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PULSED NEUTRON MEASUREMENTS IN WATER

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RESUMO

São apresentados os resultados de medidas dos parâmetros de difusão dos neutrons térmicos na água feitas recentemente pelo método da fonte pulsada de neutrons. As medidas foram realizadas em geometrias cúbicas, com curvaturas geométricas variando entre ,037 e ... .785 cm^{-2}. O efeito dos recipientes de alumínio foi estimado em têrmos de uma espessura de água equivalente. O método usado para a correção das perdas de contagem do detetor e do sistema de contagens é apresentado.

As curvas de decaimento foram analisadas por método de mínimos quadrados levando-se em conta a influência, nos parâmetros de difusão, de vários valores da distância extrapolada e de diferentes tipos de ponderação no ajuste da função λ versus B^2.

Os resultados indicam que os fatôres de ponderação podem ter considerável influência sôbre os valores do coeficiente de difusão e do coeficiente de resfriamento de difusão. Além disso, a comparação dos resultados apresentados neste trabalho com os obtidos em experimentos realizados com geometrias chatas indica que o efeito de forma pode ser menos importante do que tem sido indicado.

RESUMÉ

Les auteurs présentent les résultats de mesures des paramètres de diffusion des neutrons thermiques dans l'eau, faites récemment par la méthode des neutrons pulsés dans des géométries cubiques avec des laplaciens compris entre 0,037 et 0,785 cm^{-2}. Ils ont estimé l'effet des parois d'aluminium des récipients. Ils exposent la méthode qu'ils ont utilisée pour corriger les pertes de comptage du détecteur et de l'ensemble de comptage.

Ils ont analysé les courbes de décroissance par la méthode des moindres carrés et ont tenu compte de l'influence, sur les paramètres de diffusion, de diverses valeurs pour la distance d'extrapolation et de différents facteurs de pondération pour les points de la courbe de λ en fonction de B^2 ajustée aux résultats expérimentaux.

Les résultats montrent que les facteurs de pondération peuvent avoir une grande influence sur les valeurs de la constante de diffusion et du coefficient de refroidissement par diffusion. Enfin, la comparaison des résultats expérimentaux obtenus par les auteurs et de ceux qui ont été obtenus dans des expériences utilisant une géométrie plat indique que l'effet de forme sur les paramètres de diffusion peut être moins important qu'on l'avait supposé.

ABSTRACT

The results of recent measurements of the thermal neutrons diffusion parameters of water using the pulsed neutron technique are presented in this paper. All measurements were done in cubic geometries with bucklings varying from .037 to .785 cm^{-2}. The effect of the aluminium walls of the containers was estimated in terms of an equivalent water thickness. The method used to correct the counting losses of the detector and counting system is presented.

The decay curves were analysed by a least squares method and care was taken of the influence, on the diffusion parameters, of various values for the extrapolation distance and of different weighting factors for the experimental points of the λ versus B^2 curve fitting.

Our results indicate that the weighting factors may have a considerable influence on the values of the diffusion constant and diffusion cooling coefficient. Furthermore, the comparison of our experimental results with the ones obtained in experiments using flat and cubic geometries indicates that the shape effect on the diffusion parameters can be less important than has been considered.

1. INTRODUCTION

Many workers have measured the thermal neutrons diffusion parameters of water by the pulsed technique /1/ - /17/, but considerable discrepancies persist between the results, even considering the most recent experiments /10/, /12/, /17/. The published experimental values of the diffusion cooling coefficient C ranges from 0 to 8000 $cm^4.sec^{-1}$ and the diffusion coefficient D from 34000 to 39000 $cm^2.sec^{-1}$ (Table I). These discrepancies have been ascribed to several factors, like contamination from higher harmonics in the neutron flux, interference of the container materials, the existence of a time dependent background due to neutron scattering in the laboratory walls /12/, /20/. Recently, it has been emphazized /13/, /19/, that "flat" and "square" geometries give different values to decay constants corresponding to the same buckling: it would mean that a "shape effect" exists in the buckling, and it must be taken as another possible cause of lack of agreement in different experiments. The use of different data evaluation schemes for reduction of the experimental data, including the value of the extrapolation distance, has also been appointed as a source of discrepancies /19/, /21/, /31/.

Considering the published theoretical values, one finds for the diffusion coefficient values on the range 34000 - 39000 $cm^2.sec^{-1}$ and 2700 - 3400 $cm^4.sec^{-1}$ for the diffusion cooling coefficient /39/. The differences here come from the use of different thermalization models, or from particularities introduced into a same model, by different authors. Table II gives some values of theoretical calculations of the thermal neutron diffusion parameters for water.

Evaluations /18/ of the diffusion parameters from measurements performed in three different types of experiments - pulsed source, exponential source and infinite poisoned media - should increase the precision attainable in any of these experiments. Nevertheless, it is clear that the imprecision in the results coming from pulsed measurements may distort the value of the parameters obtained in this kind of evaluation. So, further work must be done in this field, at least concerning the shape effect and the problem of data evaluation schemes.

TABLE I

MEASURED NEUTRON DIFFUSION PARAMETERS OF WATER

BY PULSED METHOD

Ref.	$v\Sigma_a$ (sec^{-1})	D(cm^2 sec^{-1})	C(cm^4 sec^{-1})	B^2 range (cm^{-2})	T ^{o}C
/1/	4878 ± 238	-	-	Large geometry	
/2/	4716 ± 67	-	-	Infinite geometry	
/3/	4695 ± 88	38500 ± 800	-	.006 - .018	
/4/	4892 ± 50	36340 ± 750	7268 ± 1047	.100 - .700	22
/5/	4831 ± 140	35000 ± 1000	4000 ± 1000	.090 - .930	23 ± 0.5
/6/	4950 ± 147	34850 ± 1100	3136 ± 1220	.090 - .960	22
/7/	4919 ± 63	-	-	Infinite geometry	-
/8/	4808	34800	0	.080 - 1.100	-
/9/	4800 ± 90	35050 ± 600	3600 ± 700	.093 - .867	19
/10/	4780 ± 80	35400 ± 700	4200 ± 800	.110 - .750	22
/11/	-	35660 ± 600	6169 ± 1000	.100 - .650	24
/12/	4768 ± 24	37503 ± 366	5116 ± 776	.014 - .590	26.7 ± 1
	-	36892 ± 400	-	"	22**
/13/	4880 ± 160	35020 ± 1300	3620 ± 2090	.100 - .800 square	21
/	4900 ± 150	35630 ± 1170	8050 ± 1670	" flat	"
/14/	4831 ± 93	34600 ± 800	4000 ± 800	.100 - .800	20
/15/	4817 ± 29	36306 ± 650	3700 ± 620	.089 - 2.900	24.5
/16/	4883	36591	2770	.090 - .700 flat	room
	4834	35306	1901	" simet.	"
	4875	34033	659	" tall	"
/17/	4878 ± 95	34120 ± 610	3350 ± 560	.093 - 1.36	10
	-	35500 ± 630	-	"	22**
/18/ *	4782 ± 15	35630 ± 80	3420 ± 170	-1.872 - .800 *	20
This	4869 ± 29	36578 ± 279	6092 ± 464	.038 - .785	22 ± 1.5
Rep.***	4883 ± 16	36414 ± 202	5463 ± 425	"	"

* Evaluation from three different experiments

** Parameters were corrected to this temperature

*** Results from two different weighting factors

TABLE II

CALCULATED DIFFUSION PARAMETERS OF WATER

$v\Sigma_a$	D	C	F	Temperature	Author
	37045	3361	169		GHATAK & HONECK Nelkin model /32/
4886.5	37970	3380	210	$23^{o}C$	CLENDENIN /33/ Nelkin model
4887.3	38630	2730	250	$23^{o}C$	CLENDENIN /33/ Radkowski model
4889.7	37780	2900	-	$23^{o}C$	KOBAYASHI et al Nelkin model /17/
4889.8	39170	3340	-	$23^{o}C$	KOBAYASHI et al /17/ Radkowski model

In the work described in this paper we attempted to eliminate all the sources of errors mentioned above. All the samples had cubic geometries which were considered more suitable than the ones where the base/height ratio is changed to give the several B^2 values, since the shape effect will probably be a function of this quotient. Moreover, with cubic geometries, we can atribute the same extrapolation distance to all sides of the samples and analyse the compatibility of the decay constants and a particular Z_o value in a very simple way.

2. PRINCIPLE OF THE MEASUREMENT IN THE PULSED NEUTRON SOURCE METHOD

The asymptotic solution of the Boltzmann equation , assuming that the energy spectrum does not change with time is

$$\phi(r,E,t) = \phi(r,E)\, e^{-\lambda t} \qquad (1)$$

The asymptotic (fundamental) decay constant is usually written as:

$$\lambda = \bar{v}\,\Sigma_a + DB^2 - CB^4 + FB^6 + o(B^8) \qquad (2)$$

where,

$1/\bar{v}\,\Sigma_a$, neutron lifetime in a infinite media of average macroscopic absorption cross section Σ_a

D , diffusion coefficient (averaged over the spectrum)

C , diffusion cooling coefficient

For a parallelepiped system of sides a, b, c, the buckling B^2 is given by

$$B^2 = \left[\frac{\pi}{a + 2Z_o}\right]^2 + \left[\frac{\pi}{b + 2Z_o}\right]^2 + \left[\frac{\pi}{c + 2Z_o}\right]^2$$

The extrapolation distance Z_o is rigorously also a function of the buckling /22/, /35/.

The pulsed neutron method consists in the experimental determination of a series of (λ, B^2) values and in the calculation of the parameters, $\bar{v}\,\Sigma_a$, D, C. Z_o must be taken from theoretical calculations, /22/, /35/ from direct measurements (Table III) or estimated from the experimental data. The condition $\lambda \lesssim (v\,\Sigma_s)_{min}$ must be satisfied, in order that equation (1), be an asymptotic solution of the Boltzmann equation /23/, /24/; but in the case of water this limit has no experimental implications since all the decay constants are much smaller than $(\bar{v}\,\Sigma_s)_{min}$.

3. EXPERIMENTAL PROCEDURE

3.1 - Physical Arrangement

Neutrons were produced by bombarding a tritium target with 300 kev deuteron pulses from a 400 kev Van de Graaff accelerator. With the mean current of 1 μA a reasonable neutron intensity was obtained and the necessity of target cooling was minimal.

TABLE III

EXTRAPOLATION DISTANCE IN WATER

Ref.	Z_0 (cm)	Remarks
/22/	0.32 - 0.35 *	Theoretical
/ 8/	0.46 $\pm$ 0.05	Pulsed
/23/	0.36 $\pm$ 0.03	Pulsed
/36/	0.40	Pulsed
/37/	0.556 $\pm$ 0.05	Pulsed-chopper
/40/	0.35 $\pm$ 0.03	Steady State

* Slab geometry, $0 \leq B^2 \leq 0.8\ cm^{-2}$

The following neutron pulses were used:

duration	period
80 - 100 μsec	2400 μsec
40 - 50	1200
30 - 40	600

For very large geometries the ion source r-f oscillator was continuosly operated, and the deuteron pulses were obtained by varying the control electrode potential in the ion source base; for small and medium geometries the r-f oscillator was modified, to operate only during the pulse duration and the control electrode was grounded. This allowed a reduction of about 300 - 400 times in the constant background. This system increases the rise and the fall time of the deuteron pulse by a few micro seconds.

The irradiation facility was located in a building with extremely thin walls in order to prevent wall room-return neutrons.

The water tanks were 2S aluminium parallelepipeds; the lateral and bottom wall thickness were 3.17 and 1.59 mm, respectivel ly. The container's height surpassed about 1 cm the water level.

The samples and detector were surrounded by 0.8 mm Cd foils and enclosed in a 20 cm thick borated paraffin box.

The mean temperature of the deionized water (resistivity 1.3×10^5 Ω/cm) was 22.0 ± 1.5^oC.

3.2 - Data Collection

A block diagram of the apparatus is shown in figure 1. A 256 channels digital time analyser was used with 2, 4, 8, 16 μsec channel widths. Even with the small neutron current of 1 μA it was necessary to correct for counting losses.

The pulses from the BF_3 detector were amplified and analysed against noise in a conventional one channel analyser which supplied pulses to the time analyser. The 30 cm of active lenght BF_3 detector was inserted in the shielding, on the bottom of the container and about 1 cm from it. A rectangular mask was cut off the sample and detector cadmium shields to delimitate the detector area exposed to the neutron flux.

The decay constant for each buckling was measured at least two times and in certain cases up to seven times. Some measurements were repeated after a month, and the results presented a very good reproducibility. For each measurement the number of neutron bursts varied between 4×10^6 to 25×10^6.

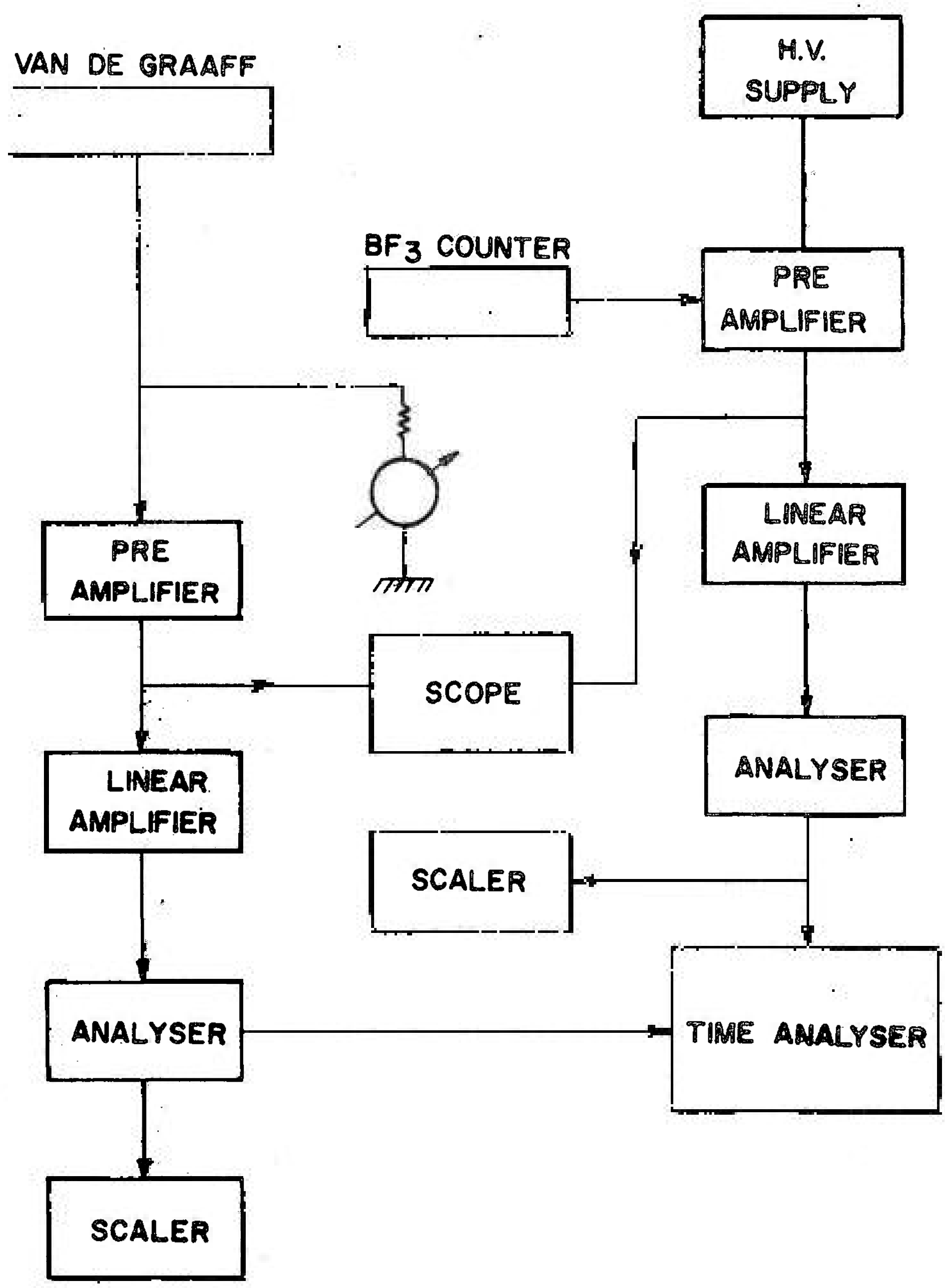

Fig. 1

BLOCK DIAGRAM OF THE ELECTRONIC SYSTEM

The error introduced by the walls of the container was evaluated by applying the method described by WIGNER and WEINBERG /26/ to calculate the saving in a reactor with thin reflector. The aluminium walls 3.17 mm thick are equivalent to 0.1 mm of water, which is always smaller than the error atributed to the sample dimensions. We tried detect this wall thickness effect in a small water volume by increasing the aluminium walls thickness but no variation of the decay constant was found.

4. DATA ANALYSIS

4.1 - Counting Losses Correction

When a count is received in any channel the time analyser needs 16 μsec to store it. The remaining time of this channel and the subsequent channels within 16 μsec are closed. Hence, we have two types of losses in the analyser.

The first one is due to the fact that owing to that 16 μsec storage time the effective number of bursts is not the same to all the channels. In this case we applied a correction factor that normalized each channel counting to the real number of cycles of analysis. If,

NB = total number of bursts
N'_i = number of observed counts in channel i.
Δt = channel width in μsec.
NB_i = effective number of analysis cycles for the channel i

$$NB_i = NB - \sum_{j=1}^{16/\Delta t} N'_{i-j}$$

then the corrected counting N_{Ci} in the channel i is given by:

$$N_{Ci} = (NB \times N_i') / NB_i$$

A similar correction is given by LALANDE /27/.

The second correction in the time analyser comes from the losses due to multiple incidences into a same channel.

In this case we have:

$$N_{2i} = NB.P_{2i}$$

$$N_{3i} = NB.P_{3i}$$

where P_{2i}, P_{3i} are the probabilities of two, three ... incidences into a same channel i, and N_{2i}, N_{3i} ... are the number of double, triple, ... incidences. The corrected counting N_i is then given by

$$N_i = N_{Ci} + \frac{1}{2} N_{2i} + \frac{2}{3} N_{3i} + \ldots$$

Supposing that the counting rate a_i into the channel i is constant, the above probabilities are given by the following expressions /28/:

$$P_{2i} = \exp(-a_i \Delta t) \quad a_i^2 (\Delta t - \tau)^2 / 2!$$

$$P_{3i} = \exp(-a_i \Delta t) \quad a_i^3 (\Delta t - 2\tau)^3 / 3!$$

The losses due to the dead time of the system preceding the multi-channel analyser ($\tau = 1.5 \pm 0.2$ μsec) were calculated by the usual procedure /38/.

4.2 - Analysis of Decay Curves

The decay curves were adjusted by a least squares method for a linear combination of exponentials plus a constant background /29/.

The minimized expression was

$$S = \frac{1}{n-p} \sum_{i=1}^{n} (Y_i - Y(t_i))^2 \, w_i \qquad (3)$$

where

n = number of experimental points

p = number of parameters, $p = 2k + 1$

k = number of exponentials

$Y(t_i)$ = points on the calculated curve; $Y(t_i) = A_o + \sum_{j=1}^{k} A_j e^{-\lambda_j t_i}$

Y_i = experimental values

w_i = weighting factors; $w_i = 1/\sigma^2_{Y_i}$

In the larger water geometries higher harmonics were detected besides the fundamental one; but for the majority of the experimental curves, after a delay of about 100 μsec after the pulse, the fitting was obtained with a single exponential plus a constant. To all the curves, fittings were also made with one exponential plus a constant by varying the time where the analysis begins (fig. 2). The values for the fundamental decay constant obtained with two exponentials plus a constant agreed quite well with the asymptotic values obtained for λ when one exponential plus a constant was used in the fitting with variable delay. Table IV shows the values of the decay constants and the corresponding geometric bucklings.

4.3 - <u>Diffusion Parameters Evaluation</u>

In the derivation of the diffusion parameters from the equation

$$\lambda(B^2) = \bar{v}\,\Sigma_a + DB^2 - CB^4 \qquad (4)$$

we considered

a) the method of weighting the experimental points,

b) the extrapolation distance to be used in calculating the geometric bucklings.

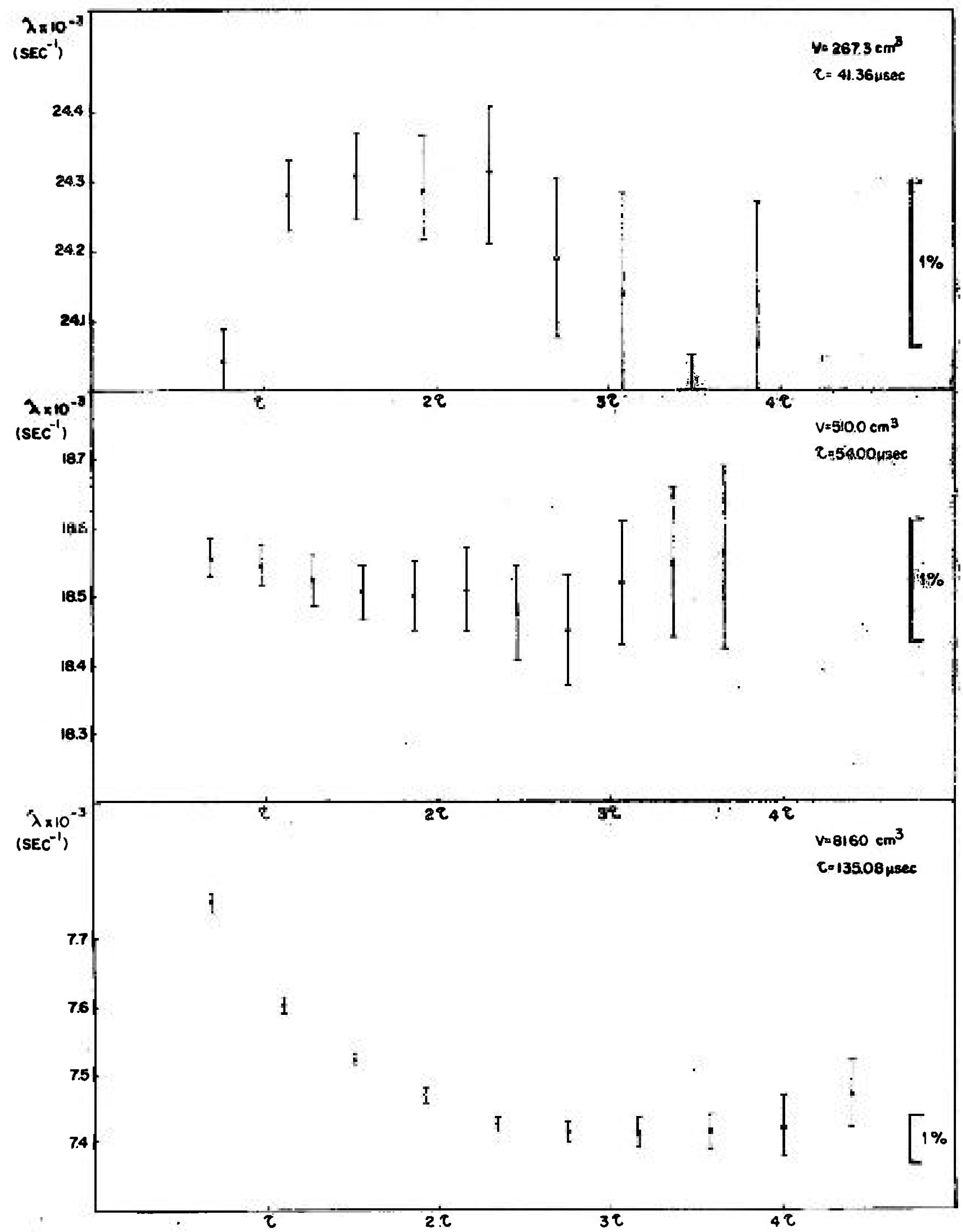

FIG.2

VARIATION OF DECAY CONSTANTS WITH DELAY TIME (UNITS OF LIFETIME)

TABLE IV

EXPERIMENTAL POINTS

Size (cm)	B^2 (cm^{-2}) Z_o = 0.32 cm		λ (sec^{-1}) *	
27.070 x 27.000 x 27.170	0.0385	± 0.0001	6271	± 22
20.110 x 20.151 x 20.136	0.0685	0.0001	7380	22
15.954 x 16.078 x 16.023	0.1065	0.0002	8661	39
12.973 x 12.997 x 12.989	0.1591	0.0004	10486	50
11.308 x 11.246 x 11.309	0.2076	0.0009	12210	50
9.971 x 9.949 x 9.980	0.2624	0.0008	14050	100
8.748 x 8.767 x 8.749	0.3343	0.0011	16583	70
7.985 x 7.995 x 7.989	0.3961	0.0014	18565	70
7.137 x 7.163 x 7.159	0.4855	0.0041	21054	300
6.485 x 6.432 x 6.406	0.5878	0.0077	24060	200
5.958 x 5.955 x 5.964	0.6765	0.0036	27012	112
5.477 x 5.497 x 5.484	0.7847	0.0092	29979	130

* The λ values are averaged over the individual determinations and the errors are the greatest of the individual determinations.

Equation (4) was fitted by a least squares procedure which minimized the expression /30/.

$$S = \frac{1}{n-p} \sum_{i=1}^{n} \left(w_{x_i} V_{x_i}^2 + w_{y_i} V_{y_i}^2 \right) \qquad (5)$$

where n,p have the same meaning as in eq. (3), V_{x_i}, V_{y_i} are the distances of experimental points to the calculated curve and w_{x_i}, w_{y_i} the B_i^2 and λ_i weighting factors, respectively.

For Z_o the following values were used

1) $Z_o = 0.76\ \lambda_{tr}$ /22/ retaining the relation $D = (\lambda_{tr}\bar{v})/3$ and adjusting after each iteration the Z_o value.

2) $Z_o = 0.32$ cm

3) $Z_o = 0.40$ cm

4) $Z_o = 0.50$ cm

For each value for Z_o the experimental points were weighted in two ways:

a) $w_{x_i} = 0;\ w_{y_i} = 1/Y_i^2$ (minimization of percentual deviations)

b) $w_{x_i} = \dfrac{1}{\sigma^2_{B_i^2}}$; $w_{y_i} = \dfrac{1}{\sigma^2_{\lambda_i}}$ where $\sigma_{B_i^2}$, σ_{λ_i} are respectively errors in B_i^2 and λ_i (statistical weighting).

Table V shows the results of these fittings, and from them we can see that:

1) $\bar{v}\ \Sigma_a$ and D do not change when the experimental points are weighted as indicated in case a) or b). The parameter $\bar{v}\ \Sigma_a$ decreases about 1% when the extrapolation distance varies from 0.32 cm to 0.50 cm and the diffusion coefficient increases about 5% in the same conditions.

2) The diffusion cooling coefficient C depends strongly on the method of weighting and on the extrapolation distance. For example, its value changes radically when Z_o passes from 0.32 to 0.50 cm.

3) Retaining the relation $Z_o = 0.76\ \lambda_{tr}$ with $\lambda_{tr} = \dfrac{3\ D}{\bar{v}}$, both weighting methods are compatible with the value

$$Z_o = 0.336 \pm 0.003 \text{ cm and } \lambda_{tr} = 0.442 \pm 0.003 \text{ cm}$$

TABLE V.

DIFFUSION PARAMETERS FOR SEVERAL Z_o VALUES - THIS EXPERIMENT

	$Z_o = 0.76\,\lambda_{tr}$		$Z_o = 0.32$ cm		$Z_o = 0.40$ cm		$Z_o = 0.50$ cm	
	$w_i = 1/\sigma_i^2$	$w_i = 1/Y_i^2$	$w_i = 1/\sigma_i^2$	$w_i = 1/Y_i^2$	$w_i = 1/\sigma_i^2$	$w_i = 1/Y_i^2$	$w_i = 1/\sigma_i^2$	$w_i = 1/Y_i^2$
$\bar{v}\,\Sigma_a$ (sec^{-1})	4880 ± 16	4864 ± 29	4883 ± 16	4869 ± 29	4864 ± 17	4844 ± 29	4842 ± 18	4814 ± 30
D ($cm^2.sec^{-1}$)	36565 ± 206	36768 ± 283	36414 ± 202	36578 ± 279	37209 ± 220	37473 ± 296	38166 ± 245	38554 ± 320
C ($cm^4.sec^{-1}$)	5273 ± 436	5927 ± 474	5463 ± 425	6092 ± 464	4357 ± 485	5227 ± 515	2643 ± 571	3847 ± 589

4) The greater is the Z_o value, the stronger the dependence of C on the weighting factors. Figure 3 shows the experimental points and the calculated parabola for $Z_o = 0.32$ cm and $Z_o = 0.50$ cm. In both the cases the maximum deviation of the experimental points from the calculated curve is about 1%.

From table II and table V it is evident that the parameters D and C can not be used to check thermalization models unless we can remove the ambiguities arisen in the least squares procedure and in the buckling calculation. The buckling calculation depends on the knowledge of Z_o. Values for this parameter found in the recent literature are given in Table III. Some values from pulsed measurements /8/, /36/, /37/ are well above the ones given by theoretical calculation /22/ and from stationary measurements /40/. In view of these discrepancies, we tried an evaluation of Z_o from the experimental data.

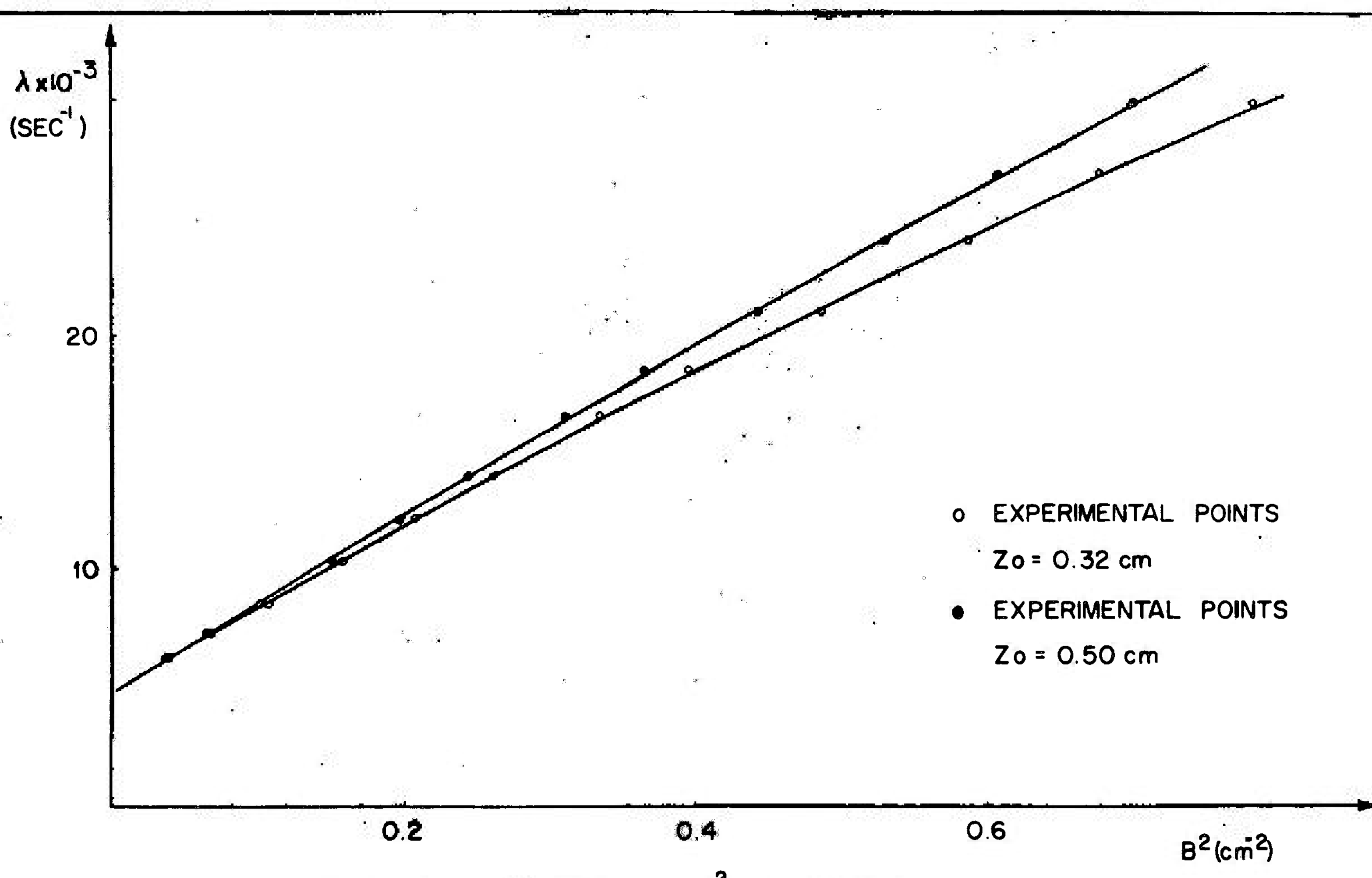

Fig. 3: λ x B^2 CURVE ($1/Y^2$ WEIGHTING)

4.4 - Z_o Evaluation

We will compute, as it was outlined by LOPEZ and KOPPEL /31/ the influence of an error δ in the extrapolated dimension $\tilde{a}$ of the samples.

If,

$\bar{a} = a + 2\Delta$, where Δ is an estimate of Z_o

$Z_o = \Delta + \delta/2$ and $(a + 2Z_o) = (\bar{a} + \delta) = \tilde{a}$

then

$$B^2(\bar{a} + \delta) = B^2(\bar{a}) + \delta \left[\frac{dB^2}{d\tilde{a}} \right]_{\bar{a}} + \ldots$$

and eq. (1) may be rewritten

$$\lambda = \bar{v}\,\Sigma_a + DB^2(\bar{a}) - \frac{2\delta}{\pi\sqrt{3}}\, DB^3(\bar{a}) - \left[C - \frac{\delta^2}{\pi^2} D \right] B^4(\bar{a}) +$$

$$+ \left[\frac{4\delta}{\pi\sqrt{3}}\, C - \frac{4\delta^3 D}{3\pi^3\sqrt{3}} \right] B^5(\bar{a}) + \left[F - \frac{10\,\delta^2}{3\pi^2}\, C + \ldots \right] B^6(\bar{a}) + \ldots \quad (6)$$

Equation (6) shows that if Z_o is estimated with an error δ, the decay constant must be fitted not more by a power series of B^2 but a power series of B, without the first order therm. If the experimental data are still fitted by equation (4) the B^3 may interfere with the B^4 term, and the calculated C value will be dependent on the B^2 range. Equation (6) on the other hand outlines a possibility of determining Z_o in a series of pulsed measurements, through the value of the B^3 coefficient. But a difficulty arises, here, namely the same that appears when one tries to introduce the FB^6 term in eq. (4): the errors in the parameters become too large. An alternative procedure was then followed. Several Δ values were introduced in the buckling calculation, and disregarding the errors, a least square fitting ($1/Y_i^2$ weighting) was made for every Δ. Then, we found the Δ which gives zero for the B^3 coefficient, that is $\Delta \equiv Z_o$. We found Z_o = 0,316 cm.

A second procedure consisted in the fitting ($1/Y_i^2$ weighting) of equation (4) for several Δ values and in assuming that the one which gives the smallest value for the sum (5) is equal to Z_o. Now we found Z_o = 0.322 cm. (fig. 4)

From these results we conclude that with the use of $1/Y_i^2$ weighting factor, our set of experimental data is consistent with an extrapolation distance Z_o = 0.320 cm.

5. COMPARISON WITH OTHER MEASUREMENTS - CONCLUSION

Our λ versus B^2 curve was compared with the ones derived from the parameters obtained by LOPEZ and BEYSTER /12/ and by KUCHLE /10/ (fig. 5). These two experiments have been indicated /19/ as an example of discrepancies between the results of cubic and flat geometry experiments. But for $B^2 \geqslant 0.40$ cm^{-2} our curve agrees well with that of KUCHLE. For $B^2 \leqslant 0.40$ cm^{-2} the agreement is better with the LOPEZ and BEYSTER curve. Hence, the shape effect seems to be smaller than it has been indicated. The reason for the desagreement in the higher part of our and the LOPEZ and BEYSTER curve should be ascribed to different causes.

We note that the curves in figure 5 were drawn from the most probable values of the parameters. If we consider an error band for these curves, the discrepancies will be considerably lowered.

We note also that the LOPEZ and BEYSTER measurements were performed at 26.7° C and the curve in figure 5 was drawn from the corrected parameters for 22° C. The D parameter was corrected by LOPEZ and BEYSTER and C was corrected by us assuming dC/dT = 20 cm^4.sec^{-1}/ °C.[1] At present, the accuracy in the coefficients dD/dT

[1] This value is in fairly good agreement with theory /33/ and with semi empirical calculations /41/.

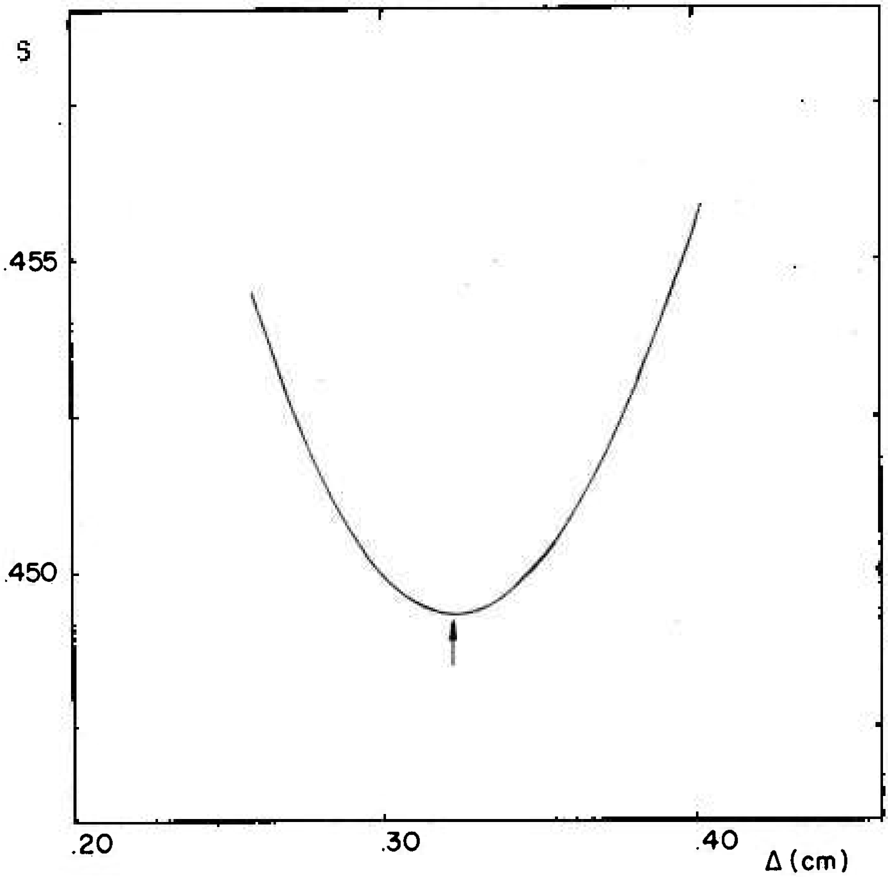

Fig. 4

EFFECT ON MINIMUM SUM S OF VARYING EXTRAPOLATION DISTANCE ($1/Y^2$ WEIGHTING) and especially dC/dT is not very high, but this does not suffice to explain the discrepancies.

The comparison of our parameters with the ones coming from theoretical calculation can be made if Z_o = 0.32 cm since in this case $\bar{v}\ \Sigma_a$ and D are practically unaffected by the weighting factors and a change of no more than 10% migh be introduced in the diffusion cooling coefficient. Our D and C coefficients are respectively smaller and higher than the theoretical ones predicted from the Nelkin or Radkowski kernels (Table II). The diffusion coefficient calculated by KOPPEL and YOUNG /34/ which takes into account the effects of the anisotropy of the molecular vibrations is in very good agreement with the D value derived in this experiment. Our value Z_o = 0.32 cm does not agree with some measurements /8/, /36/, /37/ but agrees with others /23/, /40/. The fact that the value Z_o = 0.32 cm is in very good agreement with the one coming from the expression 0.76 $(3D/\bar{v})$ (Sect. 4.3) is very encouraging but we must say that the evaluations described in Section 4.4 also depends on the weighting factor. By using statistical weighting we found there Z_o = 0.10 cm, which is absurd in view of the actual theoretical and experimental knowledge of this parameter. So, although Z_o = 0.32 cm seems to be more reasonable, there is no doubt that a bias has been introduced in our choice.

The Z_o dependence on B^2 /22/, /35/, was not introduced in our buckling calculations since the predicted variation of Z_o is of the same order or still lower than the incertitude in this parameter. It can be shown /10/ that such a dependence may be accounted for through the introduction of a B^5 term in equation (4).

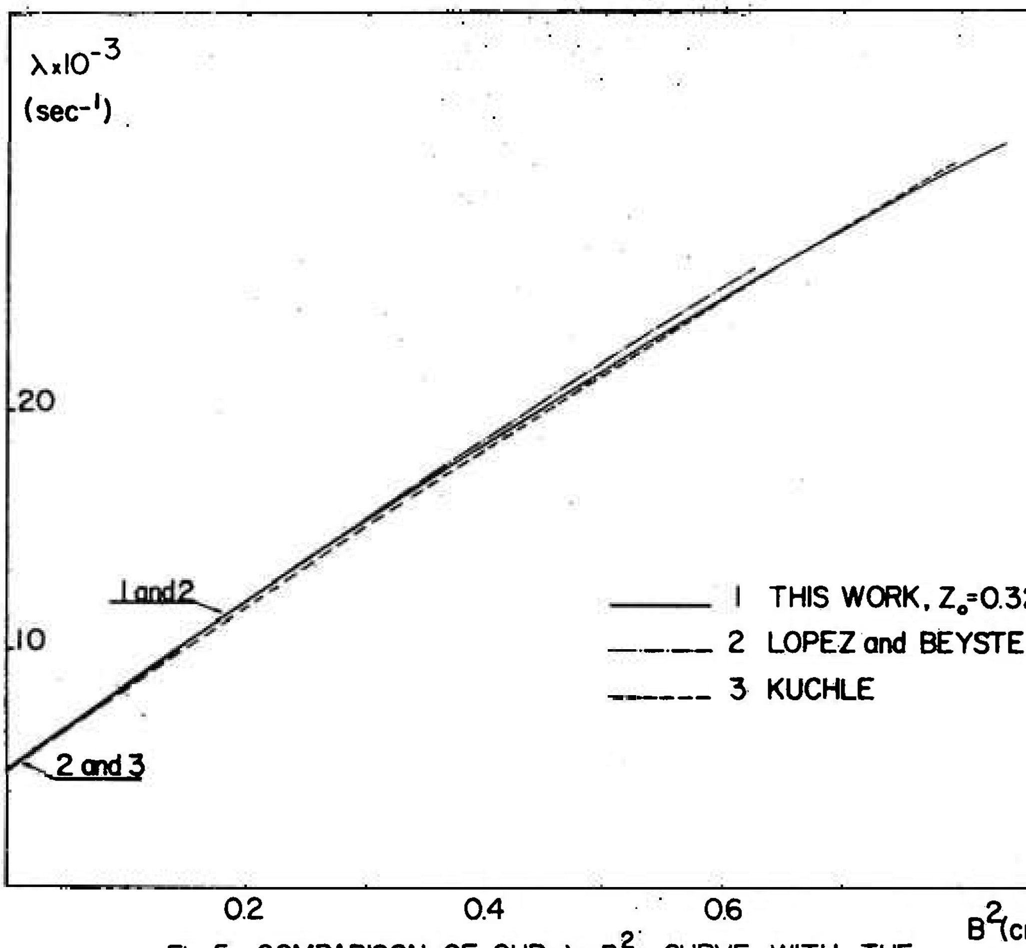

Fig. 5: COMPARISON OF OUR $\lambda \times B^2$ CURVE WITH THE ONES OF KUCHLE [10] AND LOPEZ and BEYSTER [12]

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REFERENCES

/1/ MANLEY, J.H., HAWORTH, L.J., LUEBKE, E.A., Phys. Rev. 61 (1942) 152.

/2/ VON DARDEL, G.F., WALTNER, A.W., Phys. Rev. 91 (1953) 1284.

/3/ SCOTT, F.R., THOMSON, D.B., WRIGHT, W., Phys. Rev. 95 (1954) 582.

/4/ VON DARDEL, G.F., SJÖSTRAND, N.G., Phys. Rev. 96 (1954) 1245.

/5/ ANTONOV, A.V., et al., Proc. 1st UN Int. Conf. PUAE 5 (1955) 3.

/6/ BRACCI, A., COCEVA, C., Nuovo Cim. 4 (1956) 59.

/7/ MEADS, R.E. et al, Proc. Phys. Soc. Lond. A69 (1956) 469.

/8/ CAMPBELL, E.C., STELSON, P.H., Rep. ORNL-2076 (1956) 32.

/9/ DIO, W.H., Nukleonik 1 (1958) 13.

/10/ KÜCHLE, M., Nukleonik 2 (1960) 131.

/11/ MEADOWS, J.W., WHALEN, J.F., Nucl.Sci. Engng 9 (1961) 132.

/12/ LOPEZ, W.M., BEYSTER, J.R., Nucl. Sci. Engng 12 (1962) 190.

/13/ HALL, R. S., SCOTT, S.A., WALKER, J., Proc. Phys. Soc. London 79 (1962) 257.

/14/ DLOUHY, Z., KVITEK, J., J. Nucl. Energy, 16 (1962) 376.

/15/ GON, E., LIDOFSKY, L., GOLDSTEIN, H., Trans. Am. Nucl. Soc. 8 (1965) 274.

/16/ FREED, D., WHAN, G.A., Trans. Am. Nucl. Soc. 8 (1965) 433

/17/ KOBAYASHI, K. et al., J. Nucl. Sci. Techn. (Japan) 3 (1966) 275.

/18/ ARAI, E., KÜCHLE, M., Nukleonik 7 (1965) 416.

/19/ BECKURTS, K.H., Rep. BNL-719 3 (1962) RE-1.

/20/ PAIANO, S., CARVALHO, M., Proc. Study Group Meeting on the the Utilization of Research Reactors (IAEA) 2 (1963) 319.

/21/ STARR, E., PRICE, G.A., BNL-719 3 (1962) 1034.

/22/ GELBARD, E.M., DAVIS, J.A., Nucl. Sci. Engng 12 (1962) 237.

/23/ WALKER, J., BROWN, J.B.C., WOOD, J., Proc. Karlsruse Symp. on Pulsed Neutron Research 1 (1965) 49.

/24/ NELKIN, M., BNL-719 4 (1962) RF 1

/25/ CORNGOLD, N., MICHAEL, P., WOLLMAN, W., BNL 719 4 (1962) 1103.

/26/ WEINBERG, A.M., WIGNER, E.P., The Physical Theory of Neutron Chain Reactors, The University of Chicago Press, Ill., (1958).

/27/ LALANDE, J., Inds. Atoms. 5/6 (1961) 71.

/28/ PAIANO, S., PAIANO, M.C., to be published.

/29/ PAIANO, M.C., COHENCA, J.M., Rep. IEA-106 (1965).

/30/ DEMING, W.E., Statistical Adjustment of Data, John Wiley & Sons, Inc., N.Y. (1948).

/31/ KOPPEL, J.U., LOPEZ, W.M., BNL-719 3 (1962) 946.

/32/ GHATAK, A.K., HONECK, H.C., Nucl. Sci. Engng 21 (1965) 227.

/33/ CLENDENIN, W.W., Nucl. Sci. Engng 18 (1964) 351.

/34/ KOPPEL, J.U., YOUNG, J.A., Nucl. Sci. Engng 19 (1964) 412.

/35/ WILLIAMS, M.M.R., Rep. BNL-719 4 (1962) 1331.

/36/ DeJUREN, J.A., STOOKSBERRY, R., CARROL, E.E., Rep. BNL-719 3 (1962) 895.

/37/ GAERTTNER, E.R., et al, Proc. Karlsruhe Symposium on Pulsed Neutron Research 1 (1965) 501.

/38/ EVANS, R.D., The Atomic Nucleus, McGraw-Hill Book Co. Inc., (1955).

/39/ BECKURTS, K.H., Proc. Karlsruhe Symposium on Pulsed Neutron Research 1 (1965) 3.

/40/ CARROL, E.E., STOOKSBERRY, R.W., Nucl. Sci. Engng 20 (1964) 455.

/41/ JARVIS, R.G., Rep. CRRP 1203 - AECL 2072 (1964).

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