Z-Scan Analytical Description for On-Axis Approximation

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Abstract

We describe here a formal analysis for the Z-Scan theory based on the Huygens-Fresnel principle to calculate the on-axis normalized intensity for an arbitrary nonlinear phase. With this theory we could describe the behavior of the relevant parameters of the Z-Scan curve.

1. Introduction

The Z-Scan technique, introduced in 1990 by M. Sheik-Bahae *et al.*[1], provided a simple and sensitive method to measure the complex nonlinear refractive index of a sample[2], by means of a single laser beam. In this technique the sample is moved throughout the waist of a TEM_{00} laser beam. The gaussian intensity profile induces a refractive index change, which adds a nonlinear phase with gaussian profile to the laser beam. The detection of the laser intensity, through an iris in the far field, gives the value of the nonlinear phase.

The intensity variation, and the nonlinear phase introduced by the sample are given by [1] :

$$\frac{dI}{dz'} = -\alpha I \qquad (1)$$

$$\frac{d(\Delta\phi)}{dz'} = kn_2 I \qquad (2)$$

where z′ is the coordinate inside the sample, α its linear absorption coefficient and n_2 its nonlinear refractive index, and k the laser wavenumber. The solution of *(1)* and *(2)*, for a gaussian intensity profile, gives the electric field in the exit plane of the sample :

$$E_S(z,r,t) = E_0 e^{-\alpha L/2}\,\frac{w_0}{w(z)}\,e^{-\frac{r^2}{w(z)^2}}\,e^{-i\frac{kr^2}{2R(z)}}\,e^{-ikz}\,e^{-i\Delta\phi_0 e^{-\frac{2r^2}{w(z)^2}}} \qquad (3)$$

In expression *(3)*, E_0 is the beamwaist electric field amplitude, L length of the sample, z its position relatively to the beamwaist, r the distance to the optical axis, and w_0 the beamwaist radius. The parameters $w(z)$ and $R(z)$ are the radius and the wavefront curvature of the laser beam at the position z[3]. The nonlinear phase is given by :

$$\Delta\phi_0 = \frac{kn_2 L_{eff} I}{(1+z^2/z_0^2)} = \frac{\Delta\Phi_0}{(1+z^2/z_0^2)} \qquad (4)$$

where $L_{eff} = (1-e^{-\alpha L})/\alpha$ is the effective length of the sample, z_0 the laser beam confocal parameter, and $\Delta\Phi_0$ the nonlinear phase at the beamwaist.

The traditional Z-Scan analysis is based in the Gaussian Decomposition method (GD) proposed by Weaire *et al.*[4], which gives the iris electric field as a summation over infinite gaussian beams. For small nonlinear phases and in the optical axis, the summation may be reduced to only two terms, resulting in a function symmetric around the origin for the normalized transmittance, as can be seen in Figure 1. From the obtained function we then calculate de peak-valley distance, and the transmittance variation between peak and valley, which are given by $\Delta z = 1{,}72 z_0$ and

$\Delta T_{pv} = 0{,}406|\Delta\Phi_0|$, respectively. These results are valid for peak-valley transmittance variations up to ~20%.

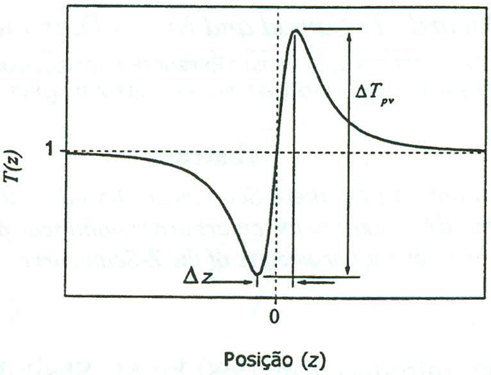

Figure 1 - Parameters for a Z-Scan curve for a positive nonlinear sample. For small phases the peak-valley distance is $\Delta z = 1{,}72 z_0$, and the transmittance variation between peak and valley is $\Delta T_{pv} = 0{,}406|\Delta\Phi_0|$.

2. Theory

We present here a model developed that deals with any gaussian profile nonlinear phase. The proposed model gives the on-axis electric field, and for its deduction we utilize the scheme of Figure 2. The model is based in the wavefront propagation by the Fresnel Integral, which is based in the Huygens-Fresnel principle[5]. This principle states that each point in a wavefront may be regarded as the center of a perturbation that originates a spherical wavefront (Huygens Principle), and all this secondary waves interfere to define the wavefront in a posterior time (Fresnel principle).

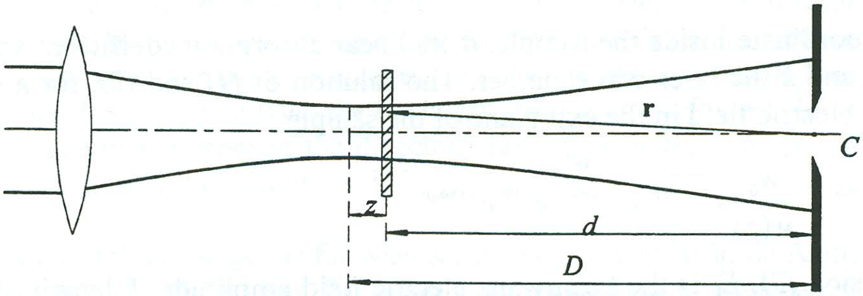

Figure 2 - Coordinates utilized in the Fresnel Diffraction Integral.

In this model we consider a nonlinear sample at the *z* position, under the influence of a TEM$_{00}$ gaussian beam, originating a gaussian profile refractive index change that adds a gaussian profile nonlinear phase to the laser beam. We want to know the electric field at the point *C*, after the propagation by a distance *d*. The electric field at the exit plane of the sample, which is given by expression *(3)*, is multiplied by the Huygens propagation term. The field at *C* is given by the sum over all spherical waves :

$$E_C = \frac{ik}{2\pi}\int_0^{2\pi} d\theta \int_0^{\infty} d\rho\, \rho E_S(z,\rho,t)\frac{e^{-ikr}}{r} \qquad (5)$$

where ρ is the radial integration variable.

For the exponential, the distance *r* is approximated by :

$$r = \sqrt{d^2 + \rho^2} \cong d + \frac{1}{2}\frac{\rho^2}{d} \qquad (6)$$

and in the denominator of the expression, *r* is substituted by *d*.

Making the variables substitution

$$\alpha = \left[\frac{1}{w(z)^2} + i\frac{k}{2}\left(\frac{1}{R(z)} + \frac{1}{d}\right)\right]$$

$$\beta = \frac{2}{w(z)^2} \qquad (7)$$

$$\Lambda = ikE_0(t)\frac{w_0}{w(z)}e^{-ikD}\frac{1}{d}$$

we must solve the following integral :

$$E_C = \Lambda\int_0^\infty d\rho\,\rho e^{-\alpha\rho^2}e^{-i\Delta\phi_0 e^{-\beta\rho^2}} \qquad (8)$$

Substituting $\xi = \beta\rho^2$, $e^{-\xi} = \zeta$ and $\mu(z) = (\alpha/\beta)-1$, we obtain

$$E_C = \frac{\Lambda}{2\beta(i\Delta\phi_0)^{\mu+1}}\int_0^{-\Delta\phi_0} d\nu\,\nu^\mu e^{-\nu} \qquad (9)$$

The integral in *(9)* is the integral representation of the Incomplete Gamma Function[6]. Defining the variable $\gamma(z) = \mu(z) + 1$, we calculate I_C, the electric field amplitude at C. The limit $I_C(\Delta\phi_0)/I_C(\Delta\phi_0 \rightarrow 0)$ gives the normalized transmittance (intensity) at the point C :

$$I_C^N = \left|\gamma(z)\frac{\Gamma(\gamma,0,i\Delta\phi_0)}{(i\Delta\phi_0)^\gamma}\right|^2 \quad , \quad \text{with} \quad \gamma(z) = \frac{1}{2}\left[\frac{i}{z_0}\left(z + \frac{z^2+z_0^2}{D-z}\right)+1\right] \qquad (10)$$

It is important to say that if the nonlinear phase temporal evolution is known, it is possible to calculate the normalized transmittance temporal evolution.

3. Simulations

With the Gamma Function series expansion[7] we show that expression *(10)* converges to the expression deduced by Sheik-Bahae[1] for small nonlinear phases. For greater phases, Figure 3 shows the normalized transmittance curves for $\Delta\Phi_0 = \pi/10, \pi/4, \pi/2, \pi, 2\pi$ e 3π. With the increasing of the nonlinear phase, the curve becomes asymmetric, and the point where $T(z) = 1$ is shifted towards $+z$, as we see in Figure 4a. The distance between peak and valley also increases, as shown in Figure 4b. The transmittance variation between peak and valley is given by $\Delta T_{pv} = 0{,}406|\Delta\Phi_0|$ for phases with modulus smaller than π, as seen in Figure 4c. For greater phases this behavior is no more linear. This is due to the fact that the Z-Scan curve lost the characteristic valley, as seen in Figure 3b. These profile changes are due to destructive interference for phases greater than 2π.

4. Conclusions

We have introduced a new theory capable of calculate the propagation of a gaussian profile nonlinear phase, resulting in the on-axis electric field. The theory may be applied to any nonlinear gaussian phase, not being restricted to the Z-Scan case. The theory can also deals with the temporal evolution of the normalized transmittance, once the temporal evolution of the nonlinear phase is known.

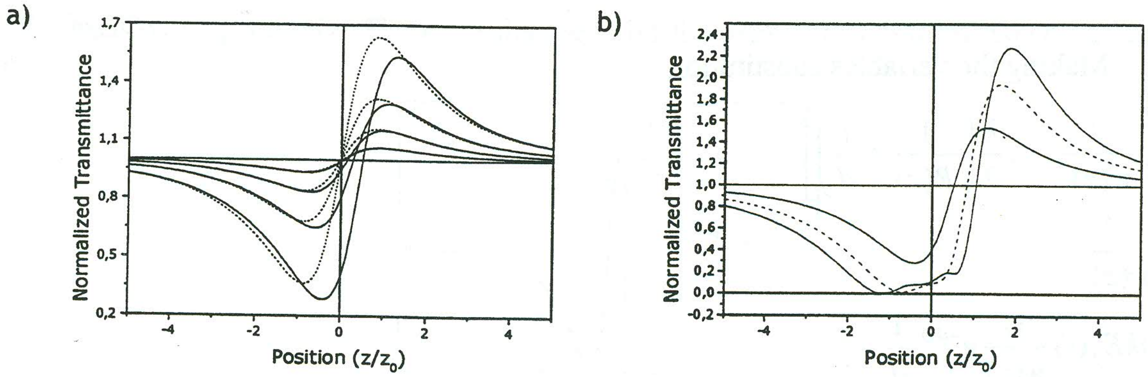

Figure 3 - **a)** GD theory results (dashed lines), and Gamma Function (straight lines), for nonlinear phases $\Delta\Phi_0 = \pi/10, \pi/4, \pi/2$ e π. **b)** Previsions from Gamma Function model to nonlinear phases $\Delta\Phi_0 = \pi, 2\pi$ e 3π.

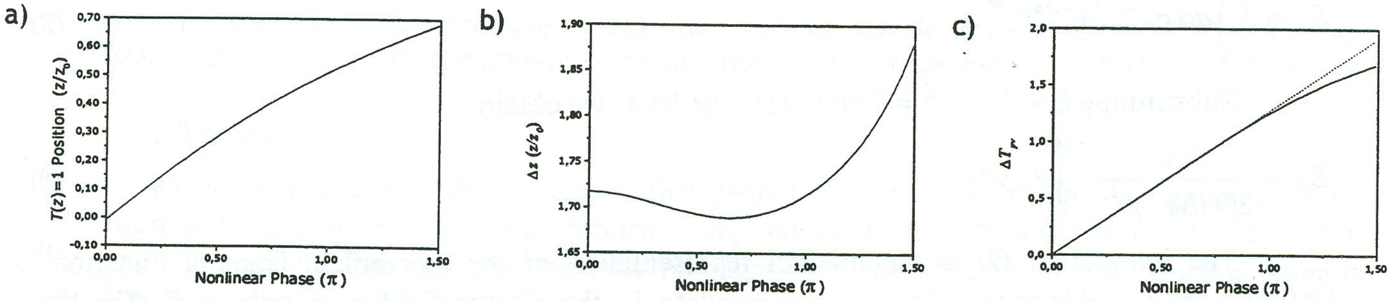

Figure 4 - Z-Scan curve parameters as function of the nonlinear phase : **a)** Position of the point $T(z) = 1$. **b)** Distance between peak and valley. **c)** Transmittance variation between peak and valley.

5. Acknowledgments

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6. References

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