

# **A Simplified Transformation Approach to Obtain Structural Calibration Functions**

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**ABSTRACT:** In a ductile fracture methodology developed by Landes et al., the result from a laboratory test is used to predict the behavior of a structural component. Within the framework of this methodology, a critical step consists in obtaining the calibration function for the structural component. For those cases where the limit load solution for the structural component is known, a transformation procedure has been used to get the calibration function for the structure directly from that of a fracture toughness specimen. Although the transformation procedure does not involve complex calculations, it does require the user to follow several steps where point to point computations are necessary. This makes the whole process laborious and time consuming. It will be shown in this paper that, with an additional assumption and without loss of accuracy, the transformation procedure can be greatly simplified. In the alternative procedure proposed here, the coefficients of the calibration function for the structure are obtained by simply scaling their counterparts for the fracture toughness specimen. This is accomplished with the use of two factors: a load factor and a deformation factor. Some examples are presented to demonstrate the convenience of this new procedure.

**KEY WORDS:** ductile fracture, failure assessment, cracked structures, fracture toughness, calibration function

The prediction of the loading behavior during ductile fracture needs a scheme to incorporate both the deformation behavior of the structure as well as the fracture or cracking behavior, usually the former is more important. The Ductile Fracture Method (DFM), based on an idea originally discussed by Ernst and Landes [1,2] and further developed by Landes and coworkers [3-5], uses a load separation concept to represent the loading of a cracked body by two separate and multiplicative functions. The first one is a geometry function, which describes the effect of crack growth. This function depends only on the geometry of the cracked body. The second one is a deformation function, which represents the plastic flow character of the material and the specific geometry/loading mode of the cracked body. The DFM makes use of these two functions

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and fracture toughness given in terms of a  $J$ - $R$  curve to predict the load versus displacement behavior of a structural component during the ductile fracture process.

The geometry function is known for several geometries or can be obtained in a relatively easy way. On the other hand, the determination of the deformation function for the structural component represents a critical step in the DFM. Two procedures can be used for that. One is a transformation procedure that determines the deformation function for the structural component directly from that for a fracture toughness specimen. The other is a numerically based one in which the deformation pattern is numerically simulated and the load separation method is applied to define the calibration parameters. The former is easier to apply for the case in which the limit load solution for the structural component is known. The latter is more general and can be used for complex structural components if nonlinear finite element analysis can be conducted for the component.

Since the major concern here is with analytical methods that can be used for quick failure assessments of cracked structures, this work concentrates in the transformation procedure. As will be seen later, with an additional assumption and without loss of accuracy, the transformation procedure can be accomplished in a much simpler way and consequently the predictions done by the DFM can be completed in a shorter time. In the next section, a review of the DFM is presented. Then the original transformation procedure as proposed by Landes et al. [3] is described. Following this, an alternative approach is suggested which greatly simplifies the transformation process. Some examples are presented in which deformation functions obtained with the original transformation procedure are confronted with those derived through the simplified approach proposed here. Also, the application of the DFM using the simplified transformation is illustrated with the prediction of load versus displacement curves. The paper is concluded with a discussion about the benefits that the new transformation procedure brings to the DFM as a whole.

## A Review of the Ductile Fracture Method

The DFM uses a calibration function and fracture toughness, given in terms of a  $J$ - $R$  curve, as the inputs to predict the load versus displacement behavior of a structural component during the fracture process. The general approach for using the DFM is illustrated in Fig. 1. The calibration function gives the relationship between load and displacement for constant values of crack length. The fracture toughness describes how the crack length changes as a function of  $J$ . To apply the method to a structure with a given crack size, the loading is represented by the load,  $P$ , versus displacement,  $v$ , for that defect size. During the loading process, the value of  $J$  is also determined. When  $J$  has increased to the point where a crack length change is indicated, the  $P$  versus  $v$  curve is taken a step down to the one for that new crack length. The loading proceeds with the calibration function giving the relationship between  $P$  and  $v$  for a given crack length and the  $J$ - $R$  curve fracture toughness indicating what current value of crack length should be used. When small increments in crack length are used, the loading follows a smooth path.

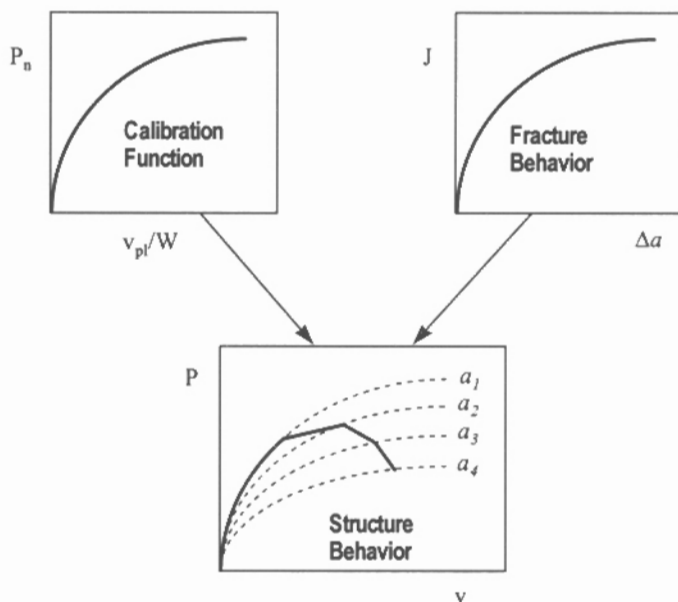


FIG. 1—Schematic of the Ductile Fracture Method

The DFM is founded on the load separation concept [6,7]. According to this concept, the relationship between load,  $P$ , crack length,  $a$ , and plastic displacement,  $v_{pl}$  for the cracked structure can be expressed as a multiplication of two separable functions

$$P = G(a/W) \cdot H(v_{pl}/W) \quad (1)$$

where  $G(a/W)$  is a function of geometry only and  $H(v_{pl}/W)$  is a function of plastic deformation only.  $W$  is a length dimension parameter; for test specimen geometries,  $W$  is usually the width, but for a structural component, it could be another dimension, such as the thickness. When the load  $P$  is divided by the  $G(a/W)$  function, the result is a normalized load

$$P_N = \frac{P}{G(a/W)} = H(v_{pl}/W) \quad (2)$$

The information in Eq. 2 is often referred to as the calibration function. It can describe the deformation behavior of a structure for a certain value of crack length. It was found that the global deformation pattern for many structures and for most materials can be accurately fitted by the following functional form

$$P_N = H(v_{pl}/W) = \frac{\left( L + M \frac{v_{pl}}{W} \right)}{\left( N + \frac{v_{pl}}{W} \right)} \left( \frac{v_{pl}}{W} \right) \quad (3)$$

This functional form came from the work of Orange [8] and is now known as LMN function [9]. The normalized load versus plastic displacement behavior as represented by this function is such that when the plastic displacement is small, the relationship between load and displacement is approximated by a power law, and when it is large, the behavior comes near to a straight line representation.

The  $G$  function is known for several geometries or can be obtained in a relatively easy way from a series of blunt notched specimens that model the structure [10]. Thus, the critical step in the DFM is the determination of the calibration function,  $H(v_{pl}/W)$ , for the structural component. For those cases in which the limit load solution for the structural component is known, a transformation procedure can be used to determine the calibration function for the structural component directly from that for a fracture toughness specimen. This procedure is described in the next section.

### The Original Transformation Procedure

The transformation procedure was derived from experimental results on an A533B steel in which four specimen geometries were tested (CT, DENT, CCT, SENT). It was verified that when the normalized loads for these geometries were plotted against the normalized displacement,  $v_{pl}/v_{el}$ , all the curves appeared to have the same trend and the differences in the normalized load between the specimens could be related to the ratios of their limit loads. If the CT specimen is taken as a fracture toughness test specimen, and the other geometries are considered as structural components, it is then possible to predict the calibration functions for the structural components from the test specimen.

Starting from the calibration function for the test specimen as represented by Eq. 3, the same functional form is assumed for the structural component, that is

$$P_n = h(v_{pl}/W) = \frac{\left(l + m \frac{v_{pl}}{W}\right)}{\left(n + \frac{v_{pl}}{W}\right)} \left(\frac{v_{pl}}{W}\right) \quad (4)$$

Thus, to find the news constants  $l$ ,  $m$ , and  $n$  that define the calibration function,  $h(v_{pl}/W)$ , for the component, the transformation procedure follows the steps described below and illustrated on Fig. 2:

- (a) First, the load versus displacement record,  $P$ - $v$  curve, for a fracture toughness test specimen needs to be obtained from an experiment (Fig. 2a);
- (b) Then this  $P$ - $v$  curve is converted into a normalized load,  $P_{ns}$ , versus normalized displacement,  $v_{pl}/W$ , Fig. 2b, which is the calibration function for the specimen,  $H(v_{pl}/W)$ ;
- (c) The abscissa,  $v_{pl}/W$  (Fig. 2b), is divided by  $v_{el}/W$  so that it becomes  $v_{pl}/v_{el}$  (Fig. 2c);
- (d) Then each point on that curve is multiplied by a factor  $f$ , which is defined as

$$f = \frac{P_{Ls}(a_{os})/G_s(a_{os}/W_s)}{P_L(a_o)/G(a_o/W)} \quad (5)$$

The result is a curve for the structural component given in terms of its normalized load,  $P_n$ , and the displacement ratio,  $v_{pl}/v_{el}$ . In the above equation,  $P_{Ls}(a_{os})$  and  $G_s(a_{os}/W_s)$  are the limit load and geometry function for the structure;  $P_L(a_o)$  and  $G(a_o/W)$  are the limit load and geometry function for the toughness specimen and  $a_o$  is the initial crack length.

- (e) The displacement ratio in the abscissa,  $v_{pl}/v_{el}$ , is converted back to normalized plastic displacement, by multiplying  $v_{pl}/v_{el}$  (Fig. 2c) by  $v_{el}/W$  for the structure. The resulting  $P_n$  versus  $v_{pl}/W$  curve in Fig. 2d is the representation of the desired calibration curve for the structure.
- (f) The  $l$ ,  $m$ , and  $n$  constants can be determined by fitting the transformed points. This is done by choosing  $l = f * L$ . Then, only  $m$  and  $n$  need to be determined from the fitted curve. The best result comes from choosing two points, the final one and one at small  $v_{pl}/W$  and fitting Eq. 4 to them.

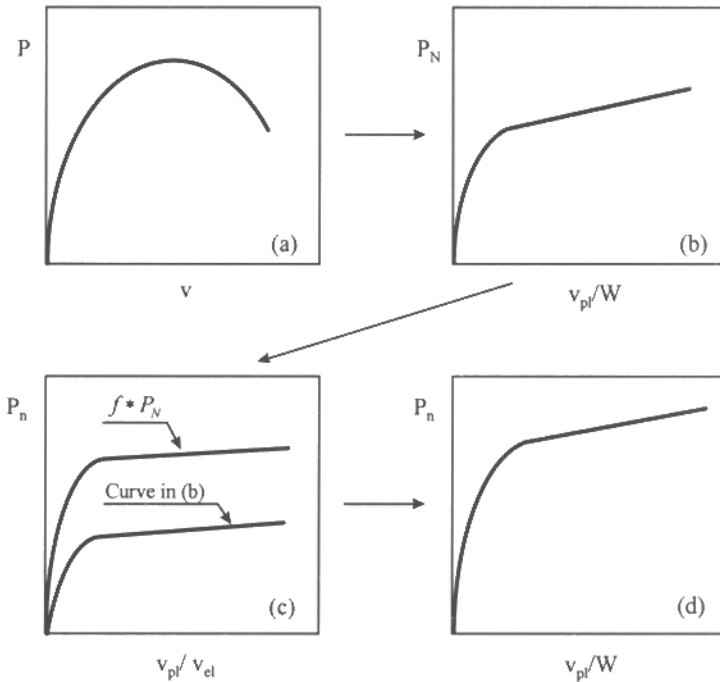


FIG. 2—Schematic of the original transformation procedure

During the process of the calibration function transformation, the elastic displacement,  $v_{el}$ , is taken as

$$v_{el} = C(a/W) \cdot P = C(a/W) \cdot G(a/W) \cdot H(v_{pl}/W) \quad (6)$$

where  $C(a/W)$  is the elastic compliance function for the structure. It can be obtained by experiment, analytical derivation or linear finite element analysis.  $v_{el}$  is calculated at each given value of  $v_{pl}/W$  (where  $H(v_{pl}/W)$  has a known value), and the product  $C(a/W)G(a/W)$  is assumed to be a constant. This is not completely true in the actual case. This product is dependent upon  $a/W$ , but in many cases it is not a strong function of  $a/W$ . Therefore, for a changing crack length, a constant value, say, the value corresponding to the initial crack length, can be used for simplicity.

## The Simplified Transformation Procedure

Even though it does not involve any complex steps, the original transformation procedure is somewhat cumbersome and time consuming, requiring the user to make one ordinate conversion (step d, above), two abscissa conversions (steps c and e), and a fitting operation to find the constants of the calibration function for the structural component (step f).

Analyzing the philosophy of the original transformation procedure, we see that it basically consists in making a normalized load and a normalized displacement adjustment on the fracture toughness specimen deformation curve to get the correspondent curve for the structural component. The load adjustment is very simple because it is based on a constant factor  $f$ , Eq. 5. On the other hand, the abscissa adjustment involves point to point computations of the elastic displacement using Eq. 6 and assuming that the product  $C(a/W)G(a/W)$  is constant.

Applying the procedure to different test geometries, in which the CT specimen was taken as the fracture toughness test specimen and the other geometries were considered as structural components, it was observed that the ratio between the normalized elastic displacement for the fracture toughness specimen,  $(v_{el}/W)$ , and the normalized elastic displacement for the structure,  $(v_{el}/W)_s$ , almost did not change during the transformation process. As will be shown in the following, if we assume that this ratio is constant, the transformation procedure can be greatly simplified.

First, define a parameter  $q$  to represent the ratio between the normalized elastic displacements

$$q = \frac{(v_{el}/W)}{(v_{el}/W)_s} \quad (7)$$

From the analysis undertaken, it can be seen that a representative value for  $q$  is obtained when the normalized elastic displacements for the fracture toughness specimen and the structure are calculated with their initial crack lengths and at their limit loads, that is

$$(v_{el})_o = C(a_o) \cdot P_L(a_o) \quad (8)$$

$$(v_{el})_{os} = C_s(a_{os}) \cdot P_{Ls}(a_{os}) \quad (9)$$

The normalized plastic displacement for the fracture toughness specimen,  $v_N = v_{p'} / W$ , and the normalized plastic displacement for the structure,  $v_n = v_{p'} / W$ , can be related by the following expression

$$v_n = \frac{v_N}{q} \quad (10)$$

Rewriting Eqs. 3 and 4 in terms of  $v_N$  and  $v_n$ , we have

$$P_N = H(v_N) = \frac{(L + Mv_N)}{(N + v_N)} v_N \quad (11)$$

$$P_n = h(v_n) = \frac{(l + mv_n)}{(n + v_n)} v_n \quad (12)$$

Multiplying Eq. 11 by the factor  $f$  (Eq. 5) is equivalent to step (d) of the original transformation procedure, which would give an intermediate representation  $P_n$  versus  $v_n$  with the constants  $l' = f.L$ ,  $m' = f.M$  and  $n' = N$ , that is

$$P_n = \frac{(l' + m'v_N)}{(n' + v_N)} v_N \quad (13)$$

Then, substituting Eq. 10 in Eq. 13, that corresponds to steps (c) and (e) of the original transformation procedure, leads to

$$P_n = \frac{l' + m'(qv_n)}{n' + (qv_n)} (qv_n) = \frac{fL + fqMv_n}{(N/q) + v_n} v_n \quad (14)$$

Now, comparing the above expression with Eq. 4, we arrive at the following expressions for the  $l$ ,  $m$ , and  $n$  constants of the structure calibration function

$$l = f.L; \quad m = f.q.M; \quad n = N/q \quad (15)$$

Therefore, the coefficients of the calibration function for the structural component can be directly obtained from their fracture toughness specimen counterparts. It would suffice to compute the factors  $f$  and  $q$  (Eqs. 5 and 7) and use the above expressions. This represents a significant simplification in the transformation procedure, eliminating the coordinates conversions and the fitting operation to get the  $l$ ,  $m$ , and  $n$  constants.

It is worth noting that the assumption that the ratio represented by Eq. 7 is constant could be inferred by the assumption that the product  $C(a/W)G(a/W)$  is constant for both the fracture toughness specimen and the structure, that is

$$q = \frac{(v_{el}/W)}{(v_{el}/W)_s} = \frac{CGP_N/W}{C_s G_s P_n/W_s} = \frac{CGP_N/W}{C_s G_s (fP_N)/W_s} = \frac{CG}{C_s G_s} \frac{W_s}{W} \frac{1}{f} \quad (16)$$

Thus, if  $CG$  is constant, not only  $q$  is constant, but also there exist a relationship between the factors  $q$  and  $f$  given by above equation.

## Examples

To compare the calibration curves obtained with the original and the simplified procedures, three different A533B steel specimen geometries were selected: CCT, SENT and DENT. For each type of geometry, two data sets were used to consider different  $a/W$  ratios. The calibration functions for these geometries were then predicted from the calibration function for a CT fracture toughness specimen of the same material. The characteristics of the CT specimen as well as of the other specimens that played the role of structural components are listed on Table 1.

TABLE 1—A533B steel specimens ( $E_{eff} = 206850$  MPa;  $\sigma_y = 468.86$  MPa;  $\sigma_{us} = 620.55$  MPa)

SAMPLE	TYPE	W (mm)	B (mm)	$a_o$ (mm)	$a_r$ (mm)	$a_o/W$ (or $2a_o/W$ )
A533-1	CT	203.2	2.54	101.85	130.02	0.50
A533CCT1	CCT	406.4	2.54	101.60	115.82	0.50
A533CCT6	CCT	203.2	2.54	60.71	71.63	0.60
A533SEN2	SENT	508.0	2.54	355.60	379.48	0.70
A533SEN6	SENT	508.0	2.54	152.40	195.88	0.30
A533DEN3	DENT	203.2	2.54	49.40	62.23	0.49
A533DEN4	DENT	203.2	2.54	35.43	47.63	0.35

Figure 3 shows the results obtained for the CCT specimens, where experimental  $P_n$  versus  $v_n$  curves are compared with those obtained using the original transformation procedure and the simplified transformation approach introduced here. The same information is presented in Figures 4 and 5 for the SENT and DENT specimens, respectively. The points represented by crosses are those obtained with the original transformation procedure and should still be fitted in order to get the coefficients of the calibration function. The solid line is the curve described by the coefficients obtained directly by Eqs. 15 of the simplified transformation approach. It can be observed that the results obtained with both procedures are practically the same.

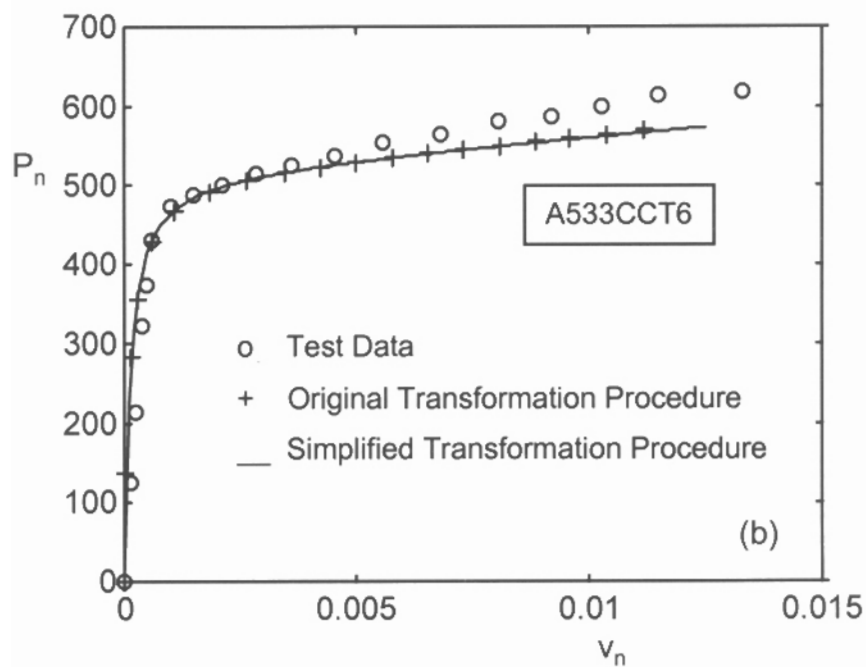
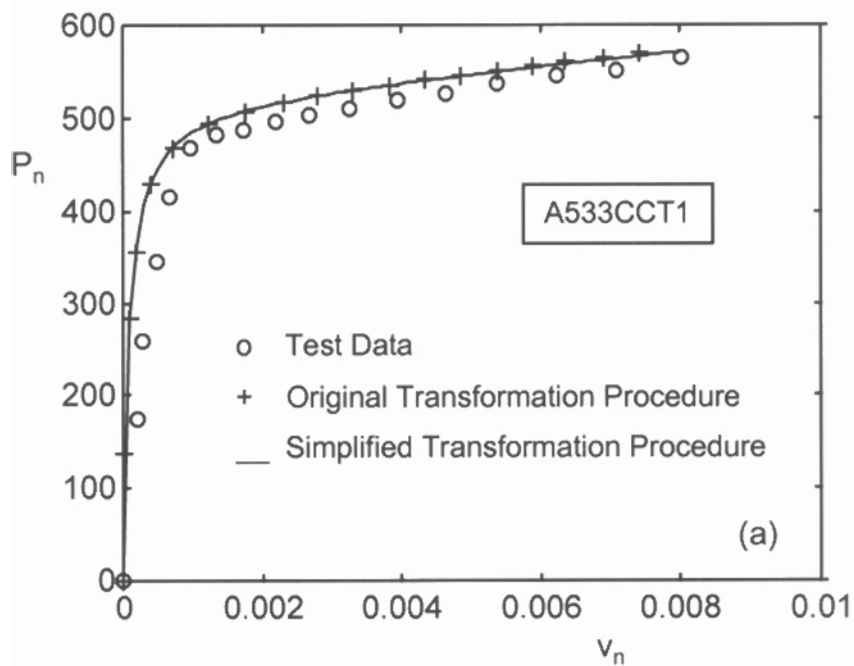


FIG. 3—Calibration curves for the CCT specimens

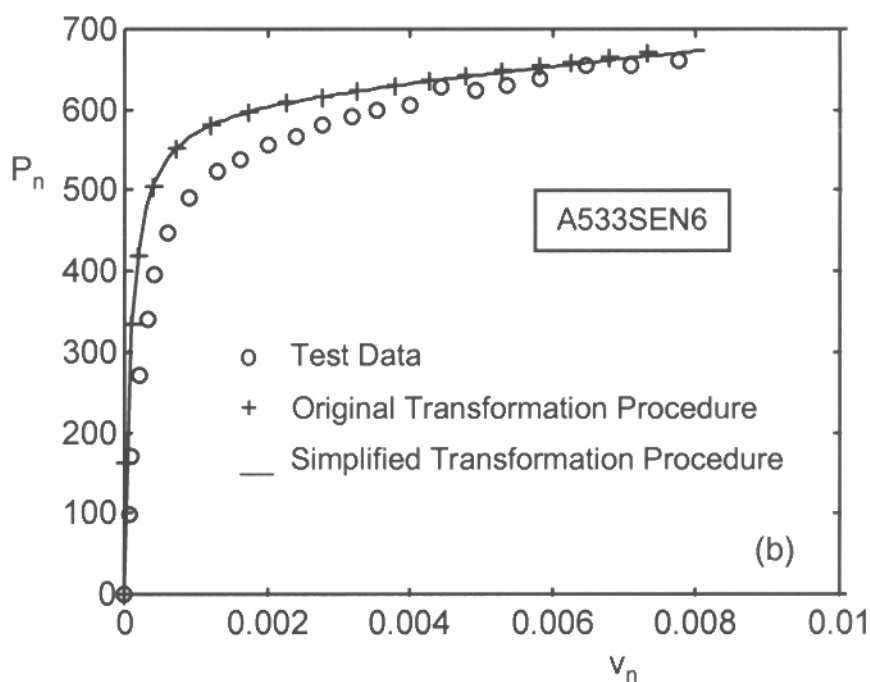
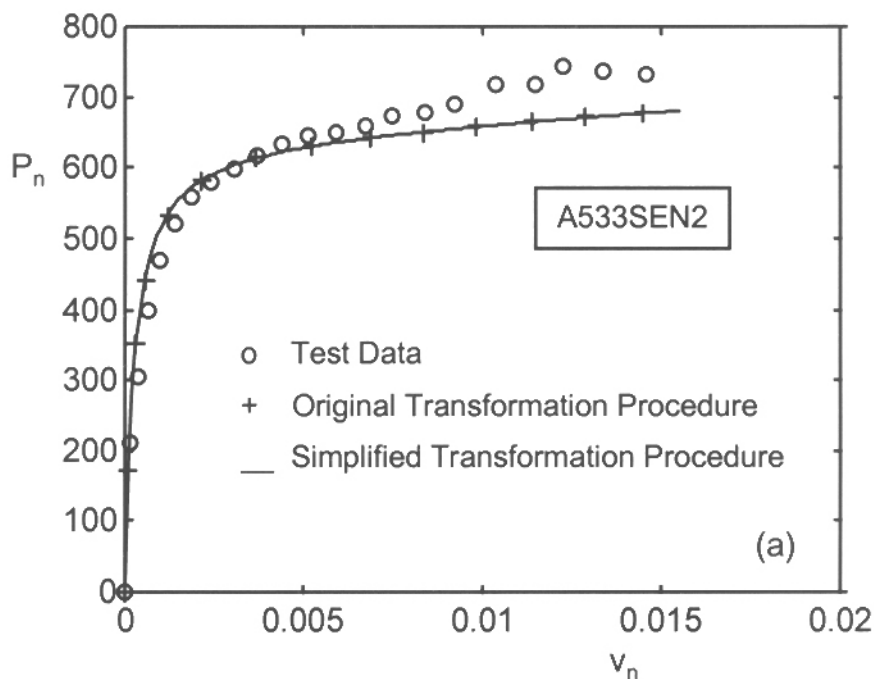


FIG. 4—Calibration curves for the SENT specimens

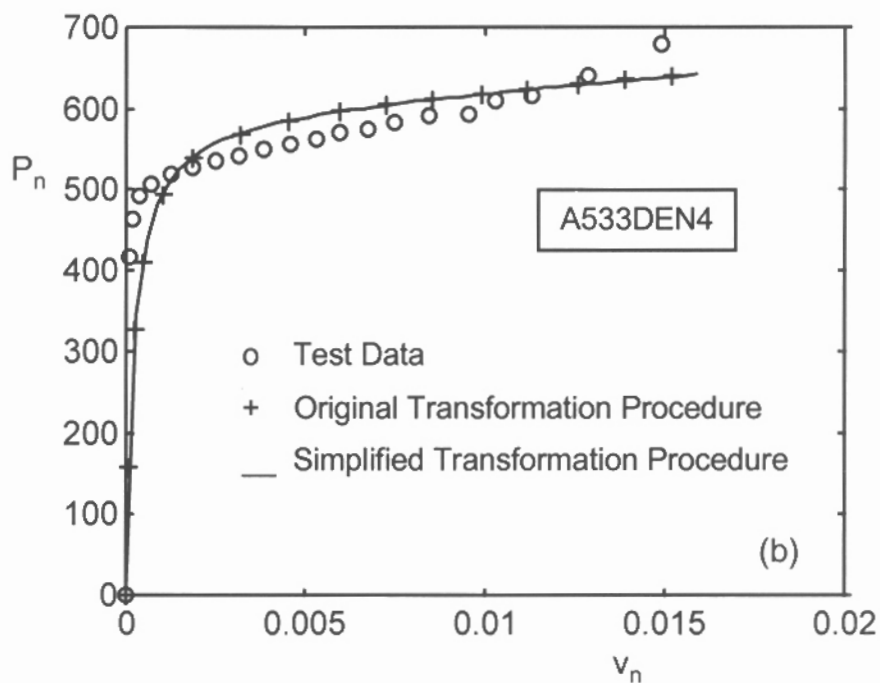
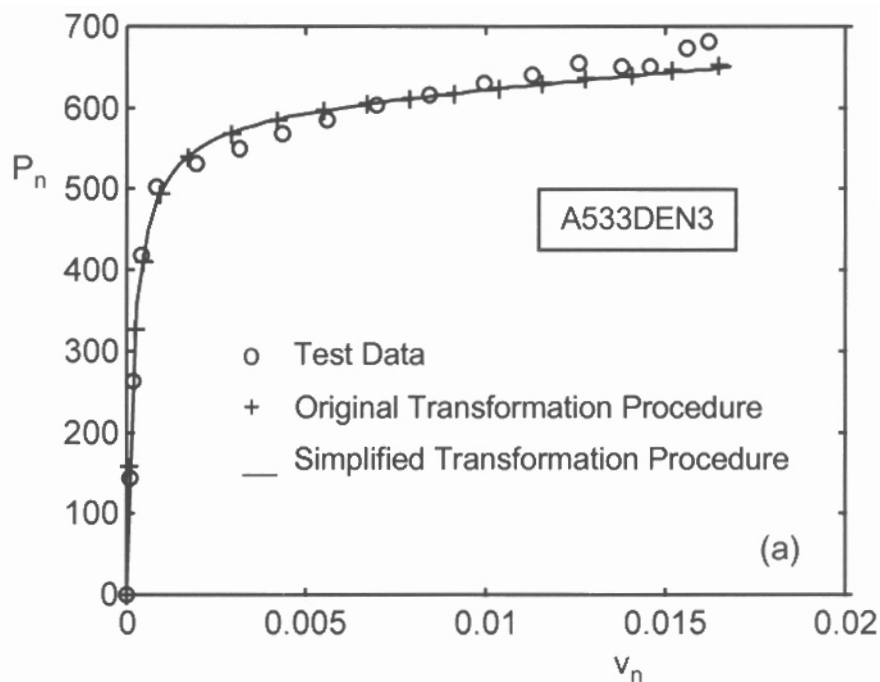


FIG. 5—Calibration curves for the DENT specimens

To illustrate the application of the DFM using the simplified transformation procedure, the steps necessary to predict the load versus displacement curve for the specimen identified in Table 1 as A533CCT1 are presented below (the basic input data come from the load versus displacement record for the CT specimen in Table 1; the  $G$  functions for CT and CCT geometries are known and the respective  $\eta_{pl}$  values are 2.15 and 1.0 [10]):

(a) First, the method of normalization [9] is used to obtain the coefficients  $L$ ,  $M$ , and  $N$  of the deformation function,  $H$ , and the  $J$ - $R$  curve for the CT specimen:

$$L = 28.42; M = 58.71; N = 6.17 \times 10^{-4}$$

$$J = 10.33(\Delta a)^{0.3801}$$

(b) The limit load and compliance for both the CT and CCT specimens, for the respective initial crack lengths, are (these values were obtained using the limit load and compliance solutions tabulated in Ref. [12]):

$$P_L = 24.33 \text{ kN}; C = 0.0702 \text{ mm/kN}$$

$$P_{Ls} = 281.11 \text{ kN}; C_s = 0.00154 \text{ mm/kN}$$

(c) Eqs. 8 and 9 are then applied to obtain  $(v_{el})_o$  for the CT and CCT specimens:

$$(v_{el})_o = 0.0702 \times 24.33 = 1.708 \text{ mm}$$

$$(v_{el})_{os} = 0.00154 \times 281.11 = 0.4329 \text{ mm}$$

(d) The next step is the computation of the factors  $f$  and  $q$  (Eqs. 5 and 7, respectively):

$$f = (281.11/0.8) / (24.33/0.1793) = 2.6$$

$$q = (1.708/203.2) / (0.4329/406.4) = 7.9$$

(e) We are now ready to calculate the coefficients  $l$ ,  $m$ ,  $n$  of the calibration function for the CCT specimen using Eqs. 15:

$$l = 2.6 \times 28.42 = 73.9$$

$$m = 2.6 \times 7.9 \times 58.71 = 1205.9$$

$$n = 6.17 \times 10^{-4} / 7.9 = 7.81 \times 10^{-5}$$

(f) The complete load versus displacement curve for the structure (in this case a CCT specimen) is then determined through a step by step procedure briefly described in the beginning of this work and depicted in Fig. 1. An independent variable is chosen to increment. The basic approach is to choose  $v_{pl}$  as the independent variable. Starting with  $a=a_o$  and with a small value for  $v_N = v_{pl}/W$ ,  $P$  (Eq. 1) and  $J_{app}$  are calculated. Follows an iterative process, where the crack length is adjusted until  $J_{app}$  matches  $J_{mat}$  from the  $J$ - $R$  curve equation. For each converged iteration, the pair of values  $P$ ,  $v$  are obtained and  $v_{pl}$  is again incremented. The process continues until the complete load versus displacement curve is achieved. The value of  $J_{app}$  is determined as a sum of an elastic and a plastic component

$$J = J_{el} + J_{pl} = \frac{K^2}{E'} + \frac{\eta_{pl}}{Bb} \int_0^{v_{pl}} P dv_{pl} =$$

$$= \frac{K^2}{E'} + \frac{\eta_{pl}W}{Bb} G(b/W) \int_0^{v_{pl}/W} H(v_{pl}/W) d(v_{pl}/W) \quad (17)$$

where  $K$  is the linear elastic stress-intensity factor,  $E'$  is the effective modulus of elasticity,  $\eta_{pl}$  is the plastic  $\eta$ -factor,  $B$  is the structural thickness, and  $b$  is an uncracked ligament length. The total displacement,  $v$ , is a sum of an elastic and a plastic component

$$v = v_{el} + v_{pl} \quad (18)$$

and the relationship between  $v_{el}$  and  $P$  is given in terms of the compliance,  $C$ , that is

$$v_{el} = C(a/W) \cdot P \quad (19)$$

The load versus displacement curve obtained for the CCT specimen (A533CCT1 in Table 1) is shown in Fig. 6, where it is compared with the experimental data. Actually, two predicted curves are presented in Fig. 6, one obtained with the  $J$ - $R$  curve from the CT specimen and the other based on the  $J$ - $R$  curve from the CCT specimen itself, which in this case was available. Analogous results are also presented in Figures 7 and 8 for the specimens A533SEN2 and A533DEN3, respectively.

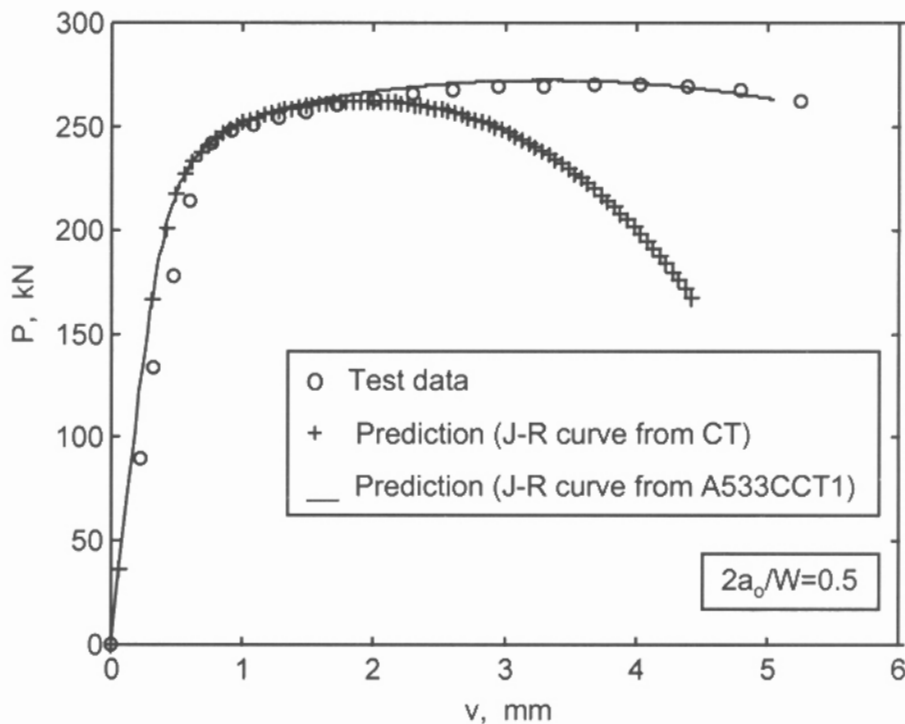


FIG. 6—Load versus displacement curve for A533CCT1

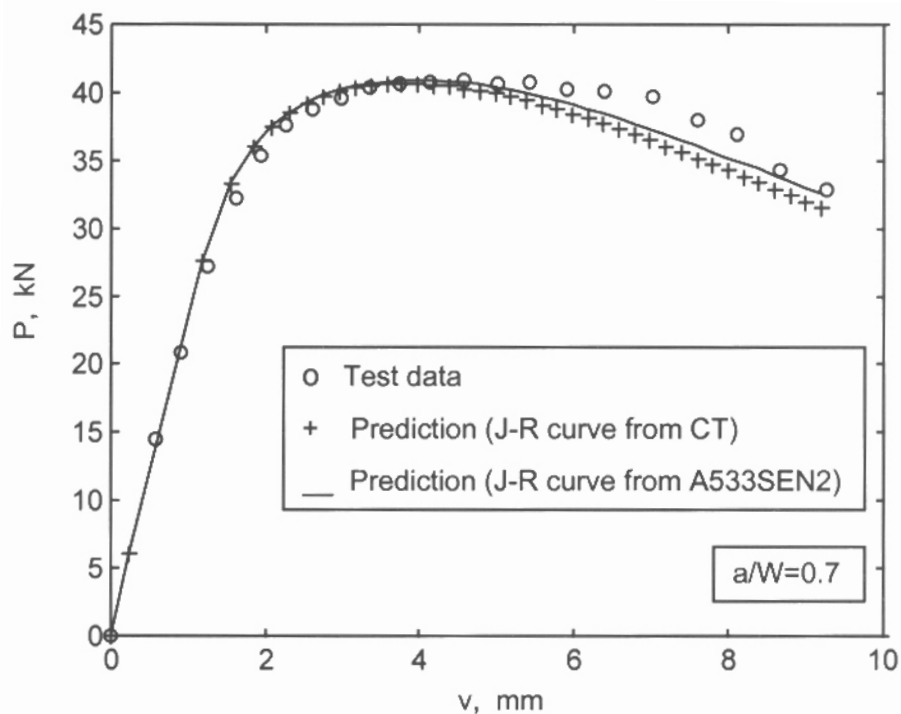


FIG. 7—Load versus displacement curve for A533SEN2

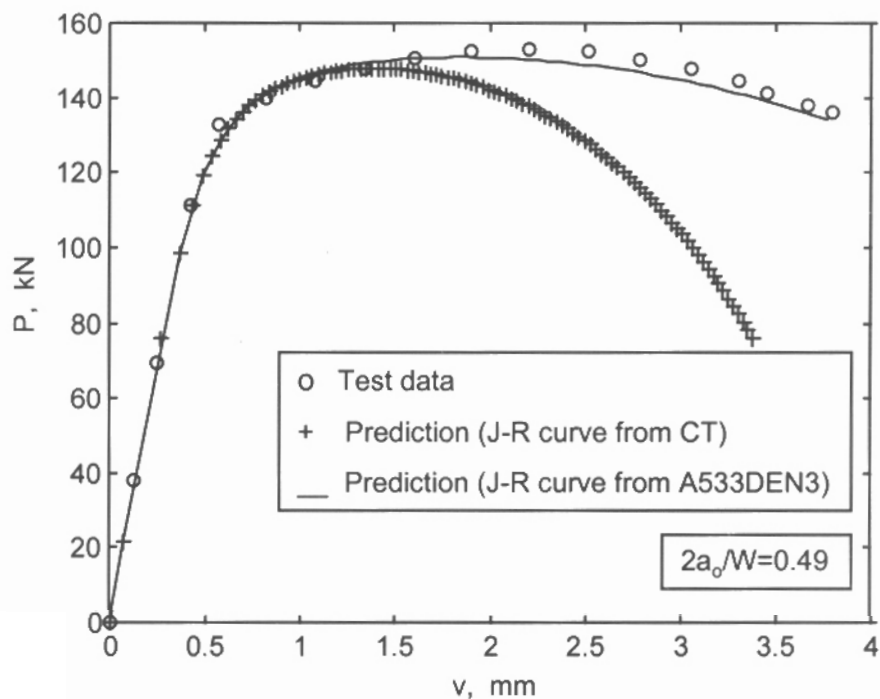


FIG. 8—Load versus displacement curve for A533DEN3

## Discussion and Conclusions

The principle of load separation, on which the DFM is founded, states that the load in cracked bodies of the same material, geometry and thickness constraint can be represented as a multiplication of two separable functions: a geometry function and a material deformation function (Eq. 1). The geometry function,  $G$ , depends on the cracked geometry only. The deformation function,  $H$ , represents the plastic flow character of the material and the specific geometry/loading mode of the cracked body.

When dealing with a fracture toughness specimen and a structural component with the same thickness constraint, it seems possible to transfer the deformation function from the specimen to the structure by considering two factors: a load factor, given by the ratio between their normalized limit loads, and a deformation factor given by the ratio between their normalized elastic displacements. Both factors had already been used in the original transformation procedure proposed by Landes et al. [3]. In the present paper, it was shown that likewise the use of a constant load factor  $f$  (Eq. 5), it is possible to define a constant deformation factor  $q$  (Eq. 7), where the normalized elastic displacements for both the specimen and the structure are calculated with their initial crack lengths and at their limit loads (Eqs. 8 and 9). With these two factors,  $f$  and  $q$ , it was demonstrated that the transformation process can be completed in a very quick and easy way.

For the cases where the thickness constraint levels of the specimen and the structure are different, an additional adjustment must be done. Donoso and Landes [11] studied the influence of the thickness constraint on the prediction of the behavior of structural components from a laboratory test specimen. They derived a constraint factor which can be used to evaluate the relative constraint in the deformation function.

Therefore, for those general cases in which both the thickness constraint and loading mode are different for the test specimen and the structural component, the approach presented here should be examined in conjunction with that from Ref. [11] to see the best and simplest way to get the correct calibration function for the structural component.

As mentioned earlier, the framework of the DFM can be used to predict the behavior of complex geometries where the calibration functions have to be determined by finite element analysis. But, the main concern of this paper was with those cases where the calibration function for the structure can be obtained directly from a fracture test specimen. In such situations, it is important to make the application of the DFM as simple as possible, so that quick failure assessments can be conducted. Considering that the determination of the calibration function for the structure is a critical step in the methodology, the transformation approach proposed here represents an important simplification for the DFM as a whole.

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