ON THE DIFFUSION OF NEUTRONS IN MODERATING AND MULTIPLYING MEDIA WITH A PERIODIC NEUTRON SOURCE

SÔBRE A DIFUSÃO DE NEUTRONS EM MEIOS MODERADORES E MULTIPLICADORES, COM UMA FONTE PERIÓDICA DE NEUTRONS

GERHARD JACOB

Publicação I E A — N.º 12
— 1958 —

INSTITUTO DE ENERGIA ATÔMICA
Caixa Postal 11049 (Pinheiros)
CIDADE UNIVERSITÁRIA "ARMANDO DE SALLES OLIVEIRA"
SÃO PAULO — BRASIL

CONSELHO NACIONAL DE PESQUISAS

Presidente — Prof. Dr. João Christovão Cardoso
Vice-Presidente — Prof. Dr. Athos da Silveira Ramos

UNIVERSIDADE DE SÃO PAULO

Reitor — Prof. Dr. Gabriel Sylvestre Teixeira de Carvalho
Vice-Reitor — Prof. Dr. Francisco João Humberto Maffei

INSTITUTO DE ENERGIA ATÔMICA

DIRETOR
Prof. Dr. Marcello Damy de Souza Santos

CONSELHO TÉCNICO-CIENTÍFICO

Representantes do Conselho Nacional de Pesquisas
Prof. Dr. Luiz Cintra do Prado
Prof. Dr. Paulus Aulus Pompéia

Representantes da Universidade de São Paulo
Prof. Dr. Francisco João Humberto Maffei
Prof. Dr. José Moura Gonçalves

CONSELHO DE PESQUISAS

Prof. Dr. Marcello Damy de Souza Santos
— Chefe da Divisão de Física Nuclear
Prof. Eng. Paulo Saraiva de Toledo
— Chefe da Divisão de Física de Reatores
Prof. Dr. Fausto Walter Lima
— Chefe da Divisão de Radioquímica
Prof. Dr. Rômulo Ribeiro Pieroni
— Chefe da Divisão de Radiobiologia

Reprint from 2nd UN Geneva Conference.
Printed by Pergamon Press, London.

P/2272 Brazil

On the Diffusion of Neutrons in Moderating and Multiplying Media with a Periodic Neutron Source

By Gerhard Jacob*

1. INTRODUCTION

Recently, Gallone, Orsoni and Salvetti in a series of articles[1–4] used symbolic calculation methods to study the diffusion and multiplication of neutrons. In particular, they studied the space and time distribution of thermal neutrons in multiplying media with several types of fast neutron source as, for instance, an instantaneous source and a constant source acting for a finite time. In this paper, we consider the case of a point source of fast neutrons with strength varying periodically in time (using Fourier analysis), and explicit calculations are carried out for triangular, half-wave sinusoidal, and rectangular fast neutron bursts in spherical, cylindrical and parallelepipedal, multiplying or moderating media.

Initially, we recall the general method developed by the authors quoted above. They consider a homogeneous multiplying medium where both fission and external source neutrons are monoenergetic. Resonance capture only is taken into account, diffusion and age theory approximations are used and the usual energy-independent boundary conditions (vanishing of neutron density at the extrapolated boundary) are supposed valid. The slowing down time is not taken into account (moderation is considered instantaneous), the mean free path of the emergent source neutrons is neglected and fission neutrons are all considered as instantaneous.

Under these assumptions, the age-diffusion equation for the neutron density in a multiplying medium, with an external fast neutron source $S_0(\mathbf{r}, t)$, assuming $\rho(\mathbf{r}, 0)=0$, may be written[4]:

$$L^2\nabla^2\rho(\mathbf{r}, t)+\{k_\infty \exp[\tau\nabla^2]-1\}\rho(\mathbf{r}, t) + pl_0 \exp[\tau_0\nabla^2]S_0(\mathbf{r}, t)=l_0\frac{\partial\rho(\mathbf{r}, t)}{\partial t}, \qquad (1)$$

where L is the diffusion length of thermal neutrons in the medium, $\rho(\mathbf{r}, t)$ the thermal neutron density, k_∞ the infinite multiplication factor, l_0 the mean life of thermal neutrons in the pure moderator, p the resonance escape probability and τ and τ_0 are the ages to thermal of fission and external source neutrons respectively. The resolvent operator $\exp[\alpha\nabla^2]$ is defined as follows: Let $f(\mathbf{r})$ be a function which may be expanded in a series of the complete orthonormal set of eigenfunctions of the equation

$$\nabla^2\phi_n(\mathbf{r})+\omega_n^2\phi_n(\mathbf{r})=0$$

relative to the boundary condition

$$[\phi_n(\mathbf{r})]_\sigma=0,$$

σ being the extrapolated boundary of the medium. Then

$$\exp[\alpha\nabla^2]f(\mathbf{r})=\exp[\alpha\nabla^2]\left\{\sum_{n=0}^{\infty}a_n\phi_n(\mathbf{r})\right\}$$

$$=\sum_{n=0}^{\infty}a_n \exp[-\alpha\omega_n^2]\phi_n(\mathbf{r}).$$

Now, to solve (1) for a medium bounded by σ, the neutron density and the external source are expanded in a series of the above $\phi_n(\mathbf{r})$:

$$\rho(\mathbf{r}, t)=\sum_{n=0}^{\infty}a_n(t)\phi_n(\mathbf{r}), \qquad (2)$$

$$S_0(\mathbf{r}, t)=\sum_{n=0}^{\infty}b_n(t)\phi_n(\mathbf{r}), \qquad (3)$$

where $a_n(t)$ and $b_n(t)$ are the Fourier transforms

$$a_n(t)=\int_{-\infty}^{\infty}A_n(\nu)\exp[i\nu t]\,d\nu, \qquad (4)$$

⑥

$$b_n(t)=\int_{-\infty}^{\infty}B_n(\nu)\exp[i\nu t]\,d\nu. \qquad (5)$$

The solution of (1) is then:

$$\rho(\mathbf{r}, t)=pl_0\sum_{n=0}^{\infty}\int_{-\infty}^{\infty}\frac{B_n(\nu)\exp[-\tau_0\omega_n^2]\phi_n(\mathbf{r})\exp[i\nu t]}{\Phi(\omega_n)+i\nu l_0}\,d\nu \qquad (6)$$

where

$$\Phi(\omega_n)=1+\omega_n^2L^2-k_\infty \exp[-\tau\omega_n^2]. \qquad (7)$$

Thus, knowing the external source $S_0(\mathbf{r}, \mathrm{t})$, it is possible to determine the expansion coefficients $b_n(t)$,

---

[*] Instituto de Energia Atomica, São Paulo. (On leave of absence from Faculdade de Filosofia da Universidade do Rio Grande do Sul.)

then $B_n(\nu)$ and afterwards the thermal neutron density $\rho(\mathbf{r}, t)$.

For the sake of completeness, we list below the normalized eigenfunctions and eigenvalues for the three more common geometries:

(*a*) Sphere of radius R and geometrical center at the origin of the coordinate system:

$$\phi_n(\mathbf{r}) = (2\pi R)^{-\frac{1}{2}} \frac{\sin \omega_n \mathbf{r}}{\mathbf{r}}, \quad \omega_n = \frac{n\pi}{R}.$$

(*b*) Circular cylinder of radius R and height $2H$ and geometrical center at the origin of the coordinate system:

$$\phi_{mn}(\mathbf{r}, z) = [RJ_1(\omega_n R)]^{-1}(\pi H)^{-\frac{1}{2}} J_0(\omega_n \mathbf{r}) \cos \omega_m z,$$

$$\omega_n = R^{-1} j_0^{(n)}, \quad \omega_m = (2H)^{-1}\pi(2m+1),$$

$$\omega_{mn}{}^2 = \omega_m{}^2 + \omega_n{}^2,$$

where $J_i(x)$ is the Bessel function of first kind and order i and $j_0^{(n)}$ is the nth root of $J_0(x) = 0$.

(*c*) Rectangular parallelepiped with dimensions $2a$, $2b$, $2c$, and geometrical center at the origin of the coordinate system:

$$\phi_{lmn}(x, y, z) = (abc)^{-\frac{1}{2}} \cos \omega_l x \cos \omega_m y \cos \omega_n z,$$

$$\omega_l = (2l+1)(2a)^{-1}\pi, \quad \omega_m = (2m+1)(2b)^{-1}\pi,$$

$$\omega_n = (2n+1)(2c)^{-1}\pi, \quad \omega_{lmn}{}^2 = \omega_l^2 + \omega_m{}^2 + \omega_n{}^2.$$

2. POINT SOURCE WITH STRENGTH OSCILLATING IN TIME

We are interested in solving (6) for an external point source of strength S_0 oscillating in time with frequency ω and starting at $t = 0$, i.e.,

$$S_0(\mathbf{r}, t) = S_0 \delta(\mathbf{r}) \exp[i\omega t]\, H(t) \tag{8}$$

where $\delta(\mathbf{r})$ is the three-dimensional delta (Dirac) function and $H(t)$ is the unit step (Heaviside) function as defined in Ref. 5. This case is a simple one and is useful to exemplify the method of calculation that will be followed in the more realistic cases considered later. Substituting (8) in (3), it follows, by the usual method of determination of coefficients and by the definition of the $\delta(\mathbf{r})$:

$$b_n(t) = S_0 \phi_n(0) \exp[i\omega t]\, H(t)$$

(we recall that $\phi_n(\mathbf{r})$ is a real function in the usual cases). Therefore, taking the inverse transform of (5) with the result obtained for $b_n(t)$ we have

$$B_n(\nu) = (2\pi)^{-1} S_0 \phi_n(0) \int_0^\infty \exp[i(\omega - \nu)t']\, dt'.$$

Substituting in (6):

$$\rho(\mathbf{r}, t) = (2\pi)^{-1} p l_0 S_0 \sum_{n=0}^{\infty} \phi_n(0)\phi_n(\mathbf{r}) \exp[-\tau_0 \omega_n{}^2]$$

$$\times \int_{-\infty}^{\infty} \left\{ \frac{\exp[i\nu t]}{(\Phi\omega_n) + i\nu l_0} \int_0^\infty \exp[i(\omega - \nu)t']\, dt' \right\} d\nu.$$

Changing the order of integration and recalling the definition of the unit step function, we have

$$\rho(\mathbf{r}, t) = p S_0 \sum_{n=0}^{\infty} \phi_n(0)\phi_n(\mathbf{r}) \exp[-\tau_0 \omega_n{}^2] \exp[i\omega t]$$

$$\times \int_{-\infty}^{t} \exp\left[-\left\{ i\omega + \frac{\Phi(\omega_n)}{l_0} \right\} y \right] H(y)\, dy$$

where $y = t - t'$. Thus, integrating:

$$\rho(\mathbf{r}, t) = p l_0 S_0 \sum_{n=0}^{\infty} \phi_n(0)\phi_n(\mathbf{r}) \frac{\exp[-\tau_0 \omega_n{}^2]}{\Phi(\omega_n) + i\omega l_0}$$

$$\times \left\{ \exp[i\omega t] - \exp\left[-\frac{\Phi(\omega_n)}{l_0} t \right] \right\}. \tag{9}$$

Equation (9) constitutes the complete solution of our problem.

In the limiting case of infinite period ($\omega = 0$), which is equivalent to considering a source of constant strength acting during an infinite time, our expression (9) agrees with the result quoted in Ref. 4 for $\Delta t \to \infty$ and $t < \Delta t$.

Now, if the medium is critical, we have $\Phi(\omega_0) = 0$ and $\Phi(\omega_n) > 0$ for $n \neq 0$ (ω_0 being the lowest non-zero eigenvalue) so that all terms of our solution (9) except the first decrease exponentially in time. Thus, after a time large enough for the higher harmonics to die out, only the first harmonic remains; and $\rho(\mathbf{r}, t)$ is given by:

$$\rho(\mathbf{r}, t) = (i\omega)^{-1} p S_0 \phi_0(0) \phi_0(\mathbf{r}) \exp[-\tau_0 \omega_0{}^2]$$

$$\times \{\exp[i\omega t] - 1\} \qquad \text{for } t \geqslant 0. \tag{10}$$

Of course, only the real parts or the coefficients of the imaginary parts of expressions (8), (9) and (10) have physical meaning.

3. POINT SOURCE OF ARBITRARY TIME PERIODICAL SHAPE

Now, we are interested in solving (6) in the case of an external point source, which is periodical in time and starts at $t = 0$.

Quite generally, we shall consider a source of the form

$$S_0(\mathbf{r}, t) = S_0 \delta(\mathbf{r}) f(t) H(t) \tag{11}$$

where $f(t)$ is any periodic function of time which can be expanded in a Fourier series. Specifically, we want to take the case of a periodic source with period T, which is "on" during a time aT ($0 \leqslant a \leqslant 1$) and is "off" during the remainder of the period $(1 - a)T$. Thus

$$f(t) = \tfrac{1}{2} c_0 + \sum_{n=1}^{\infty} [c_m \cos m\omega t + d_m \sin m\omega t]. \tag{12}$$

In the usual way, it can be shown that, for those cases which are more important in practice (because of their easier experimental realization), the coefficients have the following values:

(*a*) Triangular bursts (as shown in Fig. 1a for $a = \frac{1}{2}$)

$$
\begin{aligned}
c_0 &= a \\
c_m &= (m^2\pi^2 a)^{-1}[2\cos ma\pi - \cos 2ma\pi - 1] \qquad (13)\\
d_m &= (m^2\pi^2 a)^{-1}[2\sin ma\pi - \sin 2ma\pi].
\end{aligned}
$$

(*b*) Half-wave sinusoidal bursts (as shown in Fig. 1b for $a=\frac{1}{2}$)

$$
\begin{aligned}
c_0 &= 4a\pi^{-1} \\
c_m &= 2a[\pi(1-4m^2a^2]^{-1}[\cos 2ma\pi + 1] \qquad (14)\\
d_m &= 2a[\pi(1-4m^2a^2)]^{-1}\sin 2ma\pi.
\end{aligned}
$$

(*c*) Rectangular bursts (as shown in Fig. 1c for $a=\frac{1}{2}$)

$$
\begin{aligned}
c_0 &= 2a \\
c_m &= (m\pi)^{-1}\sin 2ma\pi \qquad (15)\\
d_m &= (m\pi)^{-1}[1-\cos 2ma\pi]
\end{aligned}
$$

Now, substituting the expression (11) for the source $S_0(\mathbf{r}, t)$ in (3) we have, by the usual method:

$$b_n(t) = S_0 H(t) f(t)\phi_n(0)$$

and thus, reversing the Fourier transform (5),

$$B_n(\nu) = (2\pi)^{-1}S_0\phi_n(0)\int_0^\infty f(t')\exp[-i\nu t']\,dt'.$$

Substituting the expression (12) for $f(t)$ we have:

$$
\begin{aligned}
B_n(\nu) = (2\pi)^{-1}S_0\phi_n(0)\Bigg\{\frac{c_0}{2}\int_0^\infty \exp[-i\nu t']\,dt' \\
+\sum_{m=1}^{\infty}\Bigg(c_m\int_0^\infty \cos m\omega t' \exp[-i\nu t']\,dt' \\
+ d_m\int_0^\infty \sin m\omega t' \exp[-i\nu t']\,dt'\Bigg)\Bigg\}.
\end{aligned}
$$

Thus, expression (6) for the neutron density becomes:

$$
\begin{aligned}
\rho(\mathbf{r}, t) = (2\pi)^{-1}pl_0S_0\sum_{n=0}^{\infty}\phi_n(0)\phi_n(\mathbf{r})\exp[-\tau_0\omega_n^2] \\
\times\left\{\frac{c_0}{2}u(\omega_n, t) + \sum_{m=1}^{\infty}[c_m v_m(\omega_n, t) + d_m w_m(\omega_n, t)]\right\} \text{ for } t \geqslant 0 \qquad (16)
\end{aligned}
$$

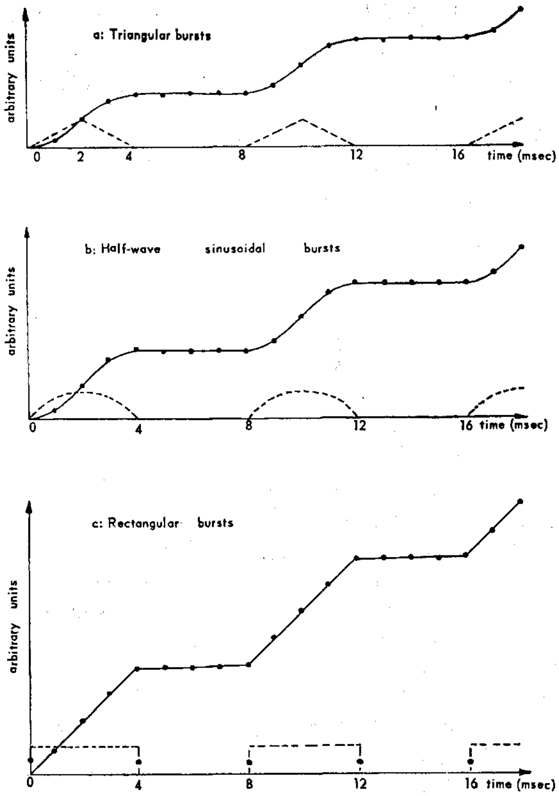
where

$$u(\omega_n, t) = \int_{-\infty}^{\infty}\left\{\frac{\exp[i\nu t]}{\Phi(\omega_n)+i\nu l_0}\int_0^\infty \exp[-i\nu t']\,dt'\right\}d\nu, \qquad (17)$$

$$
\begin{aligned}
v_m(\omega_n, t) = \int_{-\infty}^{\infty}\Bigg\{\frac{\exp[i\nu t]}{\Phi(\omega_n)+i\nu l_0} \\
\times\int_0^\infty \exp[-i\nu t']\cos m\omega t'\,dt'\Bigg\}d\nu, \qquad (18)
\end{aligned}
$$

$$
\begin{aligned}
w_m(\omega_n, t) = \int_{-\infty}^{\infty}\Bigg\{\frac{\exp[i\nu t]}{\Phi(\omega_n)+i\nu l_0} \\
\times\int_0^\infty \exp[-i\nu t']\sin m\omega t'\,dt'\Bigg\}d\nu. \qquad (19)
\end{aligned}
$$

The integrations (17), (18) and (19) may be performed in the same way as indicated before, i.e., changing the order of integration and using the definition of the unit step function; we just quote the results:



2272

Figure 1. Time behavior of source-term $S_0(t)$ and neutron density $\varrho(t)$ for critical media: $S_0(t)$, – – –, $\varrho(t)$, ———

$$u(\omega_n, t) = 2\pi[\Phi(\omega_n)]^{-1}\left\{1-\exp\left[-\frac{\Phi(\omega_n)}{l_0}t\right]\right\}, \qquad (20)$$

$$
\begin{aligned}
v_m(\omega_n, t) = 2\pi[\Phi^2(\omega_n)+m^2\omega^2 l_0^2]^{-\frac{1}{2}}\Bigg\{\cos(m\omega t-\theta_{mn}) \\
-\exp\left[-\frac{\Phi(\omega_n)}{l_0}t\right]\cos\theta_{mn}\Bigg\}, \qquad (21)
\end{aligned}
$$

$$
\begin{aligned}
w_m(\omega_n, t) = 2\pi[\Phi^2(\omega_n)+m^2\omega^2 l_0^2]^{-\frac{1}{2}} \\
\times\left\{\sin(m\omega t-\theta_{mn})+\exp\left[-\frac{\Phi(\omega_n)}{l_0}t\right]\sin\theta_{mn}\right\} \qquad (22)
\end{aligned}
$$

where

$$\cos\theta_{mn} = \Phi(\omega_n)[\Phi^2(\omega_n)+m^2\omega^2 l_0^2]^{-\frac{1}{2}} \qquad (23)$$

and

$$\sin\theta_{mn} = m\omega l_0[\Phi^2(\omega_n)+m^2\omega^2 l_0^2]^{-\frac{1}{2}}. \qquad (24)$$

Equation (16), together with Eqs. (20), (21), (22), (23), (24), constitutes the complete solution of the problem.

In the particular case in which $f(t)=\exp[i\omega t]$, i.e., in the case discussed in Section 2, an easy calculation shows that we obtain expression (9), as it had to be.

If, in the case of rectangular bursts, we let the period go to infinity ($\omega\rightarrow 0$), and take $a=1$, we arrive again at the results quoted in Ref. 4.

Finally, in the limiting case of criticality, we have, as in Section 2, $\Phi(\omega_0)=0$ and $\Phi(\omega_n)>0$ for $n\neq 0$ and so, after a time large enough for the higher harmonics to die out, the first harmonic of our solution (16) may be written:

$$\rho(\mathbf{r},t)=pS_0\phi_0(0)\phi_0(\mathbf{r})\exp[-\tau_0\omega_0^2] \times\left\{\frac{C_0}{2}t+\sum_{m=1}^{\infty}(m\omega)^{-1}[c_m\sin m\omega t + d_m(1-\cos m\omega t)]\right\} \quad \text{for } t\geqslant 0. \tag{25}$$

Figure 1 shows the time behavior of the neutron density in a critical medium, with the source term given by triangular bursts (*a*), half-wave sinusoidal bursts (*b*) and rectangular bursts (*c*) respectively, and $\alpha=\frac{1}{2}$. The period was taken as 8 milliseconds and the points indicate calculated values taking into account terms up to $m=8$ in the Fourier series.

Of course, the results obtained are also applicable to pure moderating media, taking $p=1$ and $k_\infty=0$.

4. INTERPRETATION OF RESULTS

It is of interest to analyze in some detail the time dependence of the neutron density in the expressions obtained above.

To start with, let us take the case of a critical medium with the source term given, for instance, by triangular bursts (Fig. 1a).

We see that the neutron density rises between $t=0$ and $t=\frac{1}{2}T$ (in general, between $t=0$ and $t=\alpha T$), i.e., while the source is "on", and remains constant in the remainder of the period, i.e., while the source is "off". Besides this, the curve shows an inflection point at $t=\frac{1}{4}T$ (in general, at $t=\frac{1}{2}\alpha T$), which arises because the source term has a maximum at this point.

The conclusions arrived at may also be drawn from the fact that, for a critical medium, we must have

$$\rho(t)=\int_0^t S_0(t')\,dt'$$

for each instant of time t.

Alternatively, in the case of a subcritical medium (and in particular, of a pure moderating medium), we have $\rho(t)<\int_0^t S_0(t')\,dt'$, so that the rise of the neutron density will be slower, there will be a maximum at the left of $t=\alpha T$ (this maximum being lower than the constant value in the previous case, and arising earlier because for subcritical media the neutron leakage will not be compensated by source plus fission neutrons) and afterwards, instead of remaining constant, the neutron density will decrease to a minimum (negligible for periods T large compared with the neutron lifetime l_0).

Now let us analyze, at least qualitatively, the influence of the delayed neutrons in the case of a critical and a subcritical medium with the external source considered above.

In the case of a critical medium, because there is an equilibrium at each instant of time between the neutrons which leak out of the medium and those which are produced by both fissions and external source if all neutrons are considered instantaneous, the effect of the delay of a small part of the fission neutrons will be to destroy this instantaneous equilibrium. Thus, the rise of the neutron density will be a little slower between $t=0$ and $t=\alpha T$ and from $t=\alpha T$ on, the neutron density will continue to rise instead of remaining constant, going asymptotically to the constant value shown in Fig. 1a if the period is large compared with the mean life of the longer-lived delayed neutron group.

In the case of a subcritical medium, by similar considerations, we can see that because of the delayed neutrons the increase of the neutron density will be slower, its maximum displaced to the right (eventually to the right of the point $t=\alpha T$) and the decrease of the neutron density will also be slower. And if the period is sufficiently large compared with the mean life of the longer-lived delayed neutron group, the neutron density will go asymptotically to zero. The quantitative analysis of the effect of the delayed neutrons will be the subject of a further paper.

Finally, we want to remark that the expressions obtained will be useful in the experimental study of the properties of moderating and multiplying media using periodic neutron sources instead of the impulse method used up to now.[6–9]

ACKNOWLEDGEMENTS

Thanks are due to Prof. Marcelo Damy de Souza Santos, who first suggested the study of this problem and especially to Prof. Paulo Saraiva de Toledo, for many valuable remarks and constant orientation.

I also want to thank Prof. Darcy Dillenburg for many helpful discussions and Mr. Eduardo Wilner for his collaboration in the numerical calculations.

REFERENCES

1. S. Gallone and C. Salvetti, Nuovo cimento, *7*, 482 (1950).
2. S. Gallone and C. Salvetti, Nuovo cimento, *7*, 626 (1950).
3. S. Gallone and C. Salvetti, Nuovo cimento, *7*, 901 (1950).
4. S. Gallone, L. Orsoni and C. Salvetti, Nuovo cimento, *8*, 960 (1951).
5. P. M. Morse and H. Feshbach, *Methods of Theoretical Physics*, McGraw-Hill Book Co., Inc., *1*, 414 (1953).
6. A. V. Antonov, A. I. Isakoff, I. D. Murin, B. A. Neupocoyev, I. M. Frank, F. L. Shapiro and I. V. Shtranich, *A Study of Neutron Diffusion in Beryllium, Graphite and Water by the Impulse Method*, Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva 1955, P/661, Vol. 5, p. 3, United Nations, New York (1956).
7. R. Ramanna, G. S. Mani, P. K. Iyengar, S. B. D. Iyengar and B. V. Joshi, *On the Determination of Diffusion and*

Slowing Down Constants of Ordinary Water and Beryllium Oxide Using a Pulsed Neutron Source, Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva 1955, P/872, Vol. 5, p. 24, United Nations, New York (1956).

8. N. G. Sjöstrand, *Measurements on a Subcritical Reactor with a Pulsed Neutron Source*, Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva 1955, P/789, Vol. 5, p. 52, United Nations, New York (1956).
9. F. L. Shapiro, *Record of Session 19A, Discussion of P/360, P/789, P/925*, Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva 1955, Vol. 5, p. 87, United Nations, New York (1956).