A Finite-Difference Time-Domain Analysis of Fiber Bragg Gratings

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***Abstract*—An analytical formulation and modeling of an optical fiber Bragg gratings has been developed and is reported in this paper. Supported by the Finite-Difference Time-Domain (FDTD) method, it was possible to set a 1018 nm bragg grating and simulate an electromagnetic field going through this periodic refractive-index device to obtain reflectivity, transmittivity, and bandwidth. Moreover, the model is applied to analyze the influence of structural parameters of fiber gratings, such as length, grating period, and refractive index modulation on its spectral response.**

***Index Terms*—Fiber Bragg Grating, FDTD, reflectivity**

I. Introduction

Fiber Bragg gratings are an emerging technology used in several segments of industry, telecommunication, sensors, and optical fiber lasers. It is a class of devices based on a periodic modulation of the core refractive index along the length of an optical fiber. This periodic structure acts as a selective mirror for the wavelength that satisfies the Bragg condition [1].

The FDTD method is an effective and versatile technique to simulate electromagnetic field propagation through an optical fiber. This method is based on Yee's cell, which apply finite difference operators on staggered grids in space and time for each electric and magnetic vector field component in Maxwell's curl equations [2]. Although it solves the problem in time, it can provide frequency-domain responses over a wide band using Fourier transform.

Therefore, an analytical formulation and a MATLAB simulation of an accurate selection of setting parameters have been developed and presented in this paper. In addiction, it was created a simulation with different grating lengths to compare the relation between the structural Bragg gratings parameters and its spectral responses.

II. Theoretical Model

The formalism of wave propagation in FBGs is presented by Maxwell's time-domain equations [3]:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{1}$$

$$\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \tag{2}$$

where $\vec{E}$ and $\vec{H}$ are the electric and magnetic fields, respectively, $\vec{D}$ is the electric displacement, and $\vec{B}$ is the magnetic flux density. The constitutive relations for linear, isotropic, and nondispersive materials can be written as

$$\vec{D} = \varepsilon_0 \varepsilon_r \vec{E} \tag{3}$$

$$\vec{B} = \mu_0 \vec{H} \tag{4}$$

where μ_0 is the magnetic permeability, ε_0 is the electric permittivity, and ε_r is the perturbation permittivity. Solving the Eqs. (1) and (2):

$$\hat{i}\frac{\partial H_x}{\partial t} + \hat{j}\frac{\partial H_y}{\partial t} + \hat{k}\frac{\partial H_z}{\partial t} = -\frac{1}{[\mu]} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} \tag{5}$$

$$\hat{i}\frac{\partial E_x}{\partial t} + \hat{j}\frac{\partial E_y}{\partial t} + \hat{k}\frac{\partial E_z}{\partial t} = \frac{1}{[\varepsilon]} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} \tag{6}$$

where $[\mu]$ and $[\varepsilon]$ represent the spatial distribution of permeability and permittivity.

The system of six coupled partial differential equations given by Eqs. (5) and (6) forms the basis of the FDTD numerical algorithm for electromagnetic wave interactions with general three-dimensional objects. However, it is possible to consider simplifications to two-dimensional or one-dimensional wave phenomena and can yield insight to the analytical and algorithmic features of general case.

$$\frac{\partial H_x}{\partial t} = -\frac{1}{[\mu]}(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}) \tag{7}$$

$$\frac{\partial H_y}{\partial t} = -\frac{1}{[\mu]}(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}) \tag{8}$$

$$\frac{\partial H_z}{\partial t} = -\frac{1}{[\mu]}(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}) \tag{9}$$

$$\frac{\partial E_x}{\partial t} = \frac{1}{[\varepsilon]}(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}) \tag{10}$$

$$\frac{\partial E_y}{\partial t} = \frac{1}{[\varepsilon]}(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}) \tag{11}$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{[\varepsilon]}(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}) \tag{12}$$

An algorithm to set of finite-difference equations for the time-dependent Maxwell's curl equations system was proposed by Yee [4]. The electric and magnetic field components are sampled and can be represented in discrete form, both in space and time. As shown in Fig. 1, the FDTD technique splits the three-dimensional problem geometry into cells to form a grid, where the unit cell of this grid is called a Yee cell.

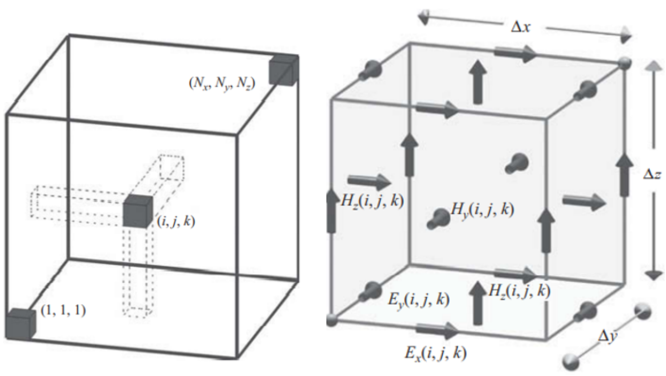

Fig. 1. A three-dimensional FDTD computational space composed of $(N_x \times N_y \times N_z)$.

The electric and magnetic fields vector components are set at the edges and centers of the face of the Yee cell, respectively. This provides a simple picture of three dimensional space being filled by an interlinked array of Faraday's and Ampere's laws contours. The material parameters are distributed over the FDTD grid and are associated with field components. Therefore, they are indexed the same as their respective field components.

The discretisation in space is performed based on the unit cell of the Yee space and the discretisation in time is obtained following the leapfrog arrangement [5]. In this way, a set of discretized equations in space (i-index) and in time (n-index) are obtained.

$$\begin{aligned} H_x|_i^{n+1} = H_x|_i^n - \big[&M_{\mu,y}\left(E_z|_{i+1}^n - E_z|_i^n\right) \\ &-M_{\mu,z}\left(E_y|_{i+1}^n - E_y|_i^n\right)\big] \end{aligned} \tag{13}$$

$$\begin{aligned} H_y|_i^{n+1} = H_y|_i^n - \big[&M_{\mu,z}\left(E_x|_{i+1}^n - E_x|_i^n\right) \\ &-M_{\mu,x}\left(E_z|_{i+1}^n - E_z|_i^n\right)\big] \end{aligned} \tag{14}$$

$$\begin{aligned} H_z|_i^{n+1} = H_z|_i^n - \big[&M_{\mu,x}\left(E_y|_{i+1}^n - E_y|_i^n\right) \\ &-M_{\mu,y}\left(E_x|_{i+1}^n - E_x|_i^n\right)\big] \end{aligned} \tag{15}$$

$$\begin{aligned} E_x|_i^{n+1} = E_x|_i^n + \big[&M_{\varepsilon,y}\left(H_z|_{i+1}^{n+1} - H_z|_i^{n+1}\right) \\ &-M_{\varepsilon,z}\left(H_y|_{i+1}^{n+1} - H_y|_i^{n+1}\right)\big] \end{aligned} \tag{16}$$

$$\begin{aligned} E_y|_i^{n+1} = E_y|_i^n + \big[&M_{\varepsilon,z}\left(H_x|_{i+1}^{n+1} - H_x|_i^{n+1}\right) \\ &-M_{\varepsilon,x}\left(H_z|_{i+1}^{n+1} - H_z|_i^{n+1}\right)\big] \end{aligned} \tag{17}$$

$$\begin{aligned} E_z|_i^{n+1} = E_z|_i^n + \big[&M_{\varepsilon,x}\left(H_y|_{i+1}^{n+1} - H_y|_i^{n+1}\right) \\ &-M_{\varepsilon,y}\left(H_x|_{i+1}^{n+1} - H_x|_i^{n+1}\right)\big] \end{aligned} \tag{18}$$

where the terms $M_{\mu,k}$ and $M_{\varepsilon,k}$ are defined according to Eqs. (19) and (20), and the k index is a general representation for the x, y and z dimensions.

$$M_{\mu,k} = \frac{\Delta t}{[\mu]\Delta k} \tag{19}$$

$$M_{\varepsilon,k} = \frac{\Delta t}{[\varepsilon]\Delta k} \tag{20}$$

In order to analyze the Bragg gratings reflectivity and transmittivity features, let us further assume that either the electromagnetic and magnetic fields are propagating in z-direction and that the periodic refractive index modulation in the fibers occurs along the longitudinal direction. Considering the refractive index doesn't change in the fiber transversal direction, it is possible to reduce the problem in a one-dimensional problem, consequently, we assume that all partial derivatives field with respect to both x and y are zero and that the propagation through Bragg grating can be written in function of the index refraction modulation represented by $[\varepsilon]$ properly modeled for the periodic case.

Reducing to one-dimensional problem, the Eqs. (13) to (18) can be rewritten as:

$$H_x|_i^{n+1} = H_x|_i^n + M_{\mu,z}\left(E_y|_{i+1}^n - E_y|_i^n\right) \tag{21}$$

$$E_y|_i^{n+1} = E_y|_i^n + M_{\varepsilon,z}\left(H_x|_{i+1}^{n+1} - H_x|_i^{n+1}\right) \tag{22}$$

An important issue in designing a time-domain numerical algorithm is the stability condition. The choice of the periodic of sampling Δt and Δz must comply with some restrictions to guarantee the stability of the solution. The correct choice of these parameters determines the solution accuracy. The numerical stability of FDTD method need to obey the Courrant-Friedrichs-Lewy (CFL) condition, which requires the time increment Δt has an specific bound relative to the lattice space increments, such that [6].

$$\Delta t \leq \frac{1}{c\sqrt{\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2} + \frac{1}{(\Delta z)^2}}} \tag{23}$$

where c is the speed of light in free space. In one-dimensional problem it reduces to

$$\Delta t \leq \frac{\Delta z}{c} \tag{24}$$

The Fast Fourier Transform (FFT) of both the incident and the reflected time-domain responses was performed in the calculation of the reflection coefficient. It is possible to calculate the FFT to discrete points using the expression [5]

$$F(f) \approx \Delta t \sum_{m=1}^{M} (e^{-i2\pi f \Delta t})^m . f(m) \tag{25}$$

where $K = e^{-i2\pi f \Delta t}$ is the kernel and can be computed to the main FDTD loop for each frequency of interest and $f(m)$ is the field value of interest at the current time step.

To calculate the coefficients $R(f)$ and $T(f)$, which represent the reflectivity and transmittivity responses of FBG, the spectra must be normalized, that is done by dividing the reflection and transmission spectrum by the source spectrum, according to Eqs. (26) and (27).

$$R(f) = \left(\frac{FFT(E_r)}{FFT(E_s)}\right)^2 \tag{26}$$

$$T(f) = \left(\frac{FFT(E_t)}{FFT(E_s)}\right)^2 \tag{27}$$

where E_r, E_t, and E_s represent the reflected, transmitted, and source electric fields, respectively. The bandwidth $\Delta\lambda$, can be defined as the first zero on either side of the main reflection peak. Consequently, it may be calculated graphically or by finding the zeros between the peak of reflection spectrum [1].

III. Results and Discussion

The algorithm was built according to flowchart represented in Fig. 2. The waveguide comprises a linear dielectric material core refractive index $n_{eff} = 1.45$ and a periodic modulation along z-direction $\delta n = 0.05$. It was considered a uniform FBG with wavelength is $\lambda_B = 1018nm$. Considering the Bragg condition, where $\lambda_B = 2n_{eff}\Lambda$, it is possible to calculate the period of gratings $\Lambda = 351.03nm$. The grating length can be calculated multiplying the number of gratings by the period.

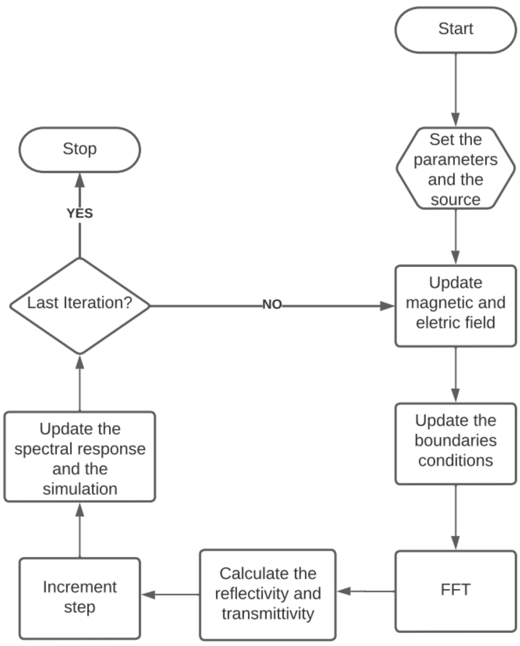

Fig. 2. Flowchart of FDTD algorithm implementation.

Firstly, it was calculated the reflectivity spectrum to the same parameters using the FTDT and the 4-th order Runge-Kutta method. Analyzing the Fig. 3, it is possible to observe that the spectrum is quite similar using the both methods.

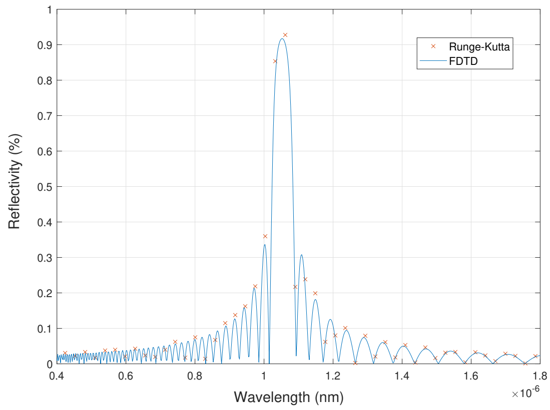

Fig. 3. Comparison between 4th-order-Runge-Kutta method and FDTD method. The red points represent the Runge-Kutta solution and the blue line represents the FDTD solution to 30 gratings.

In addiction, it was possible to calculate the reflectivity and transmittivity spectrum peak when the number of gratings are changed. The spectral responses with grid numbers ranging from 10, 20 and 30 were selected and plotted. Figs. 4 and 5 show the variation of reflectivity and transmittivity spectrum, respectively.

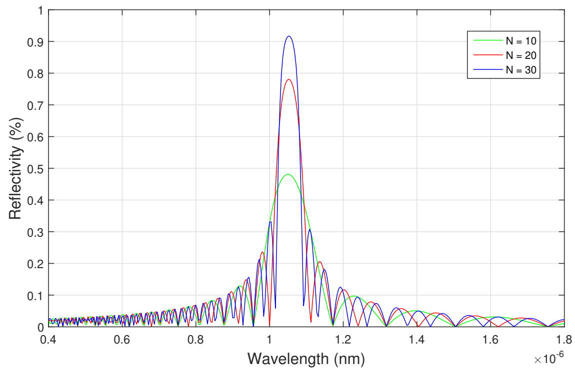

Fig. 4. Variation of the reflectivity coefficient for three different values of number of periods. It is possible to see that the greater is the gratings number, the stronger is the reflectivity.

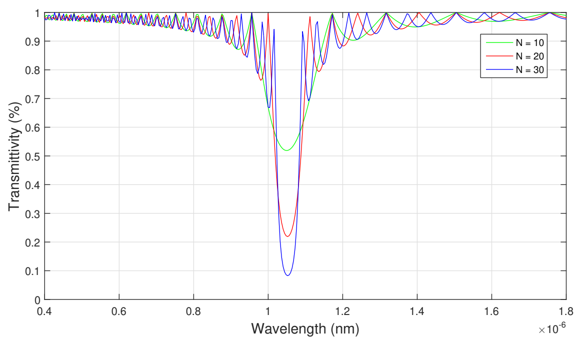

Fig. 5. Variation of the transmittivity coefficient for three different values of number of periods. It is possible to see that the greater is the gratings number, the weaker is the transmittivity.

In order to simulate the two-dimensional problem, it is possible to assume that partial derivatives field with respect to y is zero and the index refraction modulation represented by $[\varepsilon]$ can be written as a matrix 2×2. The permittivity matrix was modeled considering the periodic refractive index modulation, so that it has the same z-grid and it is filled considering the perfect match with the period of the gratings. Fig. 6 shows a schematic diagram that represents the two-dimensional approach.

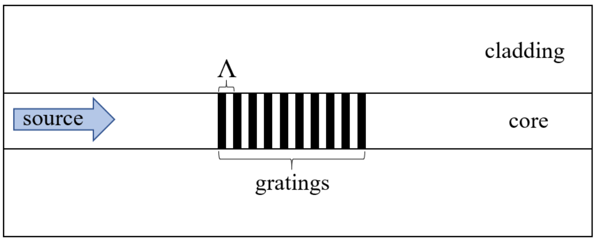

Fig. 6. Schematic diagram of fiber Bragg grating used in two-dimensional problem.

A Gaussian wave source was propagated along the z-direction and going through the gratings during $2.7497 \times 10^{-13}s$. Considering the two-dimensional structure, where the gratings were built to reflect 1018 nm with $N = 10$, it was possible to select a simulation frame represented by Fig. 6. It is noticed that when the wave propagation arrives at FBG position, it is partially reflected.

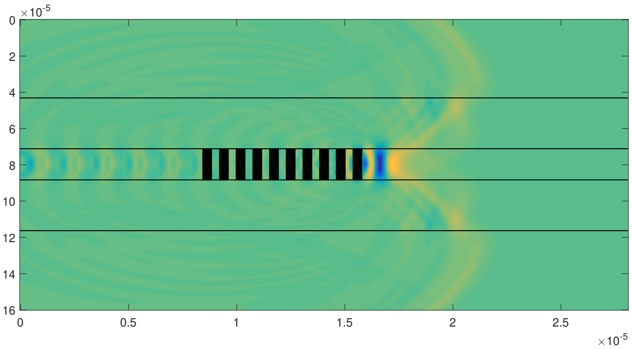

Fig. 7. Two-dimensional 1018nm FBG simulation to 10 gratings periods.

IV. Conclusion

The analytical formulation and simulation of FBG is an essential step for projecting and manufacturing these devices. An analysis of periodic structure has been presented using one-dimensional FDTD code to calculate the spectrum responses of reflectivity and transmittivity. Moreover, a two-dimensional algorithm to simulate the electromagnetic field propagation along an optical fiber has been shown.

As shown in the results, it is an effective method to simulate wave propagation in periodic refractive index devices. This method can be exploited in order to build a three-dimensional simulation considering as many parameters as necessary to the project.

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