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 $N^*N\gamma$ TIME REVERSAL VIOLATING VERTEX**

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PUBLICAÇÃO IEA N.º 296

Junho — 1973

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CONSEQUENCES FOR NUCLEAR PHYSICS OF A $NN\gamma$ AND A $N^*N\gamma$ TIME REVERSAL VIOLATING VERTEX

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Abstract

This paper studies the effects in low energy nuclear physics of a possible Time Reversal Invariance (T.R.I.) violation in the electromagnetic interaction. It is shown that this effect appears as a two body short range T.R.I. violating transition operator or as a two and three body T.R.I. violating potential. Two cases are studied. Firstly a T.R.I. violating $NN\gamma$ vertex is considered and found to have very little effect. Secondly the T.R.I. violation is assumed to occur in the $N^*N\gamma$ vertex and it is shown that if the violation is maximal the contribution to the imaginary part of the "mixing ratio" δ is $\text{Im } \delta \approx |\delta|10^{-3}$. This should be measurable.

1. Introduction

The suggestion that Time Reversal Invariance (T.R.I.) is violated by the electromagnetic interaction was put forward by Bernstein et al. and by Lee²⁻³⁾. The purpose of this paper is to examine the consequences of such an assumption in low energy nuclear physics.

Several papers have been published on this subject. For example Huffmar⁴⁾ derived a two body T.R.I. violating potential assuming a T.R.I. violating $NN\gamma$ vertex and used it to calculate T.R.I. violating effects in direct reactions. A different approach was followed by Clement and Heller⁵⁾ who calculated the effects of a four particle ($NNM\gamma$) T.R.I. violating vertex in the form (where M signifies a meson field) of a two body T.R.I. violating transition operator (Fig. 1) and in the form of a three body T.R.I. violating potential (Fig. 2).

In this paper we choose two models for T.R.I. violation and calculate their effects. In section II the effects of a T.R.I. violating $NN\gamma$ vertex are dealt with. This vertex was chosen because it has the theoretical appeal of stemming naturally from a more fundamental theory⁴⁾ such as the Lee³⁾ "a" particle theory.

In section III the T.R.I. violation is assumed to occur in the $N^*N\gamma$ vertex (where N^* is the $J = 3/2, I = 3/2, M = 1236$ MeV nucleon resonance) and its effects are calculated.

The suggestion that the $N^*N\gamma$ vertex could be T.R.I. violating is not new. In fact Barshay⁶⁾ calculated the effects of such an assumption in modifying the reciprocity relationship between the differential cross sections for the reactions $\gamma + d \rightarrow n + p$ and $n + p \rightarrow \gamma + d$.

2. The "Lee" vertex

The most general expression for the matrix element of a T.R.I. violating electromagnetic current with respect to nucleon states has been written down by Bincer⁷⁾ and Lipshutz⁸⁾. Assuming parity conservation there are twelve independent terms if both nucleons are off the

mass shell, six if only one nucleon is off the mass shell and the matrix element vanishes if both nucleons are on shell. This latter feature is very important since, because of it, the T.R.I. violating part of the current only contributes to nuclear phenomena when other nucleons are involved, that is as an exchange effect. For our purposes we take the simplest possible form for the T.R.I. violating matrix element of the current, namely,

$$\langle p' | K_{\mu} | p \rangle = (\bar{u}_{p'} | F'(q') | \gamma_{\mu} P_{\mu} - \not{q} P_{\mu} | u_p) \quad (1)$$

where

$$F'(q') = F'^S(q^2) + F'^V(q^2) \not{q}$$

and

$$q = p' - p \quad \text{and} \quad P = p' + p$$

The expression (1) is parity conserving, satisfies the Ward identity and, as mentioned earlier, stems naturally from a theory such as the Lee³⁾ "a" particle theory.

Using the above matrix element it is possible to calculate a resultant T.R.I. violating electromagnetic transition operator and two and three body T.R.I. violating potentials.

The T.R.I. violating transition operators are extracted from the Feymann graphs shown in Fig. 3. In those graphs the waved line is a photon, the broken line is a pion and the bubble is the T.R.I. violating vertex.

The two body T.R.I. violating potential arises from a set of graphs a few of which are shown in Fig. 4.

In the graphs of Fig. 4 the bubble again represents the T.R.I. violating vertex and the point on the other end of the virtual photon is the normal electromagnetic vertex. Because of the extra factor e due to this normal vertex it is possible, at least as a first approximation, to assume the effect of the two body T.R.I. violating potential to be small compared with the T.R.I. violating transition operators.

The three body potential arises from the graphs of Fig. 5.

The part of the three body potential connecting the third nucleon (left hand side in the graph 5-a and right hand side in graph 5-b) is a long range one due to the fact that the virtual photon exchanged is a massless particle. This implies that its matrix elements between states diagonal with respect to (A-2) nucleon orbitals⁵⁾ will include a factor Ze (where Z is the atomic number) instead of the factor e as in the two body potential. Therefore for large nuclei (and hence large Z) it is perhaps appropriate to expect that the effect of the three body T.R.I. violating potential will dominate. This is unfortunate since calculations with three body potentials are complicated. For this reason we will focus attention mainly on light nuclei where the T.R.I. violating transition operators are expected to dominate.

The transition operator is extracted from the graphs of Fig. 3 by means of well known methods (see for example ref. 9 and 20). The results is

$$V^{TRV}(r_1, r_2, E) = \frac{f^2 F'^V}{m} \sigma_1 \cdot E(r_1) \sigma_2 \cdot (r_1 - r_2) (\tau_{(1)} \times \tau_{(2)})_z F(r_{12}) + (1 \leftrightarrow 2) \dots (2)$$

In eq. (2) $E(r_1)$ is the electric field and $F(r_{12}) = -\frac{1}{4\pi} \left(\frac{\mu}{|r_1 - r_2|^2} + \frac{1}{|r_1 - r_2|^3} \right) e^{-\mu|r_1 - r_2|}$

f is the pion nucleon coupling constant, μ is the pion mass and F'^V is the value of the vector part of the form factor for $q^2 = 0$.

To obtain the three body operator corresponding to the graphs of Fig. 5 it is sufficient to replace the electric field $E(r_1)$ by

$$E(r_1) = \frac{e(r_1 - r_3)}{|(r_1 - r_3)|^3} \frac{1}{2} (1 + \bar{\tau} \frac{(3)}{z})$$

The right hand side of this equation is of course the electric field E produced by the third nucleon in the position of the first one.

The operator given by equation (2) has exactly the same form as the one given by Clement and Heller⁵⁾. Full advantage will be taken of this fact in the estimates given below.

Expanding the operator given by equation (2) in electric ($E^{TRV}(L)$) and magnetic ($M^{TRV}(L)$) multipoles and using the same phases and overall normalization as Clement and Heller⁵⁾ we obtain

$$E^{TRV}(L) = \frac{f^2 F'^V}{4\pi m} \left(\frac{4\pi}{3}\right)^{1/2} 2 \left[L(2L+1) \right]^{1/2} \sum_{i < j} (\tau_{(i)} \times \tau_{(j)})_z \frac{d}{dr_{ij}} \left(\frac{\exp(-\mu r_{ij})}{r_{ij}} \right)$$

$$\left[\frac{1}{3} \sigma_i \cdot \sigma_j R_{ij}^{L-1} \left[Y_{L-1}(\hat{R}_{ij}) \otimes Y_1(\hat{r}_{ij}) \right]_M^{(L)*} - \sum_{L'} \left[5(2L'+1) \right]^{1/2} \begin{Bmatrix} L' & 2 & L \\ 1 & L-1 & 1 \end{Bmatrix} R_{ij}^{L-1} \right. \\ \left. \left[\left[Y_{L-1}(\hat{R}_{ij}) \otimes Y_1(\hat{r}_{ij}) \right]^{(L')} \otimes \left[\sigma_i \otimes \sigma_j \right]^{(2)} \right]_M^{(L)} \right]^* \dots (3)$$

$$M^{TRV}(L) = -\frac{f^2 F'^V}{4\pi m} \left(\frac{4\pi}{3}\right)^{1/2} 2K \left(\frac{L}{L+1}\right)^{1/2} \sum_{i < j} (\tau_{(i)} \times \tau_{(j)})_z \frac{d}{dr_{ij}} \left(\frac{\exp(-\mu r_{ij})}{r_{ij}} \right)$$

$$\left[\frac{1}{3} R_{ij}^L \sigma_i \cdot \sigma_j \left[Y_L(\hat{R}_{ij}) \otimes Y_1(\hat{r}_{ij}) \right]_M^{(L)*} + R_{ij}^L \sum_{L'} \left[5(2L'+1) \right]^{1/2} \begin{Bmatrix} L' & 2 & L \\ 1 & L & 1 \end{Bmatrix} \right]$$

$$\left[\left[Y_L(\hat{R}_1) \otimes Y_L(r_1) \right]^{(L)} \otimes \left[\sigma \otimes \sigma_1 \right]^{(2)} \right]^{(L)} + \left[\frac{(2L+1)^2 L}{16\pi} \right] \begin{pmatrix} L-1 & 1 & L \\ 1 & L & 1 \end{pmatrix} \\
 \left[Y_{L-1}(\hat{R}_1) \otimes \left[\sigma \otimes \sigma_1 \right]^{(1)} \right]^{(L)} \quad (4)$$

In equations (3) and (4) $r_1 = r - r_0$, $R_1 = 1/2(r + r_0)$, K is the photon energy, $[a \otimes b]_K^{(L)}$ means the tensor product of the vectors a and b and Y_L is a spherical harmonic. Details of the techniques used can be found in the paper by Clement¹⁰.

The effects of (3) and (4) will now be estimated. We will consider only the effects of possible T R I violation in γ ray oriented angular correlation tests (Jacobson and Henley¹ and Lobov¹²). Those effects are proportional to the imaginary part of the mixing ratio defined as follows

$$\delta = \frac{\langle 1 || E(L+1) || 1 \rangle}{\langle 1 || M(L) || 1 \rangle}$$

The imaginary part of δ , introduced by the operators given by equations (3) and (4), can be calculated easily noting that $E^{NOR}(L)$ ($M^{NOR}(L)$) the T R I conserving Electric (Magnet multipole) has real reduced matrix elements

$$\text{Im } \delta = \delta_R (-1) \left[\frac{\langle 1 || E^{TRV}(L+1) || 1 \rangle}{\langle 1 || E^{NOR}(L+1) || 1 \rangle} - \frac{\langle 1 || M^{TRV}(L) || 1 \rangle}{\langle 1 || M^{NOR}(L) || 1 \rangle} \right] \quad (5)$$

From equation (5) it is seen that the effect depends on the ratio between the T R I violating multipole and the corresponding normal ones, and on

$$\delta_R = \frac{\langle 1 || E^{NOR}(L+1) || 1 \rangle}{\langle 1 || M^{NOR}(L) || 1 \rangle}$$

This is useful since it implies that it is possible to use any one of the many definition multipole operators found in the literature (see Brink and Rose¹³ for a review) to calculate these ratios without worrying about phase and normalization problems. One must however note of these conventions when using values of δ from experimental data.

Our task therefore is to estimate the two ratios

$$\frac{\langle 1 || E^{TRV}(L+1) || 1 \rangle}{\langle 1 || E^{NOR}(L+1) || 1 \rangle} \quad \text{and} \quad \frac{\langle 1 || M^{TRV}(L) || 1 \rangle}{\langle 1 || M^{NOR}(L) || 1 \rangle} \quad \text{in equation}$$

(5) Now a close look at equations (3) and (4) will reveal that all the multipole operators are such that as with the usual multipole operators an additional factor $K R_0$ (where R_0 is the nuclear radius) is introduced when there is an unit increase in multipolarity. Thus to compare to the normal transition operators, it is sufficient to consider the ratio of the lowest multipole operators. As remarked before the transition operator given by equation (2) has the same form as the operator obtained by Clement and Heller⁵⁾. Using their results and using the "maximal estimate" $F^{\nu} = \frac{e}{m}$ where m is the mass of the proton we obtain

$$\langle (E 1)^{TRV} \rangle \approx \frac{2ef}{4\pi\mu} \left(\frac{\mu}{m}\right) \cdot \frac{1}{(\mu R_0)^2}$$

$$\langle (M 1)^{TRV} \rangle \approx \frac{2ef}{4\pi\mu} \left(\frac{\mu}{m}\right) \cdot \frac{K R_0}{(\mu R_0)^2}$$

$$\langle (E 1)^{NOR} \rangle \approx e R_0$$

$$\langle (M 1)^{NOR} \rangle \approx \frac{e}{m}$$

where R_0 is the nuclear Radius, K is the energy of the γ -ray, μ is the mass of the pion and m is the mass of the proton.

Therefore we obtain from (5), for 1 MeV energy γ rays

$$\text{Im } \delta = \delta_R \times 10^{-7}$$

This value of $\text{Im } \delta$ is much too small to be detected at present

3. The $N^* N \gamma$ vertex

In this section a T R I violating $N^* N \gamma$ vertex is introduced and its consequences worked out. The suggestion that the $N^* N \gamma$ vertex could be T.R.I violating is not new. In fact Barshay⁶⁾ calculated the effects of such an assumption in modifying the reciprocity relationship between the differential cross sections for the reactions $\gamma + d \rightarrow n + p$ and $n + p \rightarrow \gamma + d$

The N^* ($J = 3/2$, $I = 3/2$ and $M = 1236$ MeV) is a nucleon resonance with both spin and I -spin $3/2$ and mass $M = 1236$ MeV. The Rarita-Schwinger formalism is used to describe it. To take in account the four charged states a column vector Ψ_λ is introduced, together with a spinor Ψ for the nucleon, thus

$$\Psi_{\Lambda} = \begin{bmatrix} \Psi_{\Lambda} \\ \Psi_{\Lambda} \\ \Psi_{\Lambda} \\ \Psi_{\Lambda} \end{bmatrix} \quad \text{and} \quad \Psi = \begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix}$$

The N^*NM vertex is taken to be¹⁴⁾

$$L_{N^*N\pi} = -\frac{G}{\mu} \bar{\Psi}_{\Lambda} T_{\alpha} \Psi \left(\frac{\partial A_{\alpha}}{\partial x_{\lambda}} \right) - \frac{G}{\mu} \bar{\Psi} T_{\alpha}^* \Psi_{\Lambda} \left(\frac{\partial \phi_{\alpha}}{\partial x_{\lambda}} \right) \quad (6)$$

where ϕ_{α} is the pion field and T_{α} are the following matrices

$$T_1 = \begin{bmatrix} 1 & 0 \\ 0 & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & 0 \\ 0 & 1 \end{bmatrix} \quad T_2 = \begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} & 0 \\ 0 & 1 \end{bmatrix} \quad T_3 = \begin{bmatrix} 0 & 0 \\ \frac{2}{\sqrt{3}} & 0 \\ 0 & -\frac{2}{\sqrt{3}} \\ 0 & 0 \end{bmatrix}$$

We take for the $NN^*\gamma$ vertex the form (Sallin¹⁵⁾, Barshay¹⁶⁾)

$$L_{NN^*\gamma} = \frac{1}{m} \bar{\Psi}_{\Lambda} \epsilon^{\lambda\mu\nu\sigma} \gamma_{\mu} \gamma_{\nu} \Psi F_{\sigma\lambda} - \frac{1}{m} \bar{\Psi} \epsilon^{\lambda\mu\nu\sigma} \gamma_{\mu} \gamma_{\nu} \Psi_{\Lambda} F_{\sigma\lambda} \quad (7)$$

Where $F_{\lambda\mu} = \partial_{\lambda} A_{\mu} - \partial_{\mu} A_{\lambda}$ is the electromagnetic tensor. The vertex (7) is T.R.I. violating if we take the matrix ϵ to be complex that is

$$\epsilon = \begin{bmatrix} 0 & 0 \\ \epsilon_2 - \epsilon_1^* & 0 \\ 0 & \epsilon_2 - \epsilon_1^* \\ 0 & 0 \end{bmatrix}$$

The transition operator $W(B)$ is extracted from the graphs of Fig. 6 by using the methods already mentioned in section II (for details see ref. 16).

The result is

$$W(B) = W_1(B) + W_2(B) \quad (8)$$

where

$$W_1(B) = \frac{1}{3} \frac{G f i}{\mu(M-m)2m^2} \mathbf{B}(r_1) \cdot \sigma_1 \times \sigma_2 F(r_{12}) \cdot \left[\frac{2}{\sqrt{2}} T_{20}(\tau^{(1)}, \tau^{(2)}) (\epsilon_p^* - \epsilon_p + \epsilon_N^* - \epsilon_N) \right. \\ \left. + \frac{2}{\sqrt{3}} (\epsilon_N^* - \epsilon_N - \epsilon_p^* + \epsilon_p) \tau_z^{(2)} \right] + \frac{1}{3} \frac{G f (-i)}{\mu(M-m)2m^2} \mathbf{B}(r_1) \sigma_1 \times (r_1 - r_2) \sigma_2 \cdot \frac{r_1 - r_2}{|r_1 - r_2|} \\ K(|r_1 - r_2|) \left[\frac{2}{\sqrt{2}} T_{20}(\tau^{(1)}, \tau^{(2)}) (\epsilon_p^* - \epsilon_p + \epsilon_N^* - \epsilon_N) + \frac{2}{\sqrt{3}} (\epsilon_N^* - \epsilon_N - \epsilon_p^* + \epsilon_p) \tau_z^{(2)} \right] + (1 \leftrightarrow 2) \quad (9)$$

$$W_2(B) = \frac{(-2)}{3} \frac{G f}{\mu(M-m)2m^2} \mathbf{B}(r_1) \cdot \sigma_2 F(r_{12}) \left[\frac{1}{\sqrt{3}} (\epsilon_p^* - \epsilon_p - \epsilon_N^* + \epsilon_N) (\tau^{(1)} \times \tau^{(2)})_z \right] \\ + \frac{2}{3} \frac{G f}{\mu(M-m)2m^2} \mathbf{B}(r_1) \cdot (r_1 - r_2) \sigma_2 \cdot \frac{r_1 - r_2}{|r_1 - r_2|} K(|r_1 - r_2|) \left[\frac{1}{\sqrt{3}} (\epsilon_p^* - \epsilon_p - \epsilon_N^* + \epsilon_N) \right. \\ \left. (\tau^{(1)} \times \tau^{(2)})_z \right] + (1 \leftrightarrow 2) \quad (10)$$

where

$$F(r) = -\frac{1}{4\pi} \left(\frac{\mu}{r^2} + \frac{1}{r^3} \right) e^{-\mu r}$$

and

$$K(r) = \frac{d}{dr} F(r)$$

The three body operator corresponding to the graphs of Fig. (7) can be obtained from the transition operators given by equations (8), (9) and (10) by replacing $B(r_1)$ by the magnetic field produced by the third nucleon in the position of the first, viz.

$$\mathbf{B}(r_1) = \frac{e}{m} \left[3 \frac{\mu \sigma_3 \cdot (r_1 - r_3)}{|r_1 - r_3|^3} (r_1 - r_3) - \mu \frac{\sigma_3}{|r_1 - r_3|} \right]$$

where

$$\mu = \frac{1}{2} (\mu_n - \mu_p) - \frac{1}{2} (\mu_n - \mu_p) \tau^2 \quad (3)$$

is the static magnetic dipole of the nucleon in nuclear magnetons

The operator (8) can now be expanded into multipoles as was carried out for the operator given by equation (2). The results for the electric L-multipoles stemming from $W_1(\mathbf{B})$ and $W_2(\mathbf{B})$ are respectively

$$\begin{aligned} W_1(\mathbf{B}) &= \frac{1}{2} \frac{Gf}{\mu(M-m)2m^2} \frac{2}{\sqrt{3}} (\epsilon_N^* - \epsilon_N - \epsilon_p^* + \epsilon_p) \sum_{L \geq 1} K\left(\frac{L}{L+1}\right)^{1/2} R_{ij}^L \left[Y_L(R_{ij}) \otimes (\sigma_i \times \sigma_j) \right]_M^{*L} \\ (\tau_i^{(1)} \times \tau_j^{(1)}) &+ \frac{1}{3} \frac{Gf}{\mu(M-m)2m^2} \frac{2}{\sqrt{3}} (\epsilon_N^* - \epsilon_N - \epsilon_p^* + \epsilon_p) \sum_{L \geq 1} R_{ij}^L \frac{K(|r_i - r_j|)}{|r_i - r_j|} \\ &\left[Y_L(R_{ij}) \otimes \left\{ \left[\sigma_i \times (r_i - r_j) \right] \sigma_j (r_i - r_j)_z^2 + \left[\sigma_j \times (r_i - r_j) \right] \sigma_i (r_i - r_j)_z^2 \right\} \right]_M^{*L} + \\ &\frac{1}{3} \frac{Gf}{\mu(M-m)2m^2} \frac{2}{\sqrt{2}} (\epsilon_p^* - \epsilon_p + \epsilon_N^* - \epsilon_N) K\left(\frac{L}{L+1}\right)^{1/2} \sum_{L \geq 1} R_{ij}^L \frac{K(|r_i - r_j|)}{|r_i - r_j|} T_{20}(\tau_i^{(1)}, \tau_j^{(1)}) \\ &\left[Y_L(R_{ij}) \otimes \left\{ \left[\sigma_i \times (r_i - r_j) \right] \sigma_j (r_i - r_j)_z + \left[\sigma_j \times (r_i - r_j) \right] \sigma_i (r_i - r_j)_z \right\} \right]_M^{*L} \quad (11) \end{aligned}$$

$$\begin{aligned} W_2(\mathbf{B}) &= (-1)^2 \frac{Gf}{3 \mu(M-m)2m^2 \mu} \frac{1}{\sqrt{3}} (\epsilon_p^* - \epsilon_p - \epsilon_N^* + \epsilon_N) K\left(\frac{L}{L+1}\right)^{1/2} \sum_{L \geq 1} F(r_{ij}) R_{ij}^L \left[Y_L(R_{ij}) \otimes \right. \\ &\left. (\tau_i^{(1)} \times \tau_j^{(1)})_z \right]_M^{*L} + \frac{2}{3} \frac{Gf}{(M-m)2m^2 \mu \sqrt{3}} (\epsilon_p^* - \epsilon_p - \epsilon_N^* + \epsilon_N) K\left(\frac{L}{L+1}\right)^{1/2} \sum_{L \geq 1} |r_i - r_j| K(|r_i - r_j|) \\ &R_{ij}^L \left[Y_L(R_{ij}) \otimes (\sigma_i - \sigma_j) \right]_M^{*L} (\tau_i^{(1)} \times \tau_j^{(1)})_z + \frac{2}{3} \frac{Gf(-)}{\mu(M-m)2m^2} \frac{1}{\sqrt{3}} (\epsilon_p^* - \epsilon_p - \epsilon_N^* - \epsilon_N) K \\ &\left(\frac{L}{L+1}\right)^{1/2} \sum_{L \geq 1} R_{ij}^L \left[Y_L(R_{ij}) \otimes \left\{ \left[(r_i - r_j) \times (\sigma_i - \sigma_j) \right] \times (r_i - r_j) \right\} \right]_M^{*L} \frac{K(|r_i - r_j|)}{|r_i - r_j|} (\tau_i^{(1)} \times \tau_j^{(1)})_z \quad (12) \end{aligned}$$

Analogously the magnetic multipoles are

$$\begin{aligned}
W_1(ML) &= \frac{1}{2} \frac{Gf}{\mu(M-m)2m^2} \frac{2}{\sqrt{3}} i (\epsilon_N^* - \epsilon_N - \epsilon_p^* + \epsilon_p) \left[L(2L+1) \right]^{\frac{1}{2}} \sum_{i < j} R_{ij}^{L-1} \left[Y_{L-1}(R_{ij}) \otimes \left\{ \left[\sigma_i \times \sigma_j \right] \sigma_j (r_i - r_j) \tau_2 + \left[\sigma_j \times \sigma_i \right] \sigma_i (r_i - r_j) \tau_2 \right\} \right]_M \frac{K(|r_i - r_j|)}{|r_i - r_j|} \\
&+ \frac{1}{3} \frac{Gf}{\mu(M-m)2m^2} \frac{2}{\sqrt{3}} i (\epsilon_N^* - \epsilon_N - \epsilon_p^* + \epsilon_p) \left[L(2L+1) \right]^{\frac{1}{2}} \sum_{i < j} R_{ij}^{L-1} \left[Y_{L-1}(R_{ij}) \otimes \left\{ \left[\sigma_i \times \sigma_j \right] \sigma_j (r_i - r_j) \tau_2 + \left[\sigma_j \times \sigma_i \right] \sigma_i (r_i - r_j) \tau_2 \right\} \right]_M \frac{K(|r_i - r_j|)}{|r_i - r_j|} \\
&+ \frac{1}{3} \frac{Gf}{\mu(M-m)2m^2} \frac{2}{\sqrt{3}} i (\epsilon_p^* - \epsilon_p + \epsilon_N^* - \epsilon_N) \left[L(2L+1) \right]^{\frac{1}{2}} \sum_{i < j} R_{ij}^{L-1} \left[Y_{L-1}(R_{ij}) \otimes \left\{ \left[\sigma_i \times \sigma_j \right] \sigma_j (r_i - r_j) \tau_2 + \left[\sigma_j \times \sigma_i \right] \sigma_i (r_i - r_j) \tau_2 \right\} \right]_M \frac{K(|r_i - r_j|)}{|r_i - r_j|} \\
&+ \frac{1}{3} \frac{Gf}{\mu(M-m)2m^2} \frac{2}{\sqrt{3}} i (\epsilon_p^* - \epsilon_p + \epsilon_N^* - \epsilon_N) \left[L(2L+1) \right]^{\frac{1}{2}} \sum_{i < j} R_{ij}^{L-1} \left[Y_{L-1}(R_{ij}) \otimes \left\{ \left[\sigma_i \times \sigma_j \right] \sigma_j (r_i - r_j) \tau_2 + \left[\sigma_j \times \sigma_i \right] \sigma_i (r_i - r_j) \tau_2 \right\} \right]_M \frac{K(|r_i - r_j|)}{|r_i - r_j|} \quad (13)
\end{aligned}$$

$$\begin{aligned}
W_2(ML) &= (+) \frac{2}{3} \frac{Gf}{\mu(M-m)2m^2} \left[L(2L+1) \right]^{\frac{1}{2}} \frac{i}{\sqrt{3}} (\epsilon_p^* - \epsilon_p - \epsilon_N^* + \epsilon_N) \sum_{i < j} R_{ij}^{L-1} \\
&\left[Y_{L-1}(R_{ij}) \otimes \left(\sigma_j - \sigma_i \right) \right]_M F(r_{ij}) (\tau^{(i)} \times \tau^{(j)})_2 + (-) \frac{2}{3} \frac{Gf}{\mu(M-m)2m^2} \frac{i}{\sqrt{3}} (\epsilon_p^* - \epsilon_p - \epsilon_N^* + \epsilon_N) \\
&\left[L(2L+1) \right]^{\frac{1}{2}} \sum_{i < j} |r_i - r_j| K(|r_i - r_j|) R_{ij}^{L-1} \left[Y_{L-1}(R_{ij}) \otimes \left(\sigma_j - \sigma_i \right) \right]_M (\tau^{(i)} \times \tau^{(j)})_2 + \\
&+ \frac{2}{3} \frac{Gf}{(M-m)2m^2 \mu} \frac{i}{\sqrt{3}} (\epsilon_p^* - \epsilon_p - \epsilon_N^* + \epsilon_N) \left[L(2L+1) \right]^{\frac{1}{2}} \sum_{i < j} R_{ij}^{L-1} \left[Y_{L-1}(R_{ij}) \otimes \left(\sigma_j - \sigma_i \right) \right]_M \\
&\left[(r_i - r_j) \times (\sigma_j - \sigma_i) \right]_M \frac{K(|r_i - r_j|)}{|r_i - r_j|} (\tau^{(i)} \times \tau^{(j)})_2 \quad (14)
\end{aligned}$$

Before proceeding with a more detailed calculation it is worth noting that the effects of the electric multipoles ($W(EL) = W_1(EL) + W_2(EL)$) stemming from $W(B_1)$ are negligible compared with the effects of the magnetic multipoles ($W(ML) = W_1(ML) + W_2(ML)$), for low energy photons, so that the following relation holds.

$$\frac{\langle I || W(ML) || I_1 \rangle}{\langle I || M^{NOR}(L) || I_1 \rangle} \gg \frac{\langle I || W(EL) || I \rangle}{\langle I || E^{NOR}(L) || I_1 \rangle} \quad (15)$$

To see this an estimate similar to the one carried out in section II is made below for one term only of the transition operator $W(B)$ noting that all terms have the same order of magnitude

The first term of equation (12) is

$$W_1^{(1)}(B) = (-) \frac{2}{3} \frac{Gf}{\mu m(M-m)2m} \frac{1}{\sqrt{3}} (\epsilon_D^+ - \epsilon_D - \epsilon_N^+ + \epsilon_N) \left[B(r_1) \sigma_2 - B(r_2) \sigma_1 \right]$$

$$F(r_1) = (\tau^{(1)} \times \tau^{(2)})_z$$

This can be expanded in multipoles and we call $W_2^{(a)}(EL)$ and $W_2^{(a)}(ML)$ the electric and magnetic multipoles stemming from $W_1^{(a)}(B)$. Those multipoles have the same form as the ones resulting from the special scalar meson vertex in the paper by Clement¹⁰⁾. Therefore by using the estimates given there we have

$$\frac{\langle W_2^{(a)}(E1) \rangle}{\langle (E1)^{NOR} \rangle} \propto \frac{(KR_0)}{(\mu R_0)^2} \quad \text{and} \quad \frac{\langle W_2^{(a)}(M1) \rangle}{\langle (M1)^{NOR} \rangle} \propto \frac{m R_0}{(\mu R_0)^2}$$

Since $m \gg K$ the inequality (15) is justified

4. Calculation for ${}^A F$

An estimate of the contribution of the operators given by equations (11) to (14) to the value of $\langle m \delta \rangle$ given by equation (5) was carried out for the transition from the level $(J=2, I=1, E=3.06 \text{ MeV})$ to the level $(J=3, I=0, E=0.94 \text{ MeV})$ in ${}^A F$. The wave functions for the levels were taken from the work by Elliot and Flowers¹⁷⁾, viz.

$$\Psi(J=0, J=3) = 0.59(d^2)^{13}D + 0.03(d^2)^{13}G - 0.12(d^2)^{13}F + 0.79(sd)^{13}D$$

$$\Psi(J=1, J=2) = 0.65(d^2)^{13}D + 0.33(d^2)^{13}P - 0.20(d^2)^{13}F - 0.61(sd)^{13}D +$$

$$+ 0.22(sd)^{13}D$$

It is evident that only the isospin antisymmetric part of the operators given by equations (13) and (14) will contribute. By using some recoupling of angular momentum it results that the contribution to M. 1 antisymmetric in isospin from equation (13) and (14) will be respectively

$$W_1(M,1) = W_1^{(a)}(M,1) + W_1^{(b)}(M,1) + W_1^{(c)}(M,1)$$

where

$$W_1^{(a)}(M,1) = \frac{Gf}{\mu(M-m)2m^2} \frac{1}{\sqrt{3}} i(\epsilon_N^* - \epsilon_N - \epsilon_p^* + \epsilon_p) \left(\frac{3}{4\pi}\right)^{1/2} \sum_{i < j} (\sigma_i \times \sigma_j)_M^{*1} (\tau_z^i - \tau_z^j) F(r_{ij})$$

$$W_1^{(b)}(M,1) = \frac{1}{3} \frac{Gf}{\mu(M-m)2m^2} \frac{2}{\sqrt{3}} i(\epsilon_N^* - \epsilon_N - \epsilon_p^* + \epsilon_p) \left(\frac{1}{4\pi}\right)^{1/2} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \sum_{i < j} (\sigma_i \times \sigma_j)^{*1}$$

$$(\tau_z^i - \tau_z^j) |r_i - r_j| K(|r_i - r_j|)$$

$$W_1^{(c)}(M,1) = \frac{(-)}{3} \frac{Gf}{\mu(M-m)2m^2} \frac{2}{\sqrt{3}} i(\epsilon_N^* - \epsilon_N - \epsilon_p^* + \epsilon_p) \left[6 \times 5 \times 3\right]^{1/2} \begin{Bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \end{Bmatrix} \begin{pmatrix} 1 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\sum_{i < j} (-i) \left[\left[\sigma_i \otimes \sigma_j \right]^{(1)} \otimes Y_2(r_{ij}) \right]_M^{*(1)} (\tau_z^{(i)} - \tau_z^{(j)}) |r_i - r_j| K(|r_i - r_j|)$$

and

$$W_2(M,1) = W_2^{(a)}(M,1) + W_2^{(b)}(M,1) + W_2^{(c)}(M,1) + W_2^{(d)}(M,1)$$

where

$$W_2^{(a)}(M,1) = (-) \frac{2}{3} \frac{Gf}{(M-m)2m^2 \mu} \frac{i}{\sqrt{3}} (\epsilon_p^* - \epsilon_p - \epsilon_N^* + \epsilon_N) \left(\frac{3}{4\pi}\right)^{1/2} \sum_{i < j} \left[\sigma_i - \sigma_j \right]_M^{*1} (\tau^{(i)} \times \tau^{(j)})_z F(r_{ij})$$

$$W_2^{(b)}(M,1) = (+) \frac{2}{3} \frac{Gf}{(M-m)2m^2 \mu} \frac{i}{\sqrt{3}} (\epsilon_p^* - \epsilon_p - \epsilon_N^* + \epsilon_N) \left(\frac{3}{4\pi}\right)^{1/2} \sum_{i < j} |r_i - r_j| K(|r_i - r_j|)$$

$$\left[\sigma_i - \sigma_j \right]_M^{*1} (\tau^{(i)} \times \tau^{(j)})_z$$

$$W_2^{(c)}(M 1) = (-) \frac{2}{3} \frac{G f}{\mu(M-m)2m} 3 i (\epsilon_p^* - \epsilon_p - \epsilon_N^* + \epsilon_N) W(1111;01) \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \left(\frac{1}{4\pi}\right)^{1/2} \begin{matrix} \Sigma \\ \langle \rangle \end{matrix}$$

$$(\sigma_i - \sigma_i)^* \begin{matrix} \Sigma \\ \langle \rangle \end{matrix} (\tau_i \times \tau_i)_z |r_i - r_i| K(|r_i - r_i|)$$

$$W_2^{(d)}(M 1) = (-) \frac{2}{3} \frac{G f}{\mu(M-m)2m} \sqrt{3} \times 5 i (\epsilon_p^* - \epsilon_p - \epsilon_N^* + \epsilon_N) W(1111;21) \begin{pmatrix} 1 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \begin{matrix} \Sigma \\ \langle \rangle \end{matrix}$$

$$\left[Y_2(r_i) \otimes (\sigma_i - \sigma_i) \right] \begin{matrix} \Sigma \\ \langle \rangle \end{matrix} |r_i - r_i| K(|r_i - r_i|) (\tau_i \times \tau_i)_z$$

The T R I conserving one body usual magnetic dipole transition operator is written with the same phases and overall normalization factor.

$$(M 1)^{NOR} = \left(\frac{3}{4\pi}\right)^{1/2} \frac{1}{2m} \Sigma \left[\frac{e}{2} (1 + \tau_z^{(i)}) \ell_i + \left[\frac{1}{2} (\mu_n + \mu_p) - \frac{1}{2} (\mu_n - \mu_p) \tau_z^{(i)} \right] \sigma_i \right] \begin{matrix} \Sigma \\ \langle \rangle \end{matrix} \begin{matrix} \Sigma \\ \langle \rangle \end{matrix}$$

It is now a simple matter to calculate the relevant matrix elements. Short range correlation are introduced by evaluating the radial integrals from a hard core radius r_c taken to be $r_c = 0.5$ fm. For the purposes of the estimate given below a "maximal value" $i(\epsilon_p^* - \epsilon_p - \epsilon_N^* + \epsilon_N) = e$ was taken (and the value $G = 2.07$ from Gourdin¹⁸). There results

$$\text{Im } \delta \approx i \delta_R \left[\frac{\langle || W(M 1) || \rangle}{\langle || M^{NOR}(1) || \rangle} \right] = \delta_R 45 \times 10^{-3}$$

This is a relatively large result and therefore very encouraging. However, for this particular transition the "mixing ratio" is (using Warburton et al¹⁹ convention) much too small, $\delta_R \approx 0.06$. The effect E to be observed in the oriented angular correlation experiment is

$$E = \frac{\delta_R}{1 + |\delta|^2} \text{Im } \delta \approx 27 \times 10^{-4}$$

This value is a little outside the experimental possibility at present, but not far enough to preclude the experimental investigation in a few years time

Acknowledgement

The author takes this opportunity to thank Professor R.J. Blin-Stoyle for suggesting the problem and for guidance and encouragement. The author would also like to express his sincere thanks to Prof. J.P. Elliot and to Drs. A.M. Kahn and W.D. Hamilton for valuable discussions.

Resumo

Neste trabalho são estudadas as possíveis consequências em Física Nuclear de baixa energia de uma violação da simetria de inversão temporal na interação eletromagnética. Mostra-se que o efeito aparece como um operador de transição eletromagnético de dois corpos, de curto alcance e violando a simetria de inversão temporal (T R I V) ou como potenciais T R I V de dois ou três corpos. Dois casos são considerados. Primeiramente um "vertex" $NN\gamma$ T R I V é considerado e mostra-se que seus efeitos são muito pequenos. Em segundo lugar, supõe-se que a T R I V ocorre em um $N^*N\gamma$ "vertex" e mostra-se que se a violação é "maximal" então a contribuição para a parte imaginária do "mixing ratio" δ é $\text{Im } \delta \approx |\delta| \times 10^{-3}$.

Résumé

On présente une étude des conséquences en physique nucléaire à basse énergie d'une violation de la symétrie d'inversion temporelle dans l'interaction électromagnétique. On montre que cet effet apparaît comme un opérateur de transition électromagnétique de deux corps, à courte échance, en violant la symétrie d'inversion temporelle (T R I V) ou comme T R I V potentiels de deux ou trois corps. On considère deux cas. Premièrement, un "vertex" $NN\gamma$ T R I V est considéré et on montre que ses effets sont très petits. Deuxièmement, on suppose que la T R I V a lieu dans un "vertex" $N^*N\gamma$ et on montre que, si la violation est "maximal", la contribution à la partie imaginaire du "mixing ratio" δ est $\text{Im } \delta \approx |\delta| \times 10^{-3}$, qui peut être mesuré.

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Figure Captions

- Fig 1 – Contribution to a T R I violating electromagnetic transition operator. The bubble represents a four vertex defined in Ref. 5).
- Fig 2 – Contribution to a T R I. violating three body potential
- Fig 3 – Contribution to a T.R.I. violating electromagnetic transition operator. The bubble represents the vertex given by equation (1)
- Fig 4 – Some of the graphs contributing to a T.R.I. violating two body operator.
- Fig 5 – Contribution to a three body T R.I. violating potential.
- Fig 6 – Contribution to a T R I violating electromagnetic transition operator. The heavy line in the graphs represents the $N^*(J = 3/2 \ l = 3/2 \ M = 1236 \text{ MeV})$ nucleon isobar. The effective couplings $N^*N\gamma$ and N^*NM are given by equations (6) and (7).
- Fig 7 – Contribution to a three body T R.I. violating potential. The heavy line in the graph represents the N^* nucleon isobar

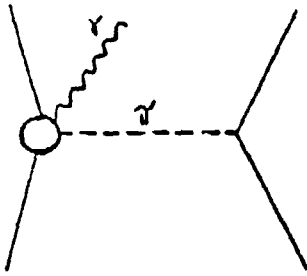


Fig. 1

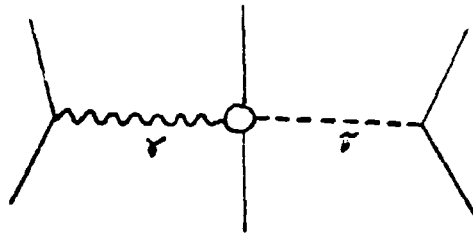
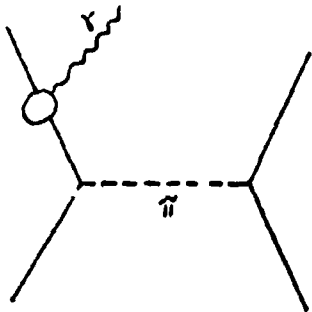
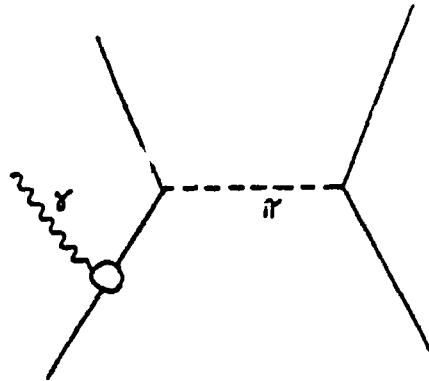


Fig. 2

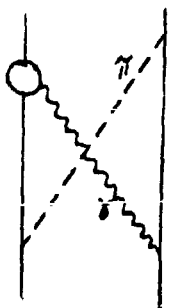


(a)

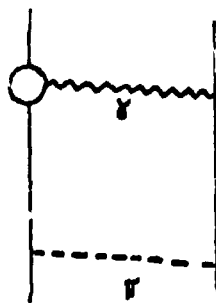


(b)

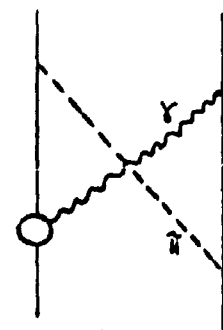
Fig. 3



(a)



(b)



(c)

Fig. 4

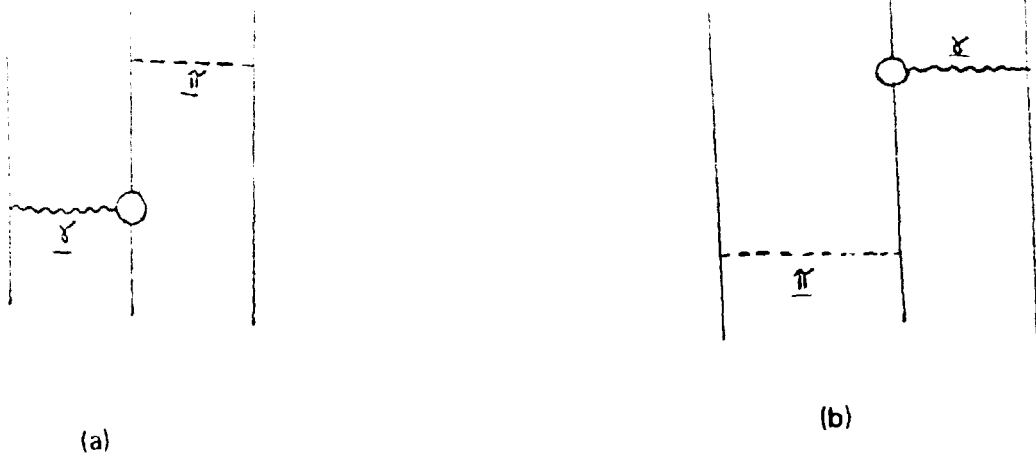


Fig. 5

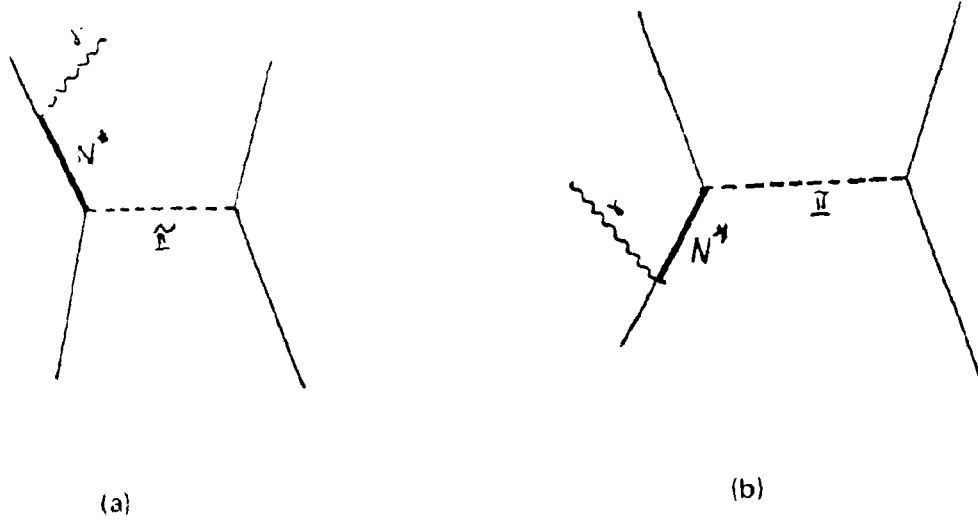


Fig. 6

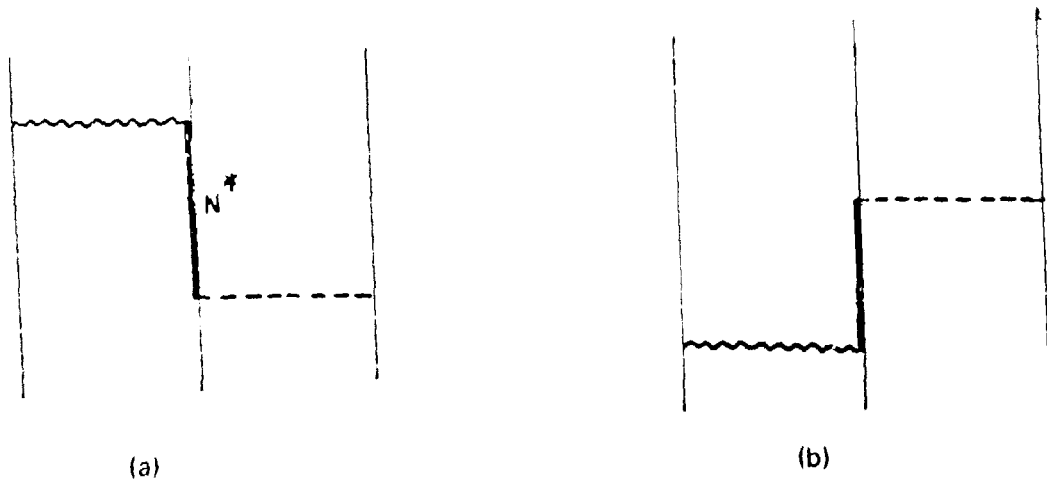


Fig. 7