



PROGRESSIVE SUBSTITUTION EXPERIMENTS IN UO_2
LATTICES MODERATED BY D_2O/H_2O MIXTURES

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PROGRESSIVE SUBSTITUTION EXPERIMENTS IN UO_2

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RESUMO

No reator NORA, Institutt for Atomenergi, Kjeller, Noruega, no período 1963-64, através do método de substituição progressiva, foram determinadas algumas curvaturas materiais de arranjo de óxido de urânio (3% enriquecido em U^{235}) moderados por misturas de água leve e água pesada.

Para uma verificação direta do método empregado, foi efetuada uma série de experiências em arranjos críticos, cujas curvaturas materiais eram conhecidas através do processo clássico, isto é, método de ativação de fôlhas no sentido axial e radial.

Outro conjunto de experiências foi efetuado com arranjos subcríticos com o propósito de se confrontarem os resultados obtidos com os calculados pelo método de substituição progressiva.

Para a análise das curvaturas materiais foi aplicada a teoria de 2 grupos - 3 regiões (moderador, teste e intermediária) na qual introduziu-se um fator de correção que leva em conta o efeito do refletor.

O mesmo processo de substituição progressiva foi aplicado para a determinação da curvatura material de um arranjo com "vazios", cuja concentração de água pesada era de ... 99.50% e espaçamento entre os elementos combustíveis era de 6.544 cm. Para este caso, a análise foi feita considerando-se a teoria de 1 grupo - 3 regiões.

Todos os resultados obtidos são bastante satisfatórios, o que mostra a eficiência do método utilizado.

RESUME

Dans la période 1963-64, au IFA Kjeller, Norvège, on a appliquée la méthode de substitution progressive pour la détermination du laplacien matière d'assemblages critiques et souscritiques (UO_2 enrichi à 3% U_{235}) modérées par moisture d'eau lourde.

Trois séries de mesures ont été effectuées.

- 1) Mesures sur des assemblages critiques ayant les laplacien matière déjà déterminés par la méthode classique.
- 2) Mesure sur des assemblages souscritique, envisageant une comparaison des résultats expérimentales avec des calculs théoriques effectués par la méthode de substitution progressive. On a utilisé une théorie à deux groupes - trois régions (modérateur, test et intermédiaire) en introduisant une correction qui tient compte du réflecteur.
- 3) Mesures sur des assemblages à régions vides, ayant une concentration d'eau lourde de 99,5% et la distance entre les barreaux de combustible étant 6,544 cm. Ici on a employé une théorie à un groupe, trois régions.

Les résultats ont démontré l'efficacité de la méthode.

ABSTRACT

Buckling measurements for cores of uranium oxide (3.00 % enriched in U^{235}) in different mixtures of D_2O/H_2O were performed in the NORA reactor by means of a progressive substitution technique.

In order to check the results, some experiments were also carried out by the substitution technique in critical lattices for which the buckling was already known. Some subcritical experiments were also performed to give additional information about the buckling obtained by substitution experiments.

The analysis has been done by three regions, two group theory and a correction was introduced in order to take into account the effect of the reflector.

For a D_2O concentration of 99.50 % and a lattice pitch of 6.544 cm, the material buckling with void was obtained by three regions, one group theory.

All the results were found to agree satisfactorily with the results from critical experiments.

1 - INTRODUCTION

A systematic study of 3% enriched UO_2 lattices moderated by mixtures of D_2O and H_2O has been undertaken in the zero power reactor NORA at the Kjeller Research Establishment [1].

Due to the rather low value of the material buckling for some of the tightly packed lattices, and to the relatively small number of fuel elements available, it was found difficult to maintain the reactor critical for several of the cores containing the tight lattices.

Progressive substitution experiments were then carried out in order to obtain the material bucklings for several of the tightly packed lattices. The technique consisted in measuring the critical water level after inserting or removing the same type of fuel elements in the centre of a reference lattice, only changing the lattice pitch in the central zone.

From the point of view of the two regions analysis, one may observe that between the interface of the two zones, there is no possibility to have uniform distribution of conventional type of cells (moderator and fuel) without having "dummy" cells, i.e., just moderator. However, this is solved by applying the three regions analysis by which the core may be divided in a reference, test and an intermediate zone.

The size of the transition region is determined directly from the definition of the unconventional type of cells.

2 - THEORY

For a composite reflected cylindrical system of i regions, assuming that all regions have the same diffusion coefficients, a two group theory analysis [2] gives the following equation

$$\left(\frac{\alpha_2^2 - \alpha_1^2}{W_r} \right) R \left[1 + (1 - W_r) \Gamma_3 \right] = (\alpha_3^2 - \alpha_1^2) + d\alpha_2^2 \left(\frac{W_{r2}}{W_r} \right) \quad (1)$$

where

α_2^2 = axial buckling of the compound core

α_1^2 = axial buckling of the reference core

R = reflector correction

$$W_r = W_{r3} + \frac{W_{r2}}{2}$$

W_{r3} = statistical weight of the test region

W_{r2} = statistical weight of the intermediate region

$$\Gamma_3 = - \left(\frac{G_3}{1 + G_3} \right)$$

$$G_3 = \left[\frac{S_{\text{ref}} - S_{\text{test}}}{S_{\text{test}} - \bar{T}} \right] \left[\frac{\left(\frac{D_f}{D_{\text{th}}} \right)_{\text{ref}} + \bar{T}}{S_{\text{ref}} + \left(\frac{D_f}{D_{\text{th}}} \right)_{\text{ref}}} \right]$$

$$\bar{T} = \frac{T_{\text{ref}} + T_{\text{test}}}{2}$$

$$S = \left(\frac{D_f}{D_{\text{th}}} \right) \left(\frac{p}{L_s^2} \right) \left(\frac{L^2}{1 + B^2 L^2} \right)$$

$$T = - \left(\frac{D_f}{D_{\text{th}}} \right) \left(\frac{p}{1 + B^2 L_s^2} \right)$$

B^2 = material buckling

L = thermal diffusion length

p = resonance escape probability

D_f, D_{th} = fast and thermal diffusion coefficients

L_s = slowing down length

For details concerning eq. (1), reference is made to Appendix A.

By plotting the left hand side of the expression (1) as a function of (W_{r2}/W_r) , one obtains a straight line. The α_3^2 value is obtained by extrapolating the straight line for full substitution, i.e., $W_{r2} = 0$. Fig. 1 shown such a plot for a typical experiment.

In Fig. 6 the unconventional cell definitions for the three regions are given.

3 - THE REFERENCE LATTICES

The cores used as reference lattices for the substitution experiments are presented in Table I.

TABLE I
Reference Lattices

Case	I	II	III	IV	V
D ₂ O (mol %)	99.50	81.03	81.03	55.00	55.00
Pitch (cm)	3.272	2.314	3.272	2.314	3.272

In case I the experiments were carried out in an aluminium tank, the inner diameter of which was 225 cm, surrounded by a graphite side reflector 50 cm thick and a graphite bottom reflector of 70 cm thickness.

For cases II - V, a small tank with an inner diameter of 120 cm was placed inside the first tank.

The fuel elements used were UO₂, 3% enriched in U²³⁵ (by weight) and with a density of 9.28 g/cm³. The diameter of the UO₂ was 11.28 mm. The cladding was 304 stainless steel with a wall thickness of 0.71 mm.

The flux distributions of the reference cores were measured by irradiating copper foils in radial and axial directions and the extrapolated dimensions were obtained by the least squares method.

4 - DESCRIPTION OF EXPERIMENTS

The neutron detection in the NORA reactor is ensured by two BF_3 counters and by two ionization chambers, symmetrically placed outside the tank.

The reactor was balanced (power \approx 1 watt) by adjusting the water level. Thus, the analysis of the progressive substitution experiments is simplified due to the fact that there is no poisoning of the control rods.

For each configuration in the central region of the reference core, four sets of critical water levels were read after making the reactor slightly subcritical or supercritical. The accuracy of the water level readings were \pm 1 mm. The temperature inside the tank was measured daily by means of thermocouples.

Table II shows the number of fuel elements inserted or removed in the central zone as well as the reference and test zone pitches. Fig. 8 shows the critical water level versus number of fuel rods in (out) for the test region.

For a D_2O concentration of 81.03%, some axial flux distributions were measured in the test and reference zones by irradiating copper foils, and for a D_2O concentration of 55.00% a small fission chamber was also used.

Results of axial neutron flux distribution for both the test and reference regions are given in Figs. 2, 3, 4, 5, 7 and 9.

TABLE II

Number of Fuel Elements; Reference and Test Zone Pitches

D ₂ O (mol %)	Reference zone pitch (cm)	Central zone pitch (cm)	No. of fuel rods
99.50	3.272	1.636	21,65,133
		2.314	5,13,25,41,61,85
81.03	2.314	1.636	5, 9,21,29,49,69
		3.272	8,18,32,50,72
	3.272	2.314	8,18,32,50,72
55.00	2.314	1.636	5, 9,21,29
		3.272	8,18,32
	3.272	2.314	8,32,50,72

5 - SUBCRITICAL EXPERIMENTS

Subcritical experiments were carried out in order to check the material buckling for a D₂O concentration of 55.00% and a lattice pitch of 1.636 cm.

The exponential decays were measured for two different radii, i.e., 21.8 and 27.9 cm.

A BF₃ proportional counter was connected to a driving unit which could be placed in any vertical position of the core. As a neutron source was used 98 mg of Ra-Be and the source was positioned at the bottom of the aluminium tank with an inner diameter of 120 cm. The radial reflector saving was estimated to be (15.00 ± 1.00) cm.

There were found pure exponential decays from 42 to 72 cm for an effective radius of 21.8 cm and from 42 to 102 cm for

a radius of 27.9 cm (Figs. 10 and 11).

From the subcritical experiments, the material buckling was found to be $(25.7 \pm 1.4)m^{-2}$ which agree satisfactorily with the value of $(26.34 \pm 1.24)m^{-2}$ from progressive substitution experiments.

6 - RESULTS

Table III shows the results obtained by means of the substitution technique and by critical experiments.

7 - STUDIES OF THE SUBSTITUTION METHOD WITH VOID - ONE GROUP THEORY

The method of analysis was suggested and described by R. Persson [2].

The material buckling with void for a D_2O concentration of 99.50% and a lattice pitch of 6.544 cm was investigated. The equivalent core radius was 49.53 cm and the radial D_2O reflector thickness was approximately 63 cm.

The material buckling obtained by means of this method was found to be $(15.04 \pm 0.46)m^{-2}$ which agrees satisfactorily with $(15.63 \pm 0.45)m^{-2}$ from critical experiments.

8 - CONCLUSIONS

The progressive substitution experiments have made it possible to extend the range of the material buckling investigated and the results obtained from the analyses by three regions, one and two group theory, gave reasonable results, when compared with buckling values from critical experiments.

For the case investigated in this report, i.e., where the fuel elements are the same as in the reference lattice, the time required to perform the substitution experiments is quite

large due to the successive approach to critical. Thus, the progressive substitution experiments offer no advantage in time over the subcritical experiments.

TABLE III
Material Bucklings

D_2O (mol %)	Pitch (cm)	Substitution Exp. $B_m^2 (m^{-2})$	Critical or Subcritical Exp. $B_m^2 (m^{-2})$
99.50	1.636	$- 11.25 \pm 0.72$	-
	2.314	2.99 ± 0.56	-
81.03	1.636	7.45 ± 0.46	-
	2.314	29.69 ± 0.32	29.91 ± 0.32 (x)
	3.272	36.49 ± 0.41	36.65 ± 0.41 (x)
55.00	1.636	26.34 ± 1.24	25.70 ± 1.40 (*)
	2.314	50.22 ± 0.73	51.89 ± 0.70 (x)
	3.272	38.53 ± 0.79	37.70 ± 0.65 (x)

(x) Critical experiments from ref. [1]

(*) Subcritical experiments.

9 - ACKNOWLEDGEMENTS

The author wishes to express his gratitude to Mr. V. O. Eriksen, Director of Physics Division, IFA, for suggesting the performance of these experiments and for stimulating discussions during the progress of this work. Acknowledgement is also given to the staff of the NORA reactor.

10 - REFERENCES

- [1] E. Andersen et al., "Experimental and Theoretical Studies of Uranium Oxide Lattices Moderated by Mixtures of Light and Heavy Water". Geneva Paper P/669, 1964.
- [2] R. Persson, Paper SM 42/47, IAEA Symposium on Exponential and Critical Experiments, 1963.
- [3] R. Persson, "One Group Perturbation Theory Applied to Substitution Measurements With Void", AE-72, 1962.

APPENDIX A

Case A

For a bare cylindrical reactor compound of i regions which have different diffusion coefficients, from two group theory one obtains: (for a purely radial perturbation)

$$\prod_i (1+G_i) \left[\frac{\delta D_{ri}^{si}}{D_{r1}^{s1}} \beta^2 (W_{r1} - U_{r1}) D_{r1}^{s1} + D_{z1}^{s1} W_{ri} \left(\frac{D_{zi}^{si}}{D_{z1}^{s1}} (\alpha_1^2 - \alpha^2) \right) \right] = 0 \quad (A1)$$

where

i = region subscript (reference $i=1$; transition $i=2$; test $i=3$)

$$1+G_i = \left(\frac{c_1}{c_i} \right) \frac{D_{k1}^{s1}}{D_{k1}^{s1}}$$

k = direction r or z

$$\delta D_{ri}^{si} = D_{ri}^{si} - D_{r1}^{s1}$$

$$D_{ki}^{si} = D_{ki}^f + S_i D_{ki}^{th}$$

W_{ri} = statistical weight

α_i^2 = axial buckling of the region i

α^2 = axial buckling of the compound core

β^2 = radial buckling of the cylindrical reactor.

The term $D_{z1}^{s1} W_{ri} (D_{zi}^{s1}/D_{z1}^{s1}) (\alpha_i^2 - \alpha_1^2)$ can be written as follows:

$$D_{z1}^{s1} \left(\frac{D_{zi}^{s1}}{D_{z1}^{s1}} \right) (\alpha_i^2 - \alpha_1^2) W_{ri} - D_{z1}^{s1} \left(\frac{D_{zi}^{s1}}{D_{z1}^{s1}} \right) (\alpha_i^2 - \alpha_1^2) W_{ri} \quad (A2)$$

Substituting the expression (A2) in (A1) and rearranging gives:

$$\sum_i (1+G_i) \left[\beta^2 (W_{ri} - U_{ri}) \frac{\delta D_{ri}^{s1}}{D_{r1}^{s1}} \frac{D_{r1}^{s1}}{D_{z1}^{s1}} + \frac{D_{zi}^{s1}}{D_{z1}^{s1}} (\alpha_i^2 - \alpha_1^2) W_{ri} - (\alpha_i^2 - \alpha_1^2) \left(1 + \frac{\delta D_{zi}^{s1}}{D_{z1}^{s1}} \right) W_{ri} \right] = 0 \quad (A3)$$

From two group theory one obtains:

$$\left(\frac{D_{ki}}{D_{k1}} \right)' = (1 + G_i) \frac{D_{ki}^{s1}}{D_{k1}^{s1}} \quad (A4)$$

$$\left(\frac{D_{k1}}{D_{k1}} - \frac{D_{k1}}{D_{k1}} \right)' = (1 + G_i) \frac{\delta D_{k1}^{s1}}{D_{k1}^{s1}} \quad (A5)$$

Substituting the expression (A4) and (A5) in (A3) gives:

$$(\alpha_i^2 - \alpha_1^2) \left[1 + \sum_i W_{ri} \left(\frac{\delta D_{z1}^{s1}}{D_{z1}^{s1}} \right)' + \sum_i G_i W_{ri} \right] -$$

$$- \beta^2 \sum_i (W_{ri} - U_{ri}) (1 + G_i) \left[\frac{D_{r1}^{s1}}{D_{z1}^{s1}} - \frac{\delta D_{r1}^{si}}{D_{r1}^{s1}} \right] = \sum_i (\alpha_i^2 - \alpha_1^2) \left(\frac{D_{zi}}{D_{z1}} \right)' W_{ri} \quad (A6)$$

The second term of the expression (A6) can be written as follows:

$$\beta^2 \sum_i (W_{ri} - U_{ri}) \left(\frac{c_1}{D_{z1}^{s1}} \frac{\delta D_{r1}^{si}}{c_i} \right)$$

or

$$\beta^2 \sum_i (W_{ri} - U_{ri}) \left(\frac{\delta D_{r1}^{si}}{D_{z1}} \right)' \quad (A7)$$

where

$$\left(\frac{\delta D_{r1}^{si}}{D_{z1}} \right)' = \left(\frac{c_1}{c_i} \right) \frac{\delta D_{r1}^{si}}{D_{z1}^{s1}}$$

Substituting the expression (A7) in (A6) one obtains:

$$(\alpha^2 - \alpha_1^2) \left[1 + \sum_i W_{ri} \left(\frac{\delta D_{zi}}{D_{z1}} \right)' + \sum_i G_i W_{ri} \right] - \beta^2 \sum_i (W_{ri} - U_{ri}) \left(\frac{\delta D_{r1}^{si}}{D_{z1}} \right)' = \sum_i (\alpha_i^2 - \alpha_1^2) \left(\frac{D_{zi}}{D_{z1}} \right)' W_{ri} \quad (A8)$$

Case B

If the core is compound of three regions, i.e., reference, intermediate and test, and assuming that all regions have the same diffusion coefficients, the expression (A8) can be written as follows:

$$(\alpha^2 - \alpha_1^2) \left[1 + \sum_i G_i W_{ri} \right] = \sum_i (\alpha_i^2 - \alpha_1^2) (1 + G_i) W_{ri} \quad (A9)$$

Expanding the last term of the expression (A9) for $i = 1, 2, 3$ and rearranging, gives:

$$\begin{aligned} (\alpha^2 - \alpha_1^2) \left[1 + \sum_i G_i W_{ri} \right] &= (\alpha_3^2 - \alpha_1^2) (1 + G_3) W_r + \\ &+ \left[\delta\alpha_2^{2'} (1 + G_2) + \frac{(\alpha_3^2 - \alpha_1^2)}{2} (G_2 - G_3) \right] W_{r2} \end{aligned} \quad (A10)$$

where

$$\delta\alpha_2^{2'} = \alpha_2^2 - \left(\frac{\alpha_3^2 + \alpha_1^2}{2} \right)$$

Assuming that $G_2 = G_3/2$ and rearranging the expression (A10), gives:

$$\frac{(\alpha^2 - \alpha_1^2)}{W_r} \left[1 + (1 - W_r) \Gamma_3 \right] = (\alpha_3^2 - \alpha_1^2) + \delta\alpha_2^{2'} \left(\frac{W_{r2}}{W_r} \right) \quad (A11)$$

where

$$\Gamma_3 = - \left(\frac{G_3}{1 + G_3} \right)$$

$$W_r = W_{r3} + \frac{W_{r2}}{2}$$

LIST OF FIGURES

- Fig. 1. Evaluation of the material buckling.
D₂O concentration: 99.50%; reference pitch = 3.272 cm;
central pitch = 2.314 cm.
- D₂O concentration: 81.03%
- Fig. 2. Axial flux distribution (foil technique); reference
pitch = 2.314 cm; central pitch = 3.272 cm; 32 fuel
elements removed.
- Fig. 3. Axial flux distribution (foil technique); reference
pitch = 2.314 cm; central pitch = 3.272 cm; 72 fuel
elements removed.
- Fig. 4. Axial flux distribution (foil technique); reference
pitch = 2.314 cm; central pitch = 1.636 cm; 69 fuel
elements inserted.
- Fig. 5. Axial flux distribution; reference pitch = 3.272 cm;
central pitch = 2.314 cm; 32 fuel elements inserted.
- D₂O concentration: 55.00%
- Fig. 6. Configurations. Reference pitch = 3.272 cm; central
pitch = 2.314 cm; number of fuel elements inserted:
8, 18, 32.
- Fig. 7. Axial flux distribution; reference pitch = 2.314 cm;
central pitch = 3.272 cm; number of fuel elements
removed: 18, 32.
- Fig. 8. Critical water levels as a function of the number of
fuel elements in the test region.

Fig. 9. Axial flux distribution; reference pitch = 3.272 cm; central pitch = 2.314 cm; number of fuel elements inserted: 18, 32, 50.

Fig. 10. Exponential decay (effective radius = 21.8 cm).

Fig. 11. Exponential decay (effective radius = 27.9 cm).

Fig. 12. Material buckling as a function of the square lattice pitch.

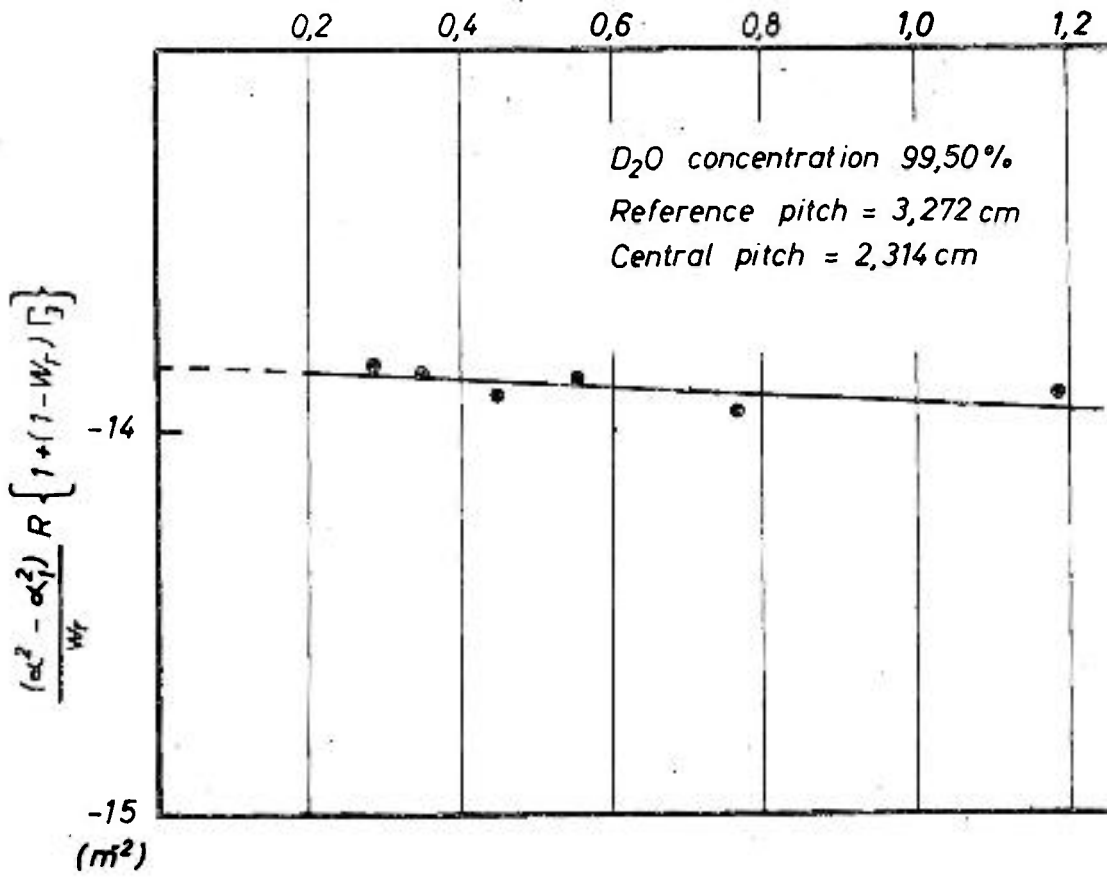


Fig.1 EVALUATION OF THE MATERIAL BUCKLING.

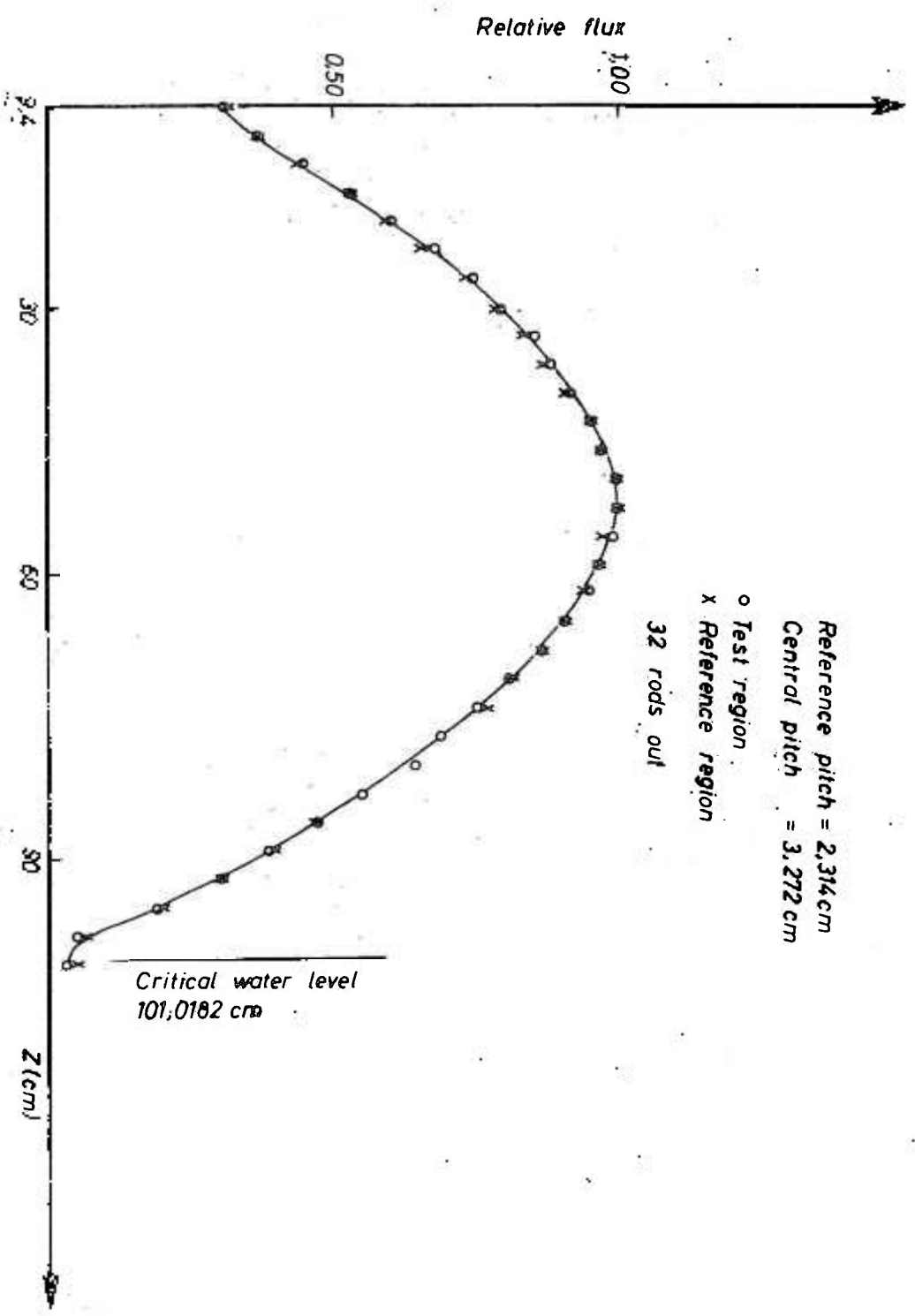


Fig. 2 AXIAL FLUX DISTRIBUTION

Reference pitch = 2,314 cm
Central pitch = 3,272 cm

○ Test region
x Reference region
72 rods out

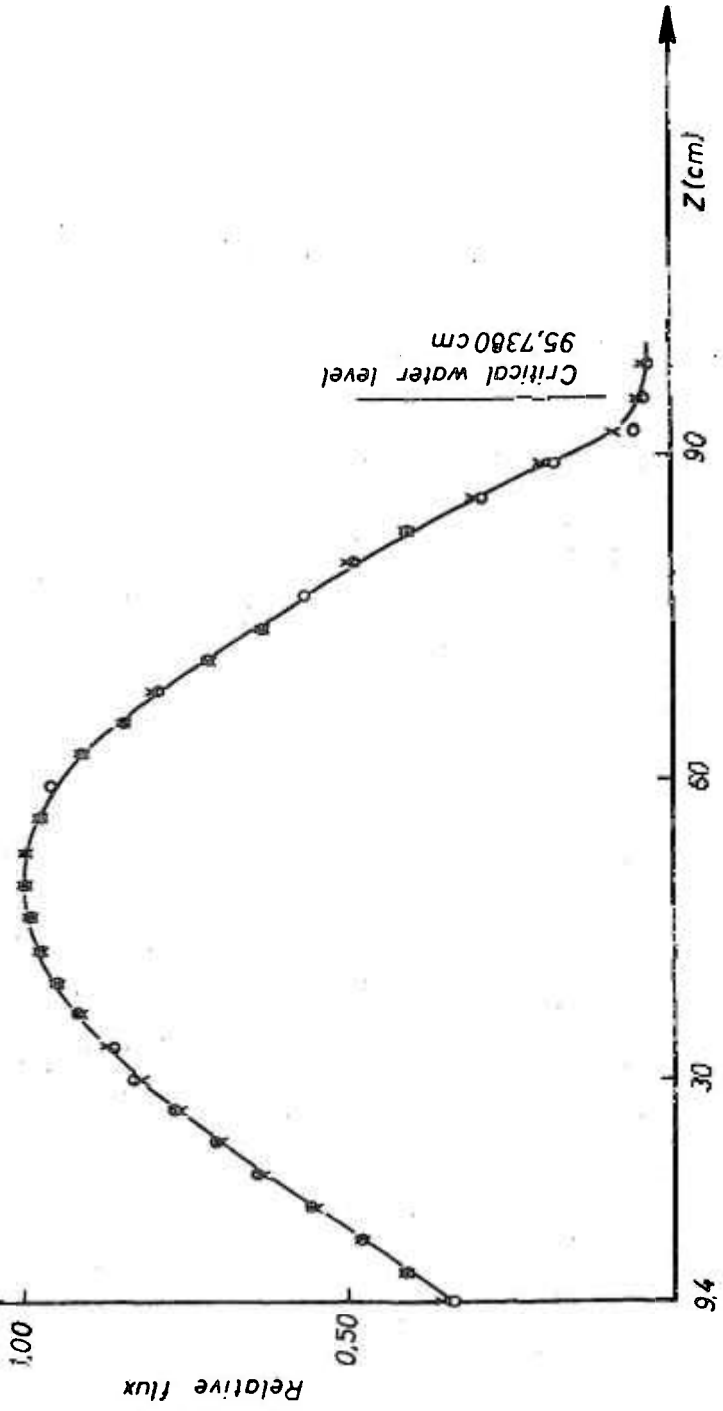


Fig. 3 AXIAL FLUX DISTRIBUTION.

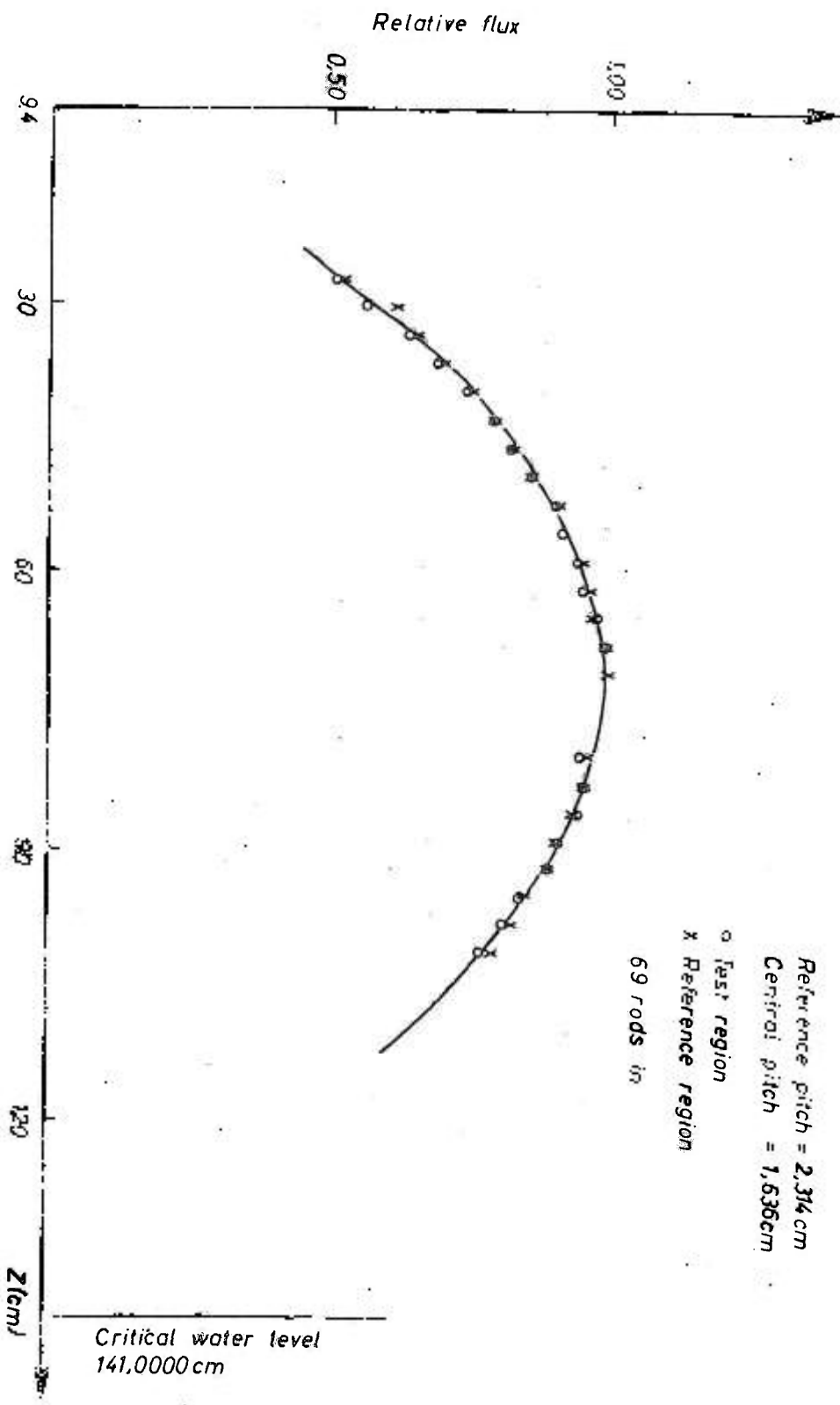


Fig. 4 AXIAL FLUX DISTRIBUTION.

Reference pitch = 3,272cm
Central pitch = 2,314 cm

o Test region
x Reference region

32 rods in

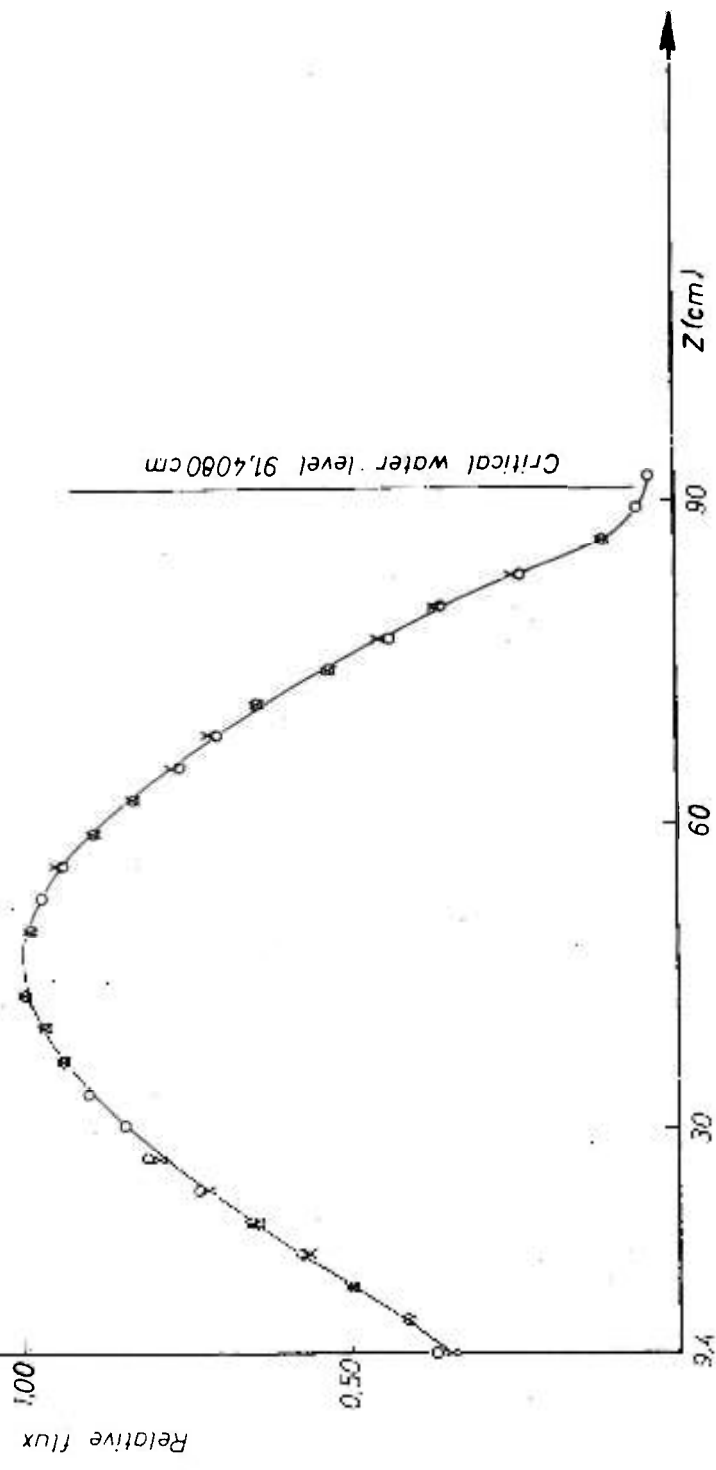
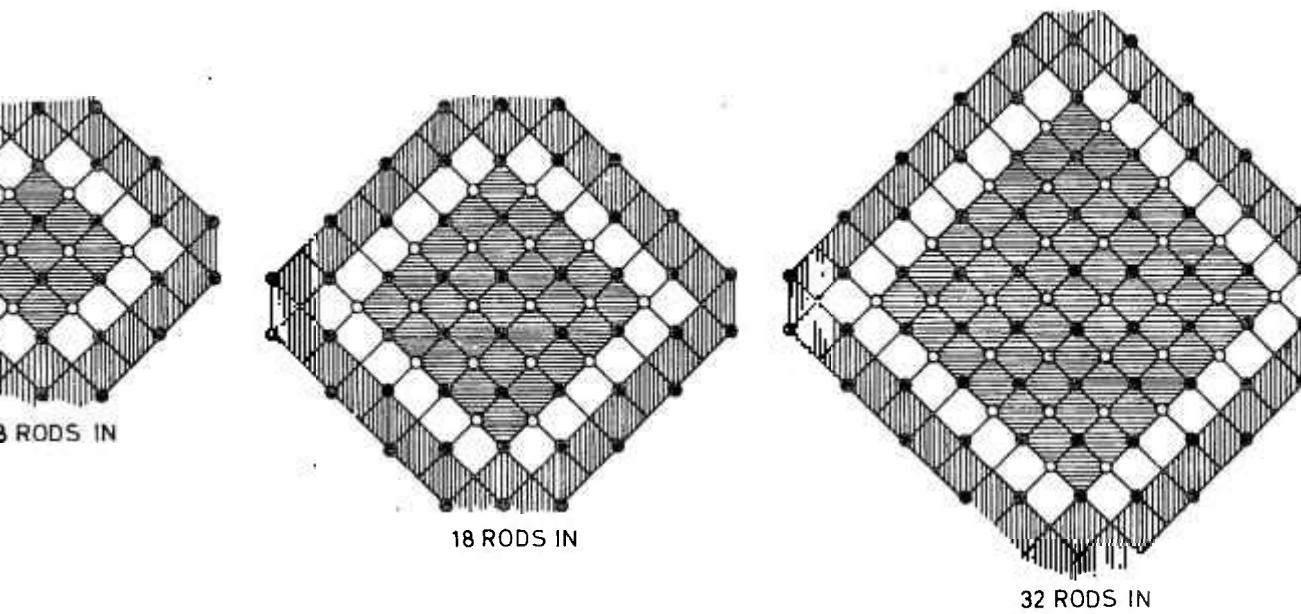


Fig.5 AXIAL FLUX DISTRIBUTION.



13 RODS IN

18 RODS IN

32 RODS IN

Fig. 6

REFERENCE CELL
 POSITION CELL
 TEST CELL



- REFERENCE FUEL ELEMENTS
- FUEL ELEMENTS (TO INSERT)

CENTRAL PITCH = 2.313 cm
 REFERENCE PITCH = 3.272 cm

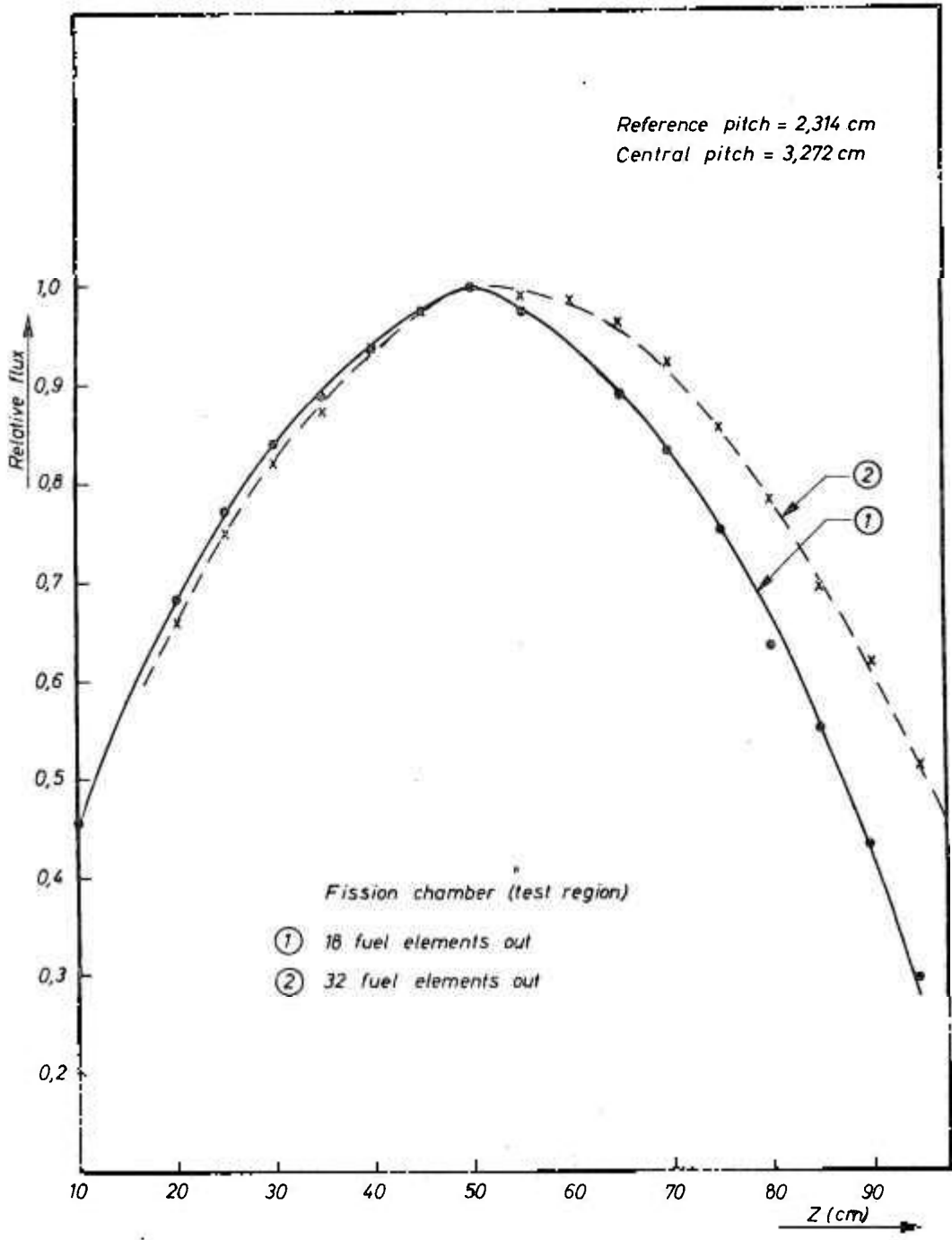


Fig.7 AXIAL FLUX DISTRIBUTIONS.

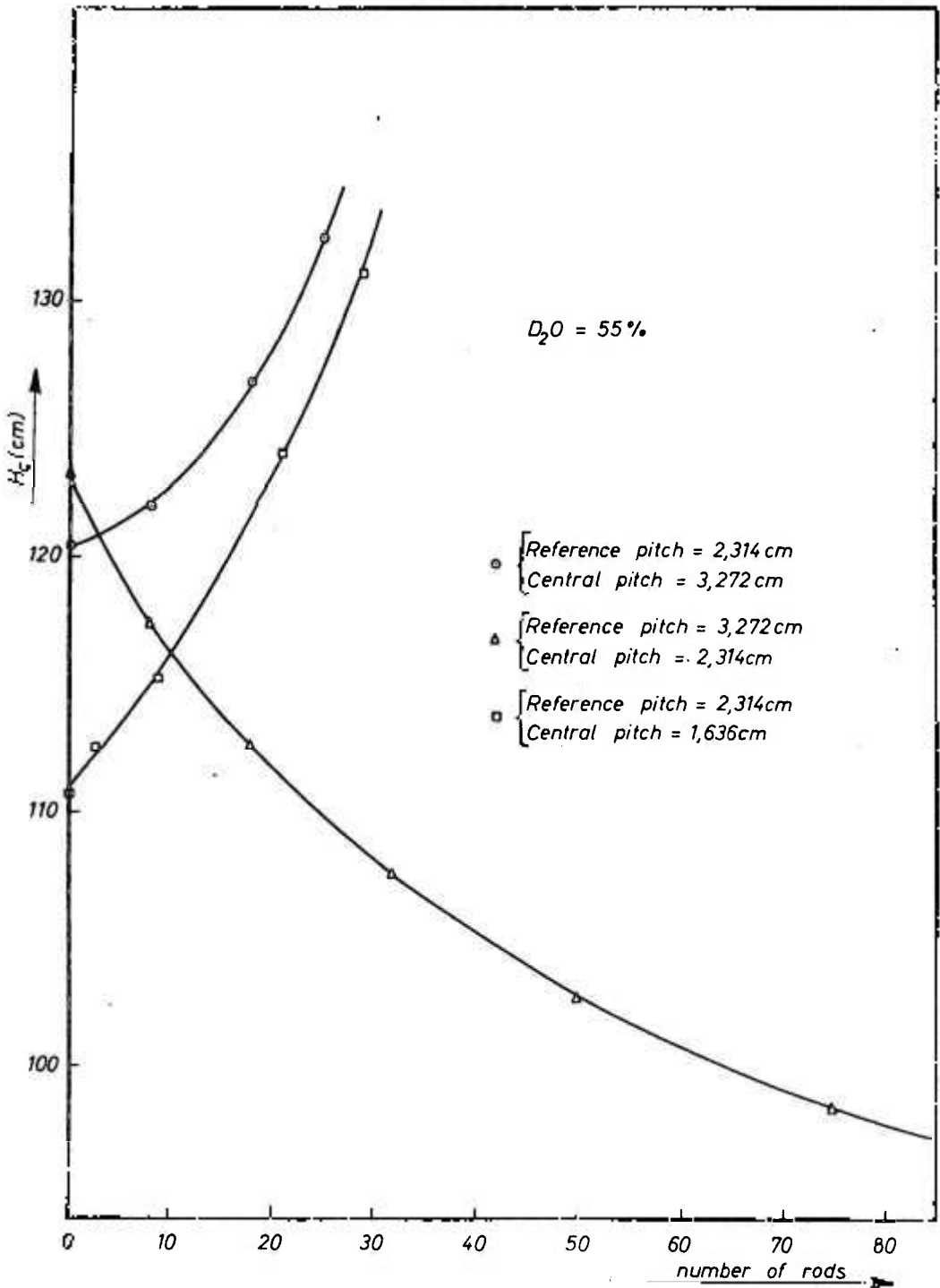


Fig. 8 CRITICAL WATER LEVELS, VERSUS NUMBER OF RODS IN (OUT) - TEST REGION.

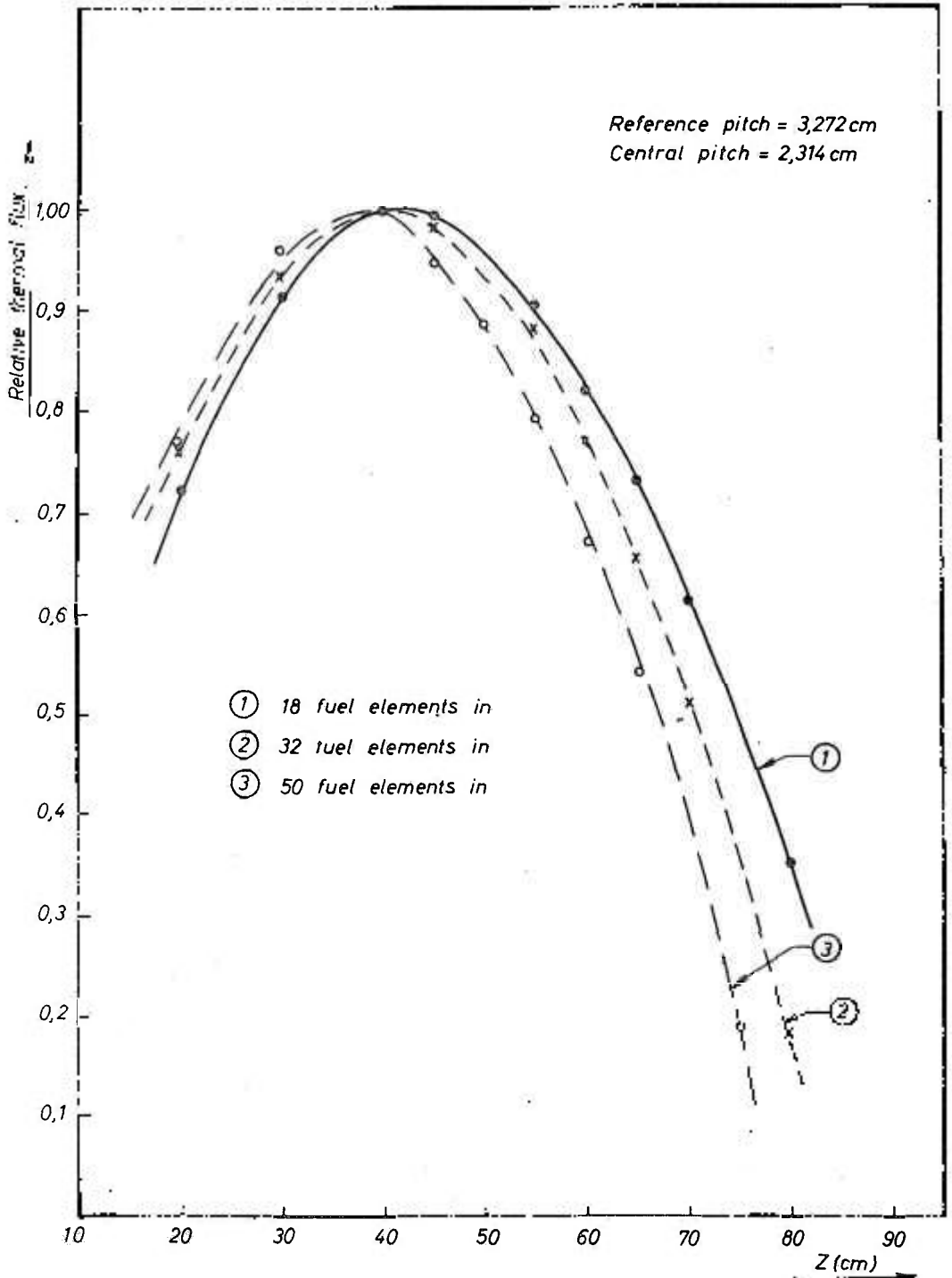


Fig. 9 AXIAL FLUX DISTRIBUTIONS.
FISSION CHAMBER.

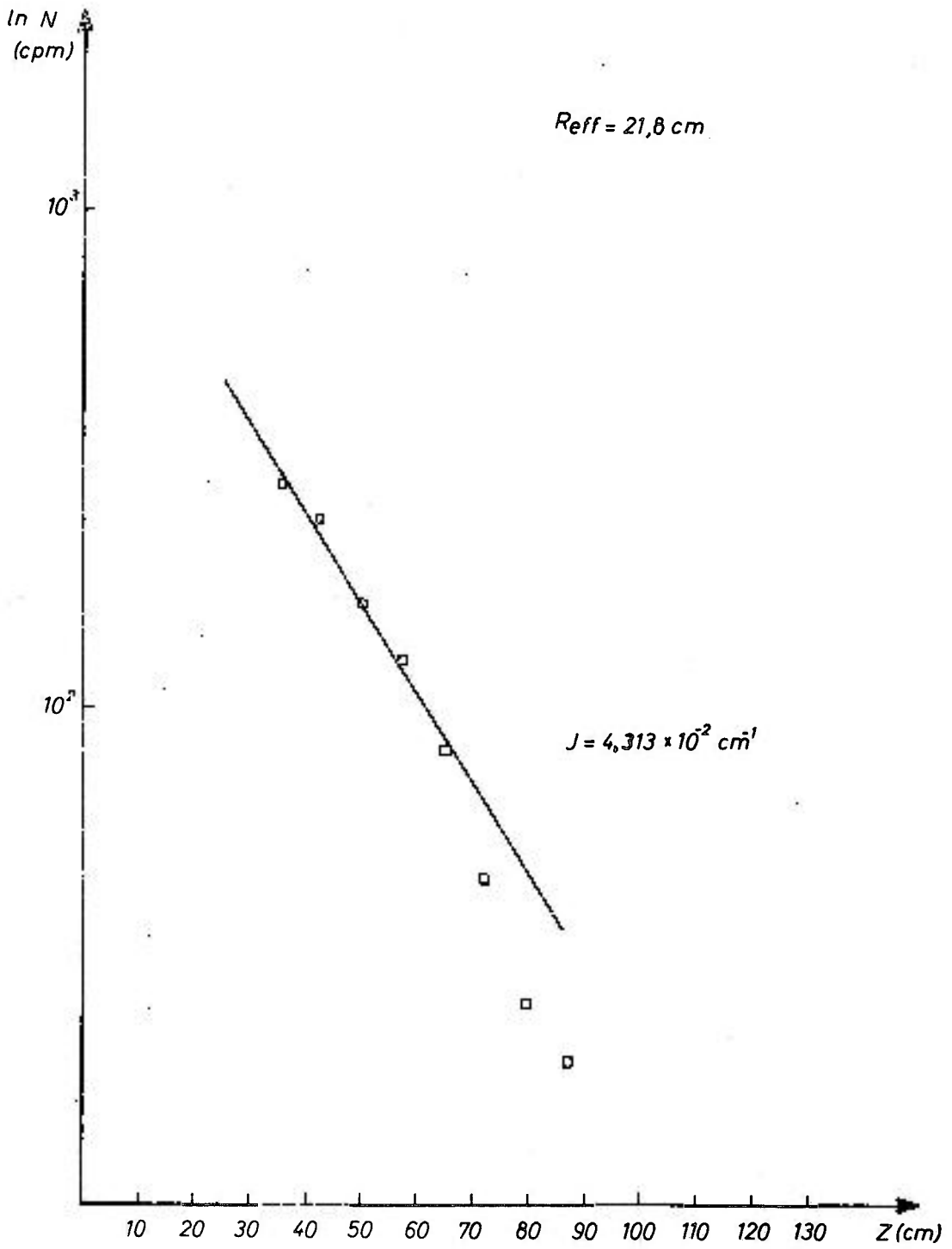


Fig. 10 EXPONENTIAL DECAY.

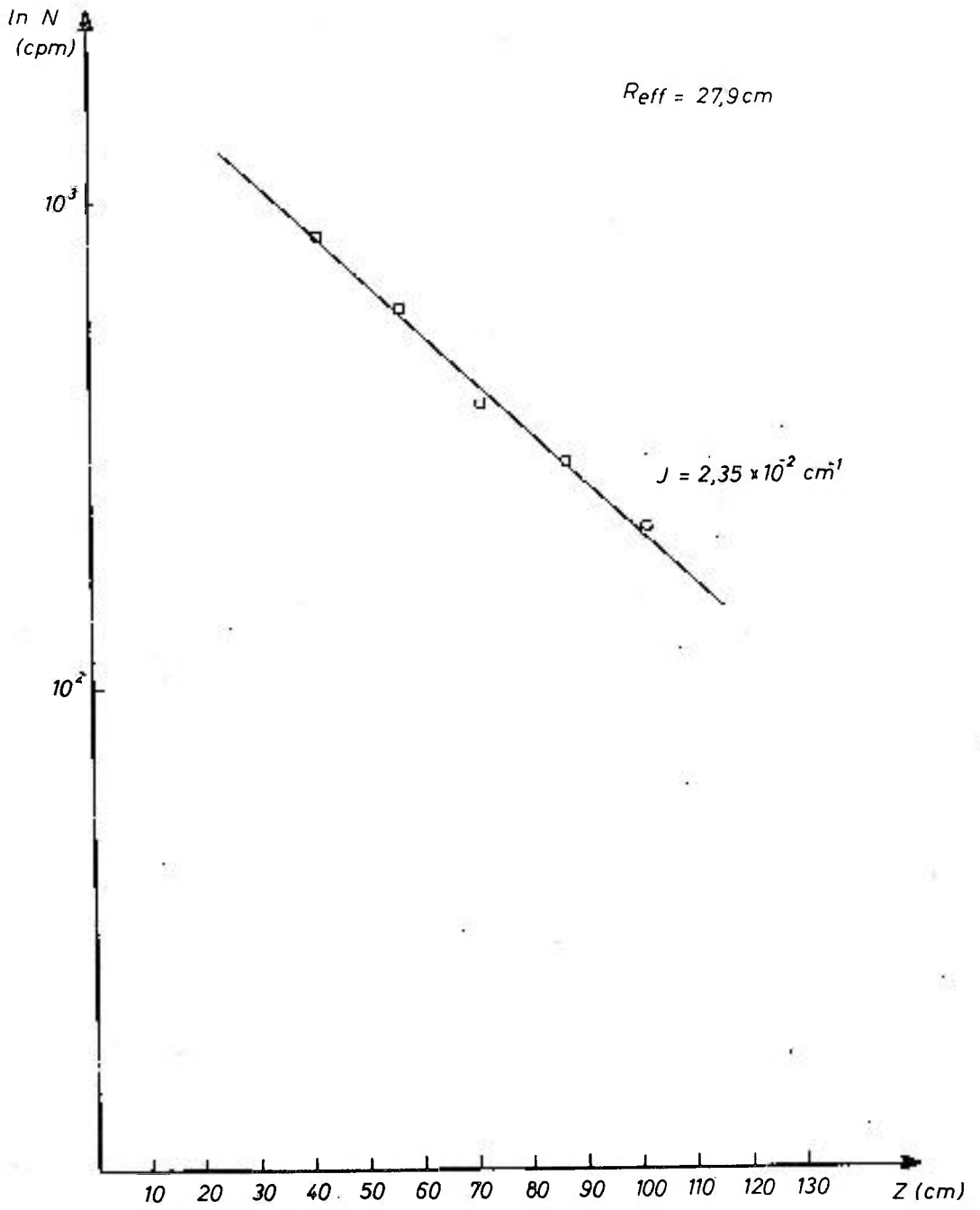


Fig:11. EXPONENTIAL DECAY.

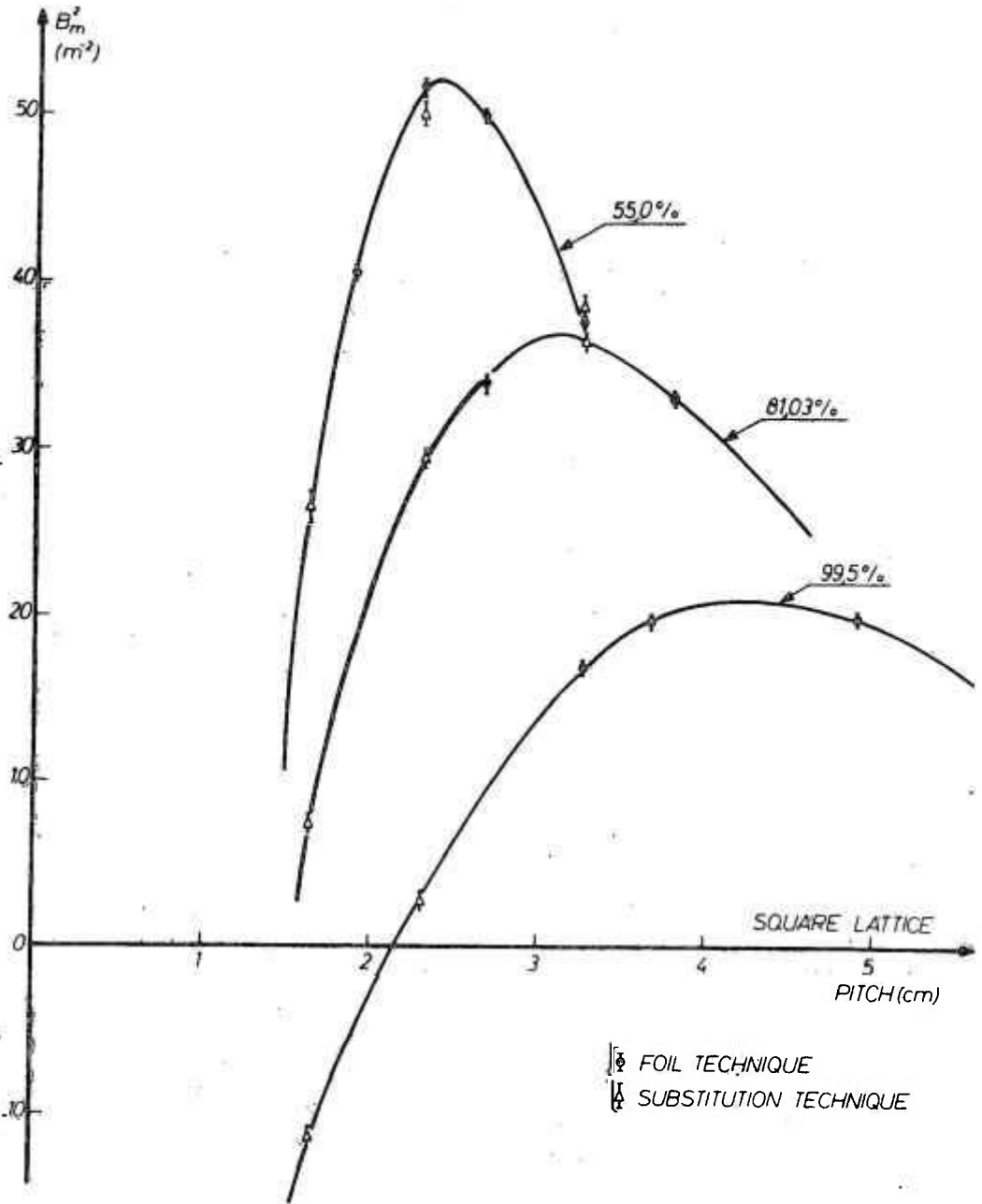


FIGURE 12
MATERIAL BUCKLING VERSUS SQUARE LATTICE PITCH