

# Excited state absorption and energy transfer in Ho<sup>3+</sup>-doped indium fluoride glass



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## ARTICLE INFO

### Article history:

Received 16 January 2017

Received in revised form

20 February 2017

Accepted 23 February 2017

Available online 9 March 2017

## ABSTRACT

This investigation examines in detail the rates of energy transfer relevant to the <sup>5</sup>I<sub>5</sub> → <sup>5</sup>I<sub>6</sub> transition (at 3930 nm) in Ho<sup>3+</sup>-doped InF<sub>3</sub> glass as a function of the Ho<sup>3+</sup> concentration. The decay times, branching ratios and rate parameters for energy transfer were measured in this investigation for Ho<sup>3+</sup> (x)-doped InF<sub>3</sub> glass with x = 2, 4 and 10 mol.% and they were used as the input parameters for a rate equation analysis. Excited state absorption (ESA) initiating from the lower laser level is included in the study. Numerical simulation of CW laser emission at 3.9 μm was performed using two pump wavelengths, one for upper laser level excitation (i.e., <sup>5</sup>I<sub>8</sub> → <sup>5</sup>I<sub>5</sub> = λ<sub>p1</sub>) and the other for lower laser level de-excitation (i.e., <sup>5</sup>I<sub>6</sub> → <sup>5</sup>S<sub>2</sub> = λ<sub>p2</sub>). The pump wavelength λ<sub>p2</sub> = 962 nm was chosen based on the measurements of ESA and the application of the McCumber method. Critically, the estimated ESA cross section at λ<sub>p2</sub> = 962 nm (σ<sub>ESA</sub> = 7.1 × 10<sup>-21</sup> cm<sup>2</sup>) is approximately sixteen times larger than ground state (<sup>3</sup>I<sub>8</sub>) absorption cross section (σ<sub>GSA</sub> = 4.3 × 10<sup>-22</sup> cm<sup>2</sup>) and ESA does not overlap with any ground state absorption process. Our calculations suggest that even for high Ho<sup>3+</sup> concentration in which cross relaxation has been shown in a previous study to quench the <sup>5</sup>I<sub>5</sub> level, ESA is nevertheless strong enough to allow a sufficient population inversion required for practical CW emission.

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## 1. Introduction

The quest longer wavelength emission from a fibre laser is largely driven by applications accessing atmospheric transmission windows and the convenient overlap with strong molecular absorption. Since the first demonstration of 3.9 μm emission from Ho<sup>3+</sup>-doped ZBLAN glass fibre quite some time ago [1,2] very little progress has been demonstrated in either increasing the output power or efficiency from rare earth doped fibres emitting in this region. This situation has resulted from two competing issues. First, the <sup>5</sup>I<sub>5</sub> → <sup>5</sup>I<sub>6</sub> transition (at 3930 nm) of Ho<sup>3+</sup> has not been easy to access because until recently, high power pump sources that directly excite the upper laser level at approximately 890 nm were not widely available. The other problem relates to the traditional use of ZBLAN glass because multiphonon absorption in this glass

limits the both the luminescence efficiency of the <sup>5</sup>I<sub>5</sub> → <sup>5</sup>I<sub>6</sub> transition and the transmission efficiency of 3.9 μm photons. This latter feature is a particular issue for a fibre geometry.

The recent demonstration [3] of mJ-level output from this transition using bulk Ho<sup>3+</sup>-doped indium fluoride glass has initiated a revival of research into this transition. Not only did this demonstration show that a fluoride-glass based host is a suitable material for efficient near 4 μm laser emission but it also represented the first use of Ho<sup>3+</sup>-doped InF<sub>3</sub> glass for laser applications. Indium fluoride glasses have a comparatively lower phonon energy allowing better infrared transmission and this glass has been shown to be an excellent source of low loss fibre [4]. Research can now be directed towards the development and optimisation of Ho<sup>3+</sup>-doped InF<sub>3</sub> glass for fibre laser emission in the 4 μm region and beyond.

Recently [5], we elucidated the primary spectroscopic parameters of Ho<sup>3+</sup>-doped InF<sub>3</sub> glass in terms of the absorption and emission cross sections and energy transfer parameters for a single

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$\text{Ho}^{3+}$  concentration of 10 mol.%. In this earlier study, a cross relaxation process was identified as having a strong quenching effect on the decay time of the  $^5I_5$  upper laser level. In the current study, we cover a wide concentration range in order to extract the rate parameters for energy transfer of all the known energy transfer pathways relevant to the  $^5I_5 \rightarrow ^5I_6$  transition. In addition, we examine excited state absorption at a wavelength of 962 nm that initiates from the lower ( $^5I_6$ ) level and show, by way of numerical simulation, that it has a beneficial effect on laser performance and allows access to much higher  $\text{Ho}^{3+}$  concentrations than thought possible given the known detrimental effects from cross relaxation at high  $\text{Ho}^{3+}$  concentration.

## 2. Experimental procedure

The composition of the glass samples used has been discussed in detail in our previous investigation [5]. All samples were prepared from 99.99% pure fluoroindate salts  $\text{GaF}_3$ ,  $\text{BaF}_2$ ,  $\text{SrF}_2$ ,  $\text{ZnF}_2$  and  $\text{HoF}_3$  based on the pioneering work found in the literature [6–9]. The samples were melted in a dry box to reduce the water content, the melt was poured into brass moulds and the samples cut and polished into rectangular prisms. The  $\text{Ho}^{3+}$  ion densities in the 2 mol.%, 4 mol.% and 10 mol.% samples were calculated to be  $4 \times 10^{20} \text{ cm}^{-3}$ ,  $8 \times 10^{20} \text{ cm}^{-3}$  and  $2 \times 10^{21} \text{ cm}^{-3}$ , respectively.

We used an optical parametric oscillator as a tunable pulsed pump source that could provide direct excitation to all the energy levels of interest. The pulse energies incident on the samples were up to 15 mJ, the pulse durations were 4 ns, and the repetition frequency was 10 Hz. A Judson model J-10 D InSb infrared detector (cooled to 77 K) was used to detect the infrared luminescence (for wavelengths longer than 1080 nm). A fast preamplifier (response time  $\sim 0.5 \mu\text{s}$ ) was used to amplify the signal. A photomultiplier tube (EMI, response time 10 ns) was employed to detect the visible and near infrared (i.e.,  $\lambda < 1100 \text{ nm}$ ) luminescence. All measurements were carried out at 300 K. Bandpass filters each with  $\sim 80\%$  transmission at 1200 nm, 2100 nm and 4000 nm with a half width of 25 nm and an extinction coefficient of approximately  $10^{-5}$  outside this band were used to isolate the relevant infrared luminescence.

## 3. Experimental results

The following equations and fitting parameters that were used to fit the measured fluorescence decay and upconversion luminescence transients of  $\text{Ho}^{3+}$ -doped  $\text{InF}_3$  glass have been discussed in detail in our previous investigation [5] and interested readers can refer to this study.

### 3.1. Luminescence decay and cross relaxation from the $^5I_5$ level

Fig. 1 shows the simplified energy level diagram of the  $\text{Ho}^{3+}$  ion used for the analysis of the experimental results and the rate equation model presented later. Fig. 2 shows measured fluorescence decay from the  $^5I_5$  level for the  $\text{Ho}^{3+}$ -doped  $\text{InF}_3$  glasses after pulsed laser excitation at 889 nm. The experimental luminescence decays were fitted using [5].

$$I(t) = I_0 \exp(-\gamma\sqrt{t} - t/t_1), \quad (1)$$

where  $\gamma$  ( $\text{s}^{-1/2}$ ) is the energy transfer (CR) parameter and  $t_1$  is the time constant of the exponential part of the decay. The integrated lifetime was obtained using

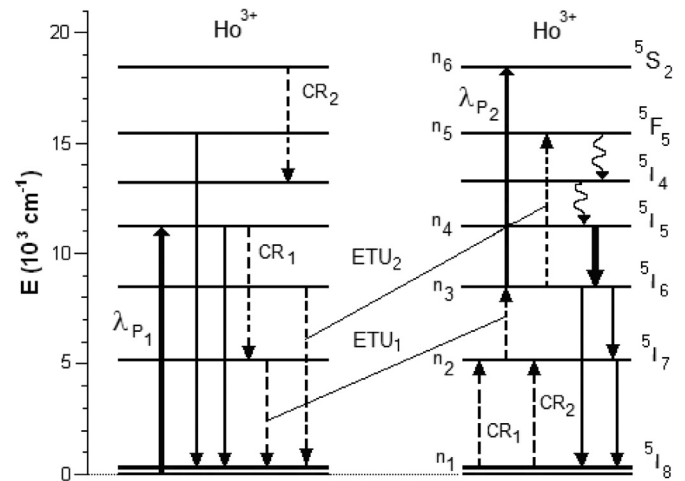


Fig. 1. Schematic energy level diagram used to show all the processes involved in the numerical model describing laser emission at  $3.93 \mu\text{m}$  (due to the  $^5I_5 \rightarrow ^5I_6$  transition) after laser pumping at  $889 \text{ nm}$  ( $\lambda_{P1}$ ) and the secondary laser pumping at  $962 \text{ nm}$  ( $\lambda_{P2}$ ).  $\text{CR}_1$  and  $\text{CR}_2$  are used to represent the cross-relaxations processes.

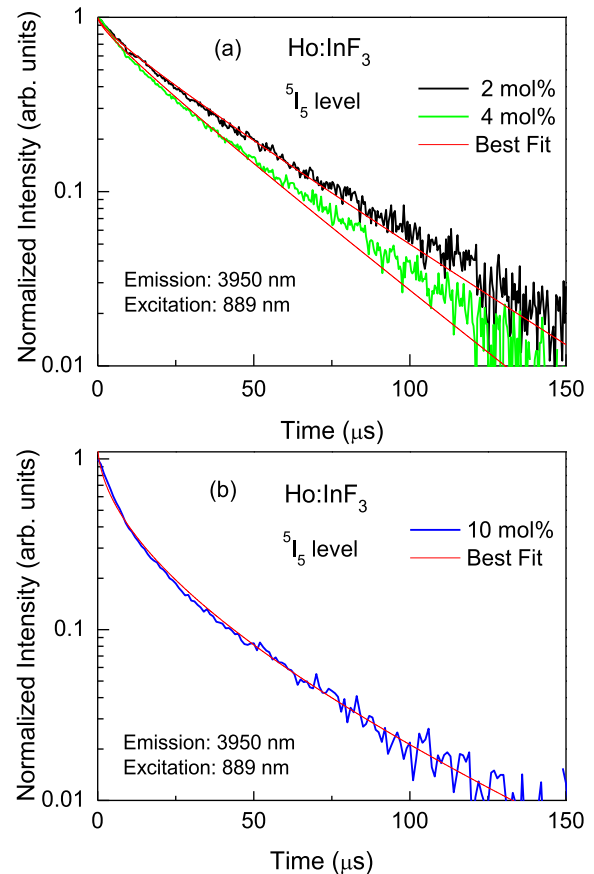


Fig. 2. Measured luminescence decay from the  $^5I_5$  level after pulsed laser excitation at  $889 \text{ nm}$  (with a mean pulse energy of 6 mJ) for the  $\text{Ho}(x):\text{InF}_3$  glasses with (a)  $x = 2, 4$  and (b) 10 mol.% - data taken from Ref. [5].

$$\tau = \frac{1}{I_0} \int_0^{\infty} I(t) dt. \quad (2)$$

The best-fit parameters using these equations are giving in Table 1.

**Table 1**The cross-relaxation rate and cross-relaxation efficiencies relevant to the luminescence decay of the  $^5I_5$  level.

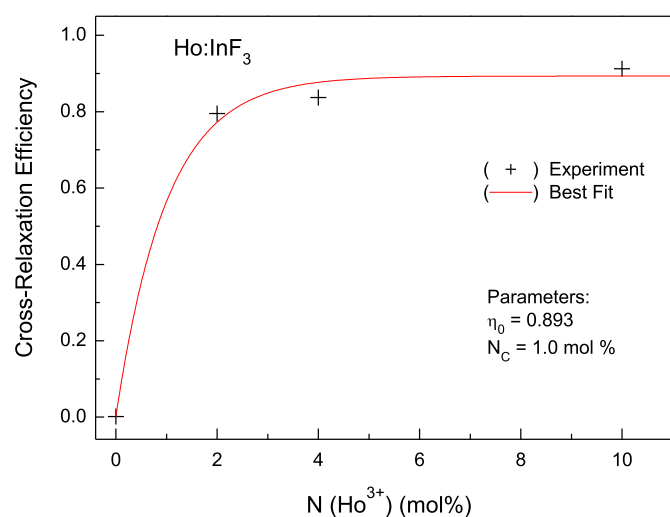
[Ho] (mol.%)	$\gamma$ ( $s^{-1/2}$ )	$\tau_1$ ( $\mu s$ )	$R^2$	$\tau_1(\text{int})$ ( $\mu s$ ) (integrated)	$W_{CR1}$ ( $s^{-1}$ )	$\eta_{CR1}$
2	66	42.4	0.996	30	25926	0.795
4	62	33.5	0.994	24	34259	0.837
10	333	135	0.992	13	69516	0.913

As observed from Fig. 2, it is clear that a nonradiative quenching process is acting on the  $^5I_5$  level with increased  $\text{Ho}^{3+}$  ion concentration. We have previously identified [5] that a cross-relaxation process (labelled  $\text{CR}_1$ ) competes with the intrinsic decay of the  $^5I_5$  level and that the cross relaxation process is a resonant interaction between two  $\text{Ho}^{3+}$  ions according to the process:  $\text{Ho}^{3+} (^5I_5) + \text{Ho}^{3+} (^5I_8) \rightarrow \text{Ho}^{3+} (^5I_7) + \text{Ho}^{3+} (^5I_7)$ . This non-radiative dipole-dipole interaction decreases the total lifetime of the  $^5I_5$  level and its respective rate (in  $s^{-1}$ ). The rate of  $\text{CR}_1$  can be obtained using the relation:  $W_{CR1} = \frac{1}{\tau_1(\text{int})} - \frac{1}{\tau_d}$ , where  $\tau_d = 135 \mu s$ , which was determined in our previous report [5]. The cross-relaxation efficiency, defined as  $\eta_{CR1} = 1 - \frac{\tau_1(\text{int})}{\tau_d}$ , can be used to determine the degree of nonradiative quenching resulting from this process. The parameters relevant to  $\text{CR}_1$  are given in Table 1.

Fig. 2 shows the efficiency of the cross-relaxation process (i.e.,  $\eta_{CR1}$ ) as a function of the  $\text{Ho}^{3+}$  ion concentration. The solid red line shows the curve of best fit using the critical radius model, which assumes the existence of a critical radius  $R_C$  for the  $\text{CR}_1$  process [5]. The use of the critical radius model is reliant on the following conditions: i)  $\eta_{CR1} = \eta_0$  (constant) for all excited  $\text{Ho}^{3+}$  ions (in the  $^5I_5$  level) having a nearby  $\text{Ho}^{3+}$  ground state ion within a distance  $R \leq R_C$ , and ii)  $\eta_{CR1} = 0$  when  $R > R_C$ . If we consider a statistical distribution of  $\text{Ho}^{3+}$  ions in the glass matrix, we can obtain the cross-relaxation efficiency as a function of the  $\text{Ho}^{3+}$  concentration using

$$\eta_{CR1} = \eta_0 \{ 1 - \exp(-N/N_C) \}, \quad (3)$$

where  $N_C = \left[ \frac{4}{3} \pi R_C^3 \right]^{-1}$ , and  $\eta_0$  is the constant efficiency,  $N$  is the  $[\text{Ho}^{3+}]$  concentration and  $N_C$  is the critical concentration (given in mol%). The results from this analysis are presented in Fig. 3. The



**Fig. 3.** Calculated cross-relaxation efficiency ( $\eta_{CR1}$ ) as a function of the  $\text{Ho}^{3+}$  ion concentration. The solid red line represents the best-fit curve using the critical radius model described by Eq. (3). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

desired parameters were obtained using a best-fit analysis and are given by  $\eta_0 = 0.893$  and  $N_C = 1.0$  mol%. The calculated dependence of the cross relaxation efficiency with the  $\text{Ho}^{3+}$  concentration for the  $^5I_5$  level decay is unusual and suggests that this process was assisted by fast energy migration. In this case, the cross-relaxation process in the current samples is more efficient than it would be if it was assisted for example, by hopping migration.

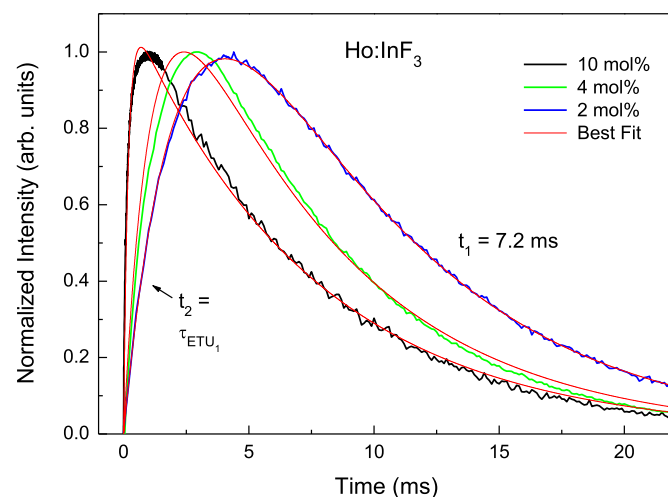
### 3.2. Energy transfer upconversion from the $^5I_7$ level ( $\text{ETU}_1$ )

It is well known [10] that a non-radiative energy transfer process involving two interacting excited  $\text{Ho}^{3+}$  ions in the  $^5I_7$  level promotes excitation to the  $^5I_6$  level according to the process:  $\text{Ho}^{3+} (^5I_7) + \text{Ho}^{3+} (^5I_7) \rightarrow \text{Ho}^{3+} (^5I_6) + \text{Ho}^{3+} (^5I_8) + \text{ph}\omega$  (p-3) (labelled the  $\text{ETU}_1$  process). The  $\text{ETU}_1$  rate constant ( $K_1$ ) was obtained for the  $\text{Ho}^{3+}(x)$ -doped  $\text{InF}_3$  glasses with  $x = 2, 4$  and  $10$  mol.%. For this experiment, we used pulsed laser excitation at  $1985$  nm with the following excitation parameters: i) mean energy ( $E$ ) of  $9.3$  mJ and focused beam area ( $A$ ) equal to  $0.0023$   $\text{cm}^2$  for the  $2$  mol% sample, ii)  $E = 8.6$  mJ and  $A = 0.0041$   $\text{cm}^2$  for the  $4$  mol.% sample and iii)  $E = 6.5$  mJ and  $A = 0.0041$   $\text{cm}^2$  for the  $10$  mol.% sample. The smaller pump spot for the lower concentration sample provided an increase in the fluorescence signal and Fig. 4 shows the upconversion measurements.

Analysis of the upconversion luminescence transient for the  $^5I_6$  level under  $^5I_8 \rightarrow ^5I_7$  laser excitation at  $1985$  nm was performed using a best-fit analysis that employed the relationship [5].

$$I(t) = I_0 \left\{ \exp(-\gamma_1 \sqrt{t} - t/t_1) - \exp(-t/t_2) \right\}, \quad (4)$$

where  $\gamma_1$ ,  $t_1$  are the fitting parameters for the decay of the  $^5I_6$  level and  $t_2$  is the upconversion time. The  $\text{ETU}_1$  rate i.e.,  $W_{\text{ETU}_1}$  ( $s^{-1}$ ) was



**Fig. 4.** Measured  $^5I_6$  luminescence transient (at  $1200$  nm) due to the  $\text{ETU}_1$  upconversion process induced by pulse laser excitation at  $1985$  nm with excitation conditions i)  $E = 9.3$  mJ and excitation volume ( $V$ ) equal to  $0.00122$   $\text{cm}^3$  for  $[\text{Ho}^{3+}] = 2$  mol.%, ii)  $E = 8.6$  mJ and  $V = 0.00217$   $\text{cm}^3$  for  $[\text{Ho}^{3+}] = 4$  mol.% and iii)  $E = 6.5$  mJ and  $V = 0.00217$   $\text{cm}^3$  for  $[\text{Ho}^{3+}] = 10$  mol.%.

**Table 2**Parameters obtained from the best-fit analysis for the upconversion luminescence from the  $^5I_6$  level after laser excitation at 1985 nm.

[Ho <sup>3+</sup> ] (mol.%)	f(abs.) ( $\lambda = 1985$ nm)	N* (photons cm <sup>-3</sup> ) (exc. density)	t <sub>1</sub> (ms) ( $\gamma_1 = 0$ )	t <sub>2</sub> (ms)	R <sup>2</sup>	W <sub>ETU1</sub> (s <sup>-1</sup> )	K <sub>1</sub> (ETU <sub>1</sub> ) (s <sup>-1</sup> )
2	0.42	$3.20 \times 10^{19}$	6.9	2.49	0.999	340	474
4	0.67	$2.65 \times 10^{19}$	7.2	1.10	0.996	848	1723
10	0.94	$2.90 \times 10^{19}$	7.3	0.184	0.997	5373	8913

f(abs.) is the absorption fraction (or absorptivity) of the  $^5I_8 \rightarrow ^5I_7$  absorption of Ho:InF<sub>3</sub> glass at the excitation wavelength of 1985 nm for an optical length of d = 5.3 mm (or sample thickness).

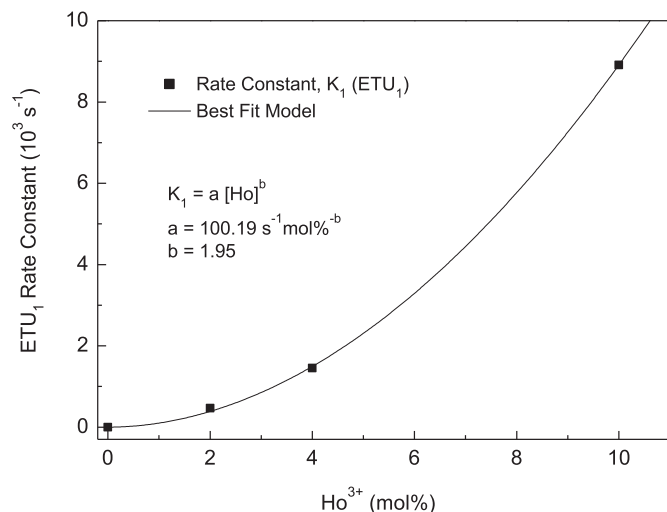
obtained using the relation  $W_{ETU1} (s^{-1}) = \frac{1}{t_2} - \frac{1}{\tau_d}$ , where  $\tau_d$  is equal to 16.3 ms, the intrinsic decay time of the  $^5I_7$  level. Table 2 shows the parameters from the best-fit analysis (see coloured curves in Fig. 4) using the measured luminescence for the three Ho<sup>3+</sup> concentrations.

Using the critical radius function given in our previous paper [5] we can obtain the ETU<sub>1</sub> rate constant K<sub>1</sub> for any Ho<sup>3+</sup> concentration using

$$K_1 = \frac{W_{ETU1}}{1 - \exp\left[-\frac{(N^* - N_0)}{N_{C1}}\right]}, \quad (5)$$

where N\* is determined measured for each new sample with differing Ho<sup>3+</sup> ion concentration, N<sub>0</sub> is equal to  $2 \times 10^{19}$  cm<sup>-3</sup> and N<sub>C1</sub> is equal to  $9.51 \times 10^{18}$  cm<sup>-3</sup>. K<sub>1</sub> values were calculated using Eq. (2) and the W<sub>ETU1</sub> rates and excitation densities (N\*) from Table 1. The calculated ETU<sub>1</sub> rates constants, K<sub>1</sub> are given in Table 2.

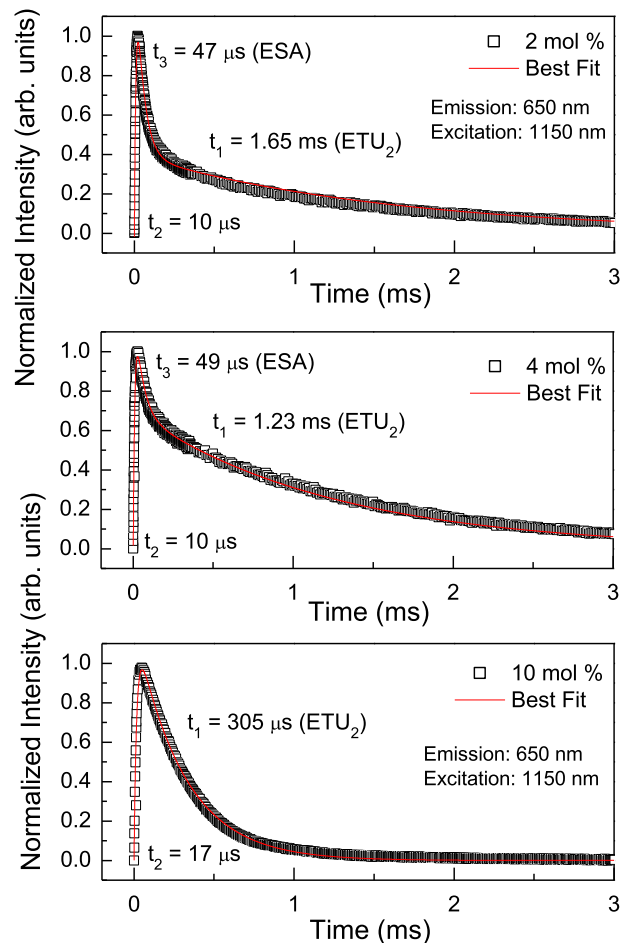
Fig. 5 shows the cross-relaxation rate constant K<sub>1</sub> as a function of the Ho<sup>3+</sup> concentration. The solid black line shows the curve of best fit using a power law function given by  $K_1 = a[Ho]^b$ , where the best-fit parameters were  $a = 100.2$  s<sup>-1</sup> (mol.%)<sup>-b</sup> and  $b = 1.95$ . The quadratic dependence of K<sub>1</sub> on the Ho<sup>3+</sup> concentration is expected for an energy transfer process involving two ions in the same excited state (i.e., the  $^5I_7$  level). If hopping migration is present, the energy transfer rate K<sub>m</sub> is proportional to  $c_A c_D$ , where c<sub>A</sub> and c<sub>D</sub> are the acceptor and donor concentrations, respectively [11]. However, in the case of the ETU<sub>1</sub> process the acceptor and donor states are identical.



**Fig. 5.** Cross-relaxation rate K<sub>1</sub> as a function of the Ho<sup>3+</sup> concentration. Solid black line shows the curve of best fit using a power law function.

### 3.3. Energy transfer upconversion from the $^5I_6$ excited level (ETU<sub>2</sub>)

A non-radiative energy transfer involving two interacting excited Ho<sup>3+</sup> ions in the  $^5I_6$  level is also well known from previous studies [10]. The ETU process is described according to: Ho<sup>3+</sup> ( $^5I_6$ ) + Ho<sup>3+</sup> ( $^5I_6$ ) → Ho<sup>3+</sup> ( $^5F_5$ ) + Ho<sup>3+</sup> ( $^5I_8$ ) +  $\text{ph}\omega$  (p~3) (labelled the ETU<sub>2</sub> process). For this experiment, pulsed laser excitation at 1150 nm and a mean pulse energy of 9.3 mJ was focused to an area A = 0.0041 cm<sup>2</sup> for the all samples. Fig. 6 shows the measured upconversion luminescence curves measured after laser excitation at 1150 nm. The comparatively long time decay component (t<sub>1</sub>, see Eq. (6)) is due to the ETU<sub>2</sub> process, which contributes to 48% of the total luminescence signal. The short decay time component (t<sub>3</sub>, see Eq. (6)) is due to ESA absorption that populates the  $^5S_2$  level, which



**Fig. 6.** Luminescence transient for emission from the  $^5F_5$  level induced by ETU<sub>2</sub> process after laser excitation at 1150 nm (with a mean energy of 9.3 mJ and excitation volume  $V = 0.00217$  cm<sup>3</sup>) for InF<sub>3</sub> glass doped with (a) 2 mol.% Ho<sup>3+</sup>, (b) 4 mol.% Ho<sup>3+</sup>, and 10 mol.%. The fast decay component (t<sub>3</sub>) relates to ESA that populates the  $^5S_2$  level and subsequently the  $^5F_5$  level.

in turn relaxes to the  $^5F_5$  level and contributes to the luminescence decay at 650 nm. We calculate that the contribution from ESA is 76% for the  $\text{Ho}^{3+}$  (2 mol.%) doped sample and 52% for the  $\text{Ho}^{3+}$  (4 mol.%) doped sample. Excited state absorption at 1150 nm is negligible for the  $\text{Ho}^{3+}$  (10 mol.%) doped sample.

A best-fit analysis to the upconversion luminescence transient due to  $\text{ETU}_2$  process after laser excitation at 1150 nm was performed using

$$I(t) = A \exp(-\gamma_1 \sqrt{t} - t/t_1) - B \exp(-t/t_2) + C \exp(-t/t_3), \quad (6)$$

where  $\gamma_1$ ,  $t_1$  are the fitting parameters for the  $\text{ETU}_2$  process,  $t_2$  is due to the  $^5F_5$  level decay and  $t_3$  is due to the  $^5S_2$  level decay after ESA. A, B and C are constants; B and C are used to determine the contributions from  $\text{ETU}_2$  and ESA, respectively. Similarly, the rate of  $\text{ETU}_2$  (in  $\text{s}^{-1}$ ) can be obtained using the relation  $W_{\text{ETU}_2}(\text{s}^{-1}) = \frac{1}{t_1} - \frac{1}{\tau_d}$ , where  $\tau_d$  is equal to 6.2 ms, the intrinsic decay of the  $^5I_6$  level, as determined from our previous study [5]. Table 3 shows the parameters from the best-fit analysis for all concentrations studied.

The critical radius model that described  $\text{ETU}_2$  as a function of the excitation density ( $N^*$ ) from our previous study [5] of the  $\text{Ho}^{3+}$  (10 mol.%) doped  $\text{InF}_3$  sample was used to obtain the  $\text{ETU}_2$  rate constant  $K_0$  the 2 mol.% and 4 mol.% samples. The rate constant  $K_2$  was obtained from

$$K_2 = \frac{W_{\text{ETU}_2}}{1 - \exp\left[-\frac{N^*}{N_{C_2}}\right]}, \quad (7)$$

where the critical concentration  $N_{C_2}$  was determined to be  $1.74 \times 10^{19} \text{ cm}^{-3}$ . Using the rate values for  $W_{\text{ETU}_2}$  given in Table 3 for the 2 mol.% and 4 mol.% and the respective excitation densities ( $N^*$ ) that were used, one can calculate the  $K_2$  values, which are given in Table 3.

### 3.4. Excited state absorption from level $^5I_6$

It was observed, see Fig. 7, that green luminescence [from the  $^5S_2$  ( $^5F_4$ ) level] after pulsed excitation at 1150 nm ( $E = 11 \text{ mJ}$ , 4 ns, 10 Hz) for all three  $\text{Ho}^{3+}$ -doped  $\text{InF}_3$  samples. The rise time of the green luminescence tracked the pulse laser duration, which indicated that ESA was present by way of two photon absorption (TPA)  $^5I_8 \rightarrow ^5I_6$ ,  $^5I_6 \rightarrow ^5S_2$ . The ESA transition ( $^5I_6 \rightarrow ^5S_2$ ) is not resonant with the 1150 nm excitation wavelength because the  $^5I_6 \rightarrow ^5S_2$  ESA process has a maximum at 962 nm (or centroid at approximately 965 nm) according to the energy level diagram for the  $\text{Ho}^{3+}$  ion. We postulate that laser excitation at 1150 nm excites the two-phonon sideband of ESA band similar to what has been observed for  $\text{Ho}^{3+}$ -doped ZBLAN glass [11].

The decay of the  $^5S_2$  excited state shown in Fig. 7 behaves with a non-exponential characteristic with the parameter in Eq. (6)  $\gamma \gg 0$  and using  $B = C = 0$ . The integrated lifetime  $\tau_1(\text{int})$ , see Fig. 7 was much shorter for all samples than the radiative lifetime i.e.,  $\tau_R = 312 \mu\text{s}$  calculated for this level. This fast decay arises from an effective

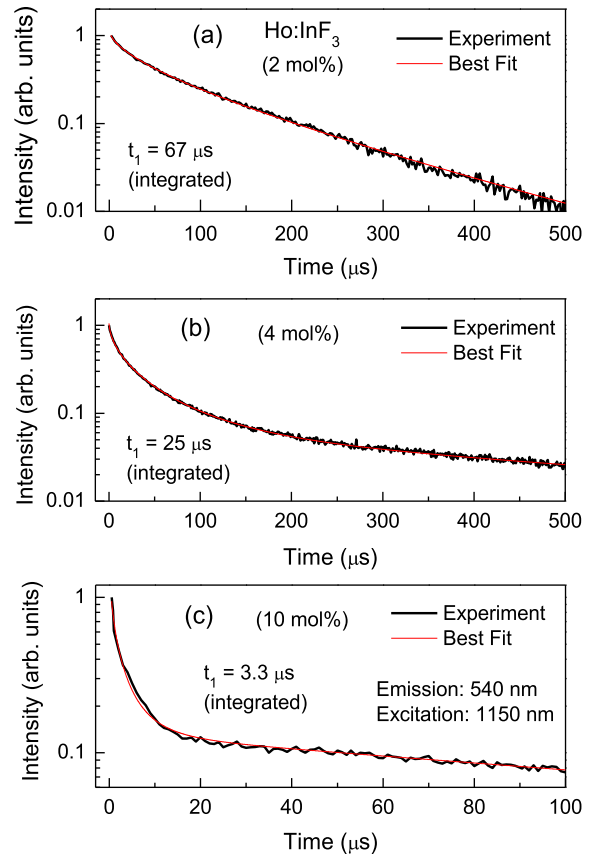


Fig. 7. Measured luminescence decay of the  $^5S_2$  ( $^5F_4$ ) level measured at 540 nm from TPA after pulsed laser excitation at 1150 nm ( $E = 11 \text{ mJ}$ , 4 ns, 10 Hz) for  $[\text{Ho}^{3+}] = 2, 4$  and 10 mol%. The fast decay component is due to the  $\text{CR}_2$  process.

Table 4  
Parameters from the best-fit analysis of the luminescence from  $^5S_2$  level.

$[\text{Ho}^{3+}]$ (mol.%)	$\gamma(\text{s}^{-1/2})$	$t_1$ ( $\mu\text{s}$ )	$\tau$	$R^2$	$W_{\text{CR}_2}(\text{s}^{-1})$
2	119	264	67	0.999	11720
4	241	200	25	0.999	36795
10	1169	60	3.3	0.993	$2.998 \times 10^5$

cross-relaxation process (labelled  $\text{CR}_2$ ) is described by  $\text{Ho}^{3+} (^5S_2) + \text{Ho}^{3+} (^5I_8) \rightarrow \text{Ho}^{3+} (^5I_4) + \text{Ho}^{3+} (^5I_7)$ . The cross-relaxation rate was calculated using the relation:  $W_{\text{CR}_2} = \frac{1}{\tau_1(\text{int})} - \frac{1}{\tau_d}$ , where  $\tau_d = \tau_R$  [calculated in this work for the  $^5S_2$  ( $^5F_4$ ) level]. Table 4 shows the parameters from the best-fit analysis using the measured luminescence in Fig. 7. The  $W_{\text{CR}_2}$  value for the 2 mol%  $\text{Ho}^{3+}$  sample can be compared with the value of  $18000 \text{ s}^{-1}$  for a 2%  $\text{Ho}^{3+}$ -doped  $\text{InF}_3$  glass studied in Ref. [12] and  $6580 \text{ s}^{-1}$  for a 2.25%  $\text{Ho}^{3+}$ -doped  $\text{InF}_3$  glass studied in Ref. [13].

The ESA absorption cross section (for  $^5I_6 \rightarrow ^5S_2$ ) was calculated using the McCumber method [14], which relates the emission and

Table 3  
Parameters from the best-fit analysis for upconversion-induced luminescence transient from the  $^5F_5$  level.

$[\text{Ho}^{3+}]$ (mol.%)	f(abs.)	$N^*$ ( $\text{cm}^{-3}$ )	$t_1$ (ms) ( $\gamma_1 = 0$ )	$t_2$ ( $\mu\text{s}$ )	$t_3$ ( $\mu\text{s}$ )	$R^2$	$W_{\text{ETU}_2}(\text{s}^{-1})$	$K_2(\text{ETU}_2)(\text{s}^{-1})$
2	0.39	$0.98 \times 10^{19}$	1.65	10	47.4	0.997	445	1018
4	0.63	$1.48 \times 10^{19}$	1.23	10	49.5	0.998	652	1138
10	0.91	$2.35 \times 10^{19}$	0.305	17	—	0.998	3117	4212

f(abs.) is the absorption fraction (or absorptivity) of the  $^5I_8 \rightarrow ^5I_6$  absorption of  $\text{Ho}:\text{InF}_3$  at  $\lambda = 1150 \text{ nm}$  for an optical length of 5.3 mm (or sample thickness).

absorption cross-sections using the relation

$$\sigma_{abs}(\lambda) = \sigma_{emis}(\lambda) \frac{N_2}{N_1} \exp\left(-\frac{\hbar\omega}{k_B T}\right), \quad (8)$$

where  $N_i = g_i \exp\left(-\frac{E_i}{k_B T}\right)$  and  $N_1$  is the equilibrium population at  $T = 300$  K for the initial level (i.e.,  ${}^5I_6$ , at  $E_1 = 8696$   $\text{cm}^{-1}$ ) and  $N_2$  is the final level (i.e.,  ${}^5S_2, {}^5F_4$  at  $E_2 = 18994$   $\text{cm}^{-1}$ ). The term  $g_i$  is the  $i$ -level degeneracy with values  $g_1 = 13$  for the  ${}^5I_6$  level and  $g_2 = 14$  for the combined  ${}^5S_2, {}^5F_4$  level.  $\hbar\omega$  is the emission photon energy (in  $\text{cm}^{-1}$ ),  $k_B$  is the Boltzmann constant and  $T = 300$  K. Fig. 8 shows the measured emission cross-section spectrum obtained using the integration relation (described in our previous study [5]) and the calculated ESA absorption cross-section for the  ${}^5I_6 \rightarrow {}^5S_2, {}^5F_4$  process. The maximum ESA cross section of  $7.1 \times 10^{-21}$   $\text{cm}^2$  occurs at 962 nm and is approximately sixteen times larger than  $\sigma_{GSA} (= 4.3 \times 10^{-22}$   $\text{cm}^2$ ) of the  ${}^5I_8 \rightarrow {}^5I_5$  transition. Note that an ESA wavelength of 962 nm does not coincide with any GSA.

#### 4. Rate equation analysis

Fig. 1 shows the energy level schematic presenting all the intrinsic and extrinsic processes involved when two pump wavelengths, at 889 and 962 nm are employed for laser emission at 3.93  $\mu\text{m}$ . Note that the use of a two-wavelength pump scheme has been successfully employed for mid-infrared fibre lasers based on the  $\text{Er}^{3+}$  ion [15–17]. We label the pump wavelength at 889 nm ( $\lambda_{P1}$ ) the primary pump and the excitation wavelength at 962 nm ( $\lambda_{P2}$ ) the secondary pump.

In the model, we assume CW excitation at both pump wavelengths. The parameters  $n_1, n_2, n_3, n_4, n_5$  and  $n_6$  are used to represent the populations in the  ${}^5I_8, {}^5I_7, {}^5I_6, {}^5I_5, {}^5F_5$  and  ${}^5S_2$  energy levels of  $\text{Ho}^{3+}$ , respectively. These levels are considered the most important and relevant to the simulation of lasing at 3.93  $\mu\text{m}$ . The  ${}^5I_4$  level is strongly quenched by fast multiphonon decay and is therefore ignored. The rate equations for 889 nm pumping (assuming  $n_1 + n_2 + n_3 + n_4 + n_5 + n_6 = 1$ ) are given by

$$\frac{dn_1}{dt} = -R_{P1} n_1 + \frac{\beta_{51} n_5}{\tau_{R5}} + \frac{\beta_{41} n_4}{\tau_{R4}} + \frac{\beta_{31} n_3}{\tau_{R3}} + \frac{\beta_{21} n_2}{\tau_{R2}} + \frac{\beta_{61} n_6}{\tau_{R6}} + K_1 n_2^2 + K_2 n_3^2 - W_{CR1} n_1 n_4 - W_{CR2} n_1 n_6 \quad (9)$$

$$\frac{dn_2}{dt} = \frac{\beta_{52} n_5}{\tau_{R5}} + \frac{\beta_{42} n_4}{\tau_{R4}} + \frac{\beta_{32} n_3}{\tau_{R3}} + \frac{\beta_{62} n_6}{\tau_{R6}} - \frac{1}{\tau_{R2}} n_2 - 2 K_1 n_2^2 + 2 W_{CR1} n_1 n_4 + W_{CR2} n_1 n_6 + W_{NR}(32) n_3 \quad (10)$$

$$\frac{dn_3}{dt} = -\frac{n_3}{\tau_{R3}} - W_{NR}(32) n_3 + W_{NR}(43) n_4 + \frac{\beta_{43} n_4}{\tau_{R4}} + \frac{\beta_{53} n_5}{\tau_{R5}} n_5 + \frac{\beta_{63} n_6}{\tau_{R6}} + K_1 n_2^2 - 2 K_2 n_3^2 - R_{P2} n_3 \quad (11)$$

$$\frac{dn_4}{dt} = R_{P1} n_1 - \frac{n_4}{\tau_{R4}} + W_{NR}(54) n_5 - W_{NR}(43) n_4 - W_{CR1} n_1 n_4 + W_{CR2} n_1 n_6 \quad (12)$$

$$\frac{dn_5}{dt} = -\frac{n_5}{\tau_{R5}} - W_{NR}(54) n_5 + K_2 n_3^2 \quad (13)$$

$$\frac{dn_6}{dt} = R_{P2} n_3 - W_{CR2} n_1 n_6 - \frac{n_6}{\tau_{R6}} \quad (14)$$

where  $R_{Pi} = \sigma_{14} \frac{I_{Pi}}{(h\nu)_{Pi}}$  is the pump rate (in  $\text{s}^{-1}$ ),  $I_{Pi}$  is the pump light (in  $\text{W cm}^{-2}$ ), and  $h\nu$  is the pump photon energy.  $\beta_{ij}$  represents the luminescence branching ratio and  $\tau_{Ri}$  is the radiative lifetime of the excited states of  $\text{Ho}^{3+}$  (labelled as  $i = 2, 3, 4, 5$  and 6). The absorption and emission cross-sections are  $\sigma_{14} = 4.3 \times 10^{-22}$   $\text{cm}^2$  (GSA at 889 nm),  $\sigma_{36} = 7.1 \times 10^{-21}$   $\text{cm}^2$  (ESA at 962 nm) and  $\sigma_{43} = 3.7 \times 10^{-21}$   $\text{cm}^2$  (at 3925 nm). All other parameters are given in Table 5. Note that the value for  $\tau_d$  for the  ${}^5S_2$  level in Table 5 is longer than the value of 105  $\mu\text{s}$  for the same energy level given in Ref. [13].

Numerical solution of the rate equations was carried out to understand the potential for efficient laser emission at 3.93  $\mu\text{m}$  under two wavelength CW pumping. The calculated evolution of the population inversion  $[(n_4 - n_3), \text{in mol.\%}]$  was obtained by numerical simulation of the rate equations for  $[\text{Ho}^{3+}] = 2, 4$  and 10 mol.%. Using the primary pumping rate ( $R_{P1}$ ) of 2000  $\text{s}^{-1}$  and several secondary-pumping rates ( $R_{P2}$ , is the secondary pump rate) from  $2 \times 10^3$  to  $6 \times 10^4$   $\text{s}^{-1}$  the population inversion was calculated as shown in Figs. 9–11. The calculations show that a positive steady state population inversion is obtained 2 ms after initiation of the primary pump for values of the secondary pumping rate  $R_{P2} > 1 \times 10^4$   $\text{s}^{-1}$ . The secondary pumping at 962 nm first depopulates the lower ( ${}^5I_6$ ) laser level and for higher values for  $R_{P2}$ , it contributes to excitation of the upper ( ${}^5I_5$ ) laser level population.

Fig. 12 shows the calculated population inversion ( $n_4 - n_3$ ) obtained at equilibrium (after 3 ms) as a function of  $R_{P2}$  for constant  $R_{P1} = 2000$   $\text{s}^{-1}$  and  $[\text{Ho}^{3+}] = 2, 4$  and 10 mol.%. The results suggest that the  $[\text{Ho}^{3+}] = 10$  mol.% is the best concentration for achieving CW laser operation at 3.93  $\mu\text{m}$  with double wavelength pumping because the population inversion fraction of 15% (when  $R_{P2} = 6 \times 10^4$   $\text{s}^{-1}$ ) is certainly sufficient to overcome the laser cavity losses relevant to a practical system. The pump intensity ( $I_{Pi}$ ) can be calculated using  $I_{Pi} = \frac{h\nu R_{Pi}}{\sigma_{abs}}$ , where  $h\nu$  is the photon excitation energy (in joules) and  $\sigma_{abs}$  is the absorption cross-section ( $\text{cm}^2$ ) (at wavelength excitation  $\lambda_{Pi}$ ). Table 6 shows the values used for this

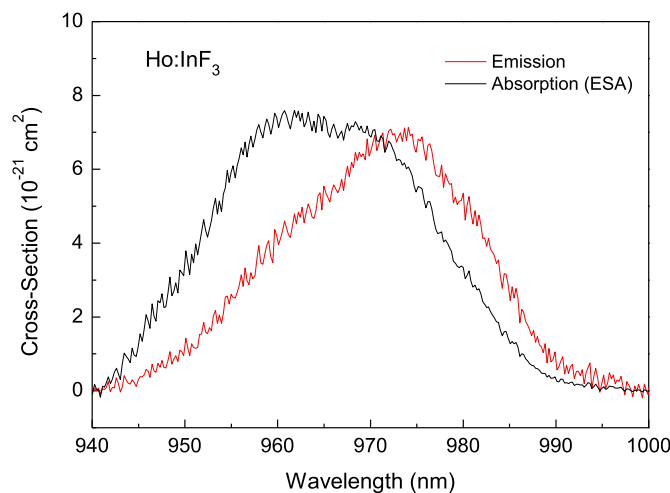


Fig. 8. Measured emission cross-section for the ( ${}^5S_2, {}^5F_4$ )  $\rightarrow$   ${}^5I_6$  transition after pulsed laser excitation at 1150 nm ( $E_p = 11$  mJ). The corresponding ESA cross-section (black curve) was obtained using the McCumber method given by Eq. (8).

**Table 5**  
Parameters used in the rate equation modelling.

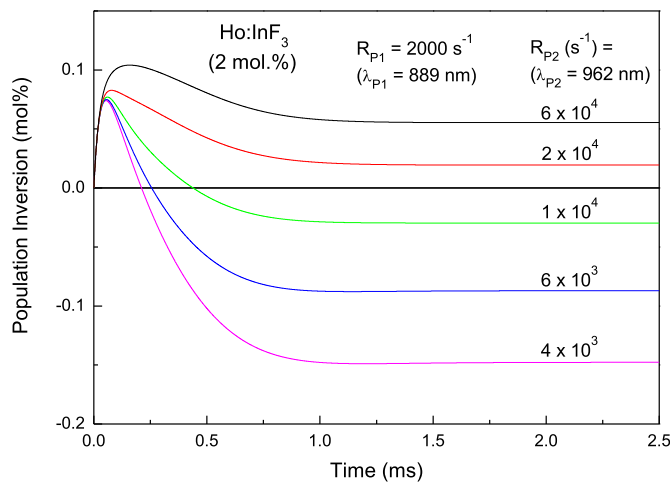
Luminescence branching ratio and radiative and intrinsic (total) lifetimes of Ho <sup>3+</sup>				
Transition (level#)	$\beta^a$	$\tau_R^b$	$\tau_d = \tau(\text{intrinsic})^c$ (W <sub>NR</sub> (s <sup>-1</sup> ))	
<sup>5</sup> S <sub>2</sub> (6) →		312 μs	$\tau_d = 312 \mu\text{s}$ (W <sub>NR</sub> (65) = 0)	
<sup>5</sup> I <sub>6</sub> (3)	0.10			
<sup>5</sup> I <sub>7</sub> (2)	0.40			
<sup>5</sup> I <sub>8</sub> (1)	0.50			
<sup>5</sup> F <sub>5</sub> (5) →		321 μs	$\tau_d = \tau = 16.3 \mu\text{s}$ (γ = 0) (W <sub>NR</sub> (54) = 58234 s <sup>-1</sup> )	
<sup>5</sup> I <sub>5</sub> (4)	0.004			
<sup>5</sup> I <sub>6</sub> (3)	0.046			
<sup>5</sup> I <sub>7</sub> (2)	0.192			
<sup>5</sup> I <sub>8</sub> (1)	0.758			
<sup>5</sup> I <sub>5</sub> (4) →		6.8 ms	$\tau_d = 135 \mu\text{s}$ (W <sub>NR</sub> (43) = 7260 s <sup>-1</sup> )	
<sup>5</sup> I <sub>6</sub> (3)	0.013			
<sup>5</sup> I <sub>7</sub> (2)	0.430			
<sup>5</sup> I <sub>8</sub> (1)	0.557			
<sup>5</sup> I <sub>6</sub> (3) →		6.2 ms	$\tau_d = \tau = 6.2 \text{ ms}$ (γ = 0) (W <sub>NR</sub> (32) = 0)	
<sup>5</sup> I <sub>7</sub> (2)	0.058			
<sup>5</sup> I <sub>8</sub> (1)	0.942			
<sup>5</sup> I <sub>7</sub> (2) → <sup>5</sup> I <sub>8</sub> (1)	1	16.2 ms	$\tau_d = 16.2 \text{ ms}$ (W <sub>NR</sub> (21) = 0)	
Energy transfer rate parameters (s <sup>-1</sup> ) (expt.) <sup>d</sup>				
Ho:InF <sub>3</sub> (mol.%)	ETU <sub>1</sub> K <sub>1</sub> (s <sup>-1</sup> )	ETU <sub>2</sub> K <sub>2</sub> (s <sup>-1</sup> )	CR <sub>1</sub> W <sub>CR1</sub> (s <sup>-1</sup> )	CR <sub>2</sub> W <sub>CR2</sub> (s <sup>-1</sup> )
2%	474	1018	25926	11720
4%	1723	1138	34259	36795
10%	8913	4212	69516	2.998 × 10 <sup>5</sup>

<sup>a</sup> Branching ratios calculated in this work.  
<sup>b</sup> Radiative lifetimes calculated using Judd-Ofelt theory in this work.  
<sup>c</sup> Experimental lifetime (intrinsic) obtained from the best fit to the luminescence done in this work.  
<sup>d</sup> Experimental transfer rates obtained in this work.

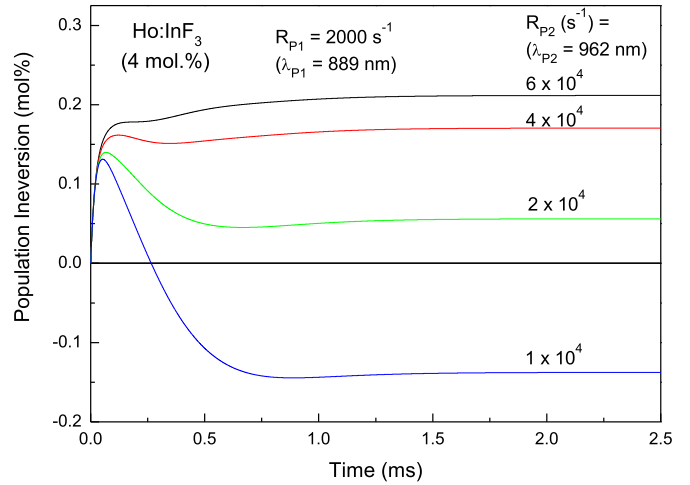
numerical simulation.

**5. Discussion**

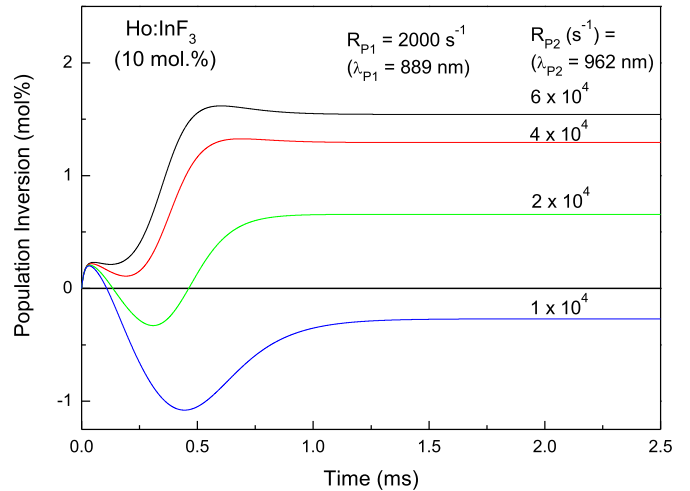
This study has quantified all the important energy transfer



**Fig. 9.** Calculated population inversion (in mol.%) for the laser transition <sup>5</sup>I<sub>5</sub> → <sup>5</sup>I<sub>6</sub> (at 3925 nm) as a function of the pumping intensities for [Ho<sup>3+</sup>] = 2 mol.% doped fluoroindate glass under CW pumping at 889 nm (λ<sub>p1</sub>) and CW pumping at 962 nm (λ<sub>p2</sub>) (ESA) at 300 K. R<sub>p1</sub> = 100 s<sup>-1</sup> corresponds to a pump intensity (I<sub>p1</sub>) equal to 51 KW cm<sup>-2</sup> and R<sub>p2</sub> = 100 s<sup>-1</sup> corresponds to 3.25 KW cm<sup>-2</sup> (I<sub>p2</sub>).



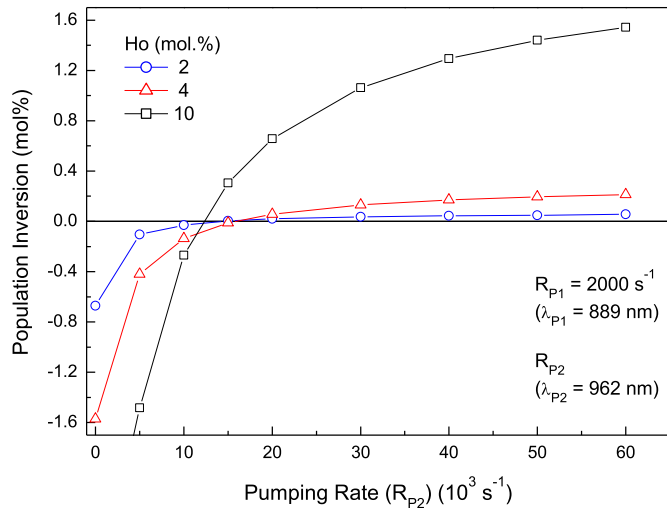
**Fig. 10.** Calculated population inversion (in mol.%) for the laser transition <sup>5</sup>I<sub>5</sub> → <sup>5</sup>I<sub>6</sub> (at 3925 nm) as a function of the pumping intensities for [Ho<sup>3+</sup>] = 4 mol.% doped fluoroindate glass under CW pumping at 889 nm (λ<sub>p1</sub>) and CW pumping at 962 nm (λ<sub>p2</sub>) (ESA) at 300 K.



**Fig. 11.** Calculated population inversion (in mol.%) for the laser transition <sup>5</sup>I<sub>5</sub> → <sup>5</sup>I<sub>6</sub> (at 3925 nm) as a function of the pumping intensities for [Ho<sup>3+</sup>] = 10 mol.% doped fluoroindate glass under CW pumping at 889 nm (λ<sub>p1</sub>) and CW pumping at 962 nm (λ<sub>p2</sub>) (ESA) at 300 K.

processes relevant to the operation of the <sup>5</sup>I<sub>5</sub> → <sup>5</sup>I<sub>6</sub> transition at 3930 nm. Accessing transitions between high lying energy levels typically involves a greater number of energy transfer processes because a greater number of energy levels are potentially populated. The <sup>5</sup>I<sub>5</sub> → <sup>5</sup>I<sub>6</sub> transition is no exception and the present study has established that at least two energy transfer upconversion and two cross relaxation processes contribute to the emission from the <sup>5</sup>I<sub>5</sub> → <sup>5</sup>I<sub>6</sub> transition. This investigation also comprises the most detailed spectroscopic study of a rare earth doped indium fluoride glass and allows future more detailed numerical simulation of laser systems based on this optical material.

The numerical simulations presented in this study suggest that double clad indium fluoride fibre that contains high concentrations of Ho<sup>3+</sup> ions can be useful for high power CW emission at 3.93 μm. In our previous study [5], we showed that CR has the effect of significantly shortening the lifetime of the <sup>5</sup>I<sub>5</sub> level which prevents, from a basic numerical simulation perspective, CW emission at high Ho<sup>3+</sup> concentrations. We have shown in this study, however, that



**Fig. 12.** Calculated population inversion (in mol.%) as a function of the pumping rate  $R_{P2}$  at  $\lambda_{P2} = 962$  nm for constant pumping ( $R_{P1}$ ) at  $\lambda_{P1} = 889$  nm for  $[\text{Ho}^{3+}] = 2, 4$  and 10 mol.%.

**Table 6**

Wavelengths, absorption cross-sections for GSA and ESA processes, pumping rate and pumping intensity used in the numerical simulations for the 3.93  $\mu\text{m}$  laser emission in  $\text{Ho}^{3+}$ -doped  $\text{InF}_3$  glass.

$\lambda_{P1} = 889$ nm	$\sigma_{\text{abs}} = 4.4 \times 10^{-22}$ $\text{cm}^{-2}$
$\lambda_{P2} = 962$ nm	$\sigma_{\text{ESA}} = 7.1 \times 10^{-21}$ $\text{cm}^{-2}$
$R_{P1} = 2 \times 10^3$ $\text{s}^{-1}$	$I_{P1} = 1020$ $\text{KW cm}^{-2}$
$R_{P2}$ ( $\text{s}^{-1}$ )	$I_{P2}$ ( $\text{KW cm}^{-2}$ )
$2 \times 10^3$	65
$4 \times 10^3$	130
$6 \times 10^3$	195
$1 \times 10^4$	324
$2 \times 10^4$	648
$6 \times 10^4$	1945

with an additional pump characterised by a centre wavelength close to the ESA peak at 962 nm, that efficient CW laser emission is possible even for  $\text{Ho}^{3+}$  concentrations as high as 10 mol.%. This is a significant step forward because high power diode lasers at both 976 nm and 899 nm are available. The present study indicates that CW emission at 3930 nm beyond the one-watt level is a strong possibility, however, given the high pump rates used in the calculations, core pumping may be necessary. A complete fibre and experimental layout must wait, however, until a more accurate and complete numerical modelling study of dual wavelength pumping of  $\text{Ho}^{3+}$ -doped  $\text{InF}_3$  glass optical fibre has been carried out.

## 6. Conclusions

We have reported a detailed time-resolved spectroscopic study of  $\text{Ho}^{3+}$ -doped indium fluoride glass relevant to the emission from the  $^5I_5 \rightarrow ^5I_6$  transition at 3930 nm. The rate parameters for all the relevant energy transfer processes have been determined. These rates are useful for modelling laser systems based on either fibre or bulk geometries. We have shown that under dual wavelength pumping,  $\text{Ho}^{3+}$ -doped  $\text{InF}_3$  glass fibres have the potential to enable CW laser emission at 3930 nm for concentrations ranging from 2 mol.% to 10 mol.%, despite the presence of strong cross relaxation that can effectively quench the upper laser level. We have established that a  $\text{Ho}^{3+}$  concentration of 10 mol.% provides the highest fractional population inversion (of 15%) for an ESA pumping rate  $R_{P2} = 6 \times 10^4$   $\text{s}^{-1}$ . Given the low background loss at 3.9  $\mu\text{m}$  (of approximately 20  $\text{dB km}^{-1}$ ) combined with secondary pumping in the 970 nm region, we expect that a fibre doped with 10 mol.%  $\text{Ho}^{3+}$  will provide a viable route to moderate power generation at 3.9  $\mu\text{m}$ .

## Acknowledgments

The authors thank the financial support from São Paulo State Research Foundation (FAPESP, Grants N<sup>o</sup> 1995/4166-0 and 2000/10986-0), the Brazilian Research Foundation (CNPq), the Australian Research Council (DP140101336) and the Natural Sciences and Engineering Research Council of Canada.

## References

- [1] J. Schneider, *Electron. Lett.* 31 (1995) 1250.
- [2] J. Schneider, C. Carbonnier, U.B. Unrau, *Appl. Opt.* 36 (1997) 8595.
- [3] A. Berrou, C. Kieleck, M. Eichhorn, *Opt. Lett.* 40 (2015) 1699.
- [4] See <http://leverrefluore.com/>.
- [5] L. Gomes, V. Fortin, M. Bernier, R. Vallée, S. Poulain, M. Poulain, S.D. Jackson, *Opt. Mat.* 60 (2016) 618.
- [6] M.A. Poulain, M.J. Poulain, *Mat. Sci. Forum* 32–33 (1988) 137.
- [7] G. Mazé, M. Poulain, J. Y. Carrée, A. Soufiane, Y. Messaddeq, "Indium Fluoride glasses" US Patent N<sup>o</sup> 5480845.
- [8] M. Poulain, A. Soufiane, Y. Messaddeq, M.A. Aegerte, *Braz. J. Phys.* 22 (1992) 205.
- [9] A. Soufiane, F. Gan, H. L'Helgoualch, M. Poulain, *J. Non-Crystalline Solids* 184 (1995) 103.
- [10] A.F.H. Librantz, S.D. Jackson, L. Gomes, S.J.L. Ribeiro, Y. Messaddeq, *J. Appl. Phys.* 103 (2008) 023105.
- [11] R.C. Powell, *Physics of Solid-state Laser Materials*, Section 5.2-Nonradiative Energy Transfer: Multistep Process, AIP Press, New York, 1998, p. 203.
- [12] N. Rakov, G.S. Maclell, C.B. De Araújo, Y. Messaddeq, *Appl. Phys.* 91 (2002) 1272.
- [13] F. Lahoz, I.R. Martín, A. Briones, *J. Appl. Phys.* 95 (2004) 2957.
- [14] D.E. McCumber, *Phys. Rev.* 136 (4A) (1968) 954.
- [15] O. Henderson-Sapir, J. Munch, D.J. Ottaway, *Opt. Lett.* 39 (2014) 493–496.
- [16] V. Fortin, F. Maes, M. Bernier, S.T. Bah, M. D'Auteuil, R. Vallée, *Opt. Lett.* 41 (2016) 559–562.
- [17] O. Henderson-Sapir, S.D. Jackson, D.J. Ottaway, *Opt. Lett.* 41 (2016) 1676–1679.