

**THE DUCTILE FRACTURE METHODOLOGY WITH A NEW PROCEDURE
TO OBTAIN STRUCTURAL CALIBRATION FUNCTIONS**

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ABSTRACT

The structural integrity of nuclear reactor components has to be confirmed under postulated accidents conditions. The severe loading that comes from accident scenarios cause the stresses in structures like pressure vessels and piping to go well beyond the linear elastic limits. The integrity evaluation of the component usually includes a postulated defect so that predictions of its load carrying capacity is generally done with a ductile fracture mechanics approach. Several ductile fracture mechanics approaches have been developed and applied to structural components containing defects. This paper will focus on a particular methodology developed by Landes et al., the Ductile Fracture Method (DFM). A critical step in this methodology is the determination of the calibration function for the structural component. This work presents an alternative procedure to obtain this function which makes the application of the DFM much simpler and faster. This is an important aspect considering that in many situations a large range of geometries and loading conditions need to be evaluated, like in the application of the Leak-Before-Break (LBB) concept on nuclear piping.

I. INTRODUCTION

The integrity evaluation of structural components subjected to load levels which push the material beyond the linear elastic limits is generally done with a ductile fracture mechanics approach. Several of these approaches have been developed and applied to structural components containing defects. This paper will concentrate on a particular methodology originally proposed by Ernst and Landes [1] and further developed by Landes and coworkers [2-4], the Ductile Fracture Method (DFM).

The DFM uses the load versus displacement record from a fracture toughness test to develop inputs for predicting the behavior of the structural component. The principle of load separation is used to convert the test record into two pieces of information: calibration function which describes the deformation behavior and fracture

toughness which describes the response of a crack-like defect to the loading. The calibration function and fracture toughness relative to the test specimen are then transformed to obtain the same kind of information for the structural component. In the final step, the calibration function and fracture toughness for the structure are used to predict the load versus displacement behavior of the structure during the ductile fracture process.

The determination of the calibration function for the structural component represents a critical step in the DFM. The original transformation procedure as proposed by Landes et al. [4] is laborious and time consuming. As will be shown in this paper, with an additional assumption and without loss of accuracy, the transformation procedure can be accomplished in a much simpler way and consequently the predictions done by the DFM can be completed with much less effort.

The paper begins with a review of the DFM, including a detailed description of the original transformation procedure. Then, an alternative approach is presented where the coefficients of the calibration function for the structural component are obtained by simply scaling their counterparts for the fracture toughness specimen. The application of the DFM with this new approach is illustrated through a numerical example. The paper is concluded with a brief discussion about the convenience of the new simplified transformation procedure.

II. BACKGROUND

The Ductile Fracture Method. The general approach for using the DFM is illustrated in Fig. 1. The calibration function gives the relationship between load and displacement for constant values of crack length. This is represented by a family of curves. The fracture toughness (given in terms of a *J-R curve*) describes how the crack length changes as a function of *J*. To apply the method to a structure with a given crack size, the loading is represented by the load, *P*, versus displacement, *v*, for that defect size. During the loading process, the value of *J* applied to the structure is also determined. When *J* has increased to the point where a crack length change is indicated, the *P* versus *v* curve is taken a step down to the one for that new crack length. The loading proceeds with the calibration function giving the relationship between *P* and *v* for a given crack length and the *J-R curve* indicating what current value of crack length should be used. When small increments in crack length are used, the loading follows a smooth path.

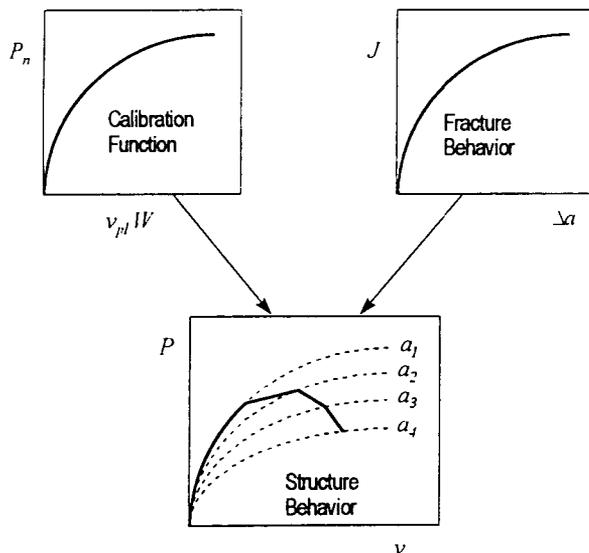


Figure 1. Schematic of the Ductile Fracture Method

The DFM is founded on the load separation concept [5,6]. According to this concept, the relationship between load, *P*, crack length, *a*, and plastic displacement, *v_{pl}*, for the cracked structure can be expressed as a multiplication of two separable functions

$$P = G(a/W)H(v_{pl}/W) \quad (1)$$

where $G(a/W)$ is a function of geometry only and $H(v_{pl}/W)$ is a function of plastic deformation only. W is a length dimension parameter; for test specimen geometries, W is usually the width, but for a structural component, it could be another dimension, such as the thickness. When the load P is divided by the $G(a/W)$ function, the result is a normalized load

$$P_N = P/G(a/W) = H(v_{pl}/W) \quad (2)$$

It was found that the global deformation pattern for many structures and for most materials can be accurately fitted by the following functional form

$$P_N = H(v_{pl}/W) = \frac{\left(L + M \frac{v_{pl}}{W}\right)}{\left(N + \frac{v_{pl}}{W}\right)} \left(\frac{v_{pl}}{W}\right) \quad (3)$$

This functional form came from the work of Orange [7] and is now known as LMN function [8]. The normalized load versus plastic displacement behavior as represented by this function is such that when the plastic displacement is small, the relationship between load and displacement is approximated by a power law, and when it is large, the behavior comes near to a straight line representation.

The G function is known for several geometries or can be obtained in a relatively easy way from a series of blunt notched specimens that model the structure [9]. Thus, the critical step in the DFM is the determination of the calibration function, $H(v_{pl}/W)$, for the structural component.

Obtaining The H Function (Original Procedure). For those cases in which the limit load solution for the structural component is known, a transformation procedure can be used to determine the calibration function for the structural component directly from that for a fracture toughness specimen. The transformation procedure was derived from experimental results on an A533B steel in which four specimen geometries were tested: CT (compact tension), DENT (double edge notched tension), CCT (center cracked tension), and SENT (single edge notched tension). It was verified that when the normalized loads for these geometries were plotted against the normalized

displacement, v_{pl}/v_{el} , all the curves appeared to have the same trend and the differences in the normalized load between the specimens could be related to the ratios of their limit loads. If the CT specimen is taken as a fracture toughness test specimen, and the other geometries are considered as structural components, it is then possible to predict the calibration function for the structural components from the test specimen.

Starting from the calibration function for the test specimen as represented by Eq. 3, the same functional form is assumed for the structural component, that is

$$P_n = h(v_{pl}/W) = \frac{\left(1 + m \frac{v_{pl}}{W}\right)}{\left(n + \frac{v_{pl}}{W}\right)} \left(\frac{v_{pl}}{W}\right) \quad (4)$$

Thus, to find the news constants l , m , and n that define the calibration function, $h(v_{pl}/W)$, for the component, the transformation procedure comprises the following steps:

- First, the load versus displacement record for a fracture toughness test specimen needs to be obtained from an experiment;
- Then this P - v curve is converted into a normalized load, P_n , versus normalized displacement, v_{pl}/W , which is the calibration function for the specimen, $H(v_{pl}/W)$;
- The abscissa, v_{pl}/W , is divided by v_{el}/W so that it becomes v_{pl}/v_{el} ;
- Then each point on that curve is multiplied by a factor f , which is defined as

$$f = \frac{P_{Ls} G_s(a_{os}, W_s)}{P_L G(a_o, W)} \quad (5)$$

The result is a curve for the structural component given in terms of its normalized load, P_n , and the displacement ratio, v_{pl}/v_{el} . In the above equation, P_{Ls} and $G_s(a_{os}, W_s)$ are the limit load and geometry function for the structure; P_L and $G(a_o, W)$ are the limit load and geometry function for the fracture toughness specimen and a_o is the initial crack length.

- The displacement ratio in the abscissa, v_{pl}/v_{el} , is converted back to normalized plastic displacement, by multiplying v_{pl}/v_{el} by v_{el}/W for the structure. The resulting P_n versus v_{pl}/W curve is the representation of the desired calibration curve for the structure.
- The l , m , and n constants can be determined by fitting the transformed points. This is done by choosing $l = f.L$. Then, only m and n need to be determined from the fitted curve. The best result comes from choosing two points, the final one and one at a small v_{pl}/W , and fitting Eq. 4 to them.

During the process of the calibration function transformation, the elastic displacement, v_{el} , is taken as

$$\begin{aligned} v_{el} &= C(a/W) P = \\ &= C(a/W) G(a/W) H(v_{pl}/W) \end{aligned} \quad (6)$$

where $C(a/W)$ is the elastic compliance function for the structure. It can be obtained by experiment, analytical derivation or linear finite element analysis. v_{el} is calculated at each given value of v_{pl}/W (where $H(v_{pl}/W)$ has a known value), and the product $C(a/W)G(a/W)$ is assumed to be a constant. This is not completely true in the actual case. This product is dependent upon a/W , but in many cases it is not a strong function of a/W . Therefore, for a changing crack length, a constant value, say, the value corresponding to the initial crack length, can be used for simplicity.

III. THE NEW TRANSFORMATION PROCEDURE

Even though it does not involve any complex steps, the original transformation procedure is somewhat cumbersome and time consuming, requiring the user to make one ordinate conversion (step d, above), two abscissa conversions (steps c and e), and to use a new fitting to find the constants of the calibration function for the structural component (step f).

Analyzing the philosophy of the original transformation procedure, we see that it basically consists in making a load and a normalized displacement adjustment on the fracture toughness specimen deformation curve to get the correspondent curve for the structural component. The load adjustment is very simple because it is based on a constant factor f , Eq. 5. On the other hand, the abscissa adjustment involves point to point computations of the elastic displacement using Eq. 6 and assuming that the product $C(a/W)G(a/W)$ is constant.

Applying the procedure to different test geometries, in which the CT specimen was taken as the fracture toughness test specimen and the other geometries were considered as structural components, it was observed that the ratio between the normalized elastic displacement for the fracture toughness specimen, (v_{el}/W) , and the normalized elastic displacement for the structure, $(v_{el}/W)_s$, almost did not change during the transformation process. As will be shown in the following, if we assume that this ratio is constant, the transformation procedure can be greatly simplified.

First, define a parameter q to represent the ratio between the normalized elastic displacements

$$q = \frac{(v_{el}/W)}{(v_{el}/W)_s} \quad (7)$$

From the analysis undertaken, it can be seen that a representative value for q is obtained when the normalized elastic displacements for the specimen and the structure are calculated with their initial crack lengths and at their limit loads, that is

$$(v_{el})_o = C(a_o)P_L \quad (8)$$

$$(v_{el})_{os} = C_s(a_{os})P_{L_s} \quad (9)$$

The normalized plastic displacement for the fracture toughness specimen, $v_N = v_{pN}/W$, and the normalized plastic displacement for the structure, $v_n = v_{pn}/W$, can be related by the following expression

$$v_n = \frac{v_N}{q} \quad (10)$$

Rewriting Eqs. 3 and 4 in terms of v_N and v_n ,

$$P_N = H(v_N) = \frac{(L + Mv_N)}{(N + v_N)} v_N \quad (11)$$

$$P_n = h(v_n) = \frac{(l + mv_n)}{(n + v_n)} v_n \quad (12)$$

Multiplying Eq. 11 by the factor f (Eq. 5) is equivalent to step (d) of the original procedure, which would give an intermediate representation P_n versus v_N with the constants $l' = fL$, $m' = fM$ and $n' = N$, that is

$$P_n = \frac{(l' + m'v_N)}{(n' + v_N)} v_N \quad (13)$$

Then, substituting Eq. 10 in Eq. 13, that corresponds to steps (c) and (e) of the original transformation procedure, leads to

$$P_n = \frac{l' + m'(qv_n)}{n' + (qv_n)} (qv_n) = \frac{fL + fqMv_n}{(N/q) + v_n} v_n \quad (14)$$

Now, comparing the above expression with Eq. 4, we arrive at the following expressions for the l , m , and n constants of the structure calibration function

$$l = f \cdot L; \quad m = f \cdot q \cdot M; \quad n = N/q \quad (15)$$

Therefore, the coefficients of the calibration function for the structural component can be directly obtained from their fracture toughness specimen

counterparts. It would suffice to compute the factors f and q (Eqs. 5 and 7) and use the above expressions.

IV. APPLYING THE DFM WITH THE NEW TRANSFORMATION PROCEDURE

In another paper from the authors [10], the effectiveness of the new transformation procedure was demonstrated by comparing calibration curves obtained using the new procedure with those obtained with the original procedure. Different A533B steel specimen geometries were used. For each type of geometry (playing the role of a structural component), the calibration function was predicted from the calibration function for a CT fracture toughness specimen of the same material. It was shown that the results obtained with both procedures were practically the same. As an example, Fig. 2 shows the results obtained for a CCT specimen (see its characteristics in Table 1); the experimental P_n versus v_n curve is compared with those obtained using the original transformation procedure and the simplified approach presented here. The points represented by crosses are those obtained with the original transformation procedure and should still be fitted in order to get the coefficients of the calibration function. The solid line is the curve described by the coefficients obtained directly by Eqs. 15 of the simplified transformation. As can be seen, the results obtained with both procedures are just about the same.

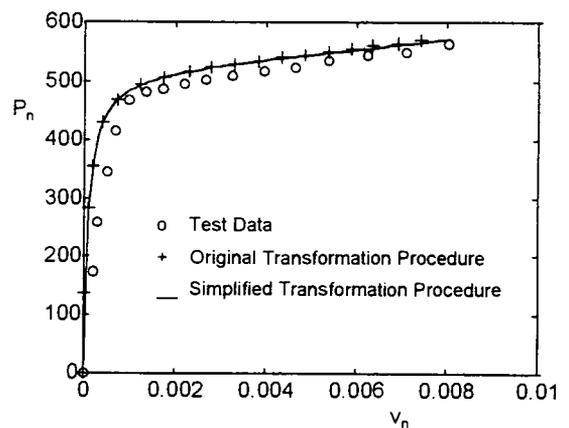


Figure 2. Calibration Curves for the CCT Specimen

TABLE 1. A533B Steel Specimens ($E_{eff} = 206850$ MPa; $\sigma_y = 468.86$ MPa; $\sigma_{uts} = 620.55$ MPa)

SPECIMEN TYPE	W (mm)	B (mm)	a_o (mm)	a_r (mm)
CT	203.20	2.54	101.85	130.02
CCT	406.40	2.54	101.60	115.82

To illustrate the application of the DFM using the simplified transformation procedure, the steps necessary to predict the load versus displacement curve for the CCT specimen in Table 1 are presented below (the basic input data come from the load versus displacement record for the CT specimen in Table 1; the G functions for CT and CCT geometries are known and the respective η_{pl} values are 2.15 and 1.0 [9]):

- (a) The method of normalization [8] is used to obtain the coefficients L , M , and N of the deformation function, H , and the J - R curve for the CT specimen:

$$L = 28.42; M = 58.71; N = 6.17 \times 10^{-4}$$

$$J = 10.33(\Delta a)^{0.3801}$$

- (b) The limit load and compliance for both the CT and CCT specimens, for the respective initial crack lengths, are (these values were obtained using the limit load and compliance solutions tabulated in Ref. [11]):

$$P_L = 24.33 \text{ kN}; C = 0.0702 \text{ mm/kN}$$

$$P_{Ls} = 281.11 \text{ kN}; C_s = 0.00154 \text{ mm/kN}$$

- (c) Eqs. 8 and 9 are applied to obtain the $(v_{el})_o$ for the CT and CCT specimens:

$$(v_{el})_o = 0.0702 \times 24.33 = 1.708 \text{ mm}$$

$$(v_{el})_{os} = 0.00154 \times 281.11 = 0.4329 \text{ mm}$$

- (d) Computation of the factors f and q (Eqs. 5 and 7, respectively):

$$f = (281.11/0.8) / (24.33/0.1793) = 2.6$$

$$q = (1.708/203.2) / (0.4329/406.4) = 7.9$$

- (e) Eqs. 15 are used to obtain the coefficients l , m , n of the calibration function for the CCT specimen:

$$l = 2.6 \times 28.42 = 73.9$$

$$m = 2.6 \times 7.9 \times 58.71 = 1205.9$$

$$n = 6.17 \times 10^{-4} / 7.9 = 7.81 \times 10^{-5}$$

- (f) The complete load versus displacement curve for the structure (in this case a CCT specimen) is then determined through the step by step procedure briefly described in Section II and depicted in Fig. 1. An independent variable is chosen to increment. The basic approach is to choose v_{pl} as the independent variable. Starting with $a=a_o$ and with a small value for $v_N = v_{pl} W$, P (Eq. 1) and J_{app} are calculated. Follows an iterative process, where the crack length is adjusted until J_{app} matches J_{mat} from the J - R curve equation. For each converged iteration, the pair of values P , v are obtained and v_{pl} is again incremented. The process continues until the complete load versus displacement curve is achieved. The value of J_{app} is determined as a sum of an elastic and a plastic component

$$J = J_{el} + J_{pl} = \frac{K^2}{E'} + \frac{\eta_{pl}}{Bb} \int_0^{v_{pl}} P dv_{pl}$$

$$= \frac{K^2}{E'} + \frac{\eta_{pl} W}{Bb} G(b/W) \cdot \int_0^{v_{pl} W} H(v_{pl}/W) d(v_{pl}/W)$$
(16)

where K is the linear elastic stress-intensity factor, E' is the effective modulus of elasticity, η_{pl} is the plastic η -factor, B is the structural thickness, and b is an uncracked ligament length. The total displacement, v , is a sum of an elastic and a plastic component

$$v = v_{el} + v_{pl} \quad (17)$$

and the relationship between v_{el} and P is given in terms of the compliance, C , that is

$$v_{el} = C(a/W)P \quad (18)$$

The load versus displacement curve obtained for the CCT specimen is shown in Fig. 3, where it is compared with the experimental data. Actually, two predicted curves are presented in Fig. 3, one obtained with the J - R curve from the CT specimen and the other based on the J - R curve from the CCT specimen itself, which in this case was available.

As we can see, the maximum load can be predicted reasonably well with the J - R curve from the CT specimen. But, using the J - R curve from the structure itself (represented here by the CCT specimen) is important, at least in this case, to capture the behavior beyond the maximum load.

The basic philosophy of the DFM [4] includes a transformation on the J - R curve to consider the specific geometry characteristics of the structural component in a similar fashion to what is done with the calibration function. Unfortunately, there is still not available any reliable method to make the correlation between the fracture toughness from a test specimen and the actual fracture toughness for the structural component and the general practice has been to confer a material property status to the J - R curve obtained from a test specimen.

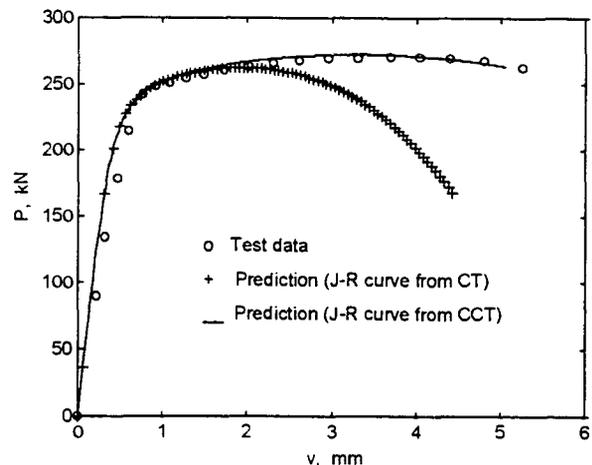


Figure 3. Load versus Displacement Curve for the CCT Specimen

V. FINAL REMARKS

When dealing with a fracture toughness specimen and a structural component with the same thickness constraint, it seems possible to transfer the deformation function from the specimen to the structure by considering two factors: a load factor, given by the ratio between their normalized limit loads (Eq. 5), and a deformation factor given by the ratio between their normalized elastic displacements. Both factors had already been used in the original transformation procedure proposed by Landes et al. [4]. In the present paper, it was shown that likewise the use of a constant load factor f (Eq. 5), it is possible to define a constant deformation factor q (Eq. 7), where the normalized elastic displacements for both the specimen and the structure are calculated with their initial crack lengths and at their limit loads (Eqs. 8 and 9). With these two factors, f and q , the transformation procedure gets much simpler since the coordinates conversions and the fitting operation to get the l , m , and n constants are eliminated.

Considering that the determination of the calibration function for the structure is a critical step in the DFM, the simplification introduced with the new transformation procedure represents an important contribution for the method. With the new procedure, the whole process of predicting the structural component behavior can be completed in a shorter time. This is a determinant factor when a systematic evaluation of a large range of geometries and loading conditions is necessary.

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