

EVALUATING A PRIMARY WASTEWATER TREATMENT PLANT, NUMERICAL ANALYSIS OF EXPERIMENTAL DATA

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ABSTRACT

The plant under study is processing the wastewater of the eastern part of Montevideo city, capital of Uruguay, its population is approximately one million people, its maximum capacity is 3 m³/s. The goal of the experiment was to determine the mean residence time, MRT, of the wastewater in the decanting tanks. The plant has four of these tanks of approximately 500 m³ each. The output of this system of decanting tanks is pumped to a 2.300 m length pipe into the River Plate. At the plant input a set of mechanical filters is used and after it the material is pumped to a central 25 m length distributor, where the entrances of the decanting tanks are placed. For evaluating the whole system, it was used about 740 MBq (20 mCi) of ¹³¹I in 10 ml as an input pulse. The analysis was performed in two steps. Initially the distributor was studied, a transfer function from the main input to the entrance of each tank was determined. In the second step, the goal of the plant evaluation, the MRT of each tank was analyzed. Considering the non-impulsive shape of the input function of the decanting tanks, a numerical deconvolution software of the output-input of each tank was developed *ab initio*. Deconvolution results of various sequences as: main input to the distributor system-decanting tanks entrances-output were compared with meaningful conclusions.

1. INTRODUCTION

This study was carried out in the Punta Carretas Wastewater Treatment Plant and Submarine Outfall of Montevideo City, capital of Uruguay. Its maximum capacity is 3 m³/s and receives a contribution of approximately one million people through the sewer system mainly from domestic uses.

The objective is to determine the Mean Residence Time, MRT in each tank and its diffusive characteristics. As decanting particles is the objective, low dispersion and laminar flow is desirable. A good fit to the analytical solution of the advection-diffusion equation should be possible under these assumptions.

A functional schematic of the plant is shown in Figure 1. The whole system is about 1000 m³, but when the experiment was carried out Tank 1 was under maintenance.

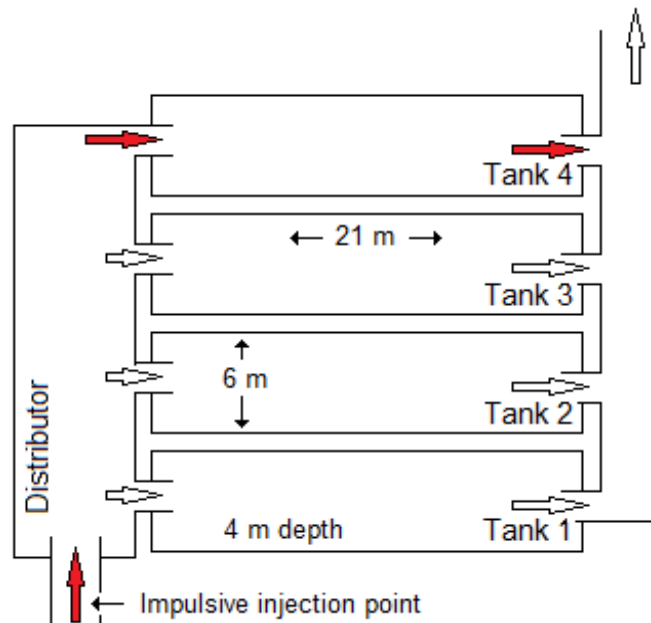


Figure 1: Functional schematic of the Wastewater Treatment Plant under study.

At the entrance of the Distributor, about 740 MBq (20 mCi) of ^{131}I in 10 ml were injected as an input pulse. Several one inch NaI detectors were used, but the calculations used in this paper will consider those located at the input and output of Tank 4.

The analysis of MRTs may be a trivial case, but if the diffusive behavior is to be analyzed, the numerical data obtained at the input and output of the tank should be fitted to an analytical equation as above mentioned.

The whole system (Tank 4 Output-Distributor Input) could be divided into two subsystems. Considering (Output-Input): Subsystem 1 is (Tank4 Input-Distributor Input) and Subsystem 2 is (Tank4 Output-Tank4 Input).

The input of the first one is impulsive, resembles a Dirac function, implying that the observed data is the Transfer Function in the time domain.

If there are no bypasses, dead volumes or any other malfunction, the values measured at the input of Tank 4 should be very similar to the analytical solution of the advection-diffusion equation. The values of its parameters, as advection and Diffusion coefficient may be calculated by any numerical fitting algorithm.

This consideration cannot be applied exactly to Subsystem 2, as the input could be of any shape, for determining the Transfer Function the numerical deconvolution seems to be the best approach.

2. NUMERICAL DECONVOLUTION

Given a system, whose transfer function is defined as $H(s)$ in the Laplace domain, the output $Y(s)$ due to an input $X(s)$, may be written in time domain as:

$$y(t) = \int_{-\infty}^{\infty} h(u)x(t - u)du \quad (1)$$

or in discrete time as

$$y_j = \sum_{i=1}^j x_i h_{j-i+1} \Delta t \quad (2)$$

It should be pointed that the discrete time situation is applied to the experiment considered. The most common methods for performing this operation are by means of Fourier Transforms and matrix algebra [1].

As this paper is focused on practical numerical deconvolution applications to complex sets of output-input of a system, and as convolution methods are studied in depth in bibliography they will not be presented here. Besides it, as practical situations involve discrete time observations, continuous time treatment is not considered.

The use of the duality between time convolution and transforms (Fourier or Laplace) product for estimating the inverse of a convolution operation has some restricting problems. Normally, additive noise and long observation periods result in very high amplitude of low frequencies components and spikes in the band limited spectrum considered. The bandwidth, established by the sampling frequency, and previous filtering operations may severely distort the results of transforms inversion.

The numerical convolution may be stated in matrix form as:

$$\vec{y} = H\vec{x} \quad \text{where} \quad \vec{y} = \begin{bmatrix} h_1 & 0 & 0 & 0 & 0 \\ h_2 & h_1 & 0 & 0 & 0 \\ h_3 & h_2 & h_1 & 0 & 0 \\ \vdots & \vdots & \vdots & \cdot & 0 \\ h_n & & h_3 & h_2 & h_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} \quad (3)$$

This method works fine for convolution. As convolution is permutable, a matrix X may also be defined but it will change with every sampling. Algebraically, the input waveform as a function of output and transfer function, or the transfer function as a relation of output and input observations could be determined by:

$$\vec{x} = H^{-1}\vec{y} \quad (4)$$

When casual systems are considered, as usual in physical systems, the inverse of H is not defined, H is singular or its condition number is very poor.

Another way for calculating the inverse of the convolution of two functions is the parameters fitting of a suitable target function.

Given an output measurement, -which is the normal case in tracers' experiments- determine the parameters of a function which physically resembles the process under study. Two cases may be considered: when an impulse injection of the tracer is applied to the system (Subsystem 1) and when an arbitrary injection shape is used (Subsystem2).

Given a function which resembles the behavior of the system, as

$$y(t) = f(t, \vec{a}) \quad (4)$$

the goal is to find the parameter vector \vec{a} which minimize some likelihood measure as the χ^2 .

$$\chi^2(\vec{a}) = \sum_{i=1}^N \left[\frac{y_i - f(t_i, \vec{a})}{\sigma_i} \right]^2 \quad (5)$$

Inside some neighborhood of the optimum parameters vector, the χ^2 may be expanded as

$$\chi^2(\vec{a}) \approx \gamma - \vec{d} \cdot \vec{a} + \frac{1}{2} \vec{a} \cdot D \cdot \vec{a} \quad (6)$$

where \vec{d} is an N -vector, D is the Hessian $N \times N$ matrix.

If the approximation is good the optimum parameters vector can be reached by:

$$\vec{a}_{min} = \vec{a}_{curr} + \frac{1}{2} D^{-1} \cdot [-\nabla \chi^2(\vec{a}_{curr})] \quad (7)$$

When the parameter vector \vec{a} is not close to its optimum Eq. (6) is no longer valid. In these cases, the steepest descent method can be used for moving on the χ^2 surface in the opposite direction of its gradient:

$$\vec{a}_{next} = \vec{a}_{curr} - \beta \times \nabla \chi^2(\vec{a}_{curr}) \quad (8)$$

Levenberg and Marquardt proposed an algorithm which changes smoothly the application of Eq. (7) and Eq. (8) [2][3], according to the distance of the current parameter vector to its optimum. Their algorithm is based in enhancing the diagonal of the Hessian matrix when the approximation is not going towards the optimum. Eq. (8) is similar to Eq. (7) if D is diagonal dominant.

3. APPLICATION

The necessary software was developed *ab initio* for analyzing Subsystem 1 and Subsystem 2 applying the Levenberg-Marquardt algorithm for performing the numerical deconvolution.

Subsystem 1 analysis: impulsive injection. When an impulsive injection of the tracer is assumed, the observed output is the target transfer function itself.

Mean residence time, variances, flow and axial dispersion may be calculated from the measured data. It is correct if zero momentum of the distribution can be assured at the injection point.

The target function used [4] was:

$$f(t, \vec{a}) = \frac{Au}{\sqrt{4\pi Dt}} \exp \left[-\frac{(L - ut)^2}{4Dt} \right] \quad (9)$$

$$\vec{a} = [A, u, D]$$

where A is the amplitude, u is the velocity, L is the length (distance between the injection point and the detector position) and D is the axial dispersion.

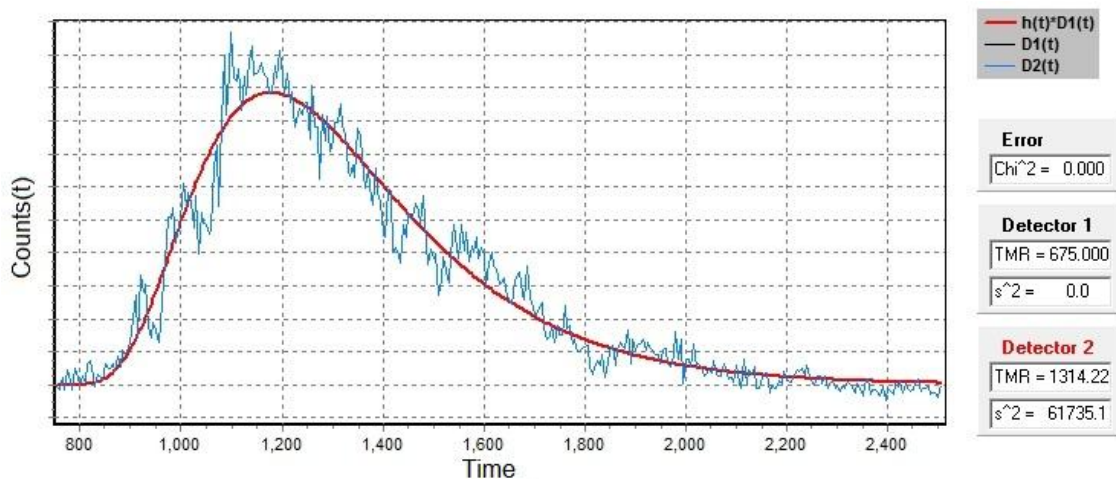


Figure 2: Deconvolution of Tank 4 Output and Distributor Input. MRT 639 s.

The parameters vector \vec{a} was calculated for a maximum likelihood of the observed data and Eq. (9).

Figure 2 shows the result of the calculation. The blue line, named Detector 1, is the input of Tank 4, assuming an impulsive input to the Distributor. The red curve is the convolution of the Dirac function and the transfer function as Eq (9) when the calculated parameters are applied.

Subsystem 2 analysis: two measures method. Using this method enables to consider any injection function, as in Subsystem 2. Even it seems quite similar to the analysis of Subsystem 1, its numerical treatment is quite different.

In the first case the output was deconvolved with a Dirac's function. The observed data is the representation of Eq. (9), so the parameters calculation demands its evaluation in time during the fitting process.

For deconvolving the output data with any input shape -distribution in time of the tracer at the injection or at any point before the detector considered as being the output- involves the calculus of the convolution of the input data with the target function of the system. The maximum likelihood is of the result of this convolution with the observed output of the system.

In this particular case the Distributor works fine, no bypasses, dead volumes or undesired feedbacks are obvious. If any of those malfunctions were present or if the impulsive injection could not be achieved, the process parameters may be difficult to obtain by other means.

The target function employed [4] was:

$$f(t, \vec{a}) = A \sqrt{\frac{Pe \cdot \tau}{4t^3}} \exp \left[-\frac{Pe(t - \tau)^2}{4t} \right]$$

$$\vec{a} = [A, Pe, \tau] \quad (10)$$

where τ is the difference of the mean residence times, it may be considered as a parameter for fine adjustments and Pe is the Péclet number, (uL/D , u is the velocity, L is the axial length and D is the diffusion coefficient).

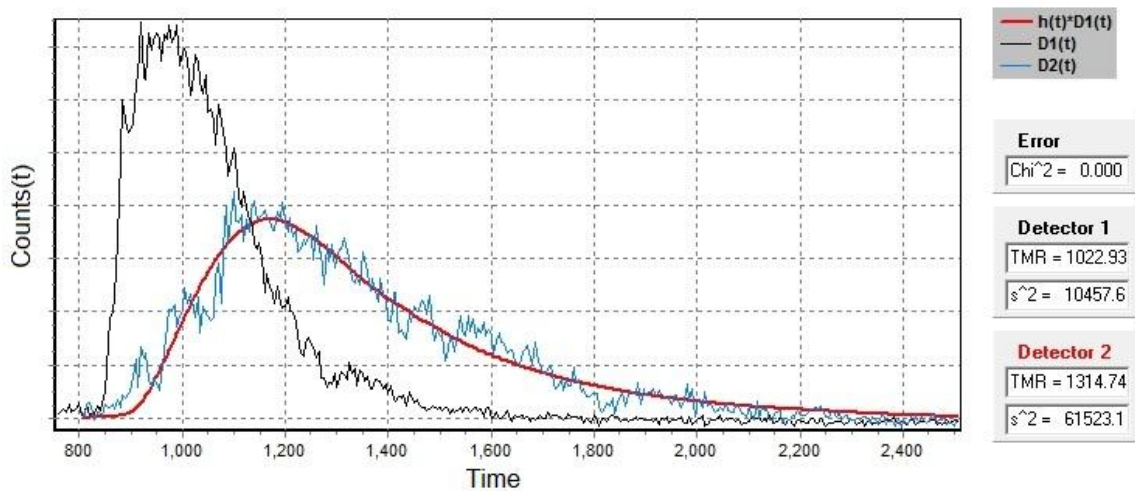


Figure 3: Analysis of Subsystem 2. Deconvolution of Tank 4 Output and Tank 4 Input. Péclet Number: 1.92, MRT 291 s.

In Figure 3 the black line, Detector 1 is the measured sequence at the input of Tank 4. The blue one, Detector 2 is the output of the tank and the red line is the convolution of the sequence shown as Detector 1 and the transfer function as Eq (9) when the calculated parameters are applied.

3. CONCLUSIONS

In many cases, as the one presented here, a meaningful physical model is available, its parameters are correlated with the system behavior. As was explained when the Subsystem1 case, if an impulsive injection of the tracer may be accomplished, the calculation may be carried out without difficulties. The calculation is done by numerical fitting of a model.

When the input of the system is complex, because of an incorrect injection of the tracer or when that input comes from a malfunctioning stage of the process, the numerical method which could be used is numerical deconvolution. This will obtain the transfer function parameters for any set of output-input data.

The method is so flexible that it could find a numerical series as the transfer function. In this case no meaningful physical parameter will be obtained, but the system behavior when an arbitrary input is applied as its input may be calculated.

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