Modeling of Fiber Bragg Gratings with Different Lengths for the Reflectivity Control for Fiber Lasers

Davi Pontes Nacaratti
University of São Paulo
São Paulo, Brazil
nacaratti@usp.br

Ricardo Elgul Samad
IPEN-CNEN/SP
São Paulo, Brazil
orcid.org/0000-0001-7762-8961

Cláudio Costa Motta
University of São Paulo
São Paulo, Brazil
orcid.org/0000-0002-2508-7320

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***Abstract*—An analytical formulation and modeling of an optical fiber Bragg gratings has been developed and is reported in this paper. Supported by the coupled-mode theory and considering that the mode fields of the unperturbed waveguide remain unchanged in the presence of weak perturbations, it is possible to obtain first-order differential equations that have solutions for some types of periodic perturbations. Finally, the model is applied to analyze the influence of structural parameters of fiber gratings, such as length, period and refractive index modulation on its reflectivity and bandwidth.**

***Index Terms*—fiber, laser, Bragg, grating**

I. Introduction

The propagation of light in optical fibers is analyzed by solving Maxwell's equations with the appropriate boundary conditions. This problem is simplified by assuming the existence of weakly guided modes, which allow their decomposition into a set of linearly polarized modes. The solutions provide the basic field distributions of the guided and radiation modes of the waveguide [1]. These modes propagate without coupling in the absence of perturbation, however, if there is a periodic modulation of the refractive index associated with phase matching phenomenon, the coupling of a specific propagation mode may occur. The technique usually applied to solve this type of problem is the coupled-mode theory.

The coupled-mode theory is based on the fact that disturbed electromagnetic fields can be expressed by a linear combination of the electromagnetic fields of the undisturbed structure [2]. This approximation provides a set of first-order differential equations, which have analytical solutions for some types of periodic perturbations. The application of a transfer matrix to solve those first-order differential equations is a method in which the waveguides are divided into short segments, and in each segment the gratings are assumed to be periodic. Thus, parameters such as the coupling coefficient, grating phase, and gain in the waveguide are independent of the propagation direction [3].

Fiber Bragg gratings (FBG) are an emerging technology used in several segments of industry, for instance, telecommunication, sensors, and optical fiber lasers. It is a class of devices based on a periodic modulation of the core refractive index along the length of an optical fiber. This periodic structure acts as a selective mirror for the wavelength that satisfies the Bragg condition [4, 5, 9]. Regarding fiber laser, the FBG is applied as the high reflector (HR) and output coupler (OC) to form the laser cavity. Typically, the reflectivity of HR and OC is about 99% and 4%, respectively [6].

Therefore, an analytical formulation and a MATLAB simulation of reflectivity and bandwidth has been developed and is presented in this paper. The main goal is to compare reflectivity results for different grating lengths and refractive index modulations.

II. Theoretical Model

The formalism of wave propagation in FBGs is presented by Maxwell's equations. It may be developed by considering the propagation of modes in an optical fiber. The general wave equation for propagation through a FBG is given by:

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} + \mu_0 \frac{\partial^2 \vec{P}}{\partial t^2} \tag{1}$$

where ε_0 is the dielectric constant, μ_0 is the magnetic permeability, $\vec{E}$ is the applied electric and $\vec{P}$ is the induced polarization.

A. Coupled-Mode Theory

The coupled mode theory is an efficient method for obtaining quantitative information about the optical characteristics of FBGs. In order to obtain the coupled-mode equations, effects of perturbations have to be included, assuming that the modes of the unperturbed waveguide remain unchanged [5]. Considering that the wave propagation occurs in a perturbed system with a dielectric grating, the total polarization $\vec{P}$ can be separated into two terms: the unperturbed polarization $\vec{P}_1$ and the perturbed polarization $\vec{P}_2$. Thus, the wave equation (1) can be rewritten as:

$$\nabla^2 \vec{E}_{\perp\mu} = \mu_0 \varepsilon_0 \varepsilon_r \frac{\partial^2}{\partial t^2} \vec{E}_{\perp\mu} + \mu_0 \frac{\partial^2}{\partial t^2} \vec{P}_{2,\mu}, \tag{2}$$

where the subscripts refer to the transverse mode $\perp$ number μ and ε_r is the relative permittivity of the unperturbed core. The perturbed polarization $\vec{P}_2$, which is driven by the propagating electric field and is due to the presence of the grating, is a detail which will be discussed in the next section.

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Moreover, to introduce the guided modes of the optical fiber into the wave equation (2) and ignoring for the moment the coupling to the radiation, according to Marcuse [5], the modes of an optical fiber can be described as a summation of L transverse guided mode with amplitudes, A_μ, with propagation constants β_μ, angular frequency ω, radial transverse field distributions of guided modes $\vec{\xi}_{\perp\mu}$ and $j = \sqrt{-1}$ [6].

$$\vec{E}_{\perp\mu} = \frac{1}{2}\sum_{\mu=1}^{L} A_\mu(z)\vec{\xi}_{\perp\mu}e^{j(\omega t-\beta_\mu z)}. \tag{3}$$

Substituting the modes of an optical fiber given by Eq. (3) into Eq. (2), and considering the weak coupling regime, further simplification is possible by applying the slowly varying envelope approximation in (4) and using $\omega^2\mu_0\varepsilon_0\varepsilon_r = \beta_\mu^2$. With these considerations, the wave equation simplifies to equation (5). [5]

$$\left|\frac{\partial^2 A_\mu}{\partial z^2}\right| << \left|\beta_\mu \frac{\partial A_\mu}{\partial z}\right| \tag{4}$$

$$\sum_{\mu=1}^{L} -j\beta_\mu \frac{\partial A_\mu}{\partial z}\vec{\xi}_{\perp\mu}e^{j(\omega t-\beta_\mu z)} = \mu_0 \frac{\partial^2}{\partial t^2}\vec{P}_{2,\perp}. \tag{5}$$

The following orthogonality relationship ensures that the power carried in the mode is $|A_\mu(z)|^2$:

$$\begin{gathered}\frac{1}{2}\int\int \hat{a}_z \cdot \left(\vec{\xi}_{\perp\mu} \times \vec{\xi}^*_{\perp\nu}\right) dxdy = \\ \frac{1}{2}\frac{\beta_\mu}{\omega\mu_0}\int\int \left(\vec{\xi}_{\perp\mu} \cdot \vec{\xi}^*_{\perp\nu}\right) dxdy = \delta_{\mu\nu},\end{gathered} \tag{6}$$

where $\hat{a}_z$ is a unit vector along the propagation direction z, $\delta_{\mu\nu}$ is the Kronecker's delta.

Multiplying both sides of (5) by $\vec{\xi}^*_{\perp\nu}$, integrating over the waveguide section and considering the orthogonality relationship (6), it becomes:

$$\frac{\partial A_\mu}{\partial z}e^{j(\omega t-\beta_\mu z)} = \frac{j}{2\omega}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\partial^2}{\partial t^2}\vec{P}_{2,\perp}\xi^*_{\perp\nu}dxdy. \tag{7}$$

The Eq. (7) may be used to describe a variety of phenomena in the coupling of modes and applies to a set of forward and backward propagating modes.

The total transverse field is described as a sum of both fields, not necessarily composed of the same mode order [5]:

$$\vec{E}_\perp = \frac{1}{2}\left(A_\nu(z)\vec{\xi}_{\perp\nu}e^{j(\omega t-\beta_\nu z)} + B_\mu(z)\vec{\xi}_{\perp\mu}e^{j(\omega t+\beta_\mu z)}\right) \tag{8}$$

Substituting the total transverse field (8) into (7) leads to (10) (shown in the next page).

B. Spatially Periodic Refractive Index Modulation

According to Kashyap [5], in a medium in which the dielectric constant varies periodically along the wave-propagation direction, the total polarization can be defined with the perturbed permittivity, $\Delta\varepsilon(z)$, and the applied field as:

$$\vec{P} = \varepsilon_0\left[\varepsilon_r - 1 + \Delta\varepsilon(z)\right]\vec{E}_\mu \tag{11}$$

The refractive index modulation of the grating is given by:

$$\delta n(z) = \overline{\Delta n}\left(1 + \frac{\nu}{2}\left\{e^{j[(2\pi N/\Lambda)z+\varphi(z)]}\right\}\right), \tag{12}$$

where $\delta n(z)$ is the refractive index change averaged over a single period of the grating, $\overline{\Delta n}$ is the refractive index perturbation over a single pitch, ν is the visibility of the fringes, Λ is the period of the perturbation, while N is an integer that denotes its harmonic order. An arbitrary spatially varying phase change of $\varphi(z)$ has been included [5].

Considering the constitutive relation between the permittivity of a material and its refractive index, $n^2 = \varepsilon$, assuming the perturbation to be a small fraction of the refractive index and considering that $\Delta n = \nu\overline{\Delta n}$, the perturbed polarization can be written as:

$$\vec{P}_2 = 2n\varepsilon_0\left(\overline{\Delta n} + \frac{\Delta n}{2}\left\{e^{j[(2\pi N/\Lambda)z+\varphi(z)]}\right\}\right)E_\mu. \tag{13}$$

Substituting (13) in (10), results in (14) (shown in the next page).

C. Coupling of Counterpropagating guided modes

The interaction between a forward-propagating mode and an identical backward-propagating mode is the simplest coupling model. By choosing the appropriate β value for identical modes $\mu = \nu$, but with opposite propagation directions, and considering the dc (17) and ac coupling constants (18):

$$\Delta\beta = \beta_\nu + \beta_\mu - \frac{2\pi N}{\Lambda} \tag{16}$$

$$\kappa_{dc} = n\omega\varepsilon_0\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\overline{\Delta n}\xi_{\perp\mu}\xi^*_{\perp\nu}dxdy \tag{17}$$

$$\kappa_{ac} = n\omega\varepsilon_0\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\Delta n}{2}\xi_{\perp\nu}\xi^*_{\perp\nu}dxdy = \frac{\nu}{2}\kappa_{dc} \tag{18}$$

So that, substituting (16), (17) and (18) into (14) is possible to obtain (19) and (20) [5]:

$$\frac{\partial B_\mu}{\partial z} = j\kappa_{dc}B_\mu + j\kappa_{ac}A_\nu e^{-j[\Delta\beta z-\varphi(z)]} \tag{19}$$

$$\frac{\partial A_\nu}{\partial z} = -j\kappa_{dc}A_\nu - j\kappa^*_{ac}B_\mu e^{j[\Delta\beta z-\varphi(z)]} \tag{20}$$

To find a solution, the following substitutions are necessary for the forward (reference) and backward propagating (signal) modes:

$$\left[\frac{\partial A_\nu}{\partial z}e^{j(\omega t-\beta_\nu z)}\right]-\left[\frac{\partial B_\mu}{\partial z}e^{j(\omega t+\beta_\mu z)}\right]=\frac{j}{2\omega}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mu_0\frac{\partial^2}{\partial t^2}\vec{P}_{2,\perp}\xi^*_{\perp\nu\mu}dxdy \tag{10}$$

$$\left[\frac{\partial A_\nu}{\partial z}e^{j(\omega t-\beta_\nu z)}\right]-\left[\frac{\partial B_\mu}{\partial z}e^{j(\omega t+\beta_\mu z)}\right]=-jn\omega\varepsilon_0 A_\nu\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\overline{\Delta n}+\frac{\Delta n}{2}\left\{e^{j[(2\pi N/\Lambda)z+\varphi(z)]}\right\}\right)\xi_{\perp\nu}e^{j(\omega t-\beta_\nu z)}\xi^*_{\perp\nu}dxdy$$
$$-jn\omega\varepsilon_0 B_\mu\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\overline{\Delta n}+\frac{\Delta n}{2}\left\{e^{j[(2\pi N/\Lambda)z+\varphi(z)]}\right\}\right)\xi_{\perp\mu}e^{j(\omega t+\beta_\mu z)}\xi^*_{\perp\nu}dxdy \tag{14}$$

$$R=A_\nu e^{-(j/2)[\Delta\beta z-\varphi(z)]} \tag{21}$$

$$S=B_\mu e^{(j/2)[\Delta\beta z-\varphi(z)]} \tag{22}$$

Differentiating the (21) and (22) and applying (19) and (20) results in the following coupled-mode equations:

$$\frac{dR}{dz}+j\left\{\kappa_{dc}+\frac{1}{2}\left[\Delta\beta-\frac{d\varphi(z)}{dz}\right]\right\}R=-j\kappa^*_{ac}S \tag{23}$$

$$\frac{dS}{dz}-j\left\{\kappa_{dc}+\frac{1}{2}\left[\Delta\beta-\frac{d\varphi(z)}{dz}\right]\right\}S=-j\kappa_{ac}R \tag{24}$$

D. Transfer Matrix Method

The transfer matrix method is an accurate technique for solving the differential equations system (23) and (24). The principle behind this method is to divide the grating structure into a multiple number N of uniform sections, so that each section can be approximately treated as uniform grating. The transfer matrix of the individual k^{th} segment is assumed to be given by T^k, which satisfies the following relation [4, 5]:

$$\begin{bmatrix}R_k\\S_k\end{bmatrix}=\begin{bmatrix}T^k_{11}&T^k_{12}\\T^k_{21}&T^k_{22}\end{bmatrix}\begin{bmatrix}R_{k+1}\\S_{k+1}\end{bmatrix} \tag{25}$$

where the transfer matrix coefficients are given by:

$$T_{11}={T_{22}}^*=\cosh(\alpha L_k)-\frac{j\delta\sinh(\alpha L_k)}{\alpha} \tag{26}$$

$$T_{12}={T_{21}}^*=\frac{j\kappa_{ac}\sinh(\alpha L_k)}{\alpha} \tag{27}$$

$$\alpha=\sqrt{|\kappa_{ac}|^2-\delta^2} \tag{28}$$

$$\delta=\kappa_{dc}+\frac{1}{2}\left(\Delta\beta-\frac{d\varphi(z)}{dz}\right). \tag{29}$$

For scattering matrix determination, some boundary conditions have been defined as $R(0)=1$ and $S(L)=0$. Thus, the reflectivity ρ can be expressed as

$$\rho=\frac{S(0)}{R(0)}=\frac{T_{21}}{T_{11}} \tag{30}$$

If the grating is uniform along z, $\frac{d\varphi(z)}{dz}=0$ and κ_{ac} is a real quantity. In (29), $\frac{\Delta\beta}{2}$ is the detuning, and it indicates how rapidly the power is exchanged between the "radiated" (generated) field and the polarization ("bound") field. [5]

The bandwidth $\Delta\lambda$, can be defined as the first zero on either side of the main reflection peak. Consequently, it may be calculated graphically or by finding the zeros between the peak of reflection spectrum [7, 8, 10].

III. Results and Discussion

For the first MATLAB simulation, the parameters are the effective refractive index $n_{eff}=1.45$, the grating length $L=5mm$, the visibility of the fringes $\nu=0.7$, the grating period $\Lambda=366.88nm$, the refractive indices modulation $\Delta n_1=4.0\times10^{-5}$, $\Delta n_2=6.0\times10^{-5}$ and $\Delta n_3=8.0\times10^{-5}$ [8]. The period $\Lambda=366.88nm$ was chosen to obtain a Bragg wavelength response close to $\lambda_B=1064nm$. Fig. 1 shows a graphic of three fiber gratings reflective spectrums with different index modulations. Table I presents the results of maximum reflectivity and bandwidth for different refractive index modulations.

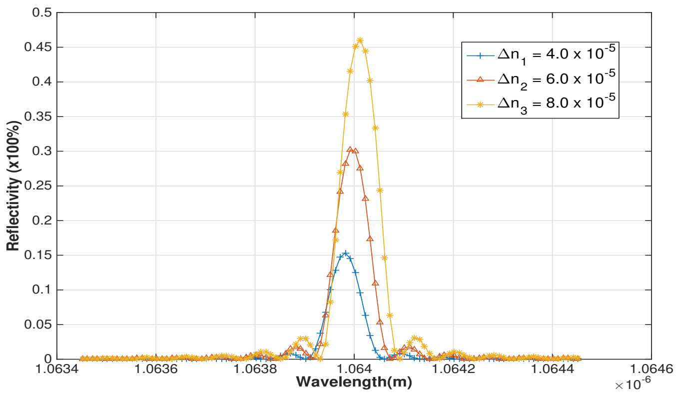

Fig. 1. Dependence of reflectivity and spectral bandwidth of FGB on refractive index modulation grating periods $\Lambda=366.88nm$.

TABLE I

SIMULATION VALUE OF REFLECTIVITY AND SPECTRAL BANDWIDTH UNDER DIFFERENT REFRACTIVE INDEX MODULATION

Refractive index modulation	4.0×10^{-5}	6.0×10^{-5}	8.0×10^{-5}
Reflectivity(%)	15.32	30.20	46.04
Bandwidth(nm)	0.1576	0.1593	0.1616

As shown in Fig. 1, the larger the refractive index modulation is, stronger is the reflectivity and bigger is the bandwidth. Furthermore, the changing of refractive index modulation changes the reflected peak wavelength. In addition, using the same parameters of grating period in [8], the same reflectivity and bandwidth values were found.

With the purpose of analyzing the dependence of reflectivity and bandwidth on fiber grating length, the parameters of simulation are $n_{eff} = 1.45$, the visibility of the fringes $\nu = 0.7$, the grating period is $\Lambda = 366.88nm$ and the refractive index modulation is $\Delta n = 6.0 \times 10^{-5}$. Fig. 2 shows the reflectivity dependence on the FBG lengths for $L_1 = 3mm$, $L_2 = 4mm$ and $L_3 = 5mm$.

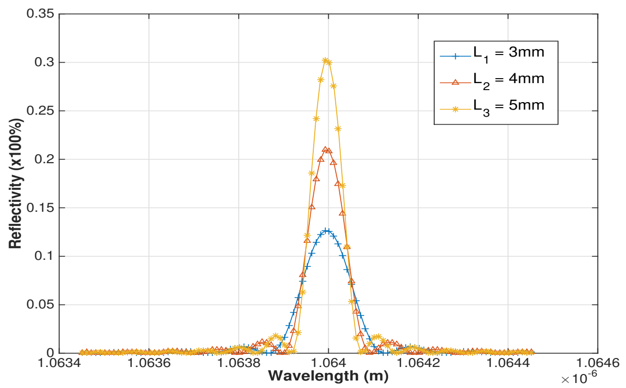

Fig. 2. Dependence of reflectivity and spectral bandwidth of FGB for grating lengths $L_1 = 3mm$, $L_2 = 4mm$ and $L_3 = 5mm$.

Table II shows that 99% reflectivity is achieved for gratings longer than 25mm. In addition, the longer is the grating, the smaller is the bandwidth. Using the model is possible to calculate the maximum reflectivity and bandwidth to all grating lengths, so that, the grating length which corresponds to 4% reflectivity, to be used as an output coupler, is $L = 1.65mm$.

TABLE II

SIMULATION VALUE OF REFLECTIVITY AND SPECTRAL BANDWIDTH UNDER DIFFERENT FIBER GRATING LENGTH

Grating Length(mm)	Reflectivity(%)	Bandwidth(nm)
1.65	4.07	0.4746
25	99.07	0.0439

Moreover, applying the simulation is possible to plot a curve that represents the relationship between the reflectivity and the grating length, presented in Fig. 3. The curve behavior presented by Fig. 3 has the same tendency reported in [4] and [9] to uniform FBGs.

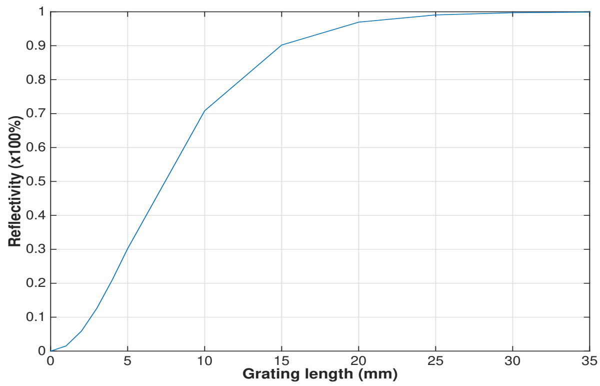

Fig. 3. Relationship between FBG reflectivity and grating length

IV. CONCLUSION

The analytical formulation and modeling of FBGs is an essential step for projecting and manufacturing these devices. As shown in the results, the reflectivity and bandwidth may be determined by changing the grating length, refractive index modulation and fiber grating period. The simulations allowed to conclude that the reflectivity rises and the bandwidth decreases with the increase of the grating length. Additionally, the reflectivity and the bandwidth increase with the growth of the refractive index modulation.

In conclusion, the proposed model for the study of FBGs reflectivity is coherent with the results presented by [4], [8] and [9]. Thus, is possible to conclude that using a refractive index modulation $\Delta n = 6.0 \times 10^{-5}$, for gratings longer than $L = 25mm$, 99% reflectivity is achieved and the grating length which corresponds to 4 % reflectivity is $L = 1.65mm$.

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