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# The luminescence parameters of Yb3+:Er3+ doped LiLa(WO4)2 single crystal grown in the form of fiber for up-conversion...



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# Journal of Luminescence



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# The luminescence parameters of  $Yb^{3+}$ : $Er^{3+}$ -doped LiLa(WO<sub>4</sub>)<sub>2</sub> single crystal grown in the form of fiber for up-conversion green emission



Laercio Gomes®, Rafael Lima Denaldi, Jair Ricardo de Moraes, Sonia Licia Baldochi

Center for Lasers and Applications, IPEN/CNEN-SP, P.O. Box 11049, São Paulo, SP 05422-970, Brazil

# ABSTRACT

This report details the first study of the luminescence properties of a single crystal grown in the form of fiber for prospective application as the gain medium for fiber laser emission at 552 nm. The excited state decay processes related with the  ${}^4S_{3/2} \rightarrow {}^4I_{15/2}$  transition in double Yb<sup>3+</sup>:Er<sup>3+</sup>-doped LiLa(WO<sub>4</sub>)<sub>2</sub> crystal have been investigated using time-resolved fluorescence spectroscopy with a  $Er^{3+}$  concentration of 0.5 mol% and Yb<sup>3+</sup> with 2, 5, 7, 10 and 15 mol%. Selective laser excitation of the <sup>2</sup>F<sub>5/2</sub> energy level of Yb<sup>3+</sup>(972 nm) and selective laser excitations of the  $^4$ I<sub>11/2</sub> and  $^4$ I<sub>13/2</sub> energy levels of Er<sup>3+</sup>(972 and 1550 nm), respectively has established that in a similar way to other optical materials, a strong energy-transfer up-conversion by way of a dipole-dipole interactions between an Yb<sup>3+</sup> excited and  $Er^{3+}$  ions, the <sup>4</sup>F<sub>5/2</sub> level (Yb<sup>3+</sup>) populates the <sup>4</sup>S<sub>3/2</sub> upper laser level of the 550 nm transition. The <sup>4</sup>S<sub>3/2</sub> energy level emits luminescence with peaks having the wavelength center at 550 nm with luminescence efficiency increasing from 7% for  $Er^{3+}$  singly doped to 36% for Yb<sup>3+</sup>(15 mol%) co-doped crystals. The  $^4S_{3/2}$  lifetime of Er $^{3+}$  is observed to increase due to the saturation of the multiphonon relaxation rate at high excited-state density of Yb<sup>3+</sup> ions. At high excited-state density, Yb<sup>3+</sup> ions saturates the accepting modes inside of a critical volume of R<sub>C</sub> = 39.4 Å centered at an excited  $Er^{3+}({}^4S_{3/2})$  ion, by the high-energy phonons generated from emission sideband of Yb<sup>3+</sup> ions in Yb(x%):Er(0.5%) crystals. It is established that the green (552 nm) upconversion luminescence of  $Er^{3+}$  is optimized using an Yb<sup>3+</sup> concentration of 11.5 mol% for Er(0.5%):LiLa  $(WO_4)_2$  crystal.

# 1. Introduction

Efficient up-conversion lasers based on  $RE^{3+}$ -doped crystals with emission at wavelengths in the visible, green to blue spectral region, are very rare and new materials are needed for development of more compact all solid state lasers pumped by IR diode lasers. Several solid optical materials have been used to demonstrate the up-conversion luminescence mainly based on the use of  $Yb^{3+}$  ions as sensitizers. The use of high  $Yb^{3+}$  concentration  $\sim$ 10 mol% and small activators concentration as  $Er^{3+}$  ions have proportioned strong visible emission under  $Yb^{3+}$  ions excitation around 972 nm where powerful diode lasers are available today. Whilst a number of Yb:Er optical materials have demonstrated visible up-conversion luminescence and few of them have been operated efficiently [\[1,2\]](#page-7-0), more luminescence spectroscopic investigation must be carried out to find new optical materials having more efficient green emission of  $Er^{3+}$  to develop up-conversion lasers.

To explore the potential of the  ${}^4S_{3/2} \rightarrow {}^4I_{15/2}$  transition of the  $Er^{3+}$ ion that emits at 552 nm in  $\mathrm{Er}^{3+}/\mathrm{Yb}^{3+}$ - doped LiLa(WO<sub>4</sub>)<sub>2</sub> crystal and more broadly to investigate the possibility of an efficient laser operating at 552 nm using a fiber single crystal, detailed spectroscopic studies are required. In this report, we investigate the luminescence properties of  $Er^{3+}/Yb^{3+}$ -doped LiLa(WO<sub>4</sub>)<sub>2</sub> single crystal grown in the form of fiber by the micro-pulling down technique [\[3\].](#page-7-1) We carry out the basic spectroscopic measurements and assess the suitability of this optical material for emission in the visible. The luminescence efficiency of these levels was determined when the experimental decay time was compared with the radiative lifetimes reported in the literature. The multiphonon decay and the cross-relaxation (Er×Er) rates involving the  ${}^{4}S_{3/2}$  in the host LiLa(WO<sub>4</sub>)<sub>2</sub> were observed to be very weak compared to the radiative decay rate (or negligible) of  ${}^{4}S_{3/2}$  excited level and perhaps this level was taken as full luminescence efficiency. Measurements of the up-conversion luminescence transient (green emission) enable to calculate the energy transfer up-conversion rate  $(U_P)$  parameters.

# 2. Experimental procedure

For this study, we grew five Er (0.5%): LiLa( $WO<sub>4</sub>$ )<sub>2</sub> single crystals in

E-mail address: [lgomes@ipen.br](mailto:lgomes@ipen.br) (L. Gomes).

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<span id="page-1-0"></span><sup>⁎</sup> Corresponding author.

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the form of fibers that were co-doped with Yb  $(x\%)$ , where  $x=0, 2, 5$ , 10 and 15 mol%. The fiber crystals were grown with lengths up to 20 mm and constant diameters of 1.0 mm using an oriented fiber seed having the c-axis in the plane perpendicular to the fiber length. All the experiments were performed using a resistive micro pulling down furnace, in an air atmosphere, with pulling rate of 0.15 mm/min. The  $Er<sup>3+</sup>$  ion density in the sample was calculated to be  $5 \times 10^{19}$  cm<sup>-3</sup> for the doped  $LiLa(WO<sub>4</sub>)<sub>2</sub>$  sample with 0.5 mol% from X-ray fluorescence measurement.

The decay characteristics of the excited states of  $Er<sup>3+</sup>$  were measured using pulsed (typically 10 mJ, 4 ns, 10 Hz) laser excitation from a tunable optical parametric oscillator (OPO) pumped by the second harmonic of a Q-switched Nd-YAG laser (Brilliant B from Quantel). Tunable laser excitation from the OPO was used to directly excite the  ${}^{2}F_{7/2}$  energy levels of the Yb<sup>3+</sup> ion at 972 nm. The infrared luminescence  $(\lambda > 1100 \text{ nm})$  was detected using an InSb infrared detector (Judson model J-10 D cooled to 77 K) in conjunction with a fast preamplifier, with a response time of approximately 0.5 μs, and analyzed using a digital 200 MHz oscilloscope (Tektronix TDS 410). The visible and near infrared ( $\lambda$  < 1100 nm) was detected using a photomultiplier tube (EMI) with a sensitive cathode of S-1 type (or S-20) (PMT EMI refrigerated at −20 °C) with a response time of 10 ns. All the fluorescence decay characteristics were measured at 300 K. To isolate the infrared luminescence signals, band pass filters each with  $\sim$ 80% transmission at 1600 and 2750 nm with a half width of 25 nm and an extinction coefficient of approximately 10−<sup>5</sup> outside this band were used. Luminescence spectra were measured using an HR 2000-VIS CCD fiber spectrometer (fiber diameter of 100 µm) from Ocean Optics with an optical resolution of 0.8 nm.

# 3. Experimental results

When  $Yb^{3+}$ :Er<sup>3+</sup>-doped LiLa(WO<sub>4</sub>)<sub>2</sub> crystal is excited at 972 nm, the main processes involving the energy levels are, see [Fig. 1:](#page-2-0) (a)  $Yb^{3+}$ ground state absorption (GSA),  ${}^{2}F_{7/2}$  + h $\nu$  (972 nm) $\rightarrow {}^{2}F_{5/2}$  and  $Er^{3+}$ ground state absorption (GSA)  ${}^{4}I_{15/2}$  (Er<sup>3+</sup>) + hv (972 nm)  $\rightarrow {}^{4}I_{11/2}$ (Er<sup>3+</sup>); (b) energy transfer up-conversion (U<sub>P1</sub>), <sup>2</sup>F<sub>5/2</sub> (Yb<sup>3+</sup>)+<sup>4</sup>I<sub>15/2</sub>  $(\text{Er}^{3+}) \rightarrow ^2\text{F}_{7/2}$   $(\text{Yb}^{3+}) + ^4\text{I}_{11/2}$   $(\text{Er}^{3+})$   $(\text{U}_{\text{P1a}})$  and  $^2\text{F}_{5/2}$   $(\text{Yb}^{3+}) + ^4\text{I}_{11/2}$  $(Er^{3+})$ →<sup>2</sup>F<sub>7/2</sub> (Yb<sup>3+</sup>)+<sup>4</sup>F<sub>7/2</sub> (Er<sup>3+</sup>) (U<sub>P1b</sub>)→<sup>4</sup>S<sub>3/2</sub>+ *ħw*; (c) energy transfer up-conversion (U<sub>P2</sub>), <sup>2</sup>F<sub>5/2</sub> (Yb<sup>3+</sup>)+<sup>4</sup>I<sub>11/2</sub> (Er<sup>3+</sup>)→<sup>2</sup>F<sub>7/2</sub>  $(Yb^{3+})+{}^{4}F_{7/2}$  (Er<sup>3+</sup>) $\rightarrow {}^{4}S_{3/2}+ \hbar w$ ; (d) Er<sup>3+</sup> excited state absorption (ESA),  ${}^{4}I_{11/2}$  + h*v* (972 nm)→ ${}^{4}F_{7/2}$ → ${}^{4}S_{3/2}$  +  $\hbar w$ . The term  $\hbar w$  relates to the phonon energy of the crystal ( $\hbar w = 880 \text{ cm}^{-1}$ ). Radiative transition rates of the main transitions of  $Er^{3+}$  in LiLa(WO<sub>4</sub>)<sub>2</sub> were calculated using Judd-Ofelt theory and the parameters values experimentally obtained for the main luminescence levels are listed in [Table 1](#page-3-0). In these calculations the intensity parameters  $\Omega_2 = 9.02 \times 10^{-20}$  cm<sup>2</sup>,  $\Omega_4 = 2.02 \times 10^{-20}$  cm<sup>2</sup> and  $\Omega_6 = 0.59 \times 10^{-20}$  cm<sup>2</sup> were taken from the literature [\[4\].](#page-7-2) For an electric dipole transition, the reduced matrix elements  $U^{(\lambda)}$  for Er<sup>3+</sup>, which are considered to vary only slightly from host to host, were considered unchanged, so we used the values from the literature [\[5\]](#page-7-3).

# 3.1. Upconversion emission of  $Er^{3+}$  in single Er- and Yb-codoped crystals

The emission spectrum from the  $Er^{3+}(0.5\%)$  -doped LiLa(WO<sub>4</sub>)<sub>2</sub> single crystal grown in the form of fiber was obtained using the HR 2000-VIS spectrometer coupled to the optical fiber bundle to collect the luminescence signal using a pulsed laser excitation at 972 nm. The measured luminescence spectra were corrected from the spectral response of the equipment (optical grating efficiency). The up-conversion luminescence spectrum was induced by up-conversion energy transfer where two excited  $Er^{3+}$  ions in the  $^{4}I_{11/2}$  interact to promote the population of the upper excited level  $^2\mathrm{H}_{11/2}$  that decays to the  $^4\mathrm{S}_{3/2}$ luminescent level. The up-conversion emission spectrum measured for the Yb(x%):Er(0.5%): LiLa(WO<sub>4</sub>)<sub>2</sub> single crystals (fiber) are shown in

<span id="page-2-0"></span>

Fig. 1. Energy level scheme showing the pump laser, energy transfer mechanisms relevant to the laser operation at 556 nm. Solid line (up): 972 nm excitation. Solid lines (down):  $Er^{3+}$  emissions (at 550, 653, 980, 1537 nm). Dashed lines (up and down):  $U_{P1}$ ,  $U_{P2}$  and ESA processes. [Fig. 1](#page-2-0)(A) shows the up-conversion ( $U_{P1}$ ) process in details what is more effective for [Yb]  $> 5$  mol%; [Fig. 1](#page-2-0)(B) shows the ESA absorption and U<sub>P2</sub> process, which are competing process to produce the up-conversion luminescence when the [Yb] concentration is between 2 and 5 mol%.  $n_0$  and  $n_1$  are the ground ( ${}^{2}F_{7/2}$ ) and excited  $(^{2}F_{5/2})$  state populations of Yb<sup>3+</sup> ions. n<sub>2</sub> and n<sub>5</sub> are the ground  $(^{4}I_{15/2})$  and excited  $(^{4}S_{3/2})$  $_2$ ) states populations of  $Er<sup>3+</sup>$  ions.

the [Fig. 2](#page-3-1), where the visible luminescence was induced by the laser excitation at 972 nm. The main up-conversion emissions are due to the  ${}^{4}S_{3/2}$ → ${}^{4}I_{15/2}$  (centroid at 550 nm),  ${}^{2}H_{11/2}$ → ${}^{4}I_{15/2}$  (527 nm) and  ${}^{4}F_{12}$   ${}^{4}I_{15/2}$  (527 nm) and  $F_{9/2} \rightarrow ^4 I_{15/2}$  (653 nm) transitions.

# 3.2. Luminescence decays from the  ${}^4S_{3/2}$ ,  ${}^4I_{11/2}$  and  ${}^4I_{13/2}$  excited levels of  $Er^{3+}$

<span id="page-2-1"></span>When the sample was excited at 972 nm both  $Yb^{3+}$  and  $Er^{3+}$  are excited and the luminescence at 527, 550 and 653 nm are observed from the <sup>2</sup>H<sub>11/2</sub>, <sup>4</sup>S<sub>3/2</sub> and <sup>4</sup>F<sub>9/2</sub> excited states of  $Er^{3+}$  strongly due to  $U_{P1}$  (or  $U_{P2}$ ) up-conversion processes shown in a schematic diagram of [Fig. 1](#page-2-0). Laser excitation at 972 nm also produces ESA (excited state absorption) from the  ${}^{4}I_{11/2}$  excited level of  $Er^{3+}$  as shown in diagram of [Fig. 1\(](#page-2-0)b). The best fit to the temporal decay characteristics relevant to the  ${}^{4}S_{3/2}$  energy decay, when directly excited by one photon or ESA absorption, was accurately described by the experimental decay function given by Eq. [\(1\)](#page-2-1)

#### <span id="page-3-0"></span>Table 1

Radiative transition rate, branching ratio and radiative lifetime of  $Er^{3+}$  calculated in this work for  $Er^{3+}$  in LiLa(WO<sub>4</sub>)<sub>2</sub> crystal. The refractive index, n = 2 was used in calculations.

Transition $(Er^{3+})$	Wavelength $\lambda$ (nm)	Radiative Rate A <sub>ed</sub> $(s^{-1})$	β	$\tau_R$ (ms)
${}^2\text{H}_{11/2}$ $\rightarrow$				0.042
$^4\mathrm{I}_{15/2}$	527	23905	0.960	
${}^4S_{3/2}$ $\rightarrow$				0.556
$^{4}I_{9/2}$	1679	91	0.050	
$^4\mathrm{I}_{11/2}$	1222	43	0.024	
$^{4}I_{13/2}$	846	491	0.273	
$^4\mathrm{I}_{15/2}$	550	1173	0.653	
${}^4\text{F}_{9/2}$ $\rightarrow$				0.324
$^4\mathrm{I}_{11/2}$	1981	102	0.033	
$^4\mathrm{I}_{13/2}$	1152	44	0.040	
$^{4}I_{15/2}$	653	2945	0.953	
${}^{4}I_{11/2} \rightarrow$				3.72
$^4\mathrm{I}_{13/2}$	2750	24	0.088	
$^4\mathrm{I}_{15/2}$	974	245	0.912	
$^{4}I_{13/2} \rightarrow$				6.92
$^4\mathrm{I}_{15/2}$	1537	145	$\mathbf{1}$	

 $\Omega_2 = 9.02 \times 10^{-20}$  cm<sup>2</sup>,  $\Omega_4 = 2.02 \times 10^{-20}$  cm<sup>2</sup> and  $\Omega_6 = 0.59 \times 10^{-20}$  cm<sup>2</sup> were obtained from Ref. [\[4\]](#page-7-2).

<span id="page-3-1"></span>

Fig. 2. Up-conversion emission spectrum of  $Er^{3+}$  in Yb(x):Er(0.5):LiLa(WO<sub>4</sub>)<sub>2</sub> single crystal (fiber) with x=2%, 5%, 7% and 10% measured after pulsed laser excitation at 972 nm with an average energy of 10 mJ. The emission intensity was normalized by the  $Yb^{3+}(x \text{ mol})$  concentration for comparison.

$$
I(t) = I_0 \exp\left(-\gamma \sqrt{t} - \frac{t}{\tau_d} - \overline{W}t\right),\tag{1}
$$

where  $\tau_d = \frac{\tau_R}{1 + W_N R \tau_R}$  is the intrinsic decay of donor,  $\tau_R$  is the radiative decay of donor and  $W_{NR}$  is the multiphonon decay rate (donor). The non-exponential term exp(-γ√t), called the classical Förster decay function, describes energy transfer  $(D \rightarrow A)$  processes without energy migration through donor states, also known as static disordered decay involving the dipole-dipole interaction [\[6\]](#page-7-4). γ (s<sup>-1/2</sup>) is the energy transfer parameter (cross-relaxation (CR) or energy-transfer by upconversion (ETU)). The last exponential term  $\exp(-\overline{W}t)$  in Eq. [\(1\)](#page-2-1) describes energy transfer ( $D \rightarrow A$ ) processes enhanced by energy migration. Considering a non-exponential decay of donor, the mean lifetime (τ) of donor excitation can be obtained by the integration of decay curve (Eq. [\(2\)](#page-3-2)) using

<span id="page-3-2"></span>
$$
\tau = \frac{1}{I_0} \int_0^\infty I(t) dt. \tag{2}
$$

<span id="page-3-5"></span>Eq.  $(1)$  was rewritten in order to have two fitting parameters,  $t_1$  and γ giving

<span id="page-3-4"></span>

Fig. 3. Emission decay curves measured at 550 nm  $({}^4S_{3/2} \rightarrow {}^4I_{15/2})$  (a) and at 1500 nm  $(^{4}I_{13/2}$ → $^4I_{15/2}$ ) after laser excitation at 972 nm in Er(0.5%):LiLa(WO<sub>4</sub>)<sub>2</sub> crystals. The  $^4S_{3/2}$ level (a) was excited by ESA (excited state absorption) process. The risetime of the 1500 nm emission (b) is due to the  ${}^{4}I_{11/2} \rightarrow {}^{4}I_{13/2}$  emission decay.

$$
I(t) = I_0 \exp\left(-\gamma \sqrt{t} - \frac{t}{t_1}\right),
$$
  
where  $t_1 = \frac{\tau d}{1 + \tau_d W}$ . (3)

Because the luminescence of  ${}^{4}F_{9/2}$  level is excited by the nonradiative multiphonon decay from the upper level  $(^{4}S_{3/2})$  we use an experimental function that precisely describes the luminescence transient given by Eq. [\(4\)](#page-3-3)

<span id="page-3-3"></span>
$$
I(t) = I_0[\exp(-\gamma_1\sqrt{t} - t/t_1) - \exp(-t/t_2)], \qquad (4)
$$

where I<sub>0</sub> is the initial amplitude,  $\gamma_1$  and  $t_1$  are the fitting parameters of  $S_{3/2}$  level decay and t<sub>2</sub> is the time decay of the <sup>4</sup>F<sub>9/2</sub> level.

Lifetime of  ${}^{4}S_{3/2}$ ,  ${}^{4}F_{9/2}$ ,  ${}^{4}I_{11/2}$  and  ${}^{4}I_{13/2}$  levels of  $Er^{3+}$  in single doped LiLa(WO4)2 crystal fiber were measured using pulsed laser excitation at 972 nm with and an average energy of 8 mJ with an approximate excitation pump power of 35 MW cm<sup>-2</sup>. Besides this high excitation pump power used in the luminescence measurements we did not see any thermal and nonlinear effects on the measurements. The experimental emission decays of  ${}^{4}S_{3/2}$  and  ${}^{4}I_{13/2}$  levels are seen in [Fig. 3](#page-3-4)(a) and (b), respectively. The emission decay of  ${}^{4}F_{9/2}$  level is indirectly excited by ESA at 972 nm is seen in [Fig. 4](#page-4-0). Best fits of the emission decays were performed using Eq. [\(4\)](#page-3-3) (or Eq. [\(3\)](#page-3-5) for the  ${}^{4}S_{3/2}$ decay) and the best-fitted parameters and lifetimes are given in [Table 2](#page-4-1).

<span id="page-4-0"></span>

Fig. 4. Emission decay curves measured at 667 nm ( ${}^4F_{9/2} {\rightarrow} {}^4I_{15/2}$ ) after laser excitation at 972 nm in Er(0.5%):LiLa(WO<sub>4</sub>)<sub>2</sub> crystals. The <sup>4</sup>F<sub>9/2</sub> level is excited by the non-radiative decay of the  ${}^4S_{3/2}$  upper level.

<span id="page-4-1"></span>Table 2 Best fit luminescence parameters, non-radiative decay and cross-relaxation rates ( $\rm s^{-1}$ ) for  $Er^{3+}(0.5\%)$  doped LiLa(WO<sub>4</sub>)<sub>2</sub> crystal.

	Level $\gamma$ (s <sup>-1/2</sup> ) $\tau$ (expt.)	(ms)			$\tau_{\rm d}$ (ms) $\tau_{\rm R}$ (ms) $W_{\rm NR}$ (s <sup>-1</sup> ) $W_{\rm CR}$	$(s^{-1})$	$\eta_e$ (%)
$^{4}S_{3/2}$ 0		0.039	0.039	0.556	23842	<b>XX</b>	
${}^{4}F_{9/2}$ 0		0.001	0.324	0.324	$9.9 \times 10^{5}$	<b>XX</b>	0.3
$^{4}I_{11/2}$ 0		0.150	3.7	3.7	6396	X	4
$^{4}I_{13/2}$ 0		4.2	6.9	6.9	93	x	61

Radiative lifetimes ( $\tau_R$ ) were calculated from Judd-Ofelt theory in this work. (x) not expected cross-relaxation to occurs. (xx) negligible cross-relaxation effect for

 $[Er<sup>3+</sup>] = 0.5$  mol%.

# 3.3. Non-radiative multiphonon decay

The non-radiative decay rate can be calculated using the decay time constant obtained from the best fit calculations and  $W_{NR}(s^{-1}) = \frac{1}{\tau_d} - \frac{1}{\tau_R}$ , where  $\tau_d$  is the intrinsic decay time (without cross-relaxation (CR)) and  $\tau_R$  is the radiative lifetime calculated from Judd-Ofelt theory [\(Table 1](#page-3-0)). The intrinsic decay constant  $\tau_d$  is equal to the value of parameter  $t_1$ obtained from best fitting of <sup>4</sup>S<sub>3/2</sub> [\(Fig. 3](#page-3-4)(a)), <sup>4</sup>I<sub>13/2</sub> (Fig. 3(b)) is equal to τ(expt.) level decay since  $W_{nr}$  is observed to be negligible for [Er<sup>3+</sup>] =0.5% for both excited levels. The risetime  $t_2$  in [Fig. 3](#page-3-4)(b) is equal τ(expt.) level decay of the  ${}^{4}I_{11/2}$  level. The intrinsic luminescence efficiency was calculated using  $\eta_{\ell} = \frac{\tau}{\tau_R}$ . [Table 2](#page-4-1) shows the resulting luminescence decay parameters. It is important to observe that the luminescence decays of the  ${}^{4}S_{3/2}$  and  ${}^{4}F_{9/2}$  levels measured are fully exponential ( $\gamma = 0$ ); which is a strong indication that multiphonon decay competes with the radiative decay in order to decrease the decays times from 556 μs to 39 μs for the  ${}^{4}S_{3/2}$  and from 324 μs to 1 μs for the  ${}^{4}F$  level for  $[Fe^{3+1} - 0.5\%]$  ${}^{4}F_{9/2}$  level for  $[Er^{3+}] = 0.5\%$ .

### 3.4. Energy transfer up-conversion due to Yb→Er energy transfer

An analysis of the up-conversion process was carried using the 550 nm emission from the  $^4S_{3/2}$  level of  $\mathrm{Er}^{3+}$  produced by excitation of the  $^2\rm{F}_{5/2}$  energy level of Yb $^{3+}$  at 972 nm. [Fig. 5](#page-5-0) shows the luminescence time transient measured for Yb(x):Er(0.5%):LiLa  $(WO<sub>4</sub>)<sub>2</sub>$  fiber crystals with x=2, 5, 10 and 15 mol% after pulsed laser excitation under an excitation intensity of 35 MW cm $^{-2}$ . It is observed in [Fig. 5](#page-5-0) that ESA process occurs for  $[Yb^{3+}]$  < 10 mol% co-doped samples, i.e. where  $A_{ESA}$  (amplitude of the ESA signal)  $>$ 0. The best-fit calculations were obtained using an experimental

<span id="page-4-2"></span>function that precisely describes the luminescence transient given by Eq. [\(5\)](#page-4-2), which has been used in Ref. [\[7\]](#page-7-5)

$$
I(t) = A_{ESA} \exp(-\gamma_1 \sqrt{t_1} - t / t_1) + A_{Up} [\exp(-\gamma_1 \sqrt{t} - t / t_1)] - \exp(-t / t_2)],
$$
\n(5)

where  $A_{ESA}$  is the amplitude of the luminescence signal due to ESA absorption and  $A_{Up}$  is the amplitude of the up-conversion signal contribution. The fitting parameter  $t_2$  (luminescence rise time) is the up-conversion characteristic time and  $\gamma_1$  and  $t_1$  are the fitting parameters for the  ${}^{4}S_{3/2}$  luminescence decay. Best-fit parameters are given in [Table 3](#page-5-1). The most important information about the rate of upconversion process is given by the rise time  $(t_2)$  that is dependent on the excitation intensity and the increasing (by ytterbium doping) acceptor  $({}^{4}S_{3/2})$  lifetime,  $t_1$ . ESA contribution that is about 60% of the upconversion ( ${}^{4}S_{3/2}$ ) luminescence signal for Yb<sup>3+</sup>(2%), decreases to 20% for  $Yb^{3+}(5%)$  codoping (see [Fig. 5](#page-5-0)(a) and (b)) and is negligible for  $[Yb^{3+}] > 5\%$ . Because the up-conversion time  $(t_2)$  remains constant (11 μs) for excitation intensities increasing from 28 MW cm−<sup>2</sup> to 64 MW cm<sup>-2</sup> we conclude that the rate probability (s<sup>-1</sup>) for U<sub>P1</sub> process has already reached its constant value  $(K_0)$  according to the critical radius model for ETU process. A critical radius model, which has been presented previously in Ref. [\[8\]](#page-7-6) explains this observation based on the existence of a critical radius  $R_C$  for the ETU interaction, which limitates the ETU rate transfer to constant value for all excited  $Yb^{3+}$ - $Er<sup>3+</sup>$  pairs having a distance separation  $R \le R_C$ . As a consequence of that model,  $K_0$  is the constant rate for the excitation density  $N_{exc} > N_C$ , where  $N_c$  is the critical concentration of excited donors (Yb<sup>3+</sup>). Therefore the rate constant of  $U_{P1}$  process (ETU) can be calculated from the relation  $W_{U_{P1}} = \frac{1}{t_2} - \frac{1}{t_4}$  using  $t_2 = 11$  µs and  $\tau_d = 3.7$  ms (the intrinsic lifetime of donor,  $\frac{4}{t_1}$ <sub>11/2</sub> level giving a rate constant  $W_{Up1}$  = 90639 s<sup>-1</sup>).

### 4. Discussion

Results of [Fig. 5](#page-5-0) have shown a lifetime increasing for the  $\frac{4}{32}{2}$  level  $(Er^{3+})$  in Yb<sup>3+</sup>(x%) co-doped LiLa(WO<sub>4</sub>)<sub>2</sub> crystals. These lifetime values are given in [Table 3](#page-5-1). [Fig. 6](#page-5-2) shows the lifetime dependence on the  $[Yb^{3+}]$ concentration. Therefore, the luminescence efficiency of green emission  $({}^{4}S_{3/2})$  increases from 7% for  $[Yb^{3+}] = 0$  to 36% for  $[Yb^{3+}] = 15$ %. This effect must be related to the multiphonon relaxation suppression at  $Er^{3+}$ site due to the high excited-state density of  $Yb^{3+}$  ions in  $Yb^{3+}(x)$ :Er<sup>3+</sup>(0.5) crystal. The phonon diffusion length for low energy phonon or acoustic phonons  $\ell_c$  is approximately equals to 11 Å for tungstate crystals. It is known that  $Yb^{3+}$  ion substitution in tungstate lattice introduces local lattice symmetry distortions [\[9\]](#page-7-7) that affect the thermal conductivity and the phonon length diffusion [\[10\].](#page-7-8) It has been observed a thermal conductivity decreasing by factor of three for  $Yb^{3+}(5%)$ -doped NaGd(WO<sub>4</sub>)<sub>2</sub> crystal in comparison to undoped tungstate crystal [\[10\]](#page-7-8) and the low energy phonon diffusion length decreasing to  $\ell_c$  = 3.7 Å. Increasing Yb<sup>3+</sup> concentration, a number of  $Yb^{3+}$  symmetry distortions increases, which will decrease the high-phonon energy diffusion abbreviating the phonons breakdown to acoustic phonons and increasing the local bath temperature. This effect associated to the high-energy ( $\sim$ 850 cm<sup>-1</sup>) phonons generated by Yb<sup>3+</sup> emission sideband in  $Yb^{3+}(x\%)$ :Er<sup>3+</sup>(0.5%) crystals, may cause the saturation effect of the excited state  ${}^{4}S_{3/2}$  multiphonon relaxation of  $Er^{3+}$  as experimentally observed for  $Yb^{3+}(x)$ :Er:LiLa(WO<sub>4</sub>)<sub>2</sub> in this work. A phonon bottleneck (accepting phonon mode saturation) in the multiphonon relaxation process for several singly doped  $RE<sup>3+</sup>$  ions in tellurite, germanate and ZBLAN glasses has been observed and described by Auzel [\[11\].](#page-7-9) However, similar effect has never been reported for  $\mathbb{E}^{3+}({}^4S_{3/2})$  luminescence induced by  $Yb^{3+} \rightarrow Er^{3+}$  up-conversion. The efficiency of the non-radiative multiphonon decay of an  $Er^{3+}(^{4}S_{3/2})$  can be calculated using the expression  $\eta_{NR} = W_{nR}(\exp)/W_{nR}(0)$ , where  $W_{nR}$  (exp) is the experimental value that is dependent on the excited state density  $N_{\text{exc}}$  and  $W_{nR}$  (0) is the intrinsic nonradiative decay of <sup>4</sup>S<sub>3/2</sub> level (23842 s<sup>-1</sup>) measured at very low density

<span id="page-5-0"></span>

Fig. 5. Measured up-conversion luminescence transient of <sup>4</sup>S<sub>3/2</sub> level measured at 550 nm for Yb(x%):Er(0.5%):LiLa(WO<sub>4</sub>)<sub>2</sub> fiber crystals (where x = 2, 5, 10 and 15 mol%) after the laser pulse excitation at 972 nm (E=10 mJ) with an intensity of 35 MW cm<sup>-2</sup> per pulse. The best-fit calculations are shown in red. The rise time t<sub>2</sub> gives the up-conversion time and t<sub>1</sub> gives the lifetime of  ${}^{4}S_{3/2}$  level.

#### <span id="page-5-1"></span>Table 3

Best fit luminescence parameters of the luminescence transient of  ${}^{4}S_{3/2}$  level (upconversion) for Yb(x%):Er(0.5%):LiLa(WO<sub>4</sub>)<sub>2</sub> single crystals (fibers). The parameter  $\gamma_1$ was observed to be zero for all the fittings.  $\eta_{\ell}$  is the luminescence efficiency of <sup>4</sup>S<sub>3/2</sub> in  $\rm Yb^{3+}/Er^{3+}$  system. The ESA contribution (fraction) is given by  $f_{ESA}\!=\!A_{ESA}/(A_{ESA}\!+\!A_{Up}).$ 

$[Yb]$ (x mol%)	$A_{ESA}$	$A_{U_D}$	t <sub>1</sub>	$t_2$	$R^2$	$f_{ESA}$	$\eta_{\ell}$ (%)
0 $\overline{2}$	0.690	$\Omega$ 0.484	$39 \mu s$ $113.5 \,\mu s$	XX $4 \mu s$	0.999 0.998	0.6	20
5	0.265	1.03	$167.5 \,\mu s$	$10 \mu s$	0.998	0.2	30
7 10	$\Omega$ $\Omega$	1.	$170 \,\mu s$ $178 \,\mu s$	$11.8 \,\mu s$ $11 \mu s$	0.997 0.997	$\Omega$ $\Omega$	31 32
15	$\Omega$		$198 \mu s$	$11 \mu s$	0.999	$\bf{0}$	36

(xx) The rise time of luminescence was following the pulse laser excitation as expected for ESA process.

Radiative lifetime of  ${}^{4}S_{3/2}$  level is  $\tau_{R}$  = 556 µs.

excitation intensity. These experimental values of  $W_{nR}$  and the respective efficiency  $\eta_{nR}$  measured for many Yb<sup>3+</sup> excitation densities ( $N_{\text{exc}}$ ) are given in [Table 4.](#page-6-0)

[Fig. 7](#page-6-1) shows the values of  $\eta_{NR}$  as a function of the excited density of ions ( $Er^{3+}$  and  $Yb^{3+}$ ) by the laser pulse excitation at 972 nm.

The excitation density  $N_{exc}$  was calculated for a pulse energy of  $E=10$  mJ with an excitation volume constant equal to V=1.15×10<sup>-3</sup> cm<sup>3</sup> and the absorbance of each Yb(x%):Er(0.5) sample used in the measurements. The effect of reduction of the multiphonon relaxation efficiency for the  $^4\text{S}_{3/2}$  level (Er $^{3+})$  with the increasing of the  $Yb^{3+}$  concentration (cm<sup>-3</sup>), observed in [Fig. 7,](#page-6-1) was modeling described assuming the existence of a critical radius  $R_C$ , which governs the multiphonon relaxation decay of  $Er^{3+}$  according to the following

<span id="page-5-2"></span>

Fig. 6. Measured lifetime of  ${}^{4}S_{3/2}$  level (Er<sup>3+</sup>) as a function of the excited density of Yb3+(and Er3+) ions (cm<sup>−</sup><sup>3</sup> ) observed after laser excitation at 972 nm in Yb(x%):Er  $(0.5\%)$ :LiLa $(WO<sub>4</sub>)<sub>2</sub>$  crystals.

assumptions: i) an excited  $Er^{3+}$  ion that has an excited  $Yb^{3+}$  ion inside of a volume defined by the critical radius  $R_C$  will have a multiphonon relaxation efficiency equal to 1 ( $\eta_{NR}=1$ ) only for the high-energy (∼800 cm<sup>-1</sup>) phonons generation (at  $Er^{3+}$  site) propagating in the opposite direction linking both  $Er^{3+}$  and  $Yb^{3+}$  excited ions (or (-z) direction). Nevertheless, the multiphonon relaxation decay will have efficiency equal to zero ( $\eta_{NR}=0$ ) for phonons generation propagating in

#### <span id="page-6-0"></span>Table 4

Multiphonon decay rate values for the  $^4S_{3/2}$  level of  $\mathrm{Er}^{3+}$  measured for many excitation density of Yb<sup>3+</sup> in Yb(x%): Er(0.5%):LiLa(WO<sub>4</sub>)<sub>2</sub> crystals.

$N_{\rm exc}$ (cm <sup>-3</sup> )	$[Yb^{3+}]$ (mol%)	$W_{NR}$ (s <sup>-1</sup> )	$\eta_{nR}$
$5\times10^{17}$	0	23842	
$7.04\times10^{18}$	2	7012	0.294
$1.55 \times 10^{19}$	5	4172	0.175
$2.03 \times 10^{19}$	7	4085	0.171
$2.53 \times 10^{19}$	10	3819	0.160
$3.17 \times 10^{19}$	15	3252	0.136

Non-radiative multiphonon efficiency was taken relatively to the intrinsic value obtained for single doped  $Er^{3+}(0.5%)$  crystal under low excitation density.

<span id="page-6-1"></span>

Fig. 7. Effect on the multiphonon relaxation efficiency for the  ${}^{4}S_{3/2}$  level (Er<sup>3+</sup>) as a function of the Yb<sup>3+</sup> concentration (cm<sup>-3</sup>) observed after laser excitation at 972 nm in Yb(x%):Er(0.5%):LiLa(WO4)2 crystals. Best fit was performed using the critical radius model.

all different (x), (-x), (y), (-y) and (z) directions; ii) an excited  $Er^{3+}$  ion having an excited Yb<sup>3+</sup> ion outside of critical volume  $V<sub>C</sub>$  will have an unchanged multiphonon relaxation process with  $\eta_{NR}=1$ . Assuming a random distribution between  $Er^{3+}$  and  $Yb^{3+}$  ions in the lattice one has the following equation, which was firstly obtained in the exponential form to describe the interaction between an excited F center and the OH<sup>-</sup> molecule in KCl [\[12\].](#page-7-10)

<span id="page-6-2"></span>
$$
\eta_{NR} = f + (1 - f) \exp(-N_{exc}/N_C), \tag{6}
$$

where  $N_{\text{exc}}$  is the excited concentration of Yb<sup>3+</sup>(cm<sup>-3</sup>) ions and  $N_C$  is the critical concentration of excited  $Yb^{3+}$  ions;  $f$  is the fraction of excited  $Er^{3+}$  ions having the intrinsic multiphonon relaxation unchanged, besides the presence of an excited  $Yb^{3+}$  inside of a critical volume  $V_c$ . [Fig. 7](#page-6-1) shows that the experimental results can be best fitted using the Eq. [\(6\)](#page-6-2) derived from the critical radius model giving  $f=0.16$ and  $N_C$  equal to 3.9×10<sup>18</sup> cm<sup>-3</sup>. A critical radius of  $R_C$ =39.4 Å is obtained. One must to observe that the fraction  $f=0.16$  is very close to ratio value 1/6 that is consistent with the initial assumption that only multiphonons generation  $(Er^{3+})$  propagating at (-z) direction do not feel the acceptor mode saturation produced by phonon emission sideband of  $Yb^{3+}$  inside of the critical interaction volume. The critical radius experimentally obtained is about 3.6 times larger than the diffusion length of the low phonon energy (acoustic phonons)  $\ell_c$  = 11 Å estimated for tungstate crystals using the following equation

$$
\ell_c = \frac{3k}{C_P v_s},\tag{7}
$$

where the thermal conductivity  $(k)$ , the specific heat  $(C_P)$  and the sound velocity ( $v_s$ ) values were obtained for KLu(WO<sub>4</sub>)<sub>2</sub> crystal [\[10\].](#page-7-8)

<span id="page-6-3"></span>

Fig. 8. Normalized up-conversion emission intensity (a) and the luminescence efficiency (b) of  ${}^{4}S_{3/2}$  level (Er<sup>3+</sup>) plotted as a function of Yb<sup>3+</sup> concentration for Yb(x%):Er  $(0.5\%)$ :LiLa $(WO<sub>4</sub>)<sub>2</sub>$  crystals.

# 4.1. Best concentration for  $Yb^{3+}(x) \rightarrow Er^{3+}$  up-conversion

[Fig. 8\(](#page-6-3)a) shows the normalized intensity, which was obtained dividing it by [Yb<sup>3+</sup>] concentration (mol%), of  ${}^{4}S_{3/2}$  up-conversion emission at 552 nm plotted as a function of  $[Yb^{3+}]$  concentration for Yb  $(x\%)$ :Er(0.5%):LiLa(WO<sub>4</sub>)<sub>2</sub> crystals. [Fig. 8](#page-6-3)(b) shows the luminescence efficiency ( $\eta$ <sub>*e*</sub>) of <sup>4</sup>S<sub>3/2</sub> up-conversion emission as a function of the Yb<sup>3+</sup> concentration (using the data of [Table 3\)](#page-5-1). Besides the luminescence efficiency showed small increase for  $[Yb^{3+}] = 15 \text{ mol}$ %, the optimization of the green up-conversion emission in Yb:Er:LiLa(WO<sub>4</sub>)<sub>2</sub> crystal is obtained by observing the intensity curve of [Fig. 8](#page-6-3)(a) that exhibits a maximum intensity for  $[Yb^{3+}]$  equals to 11.5 mol%. Numerical solution of the rate equations is carrying out to characterize the potential for laser emission at 552 nm. The calculated evolution of the population inversion  $(n_4-n_3)$ , in mol% obtained by numerical simulation of the rate equations for  $[Yb^{3+}] = 10 \text{ mol}$ % using several pumping rates for a continuous regime at 972 nm pump rates is already in course.

#### 5. Conclusions

The intrinsic luminescence efficiency of the  ${}^4S_{3/2} \rightarrow {}^4I_{15/2}$  transition with a peak wavelength of 552 nm in  $Er^{3+}$ -doped LiLa(WO<sub>4</sub>)<sub>2</sub> crystal was determined to be 7%; a value reduced from the radiative lifetime because of strong multiphonon emission process. We observed that the

decay time of the  $^4\mathsf{S}_{3/2}$  level in Yb:Er:LiLa(WO<sub>4</sub>)<sub>2</sub> crystal increases with the  $[Yb<sup>3+</sup>]$  concentration reaching the value of 198 μs and a luminescence efficiency of 36% for  $Yb^{3+}$  doping with 15 mol%. This result suggests a phonon bottleneck in the multiphonon relaxation process of the  ${}^{4}S_{3/2}$  level due to the presence of an excited Yb<sup>3+</sup> ion inside of a critical radius of R<sub>C</sub>=39.4 Å. The observation of the Yb<sup>3+</sup> $\rightarrow$ Er<sup>3+</sup> energy transfer has shown that ESA process dominates the up-conversion luminescence from <sup>4</sup>S<sub>3/2</sub> level for  $[Yb^{3+}] \le 2$  mol%. However,  $Yb^{3+} \rightarrow Er^{3+}$  energy transfer up-conversion (ETU) process dominates the visible up-conversion luminescence for  $[Yb^{3+}] \ge 5$  mol% and none ESA effect is observed for  $[Yb^{3+}] \ge 10$  mol%. We established that the green up-conversion luminescence of  $Er^{3+}$  is optimized using an Yb<sup>3+</sup> concentration of 11.5 mol% for  $Er^{3+}(0.5\%)$ :LiLa(WO<sub>4</sub>)<sub>2</sub> crystal.

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# 308736/2014-1).

#### References

- <span id="page-7-0"></span>[1] [P.E.A. Mobert, E. Heumann, G. Huber, B.H.T. Chai, Opt. Lett. 22 \(18\) \(1997\) 1412.](http://refhub.elsevier.com/S0022-2313(16)31771-9/sbref1)
- [2] [R. Brede, E. Heumann, J. Koetke, T. Danger, G. Huber, B.H.T. Chai, Appl. Phys. Lett.](http://refhub.elsevier.com/S0022-2313(16)31771-9/sbref2) [63 \(15\) \(1993\) 2030.](http://refhub.elsevier.com/S0022-2313(16)31771-9/sbref2)
- <span id="page-7-1"></span>[3] [J.R. de Moraes, S.L. Baldochi, L.R.L. Soares, V.L. Mazzocchi, C.B.R. Parente,](http://refhub.elsevier.com/S0022-2313(16)31771-9/sbref3) [L.C. Courrol, Mater. Res. Bull. 47 \(2012\) 744](http://refhub.elsevier.com/S0022-2313(16)31771-9/sbref3)–749.
- <span id="page-7-3"></span><span id="page-7-2"></span>[4] [X.Y. Huang, Z.B. Lin, L.Z. Zhang, G.F. Wang, Mater. Res. Innov. 12 \(2\) \(2008\) 94.](http://refhub.elsevier.com/S0022-2313(16)31771-9/sbref4) [5] [M.J. Weber, Phys. Rev. 157 \(2\) \(1967\) 262.](http://refhub.elsevier.com/S0022-2313(16)31771-9/sbref5)
- <span id="page-7-4"></span>[6] [A.I. Burshtein, Sov. JETP 35 \(1972\) 882.](http://refhub.elsevier.com/S0022-2313(16)31771-9/sbref6)
- <span id="page-7-5"></span>[7] [L.D. da Vila, L. Gomes, L.V.G. Tarelho, S.J.L. Ribeiro, Y. Messaddeq, J. Appl. Phys.](http://refhub.elsevier.com/S0022-2313(16)31771-9/sbref7) [93 \(2003\) 3873.](http://refhub.elsevier.com/S0022-2313(16)31771-9/sbref7)
- <span id="page-7-6"></span>[8] [S.D. Jackson, A.F.H. Librantz, F.H. Jagosich, L. Gomes, G. Poirier, S.J.L. Ribeiro,](http://refhub.elsevier.com/S0022-2313(16)31771-9/sbref8) [Y. Messaddeq, J. Appl. Phys. 101 \(2007\) 123111.](http://refhub.elsevier.com/S0022-2313(16)31771-9/sbref8)
- <span id="page-7-7"></span>[9] [J.M. Postema, W.T. Fu, D.J.W. Ijdo, J. Solid State Chem. 184 \(2011\) 2004.](http://refhub.elsevier.com/S0022-2313(16)31771-9/sbref9)
- <span id="page-7-8"></span>[10] [Jiandong Fan, Huaijin Zhang, Jiyang Wang, Zongcheng Ling, HairuiXia,](http://refhub.elsevier.com/S0022-2313(16)31771-9/sbref10) [Xiufang Chen, Yonggui Yu, Qingming Lu, Minhua Jiang, J. Phys. D: Appl. Phys. 39](http://refhub.elsevier.com/S0022-2313(16)31771-9/sbref10) [\(2006\) 1034.](http://refhub.elsevier.com/S0022-2313(16)31771-9/sbref10)
- <span id="page-7-9"></span>[11] [F. Pellé, N. Gardant, F. Auzel, J. Opt. Soc. Am. B 15 \(2\) \(1998\) 667.](http://refhub.elsevier.com/S0022-2313(16)31771-9/sbref11)
- <span id="page-7-10"></span>[12] [L. Gomes, F. Luty, Phys. Rev. B 30 \(12\) \(1984\) 7194.](http://refhub.elsevier.com/S0022-2313(16)31771-9/sbref12)