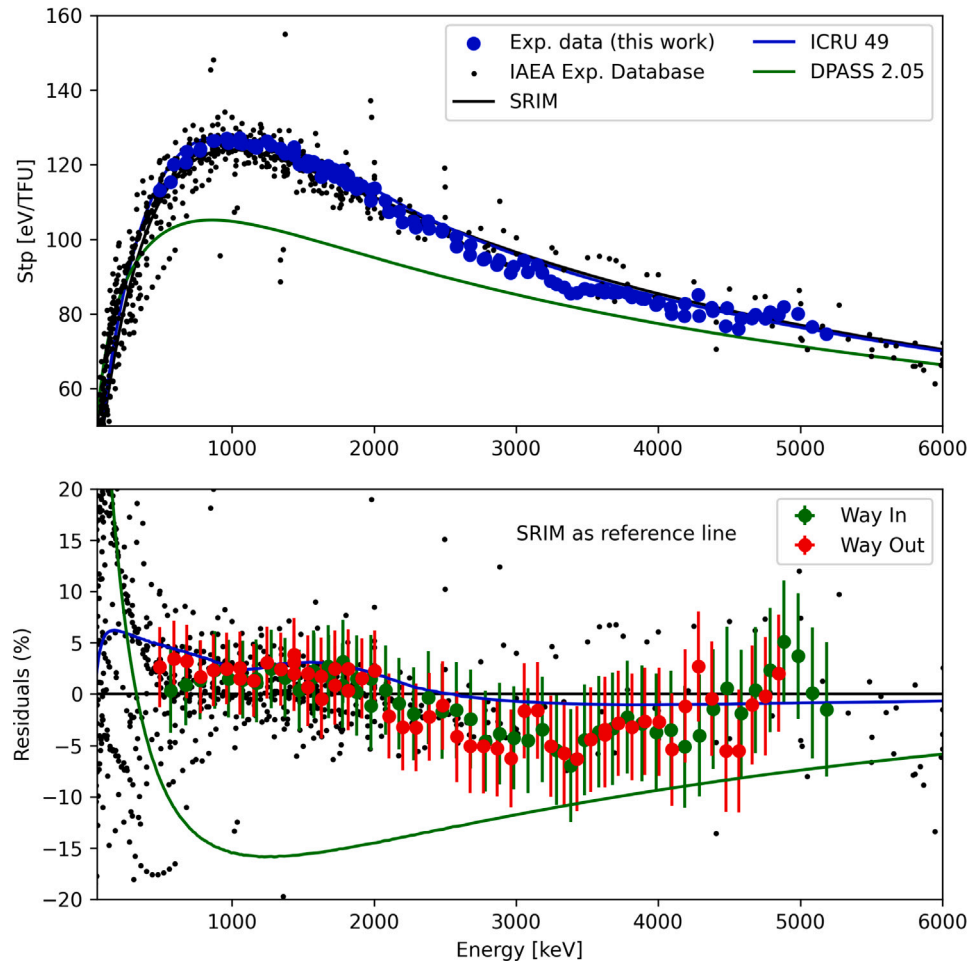


Fig. 9. Stopping cross-section measured for gold to helium ions (upper plot) and relative residuals calculated using SRIM as reference (bottom plot). The geometry used was defined using the methodology reported here. $1 \text{ TFU} = 1 \times 10^{15} \text{ at./cm}^2$. In the residuals, data was labeled according to the way used in the average, i.e. way in or way out the thin film.

4. Conclusions

In this work, we have presented a novel methodology for optimizing the measurement of stopping cross-sections using backscattering data, offering an accurate and reliable approach for determining the stopping cross-section in thin films. Our method, which is based on a systematic treatment of uncertainties, provides a robust framework for minimizing random and systematic errors while optimizing geometries for precise measurements.

It is worth noting that energy straggling, although not explicitly included, is expected to follow the same angular dependence as the systematic uncertainties; thus, its effect is indirectly minimized by the present optimization strategy.

The experimental results, which were obtained for helium in gold over a wide range of incident energies, show good agreement with well-established semi-empirical models, such as SRIM [20], DPASS [21], and ICRU-49 [22], and highlight the effectiveness of our approach in achieving uncertainties that are close to the theoretical limits. The careful selection of geometries and the use of densely spaced data points were crucial in refining the statistical accuracy of our measurements, particularly at lower energies. The final average uncertainty obtained was 4.7% with a minimum of 3.5% and a maximum of 6.6%, within the uncertainty framework defined in this work.

Although total uncertainties below 1% were not feasible in this experiment, the method provides a reliable tool for planning stopping cross-section measurements, allowing for better restriction of the spread in the obtained experimental values.

CRediT authorship contribution statement

Victor Pires: Writing – review & editing, Writing – original draft, Software, Methodology, Investigation, Formal analysis. **Arilson Silva:** Writing – review & editing, Writing – original draft, Validation, Investigation, Formal analysis. **Cleber L. Rodrigues:** Writing – review & editing, Writing – original draft, Validation, Investigation, Data curation. **Nemitala Added:** Writing – review & editing, Writing – original draft, Validation, Data curation. **Manfredo H. Tabacniks:** Writing – review & editing, Writing – original draft, Validation, Investigation, Data curation. **Tiago F. Silva:** Writing – review & editing, Writing – original draft, Validation, Supervision, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Flávio Matias:** Writing – review & editing, Writing – original draft, Validation, Investigation, Data curation. **Helio Yoriyaz:** Writing – review & editing, Writing – original draft, Validation, Investigation, Data curation. **Julian M.B. Shorto:** Writing – review & editing, Writing – original draft, Investigation, Data curation.

Declaration of Generative AI and AI-assisted technologies in the writing process

During the preparation of this work, the authors used ChatGPT by OpenAI to improve language and readability. After using this tool, the authors reviewed and edited the content as needed and assume full responsibility for the content of the publication.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Tiago F. Silva reports financial support was provided by National Council for Scientific and Technological Development. Flavio Matias reports financial support was provided by Brazilian Nuclear Energy Commission. If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

The authors thank the financial support provided by CNPq-INCT-FNA (project number 464898/2014-5). TFS acknowledges the Brazilian funding agency CNPq (project number 406982/2021-0). FM acknowledges IPEN/CNEN (Project No. 2020.06.IPEN.32). This research used resources from the Laboratory for Materials Analysis with Ion Beams - LAMFI-USP, of the University of São Paulo. The authors acknowledge the laboratory staff for their assistance during the experiments.

Data availability

Data will be made available on request.

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