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COMPARISON BETWEEN S-N-P CURVES OBTAINED FROM CONSTANT STRESS AND STEP-STRESS FATIGUE TESTS

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ABSTRACT

This Fatigue is one of the most important failure modes to be considered in many engineering applications. In actual cases, the alternate stress applied to the component may vary during its lifetime. In these situations, the direct use of S-N-P curves may be inadequate when they are based on tests where the number of cycles to failure are determined for specimens under a constant stress for a certain probability of failure.

In this paper it is shown S-N-P curves for specimens submitted each one to three different stress levels. Those curves are compared with the common S-N-P curves from constant stress tests. The Palmgren-Miner theory is applied to the obtained data from the two test methods. The results are compared and some conclusions and comments are addressed.

Keywords: Fatigue damage, cumulative damage, fatigue.

1. INTRODUCTION

Fatigue can be characterized as a phenomenon of progressive damage, being constituted basically by the nucleation and propagation of cracks, until a critical size is reached, from which the propagation happens in an unstable and uncontrollable way. Despite of the microscopic aspects have great importance for the understanding of the fatigue process, the equipment and structures designers are predominantly interested in the macroscopic aspects of the fatigue failure and in the necessary tools for its prevention during the estimated lifetime (COLLINS, 1993 and SOBCZYK & SPENCER, 1992).

For the fatigue failure prevention, one must consider the material behavior under several cyclic loads during the component lifetime. However, the fatigue characteristics of a material cannot be established from other mechanical properties. They should be measured directly, so specific mechanical tests are used to obtain the fatigue curves, denominated S-N curves. In these curves, the material life or the number of cycles to the failure, N, is expressed as function of the alternate stress range (called in this work as stress level, also) applied to the

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component, Sa. In general, to obtain S-N curves, a relatively high alternate stress range is applied to a specimen, kept in a constant level up to the material failure (COLLINS, 1993 and SOBCZYK & SPENCER, 1992). This procedure is repeated for several specimens, with different stress levels, progressively descending or ascending. The data are registered in a graph, with the alternate stress range (S) in the ordinate represents and the number of cycles to failure (N) in the abscissa.

However, the results of fatigue tests in specimens conducted in laboratory show significant dispersion, what even can commit the reliability of the obtained information of the curves. The random nature of fatigue would be obvious, if the structure is submitted to loads that vary randomly; but, even in laboratory conditions strictly controlled and under deterministic cyclic loads, the obtained results show considerable statistical dispersion. It occurs due to several random factors producing a huge variability in the results.

Thus, due to the inherent random nature of the fatigue data, the stochastic modeling is not only adequate but necessary. This point of view has been accepted thoroughly and, consequently, the problems associated to the fatigue stochastic theory are generally considered important and defiant in mechanics and in applied stochastic.

Due to the dispersion of the fatigue data at any level of alternate stress range, there is not only a single S-N curve for a certain material but a family of curves with the probability of failure as a related parameter. Those curves are called S-N-P curves or curves of constant probability of failure in a plot of alternate stress range versus number of cycles to failure (COLLINS, 1993). In statistics, the constant probability life is denominated percentile. The percentile 100p% of a distribution of probability F(x) is the time xp where a part p of the population will fail to this time, that is, the solution of p = F(xp).

The stochastic modeling of the fatigue is usually approached inside the reliability theory. For structures that need to operate without risk of failure, even with uncertainties, during their lifetimes, it is necessary to have an adequate probability measurement of the failure due to fatigue. For this reason, the problems of structures reliability from the point of view of fatigue are of great interest in recent research.

Furthermore, in practically all the engineering applications where fatigue is an important failure mode, the alternate stress range is not constant and changes in some way during lifetime. Such variations or load range changes, often called as load spectra, can make inadequate the direct use of the standard S-N-P curves, obtained from tests performed with constant alternate stress ranges. So, it is important to the designer to have a model, checked against experimental data, which allows him to do good estimates of the design parameters for operation under load spectra conditions. To develop this model, it is used a linear cumulative exposure method (NELSON, 1980) associated with inverse power relationship plus log-normal distribution.

This work looks for comparing S-N-P curves obtained from constant stress range fatigue tests with decreasing step-stress range fatigue tests. The comparison of the cumulative damage based on Palmgren-Miner theory is also presented.

2. S-N-P CURVES OBTAINED FROM DECREASING STEP-STRESS FATIGUE TESTS

2.1 The Experimental Data

The results of the cooled rotating bending fatigue tests performed in SAE 8620 steel specimens are presented in TABLE 1. Three alternate stress ranges were applied to each specimen, in decreasing order. In the first two steps, each specimen was submitted to 35,000 cycles of an alternate stress range of 258 MPa and to 65,000 cycles of an alternate stress range of 238 MPa. In the third step, the alternate stress ranges of 218 MPa, 198 MPa, 178 MPa and 158 MPa were applied to different specimens in a number of cycles sufficient to cause the material failure or to reach 2×10^6 cycles, characterizing the censoring type I according to NELSON, 1990. The failure was defined as the specimen full fracture. N is the number of cycles to failure or to censoring, i is the specimen number, and S is the alternate stress range or stress level.

TABLE 1: Experimental results for step-stress fatigue tests of SAE 8620 steel specimens.

i	3rd step		i	3rd step		i	3rd step		i	3rd step	
	S	N		S	N		S	N		S	N
1	218 MPa	212,848	12	198 MPa	402,834	23	178 MPa	900,001	34	158 MPa	1,000,002
2		779,027	13		992,657	24		1,309,463	35		1,802,316
3		687,887	14		948,233	25		1,587,256	36		1,589,964
4		773,889	15		833,713	26		1,624,848	37		1,445,535
5		358,453	16		815,698	27		1,534,548	38		1,721,045
6		708,348	17		866,419	28		1,616,273	39		2,000,000
7		916,372	18		1,336,343	29		1,596,508	40		2,000,000
8		952,438	19		1,118,548	30		1,606,979	41		2,000,000
9		702,855	20		871,999	31		2,000,000	42		2,000,000
10		493,165	21		909,221	32		2,000,000	43		2,000,000
11		681,238	22		825,585	33		2,000,000	44		2,000,000

2.2 Development of the Physical – Statistical Model

The model that establishes the relationship between alternate stress range and number of cycles to failure is based on two components:

(1) the deterministic one, which is the stress-time to failure relationship;

(2) the stochastic one, which is characterized by the time to failure distribution of probability.

The deterministic component of the model is the inverse power relationship (FREITAS & COLOSIMO, 1997) and, for the stochastic component, the adopted distribution to represent the time to failure data under constant alternate stress levels is the log-normal distribution PINTO (2004).

Supposing that x is the time to failure, the function of reliability of the log-normal distribution of the variable ln(x) is given by

$$R(x|S) = \Phi\left[-\frac{\ln(x) - \mu(S)}{\sigma}\right] \tag{1}$$

The shape parameter (σ) is supposed constant and the location one (μ) is function of the stress level S according to the inverse power relationship $\mu(S) = \ln\left[\left(\frac{A}{S}\right)^{\omega}\right]$.

The parameters A and ω are characteristic for the product, unit, geometry, manufacture, test method, etc.

In the case of data obtained from step-stress fatigue tests, it is adopted, for the effect of change of the stress level, the linear cumulative exposure model (NELSON, 1980). The inverse power relationship and the log-normal distributions are applied to this model. Its objective is to relate the time to failure distribution obtained with step-stress fatigue tests with the time to failure distribution under the operational cyclic loads for actual structures and components.

The cumulative exposure model supposes that:

(1) the remaining life time of a specimen only depends on the current cumulative damage fraction and on the applied alternate stress level, and is not dependent in how the damage fraction has been cumulated (Markov property (SOBCZYK & SPENCER, 1992));

(2) if the applied alternate stress level is maintained, the not failed specimens will fail according to time to failure distribution for this stress level. However, the initial point is kept at the value of the previous fraction of the cumulative failure.

Mathematically, this model is expressed below, where the time to failure distribution $F_o(x)$ under a particular configuration of the alternate step stresses applied in m levels will be obtained. Let's suppose that, for a certain configuration, the level j corresponds to the S_j level of the alternate stress range, from the instant x_{j-1} to the instant x_j, where j = 1, 2,..., m and $x_0 = 0$. The time to failure distribution at the constant level S_j of the alternate stress range is denoted by $F(S_j,x)$. The function $F_0(x)$ of the model CE is given by (NELSON, 1980):

$$F_0(x) = F(S_j, x-x_{j-1}+\xi_{j-1}),\ x_{j-1} \leq x \leq x_j \tag{2}$$

where ξ_{j-1} is the equivalent initial time at the level j of the alternate stress range. This time is defined as the one that would produced the same fraction of the population cumulative failure at the level j-1 of the alternate stress range. S_o ξ_{j-1} is the solution of:

$$F_0(S_{j-1}, \Delta x_{j-1} + \xi_{j-2}) = F_0(S_j, \xi_{j-1}) \tag{3}$$

where:

$$\Delta x_{j-1} = x_{j-1} - x_{j-2} \text{ and}$$
$$j = 1, 2, ..., m.$$

Substituting the expression (1) in the expression (2), the cumulative exposure model function is obtained for inverse power relationship and log-normal distribution, as

$$F_0(x) = 1 - \Phi\left\{-\frac{1}{\sigma}\ln\left[(x - x_{j-1} + \xi_{j-1})\frac{S_j^{\omega}}{A}\right]\right\},\quad x_{j-1} \leq x \leq x_j \tag{4}$$

The estimated parameters of the model were obtained using the maximum likelihood method (PINTO, 2004).

The TABLES 2 and 3 present the obtained estimated parameters for cumulative exposure model using the experimental data of TABLE 1.

TABLE 2: Estimated parameters of the maximum likelihood function.

Parameter	Estimator	Confidence Limit – 95%	
		Lower	Upper
A	634.82	480.31	789.33
ω	4.0977	3.2315	4.9639
σ	0.23798	0.18181	0.29415

TABLE 3: Asymptotic covariance matrix of the estimators of the maximum likelihood.

	A	ω	σ
A	6214.04	-34.7402	0.132531
ω	-34.7402	0.195293	-0.000627078
σ	0.132531	-0.000627078	0.000821348

2.3 S-N-P Curves Development

The adjusted model, that corresponds to the estimate of maximum likelihood function for the SAE 8620 steel specimens that fail after the number of cycles $x^*=10^{-4}.x$, is:

$$\hat{R}(S, x^*) = \Phi\left\{-\frac{1}{0.23798}\ln\left[x^*\left(\frac{S}{634.82}\right)^{4.0977}\right]\right\} \tag{5}$$

where $\Phi(.)$ is the cumulative distribution of a standard normal distribution (average equals 0 and standard deviation equals 1).

The expression of the adjusted model relative to the percentil 100p% is:

$$\hat{x}_p^* = \exp[0.23798 z_p + 4.0977(\ln 634.82 - \ln S)] \tag{6}$$

In expression (6), $\hat{\sigma}$ = 0.23798 is the estimate for the standard deviation of the log-normal distribution and $\hat{\omega}$ = 4.0977 and Â = 634.82 are, respectively, the estimates of the power and of the proportionality constant for the inverse power relationship. The values of these parameters are not known with so much precision as the number of significant digits seems to suggest.

Applying the adjusted model, the percentiles (x*p) confidence intervals can be obtained for the several values of the alternate stress ranges. To do that it is necessary to estimate the variance of $\hat{x}_p^*$. Using the delta method as described by FREITAS & COLOSIMO (1997), it is obtained:

$$Var(\hat{x}_p^*) = Var(\hat{A})\left(\frac{\partial \hat{x}_p^*}{\partial A}\right)^2 + Var(\hat{\omega})\left(\frac{\partial \hat{x}_p^*}{\partial \omega}\right)^2 + Var(\hat{\sigma})\left(\frac{\partial \hat{x}_p^*}{\partial \sigma}\right)^2 + 2Cov(\hat{A},\hat{\omega})\frac{\partial \hat{x}_p^*}{\partial A}\frac{\partial \hat{x}_p^*}{\partial \omega} + $$
$$+ 2Cov(\hat{A},\hat{\sigma})\frac{\partial \hat{x}_p^*}{\partial A}\frac{\partial \hat{x}_p^*}{\partial \sigma} + 2Cov(\hat{\omega},\hat{\sigma})\frac{\partial \hat{x}_p^*}{\partial \omega}\frac{\partial \hat{x}_p^*}{\partial \sigma}. \quad (7)$$

The results of the percentiles confidence intervals are presented in FIGURES 1, 2 and 3.

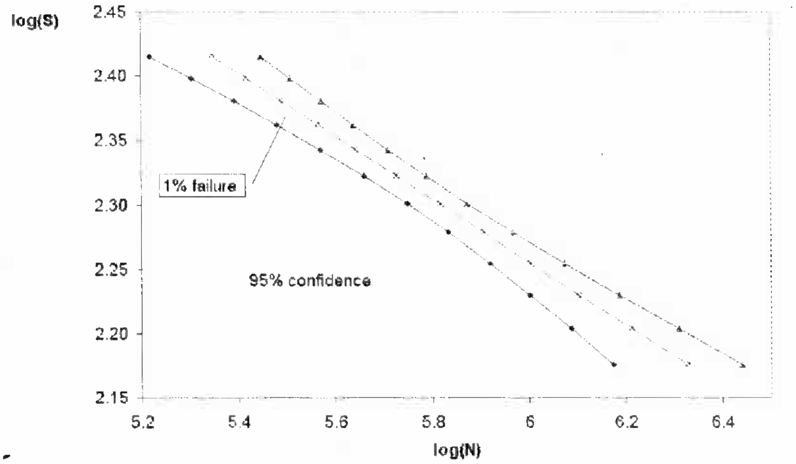

FIGURE 1: SAE 8620 steel S-N-P curve with 95% confidence limits for the percentile 1%.

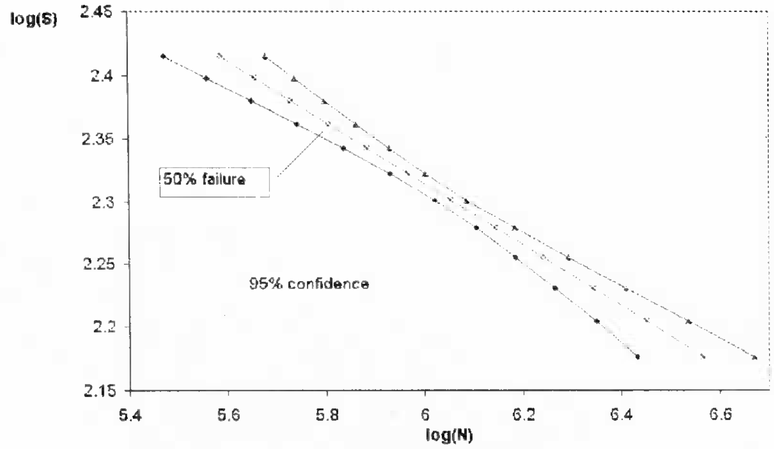

FIGURE 2: SAE 8620 steel S-N-P curve with 95% confidence limits for the percentile 50%.

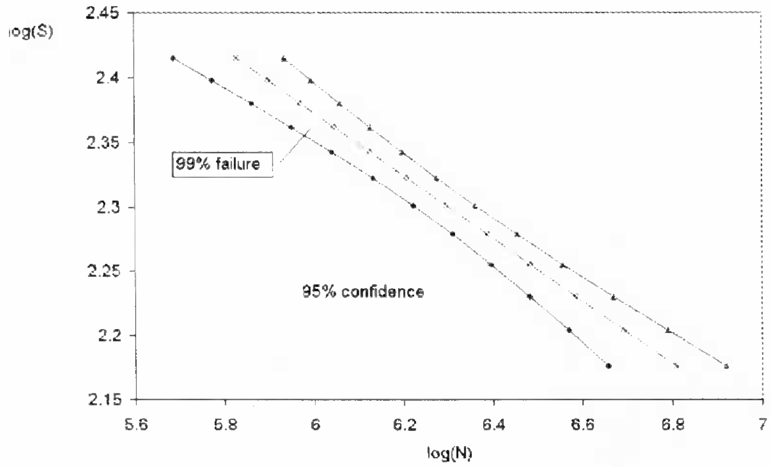

FIGURE 3: SAE 8620 steel S-N-P curve with 95% confidence limits for the percentile 99%.

The curves presented in FIGURES 1, 2 and 3 are denominated S-N-P curves. They relate the applied alternate stress level with the number of cycles to failure for a given probability of failure. As seen before, these curves were obtained from tests of specimens submitted to controlled damage (the first 2 stress steps with pre-defined number of cycles). Due to the dispersion presented by the fatigue tests each curve is presented with a range of 95% of confidence. Also, it is shown how the time to failure limits, in terms of number of cycles, change as a function of the alternate stress levels.

3. S-N-P CURVES OBTAINED FROM CONSTANT STRESS FATIGUE TESTS

3.1 The Experimental Data

The experimental data for constant stress fatigue tests are obtained from cooled rotating bending fatigue tests performed in groups of 4 or 5 specimens with 4 alternate stress levels taken between the material yielding limit and the material endurance limit.

All obtained experimental data are shown in FIGURE 4.

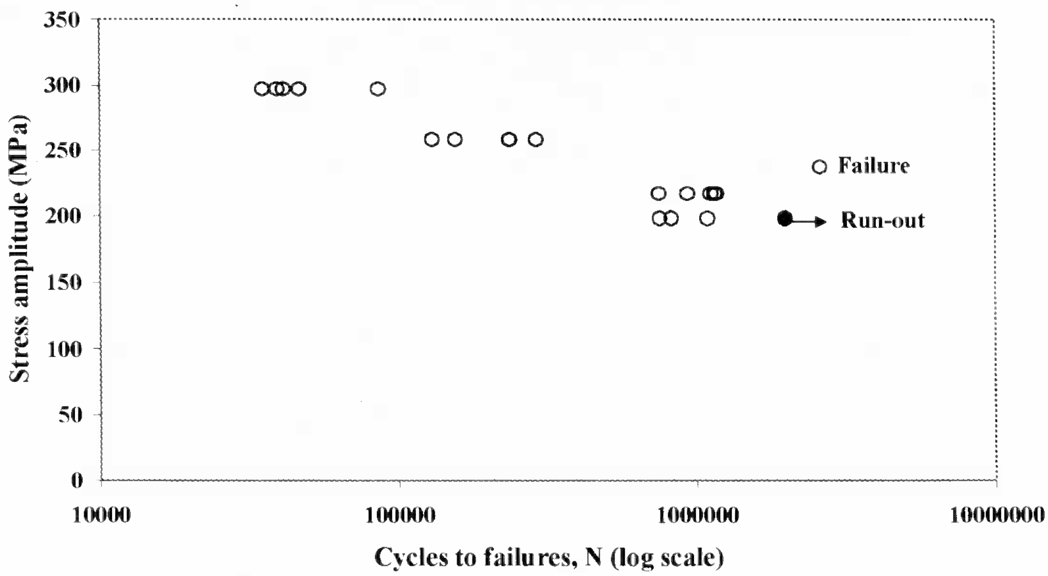

FIGURE 4: Experimental results for constant stress fatigue tests of SAE 8620 steel.

3.2 S-N-P Curves Development for Constant Alternate Stress Fatigue Tests

The S-N-P curves are obtained from a linear regression on the experimental data. The adopted equation to represent the fatigue curves is mathematically given by:

$$S = a + b \times \log[N] \quad (8)$$

where a is the linear coefficient of the straight line and b is its slope.

TABLE 4 presents the parameters of S-N-P curves obtained using the expression (8) on the experimental data of FIGURE 4.

TABLE 4: Parameters of the S-N-P curves.

Parameter	Percentiles for cooled tests		
	1%	50%	99%
a	578.9886	604.5737	630.1601
b	-65.5171	-65.5171	-65.5173

The S-N-P curves for SAE 8620 steel obtained from cooled rotating bending fatigue tests are shown in FIGURE 5. The curves are presented for three probabilities of failure, 1%, 50% and 99%. The log-normal distribution was used to obtain the curves.

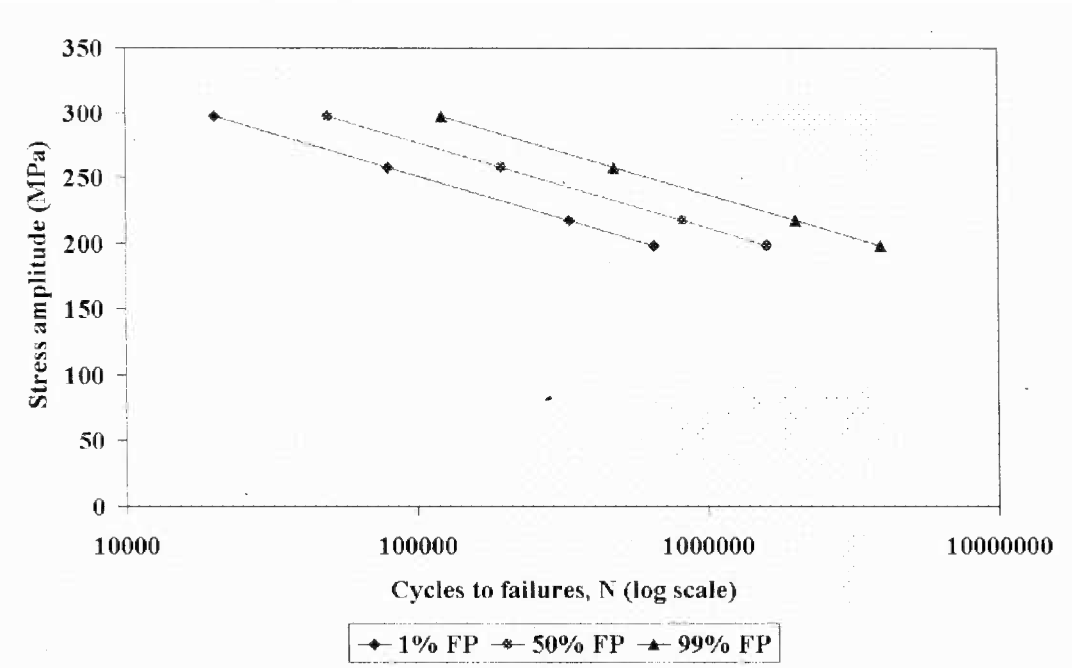

FIGURE 5: SAE 8620 steel S-N-P curves for constant stress test with probabilities of failures of 1%, 50% and 99%.

4. COMPARISON BETWEEN S-N-P CURVES OBTAINED FROM CONSTANT STRESS AND STEP-STRESS FATIGUE TESTS

Based on the developed models for alternate constant stress and step-stress fatigue tests and on the obtained results, it is shown in TABLE 5 the lifetimes as number of cycles N for 4 alternate stress ranges.

TABLE 5: SAE 8620 steel lifetimes obtained from constant stress and step-stress fatigue tests as a function of the alternate stress levels and of the probability of failure.

Alternate Stress Range (MPa)	1% PF-CS	1% 3ST	50%PF-CS	50% 3ST	99%PF-CS	99% 3ST
259	76,567	222,920	188,172	387,780	462,450	674,561
236	171,829	368,411	422,287	640,869	1,037,808	1,114,822
217	335,041	442,018	823,394	768,911	2,023,562	1,337,556
198	653,279	653,213	1,605,492	1,136,294	3,945,677	1,976,637

PF is the probability of failure CS is constant alternate stress 3ST is three steps of alternate stress

FIGURE 6 shows the plot of the TABLE 5 data of which is the comparison between SAE 8620 steel S-N-P curves obtained from constant stress and step-stress fatigue tests.

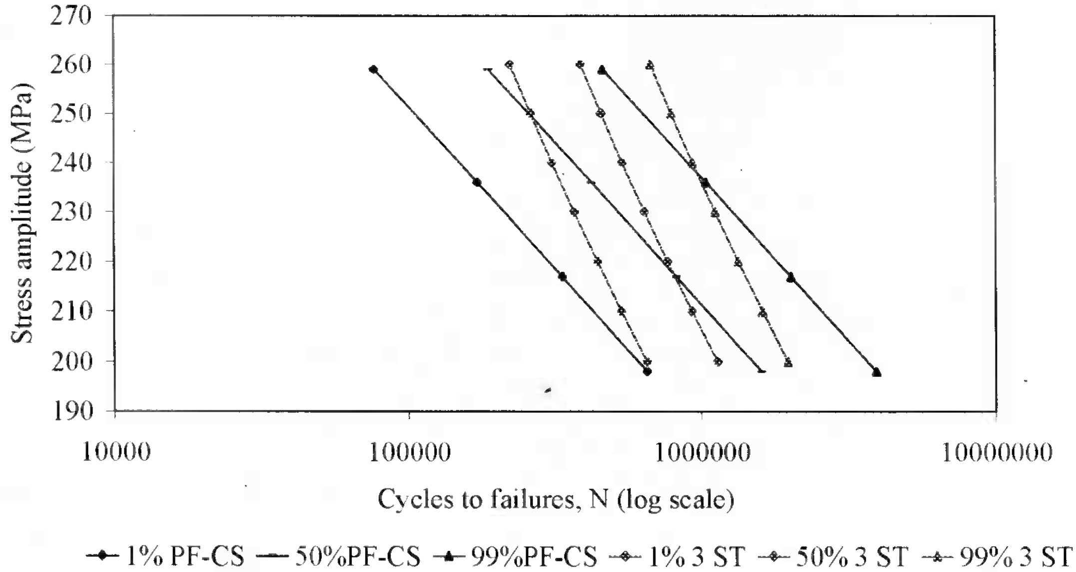

FIGURE 6: Comparison between SAE 8620 steel S-N-P curves obtained from constant stress and step-stress fatigue tests.

5. APPLICATION OF THE PALMGREN-MINER THEORY TO THE EXPERIMENTAL DATA

5.1 Introduction of damages in specimens submitted to the rotating bending fatigue

The introduction of damages in specimens submitted to the rotating bending fatigue has the purpose to assess the Palmgren-Miner damage theory.

In the TABLES 6 to 9 it is shown the number of cycles actually applied to each specimen to damage it, for each stress level.

Table 6. Stresses applied in a decreasing sequence.

Specimen number	Number of cycles			
	259 MPa	236 MPa	217 MPa	198 MPa
1	22,989	51,649	100,611	780,482
2	28,758	51,493	100,593	625,662
3	22,953	51,708	100,576	161,401
4	22,965	51,577	100,585	1,799,476
5	22,953	51,570	101,091	1,127,731
6	22,958	51,598	100,596	1,547,665
7	22,646	51,497	101,429	1,660,861
8	22,950	51,571	100,474	413,915
9	22,948	51,574	100,599	744,842

Table 7. Stresses applied in an increasing sequence.

Specimen number	Number of cycles			
	198 MPa	217 MPa	236 MPa	259 MPa
1	19,5918	100,520	51,538	89,260
2	19,5926	100,548	51,525	153,584
3	19,5931	100,519	51,499	152,921
4	19,5921	100,531	51,541	83,916
5	19,5916	100,527	51,526	128,374

Table 8. Stresses applied alternately - Sequence 1.

Specimen number	Number of cycles			
	259 MPa	198 MPa	236 MPa	217 MPa
1	25,023	200,027	60,131	588,670
2	25,070	200,050	60,118	105,560
3	25,018	200,053	60,020	396,228
4	25,042	200,050	61,270	382,600
5	25,825	201,060	61,121	273,380
6	25,024	200,138	60,050	300,334

Table 9. Stresses applied alternately - Sequence 2.

Specimen number	Number of cycles			
	198 MPa	236 MPa	217 MPa	259 MPa
1	200,326	60,051	100,252	680,063
2	200,067	60,057	100,010	29,163
3	200,212	60,034	100,051	129,483
4	200,024	60,017	100,083	185,701
5	200,022	60,014	23,638 (fractured)	XXXXXX
6	200,171	60,027	100,019	31,694

5.2 Application of the Palmgren-Miner theory

This was the first theory proposed to assess the cumulative damage. It is a linear theory and widely used due to its simplicity. The theory states (MINER, 1945 and COLLINS, 1993) that the fraction of damage for a certain stress level is

$$D_i = \frac{n_i}{N_i} \tag{9}$$

where, n_i is the number of cycles applied to the component under a stress S_i and N_i is the number of cycles obtained from the S-N-P curve for the stress S_i.

The Palmgren-Miner theory states that the failure of the component occurs when the sum of the damages is equal or greater than 1, that is,

$$D_t \geq 1 \tag{10}$$

The Palmgren-Miner proposal is very easy to apply. The main drawback of this theory are its independence on the stress levels and on the sequence of the loading and it does not take in account the interaction of the damages (MINER, 1945; COLLINS, 1993; FATEMI & YANG, 1998).

FIGURE 7 shows the application of the Palmgren-Miner theory (expression (9)) to the experimental data present in TABLES 5 to 9.

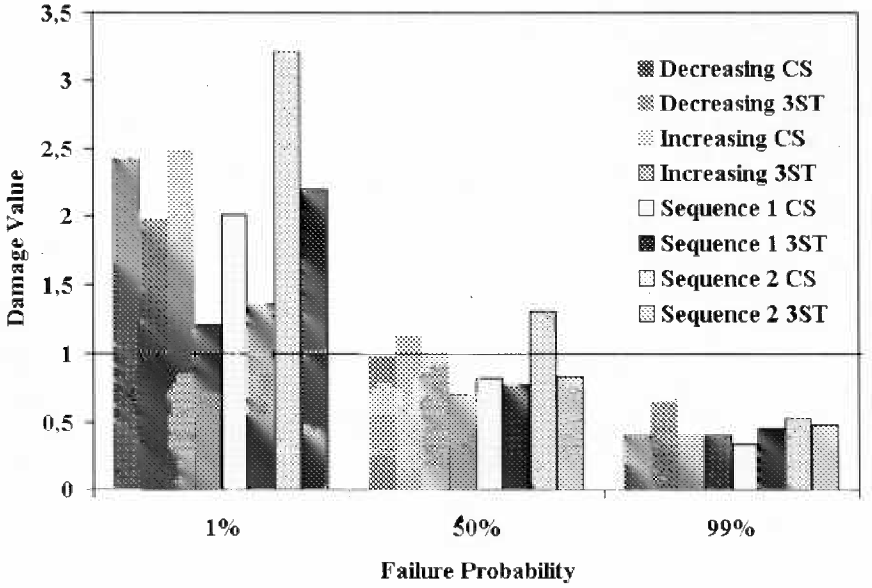

FIGURE 7: SAE 8620 steel - Application of the Palmgren-Miner theory to the fatigue tests.

6. CONCLUSION

From S-N-P curves of FIGURE 6:

(1) For higher stresses, the lifetime of undamaged material specimens is smallest than the lifetime of damaged material specimens. For lower stresses, the behavior is the opposite. The explanation of this fact requests further investigations using a methodology based on Fracture Mechanics to evaluate the lengths of stable cracks in J integral test using two sets of specimens with the same geometry, one with controlled damage and an undamaged one. From the comparison between the measured crack lengths for both sets it is expected to evaluate the crack growth in a relative way.

(2) The variability of the test results in damaged specimens is smaller than in undamaged specimens. Damaged specimens present a more homogeneous behavior than those undamaged. A possible explanation of this phenomenon is that when damaging a material micro-cracks are introduced. As the time for the crack nucleation is a random variable, when using specimens with nucleated cracks one reduces effect of this random variable and, therefore, decreases the test results variability.

(3) S-N-P curves from tests with damaged specimens have curves with greater slopes than those from tests with undamaged specimens. Thus, the damage decreases the influence of the stress level in the material lifetime.

From FIGURE 7, the following was observed, regardless how the stress was applied (monotonically increasing, monotonically decreasing, sequence 1 or sequence 2):

(1) Using the Palmgren-Miner theory, the best results for the lifetime assessment are those from 50% of probability of failure.

(2) For 1% of probability of failure, the theory is conservative, that is, when the sum of the cumulative damage is indicating 1, the specimens can stand more cycles.

(3) For 99% of probability of failure, the theory is non conservative, indicating life when the specimen is already fractured.

7. REFERENCES

1. COLLINS, J. A. ***Failure of materials in mechanical design***. 2nd ed. New York, N.Y.: J. Wiley, 1993.
2. FATEMI, A.; YANG, L. ***Cumulative fatigue damage and life prediction theories: a survey af the state of the art for homogeneous materials***. Int. J. Fatigue, Vol. 20 N° 1, pp. 9-34, Great Britain,1998.
3. FREITAS, M. A.; COLOSIMO, E. A. ***Confiabilidade: análise de tempo de falha e testes de vida acelerados***. Belo Horizonte: Fundação Christiano Ottoni, Escola de Engenharia da Universidade Federal de Minas Gerais, 1997. (Série Ferramentas da qualidade, 12).
4. MINER, M. A. **Cumulative damage in fatigue.** Journal of Applied Mechanics, 1945 A159-A164..
5. NELSON, W. ***Accelerated life testing – step-stress models and data analysis***. IEEE Transactions on Reliability, v. R-29, n. 2, p. 103-108, 1980.
6. PINTO, J. M. A. ***Modelagem de testes acelerados com esforço aplicado em níveis em um estudo de fadiga mecânica.*** 2004. Tese (Doutorado) – Instituto de Pesquisas Energéticas e Nucleares, São Paulo.
7. SOBCZYK, K.; SPENCER, Jr. B. F. ***Random fatigue: from data to theory***. Boston: Academic Press, 1992.