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A Model for Non Equilibrium, Non Homogeneous Two Phase Critical Flow

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Abstract - *Critical two phase flow is a very important phenomena in nuclear reactor technology for the analysis of loss of coolant accident. Several recent papers - Lee and Shrock(1990), Dagan(1993) and Downar(1996), among others, treat the phenomena using complex models which require heuristic parameters such as relaxation constants or interfacial transfer models. In this paper a mathematical model for one dimensional non equilibrium and non homogeneous two phase flow in constant area duct is developed. The model is three conservation equations type - mass ,momentum and energy. Two important variables are defined in the model : equilibrium constant in the energy equation and the impulse function in the momentum equation. In the energy equation, the enthalpy of the liquid phase is determined by a linear interpolation function between the liquid phase enthalpy at inlet condition and the saturated liquid enthalpy at local pressure. The interpolation coefficient is the equilibrium constant. The momentum equation is expressed in terms of the impulse function. It is considered that there is slip between the liquid and vapor phases, the liquid phase is in metastable state and the vapor phase is in saturated stable state. The model is not heuristic in nature and does not require complex interface transfer models. It is proved numerically that for the critical condition the partial derivative of two phase pressure drop with respect to the local pressure or to phase velocity must be zero. This criteria is demonstrated by numerical examples. The experimental work of Fauske(1962) and Super Moby Dick(1982) were analyzed resulting in estimated numerical values for important parameters like slip ratio, equilibrium constant and two phase frictional drop.*

Key Word Two phase critical flow , non-equilibrium

1. **Introduction**
2.

Two-phase critical flow is important to nuclear reactor safety for the analysis of loss of coolant accident scenarios (LOCA) and to many other industrial applications. During the 60's and early 70's various models were developed such as those by Fauske (1962), Moody (1965) and Henry -Fauske (1971). These models were not realistic for many practical cases. The marked departure from equilibrium when flashing occurs requires models which would include this phenomena . The idea which is widely used is to correlate the pressure under shoot or another parameter with a parameter called relaxation constant. Lee & Shrock (1990) presented a model for critical two phase flow - with boiling inception using 3 differential equations for conservation of mass , energy and momentum and a function which relates flashing pressure undershoot with relaxation constant in exponential form. They used simplifying assumptions such as homogeneous flow, vapor phase in saturated stable state , the liquid phase is in metastable state (superheated) and Kroeger's critical flow velocity criterion. Dagan (1993) presented a model for critical flashing two phase flow model with 2 continuity equations, one for each phase, 2 momentum equations, one for each phase, one energy equation for the mixture and a function which relates the void spatial distribution and bubble growth .The interfacial terms are expressed by experimental correlations. Downar (1996) presented a non equilibrium relaxation model for one-dimensional flashing liquid flow , with 3 equations of mass, momentum and energy conservation and introduced an equation relating vapor generation rate with relaxation time . Generally , the relaxation time as well as interfacial terms are heuristic in nature.

The present approach for non equilibrium two phase flow in constant area ducts treats the problem as a completely macroscopic problem independent of interface transport heuristic correlations.The model is represented by the sum of the separate equation of each phase resulting in 3 mixture conservation equations of mass, energy and momentum . The mixture is considered to be non homogeneous characterized by the slip ratio . It is assumed that the vapor phase is in saturated stable state and the enthalpy of the liquid phase is determined by a linear interpolation function between the enthalpy of liquid at the inlet condition and the enthalpy of saturated liquid at local pressure .The interpolation coefficient is called the equilibrium constant ζ.

The momentum equation is expressed in terms of the impulse function as defined by Shapiro[8]. In this form we can conclude that the change in impulse is equal to the two phase frictional pressure drop times the duct area . The terms in the three differential equations of conservation are arranged in such a manner that they can be easily integrated transforming them into three non linear algebraic equations. In addition, there are the thermodynamic relationships and the definitions for void fraction and slip ratio , resulting in a system of 12 non linear algebraic equations .

It has been proved that the critical flow condition can be expressed by the partial derivative of pressure loss with respect to static pressure is equal to zero .

3. Model

Governing physical equations :

Consider a two phase flow of steam and water in a constant area duct . The stagnation conditions at the inlet of the duct are characterized by the pressure p_o , temperature T_0 and quality x_o . At any distance z downstream , the flow variables and the thermodynamic properties are described by the one dimensional steady state continuity , energy and momentum equations obtained by the addition of the separate phase flow equations giving the mixture flow equation :

The continuity equation :

$$\frac{d}{dz}(\dot{M}_g+\dot{M}_l)=0$$

integrating along the duct , from 0 to z results :

$$\dot{M}_g+\dot{M}_l=\dot{M}_t \tag{1}$$

Where $\dot{M}_t$ is the total rate mass flow at z=0 and constant at any section z.

The energy equation:

$$\frac{d}{dz}(\dot{M}_g h_g+\dot{M}_g\frac{u_g^2}{2}+\dot{M}_l h_l+\dot{M}_l\frac{u_l^2}{2})=q$$

integrating along the duct length from 0 to z results :

$$\dot{M}_g h_g+\dot{M}_g\frac{u_g^2}{2}+\dot{M}_l h_l+\dot{M}_l\frac{u_l^2}{2}-\int_0^z q dz=\dot{M} h_o \tag{2}$$

Where ($\dot{M}_t h_o$) is the total energy per unit time at z=0 , h_g is the enthalpy of vapor phase at the saturation condition.

The non equilibrium between the phases can be characterized by the following expression for the liquid phase enthalpy h_l

$$h_l=\zeta\cdot h_{ls}+(1-\zeta)\cdot h_{lo} \tag{3}$$

h_{ls} is the liquid phase enthalpy at saturated condition , h_{lo} is liquid phase enthalpy at inlet conditions, ζ is the interpolation constant and varies between 0 and 1. To explain the physical meaning of ζ, consider a two phase flow where the friction changes the static pressure consequently changing the vapor phase temperature . If there is an instantaneous energy transfer between the vapor phase and liquid phase the equilibrium between the two phases occurs and the enthalpy of the liquid phase is h_{ls} which is represented by $\zeta=1$. However, if there is no energy transfer between the two phases, the liquid phase enthalpy is constant along the flow and the same as the inlet condition h_{lo} . This represents the limiting non-equilibrium condition and it is represented by $\zeta=0$. A real case where there is partial energy transfer between the two phases a fraction ζ of the liquid has enthalpy h_{ls} and the other fraction ($1-\zeta$) has enthalpy h_{lo}. These two fractions can be added resulting in an equivalent metastable state with enthalpy h_l.

However , the liquid phase entropy and temperature can not be obtained by the same interpolation function as for the enthalpy , but it can be estimated by thermodynamic relationships. The liquid phase specific volume is assumed to be the same as the specific volume in saturation condition since the liquid is assumed incompressible.

In the momentum equation:

the impulse function I is defined as:

$$I = p \cdot A + \dot{M}_g \cdot u_g + \dot{M}_l \cdot u_l$$

The momentum equation in terms of the impulse function is:

$$\frac{dI}{dz} = -\tau_0 \cdot \pi \cdot d$$

The impulse function I was used earlier by Shapiro[8] for single phase flow and τ_0 is the local wall frictional sheer stress . Integrating the above equation results :

$$I = I_o - \int_0^z \tau_0 \cdot \pi \cdot d \cdot dz$$

I_o is the total impulse at the inlet condition and is equal to p_oA .

Defining the total pressure drop Δp as ;

$$\Delta p = \frac{4}{d} \int_0^z \tau_o dz$$

$$I = I_o - \Delta P \cdot A \qquad (4a)$$

For the numerical calculations the variable I is split into its original form as follow:

$$p \cdot A + \dot{M}_g \cdot u_g + \dot{M}_l \cdot u_l = I_o - \Delta P \cdot A \qquad (4)$$

Definitions:

The void fraction α is defined by the following equations ;

$$\dot{M}_g = \frac{u_g \cdot (1-\alpha) \cdot A}{v_g} \qquad (5)$$

$$\dot{M}_l = \frac{u_l \cdot \alpha \cdot A}{v_l} \qquad (6)$$

the slip ratio is defined by :

$$S = \frac{u_g}{u_l} \qquad (7)$$

Thermodynamic relationships :

For the saturation conditions the vapor phase enthalpy h_g ,the liquid phase saturation enthalpy h_{ls} , vapor phase specific volume v_g, liquid phase specific volume v_l and the pressure p are given as a functions of saturation temperature T by the following polynomials :

$(C1.T - 1).(h_g - 2501) + C2 .T^2 + C3.T = 0$ (8)

$C4 + C5.T + C6.T^2 + C7.T^3 + (C8.T-1).h_{ls} = 0$ (9)

$C9 + C10/TK + C11/TK^2 + C12/TK^3 + C13/TK^4 + C14/TK^5 + C15 .v_g /TK - v_g = 0$ (10)

where $TK=T+273.15$

$C16 + C17.T + C18.T^2 + C19.T^3 + C20.v_l .T - v_l = 0$ (11)

$C21 + C22.T + C23.T^2 + C24.T^3 + (C25.T-1).p = 0$ (12)

The equations 8,9 10 ,11 and 12 are polynomials of the saturation temperature T found by curve fitting from steam tables valid from 100 °C to 330 °C. The coefficients C1,C2,......C25 are presented in Appendix A.

It has been proved as a result of numerical analysis that the critical condition can be defined by the following additional condition :

$$\left(\frac{\delta(\Delta P)}{\delta p}\right)_{S,\zeta} = 0 \qquad (13)$$

4. Method of computing

The equations (1) to (12) form a set of 12 non linear algebraic equations with total number of 13 dependent variables plus 2 model parameters resulting in 15 unknowns , which are $\dot{M}_g, \dot{M}_l, h_g, h_l, u_g, u_l, \zeta, h_{ls}, p, \Delta p, \alpha, v_g, v_l, S, T$. The known boundary conditions and geometrical parameters are $\dot{M}_t$, p_o, h_o, h_{lo} and A. So if any 3 unknown parameters have their numerical values specified , the simultaneous solution of the equations (1) to (12) can be done resulting in the numerical values for the all remaining unknowns. The 12 equations are solved by Newtom - Raphson method .

When calculating the critical conditions, the equation (13) is included so the number of unknowns to be specified are reduced to 2 instead of 3. Since the system of algebraic equations is non linear , it is expected to have more than one solution . Generally any solution is one of the following three types :

1-real solution with acceptable values where the flow variables and thermodynamic properties has physical meanings;

2- real solution with non acceptable values when the flow variables have no physical meaning, e.g. negative absolute temperature or negative absolute pressure , and in this case it is ignored.

3-Imaginary solution and in this case the Jacobean matrix is non-singular .

It is found that for all real acceptable cases which were analyzed specifying boundary values for total mass flow , total energy , total impulse and geometrical parameters, that there are two real solutions corresponding to the sub-critical and super-critical cases. When the two solution coincide, the critical condition is attained. At this condition the two phase pressure drop Δp is a maximum and this is expressed by equation (13).The following example explains this theorem.

4 Illustrative examples

The following example demonstrates how the equation (13) was obtained. Consider a constant area duct with 6.83 mm (0.269 in) diameter fed by steam water mixture in thermodynamic equilibrium. A parametric study was done varying ζand S for fixed boundary conditions as shown in Table I.

Table I - Illustrative example 1 numerical values.

Parameter	Value
total mass flow $\dot{M}_t$ kg/sec	0.0944
total energy $\dot{M}_t\, h_o$ kj/sec	189.6
total impulse I_o N	93.14
equilibrium constant ζ	0 or1
slip ratio S	2 , 6 or 20

Fig. 1 shows the calculated values for the outlet pressure for any given value of pressure drop for the above values of slip S and ζ. It can be observed that for a given fixed $\zeta$and S there is a maximum value of Δp which corresponds to the critical flow condition .This can be expressed mathematically by the equation (13) .
Mathematically, equation (13) represents an additional boundary condition , resulting in an additional model equation .Therefore , for the critical conditions we have 13 equations instead of 12 , but the total number of unknowns is still 15. Therefore only two unknown values must be specified.

Fig. (2) shows the variation of vapor phase velocity u_g with respect to ΔP . The critical conditions can be alternatively expressed mathematically by the equation ;

$$\left(\frac{\delta(\Delta P)}{\delta u_g}\right)_{S,\zeta} = 0 \quad \text{or} \quad \left(\frac{\delta(\Delta P)}{\delta u_l}\right)_{S,\zeta} = 0 \qquad (13a)$$

it can be concluded by combining equations (13a) and equation(4) that

$$\left(\frac{\delta I}{\delta p}\right)_{S,\zeta} = 0 \;,\; \left(\frac{\delta I}{\delta u_g}\right)_{S,\zeta} = 0 \quad \text{or} \quad \left(\frac{\delta I}{\delta u_l}\right)_{S,\zeta} = 0 \qquad (13b)$$

5. Analysis of experimental works :

Fauske (1962) carried out several experiments in an investigation work about two phase critical flow . The test section was a constant area duct with 6.83 mm (0.269 in) diameter and 2.794 m (110 in) length equipped with 6 pressure taps along the test section .A mixture of steam and water with various steam quality is fed through the test section attaining in all experiments critical conditions .

The boundary conditions were known , exit critical pressure is measured and the equilibrium constant is assumed to be 1, since the test section is too long .With that data, using the present model, all the flow variables are calculated including the two phase critical pressure drop and slip ratio. Table II shows the result of calculations for three cases where it can be observed that the slip ratio for high steam quality flows is about six.

Table II - Critical conditions for the Fauske experiments.

	Measured						Calculated											
Run	p_o bar	x ‰	$\dot{M}_t$ kg/sec	I_O N	h_o kj/kg	p bar	$\dot{M}_g$ kg/sec	$\dot{M}_l$ kg/sec	u_g m/sec	u_l m/sec	v_g l/kg	v_l l/kg	h_g kj/kg	h_l kj/kg	T °C	α	Δp bar	S
TSE-42	19.1	23	.1156	69.9	1334	4.87	.0366	.0789	401	66.8	387	1.09	2745	640.8	152	.9648	8.74	5.99
TSE-43	29.5	28	.1787	108.2	1508	7.24	.0655	.1132	505	83.9	271	1.11	2762	705	167	.9592	10.64	6.02
TSE-90	25.5	56	.0944	93.5	1995	5.88	.0564	.0379	511	84.9	327	1.1	2753	670.4	159	.9865	10.87	6.02

Table III- Critical conditions for the Super Moby Dick experiments .

	Measured							Calculated											
Run	p_o bar	x ‰	$\dot{M}_t$ kg/sec	I_O N	h_o kj/kg	α	p bar	$\dot{M}_g$ kg/sec	$\dot{M}_l$ kg/sec	u_g m/sec	u_l m/sec	v_g l/kg	v_l l/kg	h_g kj/kg	h_l kj/kg	T °C	ξ	Δp bar	S
234b4x	30	4	3.59	942	1080	.79	16.6	.2	3.389	93.6	62.2	117	1.17	2791	977	203	.22	6.09	1.51
20b340x	20	3.4	2.75	628	973	.87	11.2	.16	2.59	104.2	69.3	177	1.13	2778	859	185	.4	2.54	1.5

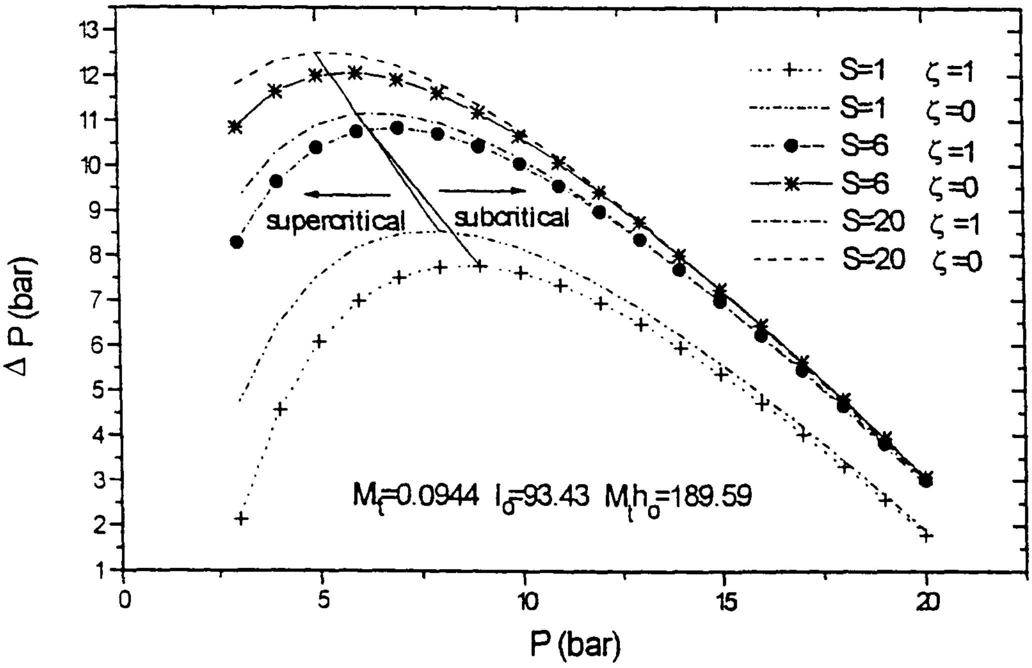

Fig. 1 Calculated pressure loss (ΔP) versus static pressure (P).

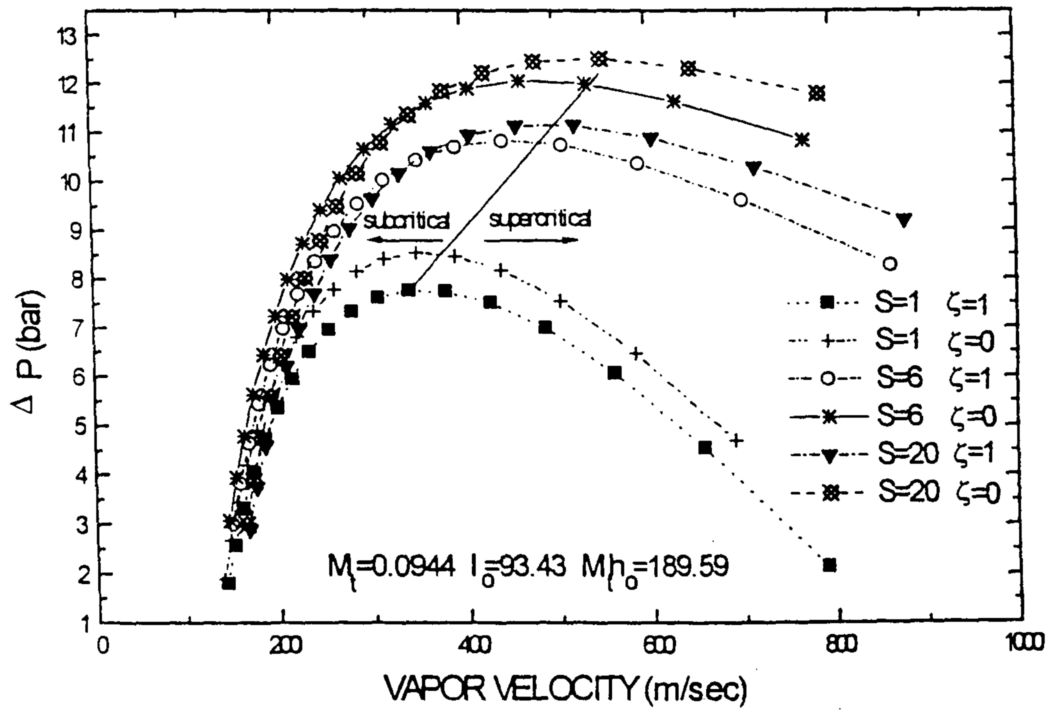

Fig. 2 – Calculated pressure loss (ΔP) versus vapor phase velocity.

Another experimental work, known as Super Moby Dick experiments (1981), were performed using a test section with inlet convergent nozzle followed by a constant area duct with 20 mm diameter and 390 mm length ending with a divergent nozzle . A low steam quality or nearly saturated steam is fed to the test section . Table III shows the calculated results of two runs.

It was stated in Super Moby Dick report that the error in the void fraction measurement is between 2% and 5% resulting in a possible lower limit for void fraction of α=0.85 for the experiment 20b340x. Using this void fraction value it was calculated ζ=0.26 and Δp=3.33 . Therefore, a 2% deviation in void fraction causes a 36% deviation in the calculated ζ value and 31% deviation in Δp value. Under this condition the sensibility of Δp and ζ upon the α values is very high. So, any correlation based upon measurement of void fraction may contain a large margin of error . From the calculated values for the critical velocity (u_g=104 m/sec and u_l=69.3 m/sec) one can observe that they are far from the sound speed in each phase. This results are not in agreement with the Kroeger's choking criteria used by Lee & Shrock [1].

6 Closure

The main characteristic of this model is its simplicity , since only one new simplifying assumption is introduced to approximate the liquid phase enthalpy through the definition of the equilibrium constant . In comparison with other models where many new simplifying assumption are used with a wide range of new unknown parameters needed , without clear advantage, this model present an alternative advantageous solution .

Due its simplicity , this model allows a complete parametric analysis with no need of any experimental correlation and their associated parametric values. Also, due to the analytical nature of the model , sensitivity analysis can be performed analytically .

The introduction of the impulse function in the momentum equation produce a form much better suited for experimental purposes. Measurement of the axial thrust (impulse) instead of void fraction can be done with very high precision resulting in correlations for equilibrium constant , slip ratio and pressure drop with low margin of error . Meanwhile, under flashing conditions, the equilibrium constant can be assumed to be 0.25 and the slip ratio can be assumed 1.5.

An important result obtained in this work is the determination of a new criteria for critical flow conditions expressed by equations (13) and (13a).

7. Nomenclature

A	cross sectional area
L	duct length
d	diameter
$\dot{M}_t$	total mass flow rate
$\dot{M}_g$	vapor phase mass flow rate
$\dot{M}_l$	liquid phase mass flow rate

u_g	vapor phase velocity
u_l	liquid phase velocity
x	steam quality
α	void fraction
p	static pressure
p_o	stagnation inlet pressure
x_o	inlet steam quality
h_o	inlet enthalpy
T	vapor phase temperature
h_g	saturated vapor enthalpy
h_{go}	inlet vapor enthalpy
h_l	liquid phase enthalpy
h_{ls}	saturated liquid enthalpy
h_{lo}	inlet liquid phase enthalpy
v_g	vapor phase specific volume
v_l	liquid phase specific volume
S	slip ratio
τ_o	local sheer stress
Δp	two phase frictional drop

8. -References

1. S. Y. LEE and V. E. SCHROCK," Critical Two-Phase Flow in Pipes for Subcooled Stagnation States with a Cavity Flooding Inception Flashing Model, " J. of Heat Transfer, Trans. of ASME, **112,** (1990).

2. R.DAGAN , E.ELIAS , E. WACHOLDER and S.OLEK, "A Two-Fluid Model for Critical Flashing Flows in Pipes," Int. Journal Multiphase Flow, **19,** (1993).

3. P. DOWNAR-ZAPOLSKI , Z. BILICK , L.BOLLE and J.FRANCO, "The Non-Equilibrium Relaxation Model for One-dimensional Flashing Liquid Flow, " Int. J. Multiphase Flow , **22** , (1996).

4. H. K. FAUSKE ,"Contribution to the Theory of Two-Phase , One-Component Critical Flow,"Argonne National Laboratory, ANL-6633, (1962).

5. CH.JEANDEY , L. GROS D'ALLON , R. BOURGINE, G, BARRIERE, "Auto Vaporisation D'ecoulements Eau/Vapeur", Rapport T.T N° 163, Commissariat a l'energie Atomique , Center d'Etudes Nucleares de Grenoble, Juillet, (1981).

6. F. J. MOODY, " Maximum Flow Rate of a Single Component , Two- Phase Mixture", J. of Heat Transfer, Trans. ASME, **87,** (1965).

7. R. E. HENRY AND H.K. FAUSKE, "The Two-Phase Critical Flow of One-Component Mixture in Nozzles , Orifices , and Short Tubes", J. of Heat Transfer, Trans.ASME, **93,** (1971).

8. A. H. SHAPIRO," The Dynamics and Thermodynamics of Compressible Fluid Flow ", The Ronald Press Company, New York, (1954).

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