THE Be^9 (n,2n) Be^8 REACTION AND ITS INFLUENCE ON THE INFINITE MULTIPLICATION FACTOR FOR BERYLLIUM MODERATED HETEROGENEOUS THERMAL REACTORS

A REAÇÃO Be^9 (n,2n) Be^8 E A SUA INFLUÊNCIA SÔBRE O FATOR DE MULTIPLICAÇÃO INFINITA EM REATORES TÉRMICOS HETEROGÊNEOS MODERADOS POR BERÍLIO

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The Be'(n, 2n)Be[8] Reaction and its Influence on the Infinite Multiplication Factor for Beryllium Moderated Heterogeneous Thermal Reactors

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The existence of a (n, 2n) reaction in Be^9 with a threshold of 1.86 Mev[1] (in the laboratory system) has been known for a long time; an analysis of the bibliography on the subject shows however that its influence on the behaviour of a thermal heterogeneous beryllium moderated reactor has been made only in an incomplete way. Indeed, the only paper on the subject which came to our knowledge was one by Mills and Smith,[2] in which the influence of this reaction on the infinite multiplication factor was calculated without taking into account the change of the fission neutron spectrum leaving the fuel elements. In this paper such an influence is analysed and an elementary theory, similar to the theory of the determination of the fast fission factor in uranium in thermal heterogeneous reactors is developed, taking into account, therefore, the change in the energy spectrum of the neutrons leaving the fuel element.

Since the threshold of the $Be^9(n, 2n)Be^8$ reaction is 1.86 Mev, only a fraction of the fast neutrons produced during the fission process will be able to produce the reaction. In order to account for the fraction of fast neutrons with an energy above the threshold, it will be necessary to know the energy spectrum of the neutrons which leave the uranium bar and penetrate in the beryllium moderator. The energy spectrum of such neutrons, however, is not the same as the fission neutron spectrum since there is a modification due to the neutron collisions inside the uranium bar.

In the first chapters of this paper the energy spectrum of the neutrons leaving the uranium bar and the influence on k_∞ of the (n, 2n) reaction in the beryllium will be calculated. An estimate of the contribution of this reaction to k_∞ in a heterogeneous reactor will finally be obtained.

ENERGY SPECTRUM OF NEUTRONS LEAVING THE URANIUM BAR

Let us consider a natural uranium bar in a thermal heterogeneous reactor in a steady state. The neutrons arising from the U^{235} fissions either escape from the bar or undergo collisions with the U^{238} nuclei (the collisions with U^{235} nuclei are negligible as compared with the ones in U^{238} due to the low concentration of U^{235}, and also because the cross sections in the fast region, in which we are interested, are comparable). In such collisions there are four possible reactions: elastic scattering, inelastic scattering, radiative capture and capture with fission of U^{238}, the last one being important only if the energy of the incoming neutron be larger than about 1 Mev. Elastic collisions theoretically lead to an energy loss of the incoming neutron; such a loss, however, is of the order of 1 ev, and its influence will be neglected since we are mainly interested in fast neutrons of an energy larger than $E_0 = 1.86$ Mev.

Inelastic collisions, in general, will produce a large energy loss and with a good approximation we will suppose—according to Glasstone and Edlund[3]—that the final neutron energy is always smaller than 1 Mev. Radiative capture reactions constitute obviously a sink for neutrons; their influence for fast neutrons is not very large, however. If we consider now the energy spectrum of U^{235} fission neutrons—which we will call primary neutrons—it is clear that the existence of inelastic scattering, radiative capture and fast fission will give rise to a change in the spectrum of the primary neutrons.

If we repeat the calculations for the determination of the fast fission factor in a uranium bar,[3] one obtains easily that for each primary fission neutron of U^{235}, a fraction β escapes from the bar either without undergoing any collision or having suffered only elastic collisions, or collisions which result in the fission of U^{238}, where β is given by:

$$\beta = (1 - P) + \frac{P}{P'}(1 - P')\left[\frac{1}{1 - P'Z} - 1\right], \quad (1)$$

In this expression:

P = probability that a primary neutron will undergo a collision inside the bar;

P' = probability that a primary neutron which has already collided at least once will undergo another collision, or the probability that a neutron from the fast fission of U^{238} will collide in the bar;

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$Z = (\nu\sigma_f + \sigma_e)/\sigma$;
σ_f = total cross section for fission of U^{238};
σ_e = cross sections for elastic scattering in U^{238};
σ = total cross section for neutrons in U^{238};
ν = average number of neutrons emitted per U^{238} fission.

All these values of the cross section are average values over the fission neutron spectrum. By using the same method it can be shown that

$$\gamma = P\frac{\sigma_i}{\sigma}\cdot\frac{1}{1-P'Z} \tag{2}$$

where γ is the fraction of neutrons, per primary U^{235} fission neutron, escaping from the bar after having suffered at least one inelastic collision and σ_i the cross section for inelastic scattering by U^{238} (also an average value for the fission neutron spectrum).

According to our hypotheses, the neutrons produced in the fission of either U^{235} or U^{238} which escape from the bar after having undergone either elastic collisions, or without colliding with uranium nuclei, have an energy spectrum identical to the fission neutron spectrum, whereas the neutrons which escape from the bar after having suffered at least one inelastic collision will have an energy smaller than E_0, the threshold of the Be^9(n, 2n)Be^8 reaction.

Let $N(E)dE$ be the probability of a fission neutron (from U^{235} or U^{238}) having an energy between E and $E + dE$ and let us divide the total energy interval in two parts: I, between 0 and E_0; and II, between E_0 and ∞.

If we consider a group of n primary fission neutrons from U^{235}, we will have a group n' produced by them, or from their secondaries, emerging from the bar, where

$$n' = n(\beta + \gamma)$$

or

$$n'/n = \beta\int_0^{\infty} N(E)dE + \gamma\int_0^{E_0} N'(E)dE$$

where $N'(E)dE$ is the normalized energy spectrum of those neutrons emerging from the bar after suffering inelastic collisions which have therefore an energy smaller than E_0 and a spectrum different from the fission neutron spectrum. If we put:

$$n'/n = \int_0^{E_0}[\beta N(E) + \gamma N'(E)]dE + \int_{E_0}^{\infty}\beta N(E)dE$$

the spectrum of the emerging neutrons will be:

$$\bar{\mathcal{N}}(E)dE = \begin{cases}[\beta N(E) + \gamma N'(E)]dE & \text{for } E \leqslant E_0\text{—group I}\\ \beta N(E)dE & \text{for } E \geqslant E_0\text{—group II}\end{cases} \tag{3}$$

The shape of the function $N'(E)$ does not interest us here since the neutrons from group I have no possibility of producing (n, 2n) reactions in beryllium and will therefore begin a purely slowing down process. The total number of neutrons in each group per primary neutron from U^{235} fission will be:

(a) in group I

$$n_1 = \int_0^{E_0}\bar{\mathcal{N}}(E)dE = [1 - \alpha(E_0)]\beta + \gamma \tag{4}$$

where

$$\alpha(E_0) = \int_{E_0}^{\infty} N(E)dE \tag{5}$$

and noting that:

$$\int_0^{\infty} N(E)dE = \int_0^{E_0} N'(E)dE = 1. \tag{6}$$

(b) in group II

$$n_2 = \int_{E_0}^{\infty}\bar{\mathcal{N}}(E)dE = \beta\alpha(E_0). \tag{7}$$

We should observe that

$$n_1 + n_2 = \beta + \gamma = \varepsilon \tag{8}$$

where ε is the fast fission factor as used in the theory of heterogeneous reactors. It follows from this consideration that the spectrum represented by $\bar{\mathcal{N}}(E)dE$ is not normalized to 1, but rather to ε. In order to facilitate the following calculations we will introduce a function $\mathcal{N}(E)dE$ which, being normalized to 1, can be interpreted in terms of probability:

$$\mathcal{N}(E)dE = \begin{cases}\frac{1}{\varepsilon}[\beta N(E) + \gamma N'(E)]dE & \text{for } E \leqslant E_0\\ \frac{1}{\varepsilon}\beta N(E)dE & \text{for } E \geqslant E_0\end{cases} \tag{9}$$

$\mathcal{N}(E)dE$ will be therefore the probability of a neutron, emerging from the uranium bar, to have an energy between E and $E + dE$; this function gives the energetic distribution of neutrons emerging from the bar.

A FAST EFFECT FACTOR IN BERYLLIUM

If the neutrons from group II, emerging from the uranium bar, enter the beryllium moderator, they can produce the reaction Be^9(n, 2n)Be^8. In order to describe such an effect, we will introduce a coefficient ε_1—which we will call fast effect factor in beryllium—defined as the ratio between the number n'' of neutrons of an energy smaller than E_0, produced in the Be, to the number n' of neutrons emerging from the uranium bar.

In the theory which will be developed here we will suppose that the probability of a neutron, of energy greater than E_0, colliding with one beryllium nucleus, be unity; by this hypothesis the possibility of a neutron from group II being scattered back into the uranium bar will be neglected. The leakage probability will be considered zero, which is equivalent to stating that we are considering an infinite reactor. The fate of a neutron of energy between E and $E + dE$ emerging from the uranium bar will be determined by the probabilities of the several reactions which can be produced after the first collision with a beryllium nucleus. Such reactions are:

(*a*) elastic scattering—cross section $\sigma_e(E)$
(*b*) inelastic scattering—cross section $\sigma_1(E)$
(*c*) radiative capture—cross section $\sigma_c(E)$
(*d*) (n, 2n) reaction—cross section $\sigma_{2n}(E)$
(*e*) (n, α) reaction—cross section $\sigma_\alpha(E)$.

The values of all these cross sections are functions of the incoming neutron energy; in particular $\sigma_{2n}(E)$ will be zero for $E \leqslant E_0$.

Let us analyse initially the influence of elastic collisions. In this case we have to take into account the energy loss of the neutron in the elastic collisions with beryllium, since these losses are of the order of a few hundred kev and they cannot be neglected as they were in the computation of the elastic collisions with uranium nuclei.

A straightforward calculation (M. M. Shapiro[4]) shows that the mean value of the ratio between the final and the initial energies in an elastic collision between a neutron and a nucleus of mass A is

$$\xi = \overline{\left(\frac{E_2}{E_1}\right)} = \frac{1}{2}(1 + \alpha)$$
$$\alpha = [(A - 1)/(A + 1)]^2$$

where the angular distribution in the center of mass system is supposed to be isotropic.

For the beryllium we have:

$$\alpha = 0.640 \therefore \xi = 0.820.$$

The change in energy of a neutron which has undergone an elastic collision is fairly high and therefore the initial energy distribution will be changed. We will suppose from now on that in each elastic collision the neutron will lose always the same fraction of its initial energy.

Let us consider now inelastic collisions; with a good approximation we can suppose that the resulting neutrons will have a final energy smaller than E_0, since all the low energy levels in beryllium are above 1.5 Mev. It should be noted, besides, that the influence of these collisions is completely negligible since the cross section for the Be^9(n, n')Be^9 reaction is smaller than 14 mb at 2.5 Mev.[5]

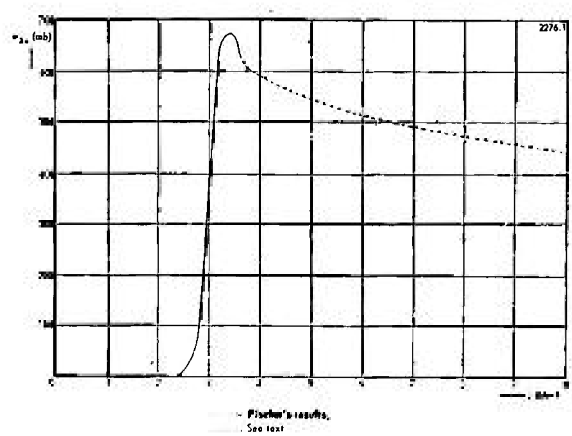

Figure 1

Radiative capture is also negligible for neutrons belonging to group II since even for thermal neutrons its value is only 9 mb.[6]

Let us consider finally the reactions Be^9(n, 2n)Be^8 and Be^9(n, α)Li^6. The threshold of the first reaction is 1.86 Mev and the threshold of the second reaction is 0.71 Mev, both in the laboratory system.[1]

The cross section of the second of those reactions has been studied between 1.8 and 4 Mev[6] whereas the behaviour of the cross section of the (n, 2n) reaction is almost unknown. As far as the calculation of the fast effect in beryllium is concerned the reaction (n, α) has the same effect as the radiative capture, both behaving as a neutron sink. In the theory which we will develop we will consider an absorption cross section $\sigma_a(E)$ given by:

$$\sigma_a(E) = \sigma_c(E) + \sigma_\alpha(E). \tag{11}$$

The neutrons produced in the (n, 2n) reaction will have energies which we will suppose to be smaller than E_0. Such a hypothesis is correct for neutrons of an energy smaller than about 4 Mev if in the (n, 2n) reaction the excess energy would be divided equally, or nearly so, between the two neutrons. Since there is no available experimental data on the energy distribution of the two neutrons arising from this reaction we will suppose the above hypothesis to hold. And, since the fraction of neutrons emerging from the bar with an energy higher than 4 Mev is very small (~5%), the hypothesis made seems to be reasonable if one takes into account all the approximations, both theoretical and experimental, which the complexity of the problem imposes.

We will suppose therefore, that in the moderator:

(*a*) in each elastic collision a neutron loses a fraction $(1 - \xi)$ of its initial energy;
(*b*) after an inelastic scattering the neutron energy is always smaller than E_0;
(*c*) the neutrons resulting from the (n, 2n) reactions have energies smaller than E_0;
(*d*) the collision probability of a neutron with a beryllium nucleus is unity.

Let us consider now n' neutrons emerging from the uranium bar. Taking into account that the neutrons inelastically scattered and those born out of the (n, 2n) reactions all have energies less than E_0, it is easy to see that

$$\nu_1 = n'\left[\int_0^\infty \frac{\mathcal{N}(E)dE[\sigma_1(E) + 2\sigma_{2n}(E)]}{\sigma(E)} + \int_0^{E_0/\xi} \frac{\mathcal{N}(E)dE \,.\, \sigma_e(E)}{\sigma(E)}\right] \tag{12}$$

is the number of neutrons with energies less than E_0 and

$$\nu_1' = n' \int_{E_0/\xi}^\infty \frac{\mathcal{N}(E)dE \,.\, \sigma_e(E)}{\sigma(E)} \tag{13}$$

is the number of neutrons with energies higher than

E_0 and are the only ones capable of giving origin to a (n, 2n) reaction in a further collision with a beryllium nucleus.

Due to the loss of energy resulting from elastic collisions with the beryllium nuclei the energy spectrum of those ν_1' neutrons is degraded by ξ times the initial value.

Repeating the same reasoning as above and taking into account this degradation in energy up to the third collision it can be shown that the number of neutrons n''' originating out of n' emergent neutrons is given by:

$$n''' = n'\varepsilon_1^{(3)}$$

with

$$\varepsilon_1^{(3)} = 1 + \frac{\beta}{\varepsilon}\left[\int_{E_0}^{\infty} \frac{N(E)dE[\sigma_{2n}(E) - \sigma_a(E)]}{\sigma(E)} + \int_{E_0/\xi}^{\infty} \frac{N(E)dE[\sigma_{2n}(\xi E) - \sigma_a(\xi E)]\sigma_c(E)}{\sigma(E)\sigma(\xi E)} + \int_{E_0/\xi^2}^{\infty} \frac{N(E)dE[\sigma_{2n}(\xi^2 E) - \sigma_a(\xi^2 E)]\sigma_e(E)\sigma_e(\xi E)}{\sigma(E)\sigma(\xi E)\sigma(\xi^2 E)}\right] - \frac{\gamma}{\varepsilon}\int_0^{E_0} \frac{N'(E)dE \,.\, \sigma_a(E)}{\sigma(E)}$$

having supposed that after the third collision all neutrons enter in a pure slowing down process, and using Eq. 9.

The physical meaning of the different terms are:

1. neutron production through the $Be^9(n, 2n)Be^8$ reaction by neutrons with a fission spectrum:

$$\frac{\beta}{\varepsilon}\left[\int_{E_0}^{\infty} \frac{N(E)dE \,.\, \sigma_{2n}(E)}{\sigma(E)} + \int_{E_0/\xi}^{\infty} \frac{N(E)dE \,.\, \sigma_e(E)\sigma_{2n}(\xi E)}{\sigma(E)\sigma(\xi E)} + \int_{E_0/\xi^2}^{\infty} \frac{N(E)dE \,.\, \sigma_e(E)\sigma_e(\xi E)\sigma_{2n}(\xi^2 E)}{\sigma(E)\sigma(\xi E)\sigma(\xi^2 E)}\right]$$

2. absorption of neutrons inelastically scattered in the uranium bar:

$$\frac{\gamma}{\varepsilon}\int_0^{E_0} \frac{N'(E)dE\sigma_a(E)}{\sigma(E)}.$$

3. absorption of neutrons with a fission spectrum:

$$\frac{\beta}{\varepsilon}\left[\int_0^{\infty} \frac{N(E)dE \,.\, \sigma_a(E)}{\sigma(E)} + \int_{E_0/\xi}^{\infty} \frac{N(E)dE \,.\, \sigma_e(E)\sigma_a(\xi E)}{\sigma(E)\sigma(\xi E)} + \int_{E_0/\xi^2}^{\infty} \frac{N(E)dE \,.\, \sigma_e(E)\sigma_e(\xi E)\sigma_a(\xi^2 E)}{\sigma(E)\sigma(\xi E)\sigma(\xi^2 E)}\right]$$

ESTIMATE OF THE $Be^9(n, 2n)Be^8$ AND $Be^9(n, \alpha)He^6$ REACTION CROSS SECTION

Those cross sections are the only important ones besides the elastic scattering cross section, since the cross sections for inelastic scattering and radiative capture in the beryllium for neutrons with energy higher than E_0 are quite small.[5, 6]

Considering the $Be^9(n, 2n)Be^8$ reaction, the most recent data are the ones given by Fischer[7] for neutrons with energy between 2.5 and 3.25 Mev (Fig. 1).

Fischer's curve was extrapolated to 11 Mev supposing that

$$\sigma_{2n}(E) = [\sigma(E) - \sigma_e(E)] - \sigma_\alpha(E). \tag{14}$$

The values of $[\sigma(E) - \sigma_e(E)]$ were taken from Hughes *et al.*[6] and for the values of $\sigma_\alpha(E)$ we just took the data of Hughes *et al.*, making a reasonable extrapolation up to 11 Mev. This extrapolation is not serious at all, since it is easy to show that the influence on ε_1 of the absorption in the energy region between 4 and 11 Mev is practically negligible, due to the rapid decrease of the fission spectrum above 3 Mev. The 11 Mev limit was taken mainly for computational convenience and it can be shown that the main contribution to ε_1 comes from fission neutrons with energy less than 3.5 Mev.

An inspection of Fischer's curve shows that for neutrons with energy less than 2.7 Mev the cross section for the reaction $Be^9(n, 2n)Be^8$ is very small; this is quite surprising, even supposing, as Fischer did, that this indicates that the mechanism of the (n, 2n) reaction proceeds mainly via inelastic scattering to the 2.43 Mev excited state of Be^9 followed by the emission of the second neutron.

An indication of the correctness of the extrapolations made is given by the mean value for the $Be^9(n, 2n)Be^8$ cross section taken over the energy spectrum of a Ra-Be source; with the curve of Fig. 1 we get for this mean value 0.48b, in reasonable agreement with the value of 0.3 ± 0.1b quoted by Ajzenberg and Lauritsen,[1] as obtained by Edge and Wilkinson.

Finally for the elastic scattering cross section $\sigma_e(E)$ we took the following values:

(*a*) up to 4 Mev, as the difference between the total cross section $\sigma(E)$ and the sum $[\sigma_\alpha(E) + \sigma_{2n}(E)]$; $\sigma(E)$ and $\sigma_\alpha(E)$ were taken from Ref. 6, and $\sigma_{2n}(E)$ from Fischer's data.

(*b*) between 4 Mev and 11 Mev, directly from Ref. 6.

NUMERICAL RESULTS

As an example, we consider the case of a natural uranium bar of 2.5 cm diameter. Taking for the various nuclear constants the values given by M. M. Shapiro,[4] we obtain:

$$\varepsilon = 1.027; \quad \beta = 0.860; \quad \gamma = 0.167.$$

The results of the calculations taking into account the collisions in the beryllium moderator up to the first, second and third orders are given in the Table 1. In this Table, $I^{(i)}$ is defined as

$$I^{(i)}(\%) = (\varepsilon_1^{(i)} - 1) \times 100$$

and gives the percentage influence of the $Be^9(n, 2n)Be^8$ reaction on the infinite multiplication factor.

In these calculations the influence of the term corresponding to the absorption of neutrons inelastically scattered in the uranium bar is not considered, since σ_c(E) is always very small as stated before.

Table 1

i	$\varepsilon_1^{(i)}$	$I^{(i)}$ %
1	1.042	4.2
2	1.062	6.1
3	1.068[a]	6.8

[a] Since the paper was written, recent values[9] of the Be^9(n, 2n) and (n, α) cross sections have come to our notice. Some rough preliminary calculations show the overall effect on $\varepsilon_1^{(3)}$ is to decrease its value from 6.8% to 5.5%, $\varepsilon_1^{(1)}$ and $\varepsilon_1^{(2)}$ being reduced in proportion.

The influence of the third collisions gives an increase of only 10% on I, and considering the rather poor precision of the experimental values it seems useless to take into account the influence of collisions of higher order.

The value obtained for I^3 agrees with the value quoted by Sanders[8] but is lower than the value of 10% quoted by Mills and Smith[2] as being obtained by the analysis of critical experiments. The origin of this discrepancy possibly lies in the values for the Be^9(n, 2n)Be^8 cross section obtained by Fischer; but a definite answer can only be given after a good experimental curve for this cross section is obtained.

Anyway, an effect that can increase the value of k_∞ by about 5 or 10% should certainly be considered in the design of reactors, and so it seems worthwhile to determine with better precision the experimental curves of $\sigma_{2n}(E)$ and $\sigma_\alpha(E)$.

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