

APPLICATION OF A MODEL BASED ON A PAIR OF LAPLACE TRANSFORMS FOR STANDARD LOW-ENERGY X-RAY BEAMS SPECTRAL RECONSTRUCTION

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ABSTRACT

The direct measurement of the spectrum of an X-ray beam by some spectroscopic method is relatively difficult and expensive. Spectra can be alternatively derived by an indirect method from measurements of transmission curve of the X-ray beam and the use of Laplace transforms. The objective of this work was the application of an indirect method that use a spectral model based on a pair of Laplace transforms to reconstruct experimental published spectra of standard low energy X-ray beams at radiation protection level and determine the mean photon energy from the reconstructed spectra for radiation quality specification. The spectral model was applied using calculated transmission curves and the reconstructed spectra provided a coarse approximation to experimental data. Even though, the mean photon energy of the X-ray beams determined from these reconstructed spectra present a satisfactory result showing the value of the analysis of transmission curves for the X-ray beam quality specification.

1. INTRODUCTION

Dosimeters are used in radiation protection dosimetry both for area and individual monitoring. The dosimeters used for measuring radiation levels are referred to as area monitors and the dosimeters used for recording the dose by individuals working with radiation are referred to as individual dosimeters. All dosimeters must be calibrated in terms of the appropriate quantities in reference radiation qualities.

X and gamma reference radiation qualities used to calibrate and to perform type testing of dosimeters used in radiation protection dosimetry are defined by the International Organization for Standardization (ISO). In the international standard ISO 4037-1 [1] the quality of a filtered X radiation is specified in terms of the mean photon energy, the first half-

value layer and the homogeneity coefficient. The homogeneity coefficient is the ratio of the first half-value layer to the second half-value layer.

The first and second half-value layer can be determined from attenuation measurements using ionization chambers. The mean photon energy can be determined only from the spectra of the X radiation qualities.

The direct measurement of the spectrum of an X-ray beam by some spectroscopic method is relatively difficult and expensive. Spectra can be alternatively derived by an indirect method from measurements of transmission curve of the X-ray beam and the use of Laplace transforms [2,3].

The objective of this work was the application of an indirect method that use a spectral model based on a pair of Laplace transforms to reconstruct experimental published spectra of low energy X-ray beams used to calibrate and to perform type testing of dosimeters used in radiation protection dosimetry [4,5] and determine the mean photon energy from the reconstructed spectra.

2. MATERIAL AND METHODS

The air kerma K for an X-ray beam is usually expressed by

$$K = \int_0^{\infty} \Phi_E E \frac{\mu_{tr}}{\rho}(E) dE . \quad (1)$$

where Φ_E is the energy distribution of fluence and μ_{tr}/ρ is the mass energy transfer coefficient of the air for photons of energy E .

If a narrow beam is attenuated with a material of mass thickness d , the energy distribution of fluence becomes

$$\Phi_E(d) = \Phi_E(0) e^{-\mu_m d} , \quad (2)$$

where μ_m is the mass attenuation coefficient of the attenuating material.

The air kerma becomes a function of d and is given by

$$K(d) = \int_0^{\infty} \Phi_E(0) E \frac{\mu_{tr}}{\rho}(E) e^{-\mu_m d} dE . \quad (3)$$

The relative transmission $T(d)$ is defined as the ratio between the total air kerma of the attenuated beam, $K(d)$, and the total air kerma of the unattenuated beam, $K(0)$:

$$T(d) = \frac{K(d)}{K(0)} . \quad (4)$$

Let

$$F(E) = \frac{\Phi_E(0) E \frac{\mu_{tr}}{\rho}(E)}{K(0)} . \quad (5)$$

Then, from equations (3), (4) and (5),

$$T(d) = \int_0^{\infty} F(E) e^{-\mu_m d} dE. \quad (6)$$

$F(E)dE$ is the fraction of $K(0)$ that is due photons of energy lying between energy E and $E+dE$.

By changing integration variable from E to μ_m and putting

$$F(E) \left(-\frac{dE}{d\mu_m} \right) = f(\mu_m), \quad (7)$$

equation (6) becomes

$$T(d) = \int_0^{\infty} f(\mu_m) e^{-\mu_m d} d\mu_m. \quad (8)$$

In this change of variable is assumed that μ_m is a differentiable and monotonically decreasing function of E ; $\mu_m \rightarrow 0$ when $E \rightarrow \infty$ and $\mu_m \rightarrow \infty$ when $E \rightarrow 0$.

The definition of a Laplace transform is [6]:

$$\mathcal{L}[f(t)] = F(s) = \int_0^{\infty} f(t) e^{-st} dt. \quad (9)$$

So, the transmission curve of an X-ray beam can be described as a Laplace transform:

$$T(d) = \mathcal{L}[f(\mu_m)]. \quad (10)$$

If the transmission curve is known, $f(\mu_m)$ is determined by the inversion of the Laplace transform,

$$f(\mu_m) = \mathcal{L}^{-1}[T(d)], \quad (11)$$

and the spectrum $F(E)$ derived from it as

$$F(E) = -f(\mu_m) \frac{d\mu_m}{dE}. \quad (12)$$

In this work, it is assumed that the transmission curve is represented by the mathematical model proposed by Greening [2]:

$$T(d) = \sqrt{\frac{a}{d+a}} e^{-b(\sqrt{d+a}-\sqrt{a})} e^{-\mu_m^0 d}, \quad (13)$$

where a and b are fitting parameters and μ_m^0 is assigned to the mass attenuation coefficient of the attenuating material for the maximum energy of the spectrum.

The corresponding inverse Laplace transform of the equation (13) is

$$f(a, b, \mu_m) = \sqrt{\frac{a}{\pi}} e^{\frac{b\sqrt{a} - (\mu_m - \mu_m^0)a - \frac{b^2}{4(\mu_m - \mu_m^0)}}{\sqrt[3]{(\mu_m - \mu_m^0)}}}. \quad (14)$$

The air kerma spectra $dK(E)/dE$ of the radiation qualities N-10, N-15, N-20, N-25 and N-30 of the ISO narrow spectrum series [1] determined from the fluence spectra measured by Ankerhold et. al [4,5] at 1,0 m distance between the focus of the X-ray tube and the spectrometer were used.

The transmission curves for aluminum were calculated using the following equation:

$$K(d) = \int_{E_{min}}^{E_{max}} \frac{dK(E)}{dE} e^{-\mu_m(E)d} dE, \quad (15)$$

where E_{min} is the minimum energy of the spectra and E_{max} is the maximum energy of the spectra. The monoenergetic mass attenuation coefficients $\mu_m(E)$ for aluminum were computed interpolating between two-points on a log-log scale within values taken from Hubbel and Seltzer [7].

The parameters a and b for each transmission curve were obtained from a nonlinear least-square curve fitting to these data using the equation (13).

The fitting parameters a and b were applied to the mathematical model given in the equation (14) to reconstruct the experimental published air kerma spectra [4,5]:

$$F(E) = -\sqrt{\frac{a}{\pi}} e^{\frac{b\sqrt{a} - (\mu_m - \mu_m^0)a - \frac{b^2}{4(\mu_m - \mu_m^0)}}{\sqrt[3]{(\mu_m - \mu_m^0)}}} \frac{d\mu_m}{dE}. \quad (16)$$

Values of $d\mu_m/dE$ were calculated from the differentiation of the interpolation equations used.

These reconstructed air kerma spectra were used to derive the fluence spectra from equation (5) and then calculate the mean photon energy defined by [1,4]:

$$\bar{E} = \frac{\int_{E_{min}}^{E_{max}} \Phi_E E dE}{\int_{E_{min}}^{E_{max}} \Phi_E dE}. \quad (17)$$

The monoenergetic mass energy transfer coefficient μ_{tr}/ρ for air in equation (5) were computed interpolating between two-points on a log-log scale within values of monoenergetic mass energy absorption coefficients μ_{en}/ρ for air taken from Hubbel and Seltzer [7] since the differences are negligible for photon energies considered in this work.

3. RESULTS

Figures 1 to 5 shown the transmission curves calculated using the equation (15) for the air kerma spectra of the radiation qualities N-10, N-15, N-20, N-25 and N-30 used [4,5]. The lines in the figures are the results of a nonlinear least-square curve fitting to these transmission data using the equation (13).

Measured [4,5] and reconstructed fluence spectra using equations (5) and (16) are presented in figures 6 to 10.

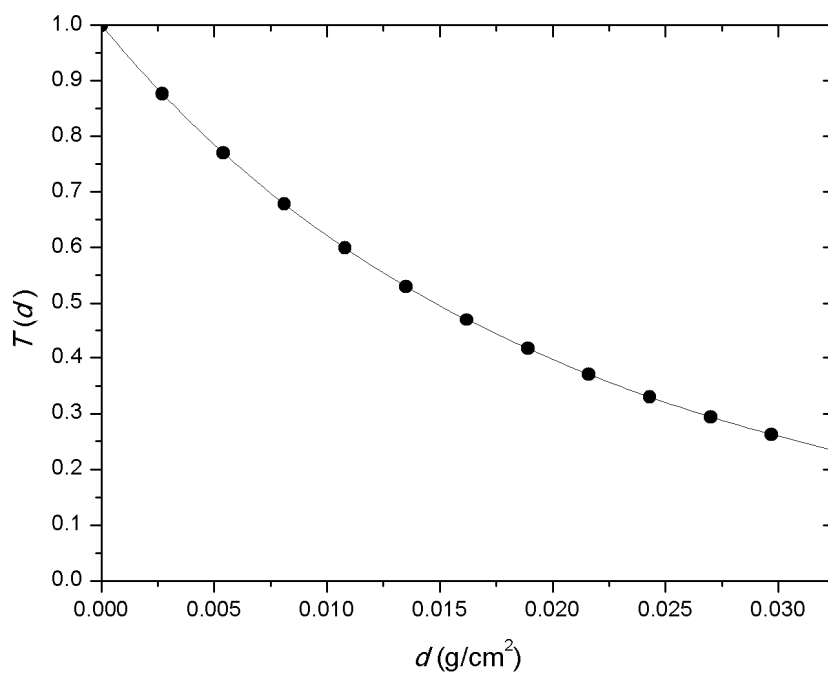


Figure 1. Transmission curve for radiation quality N-10. ●, calculated points; the solid line is the result of a nonlinear least-square curve fitting to calculated points.

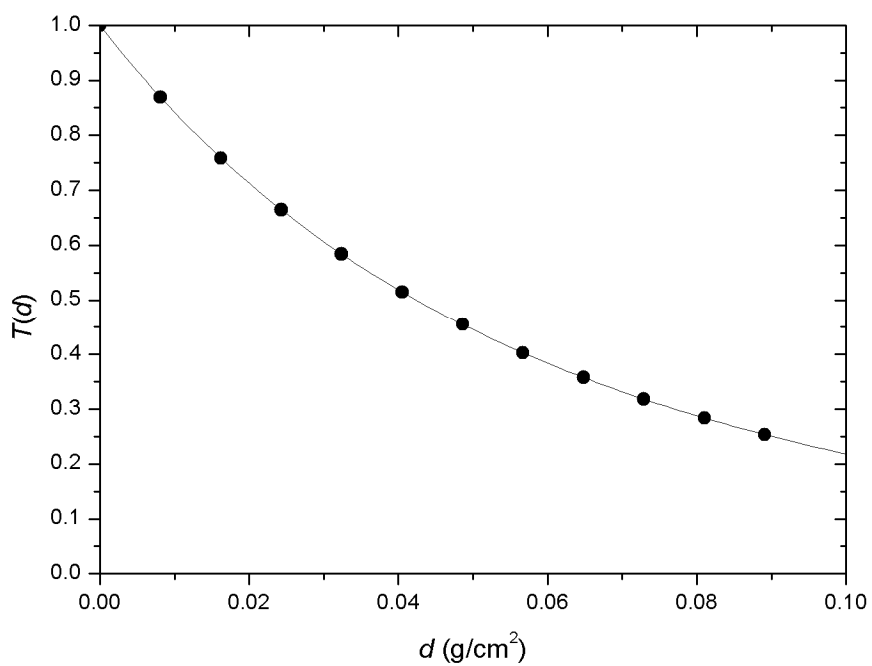


Figure 2. Transmission curve for radiation quality N-15. ●, calculated points; the solid line is the result of a nonlinear least-square curve fitting to calculated points.

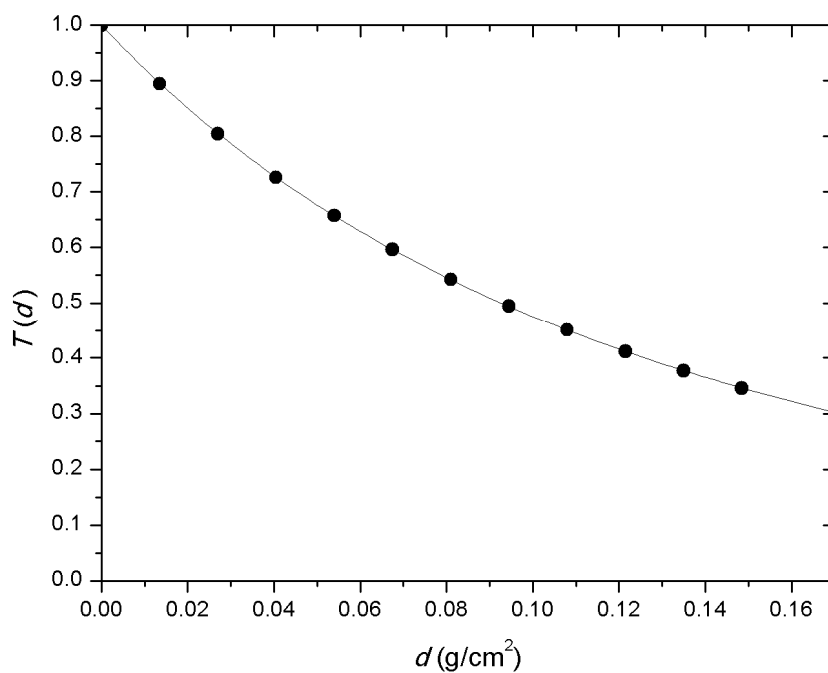


Figure 3. Transmission curve for radiation quality N-20. ●, calculated points; the solid line is the result of a nonlinear least-square curve fitting to calculated points.

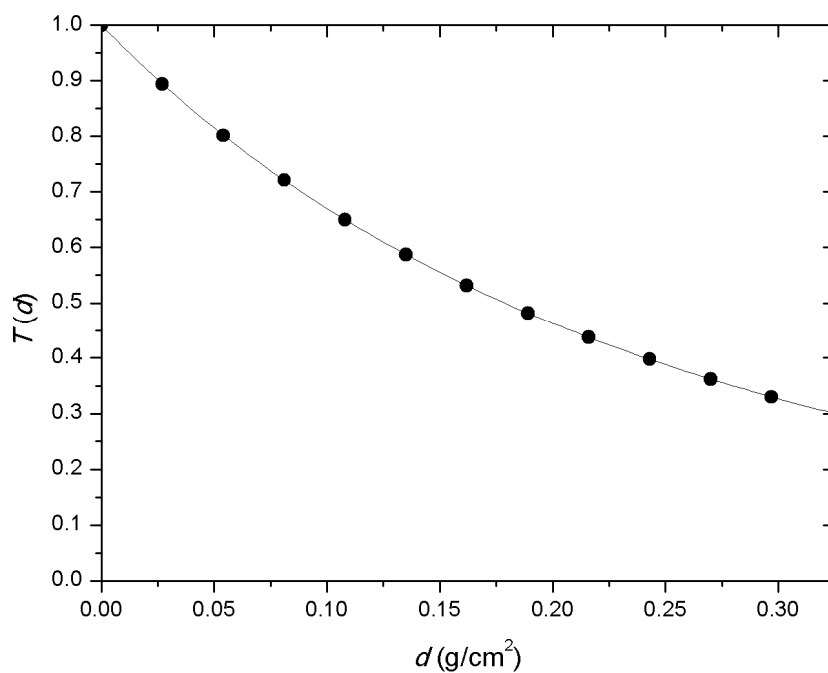


Figure 4. Transmission curve for radiation quality N-25. ●, calculated points; the solid line is the result of a nonlinear least-square curve fitting to calculated points.

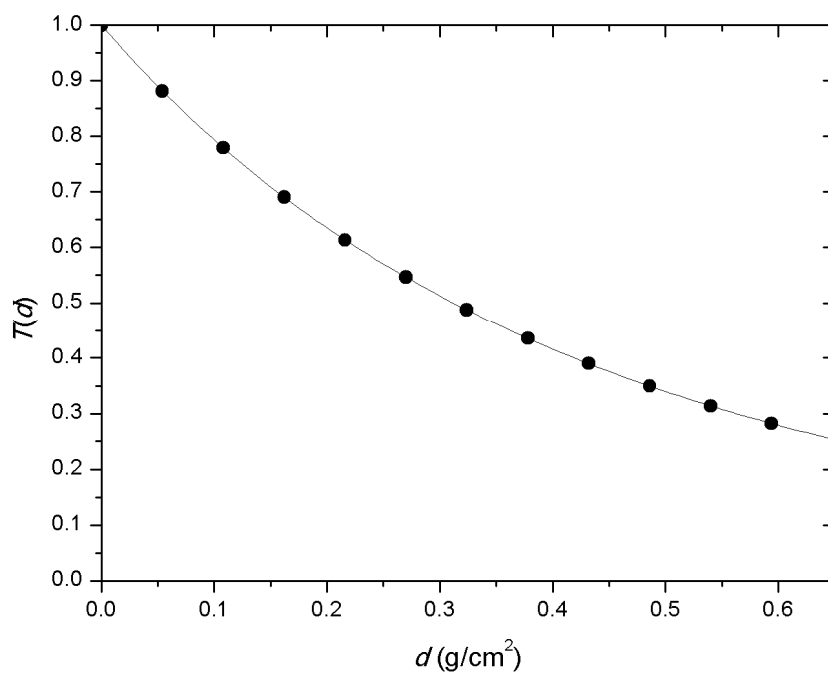


Figure 5. Transmission curve for radiation quality N-30. ●, calculated points; the solid line is the result of a nonlinear least-square curve fitting to calculated points.

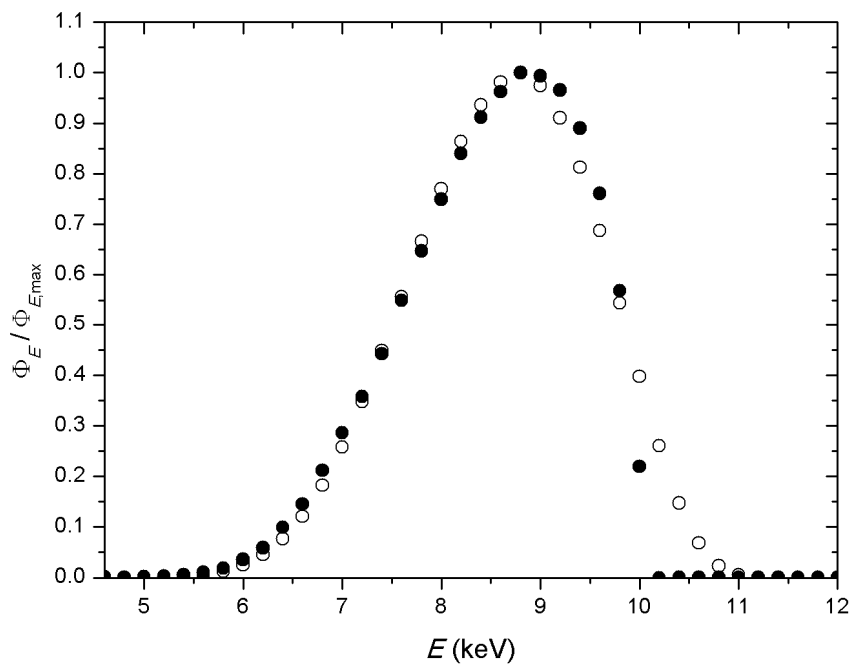


Figure 6. Fluence spectra normalized to maximum value for radiation quality N-10. ●, measured points; ○ reconstructed points.

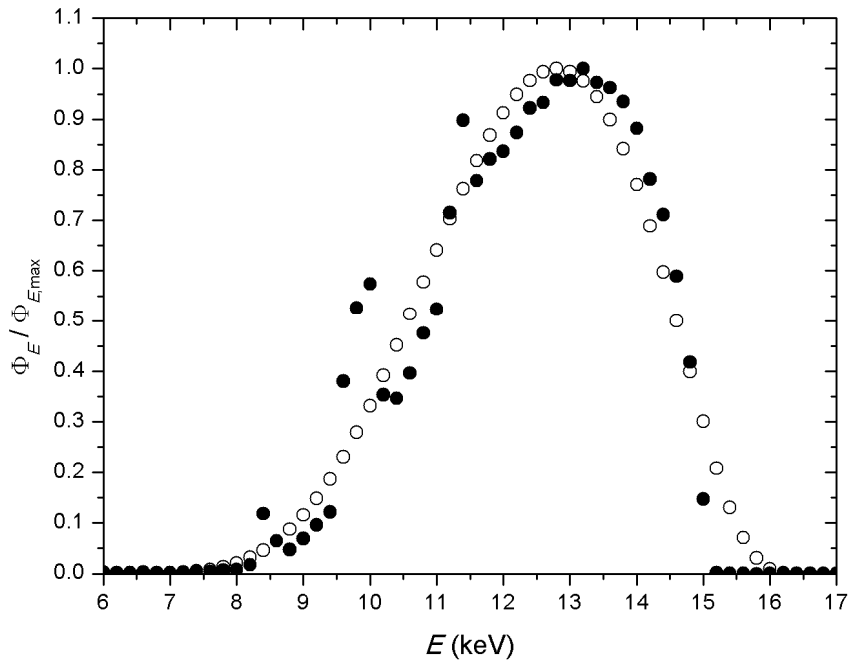


Figure 7. Fluence spectra normalized to maximum value for radiation quality N-15.
●, measured points; ○ reconstructed points.

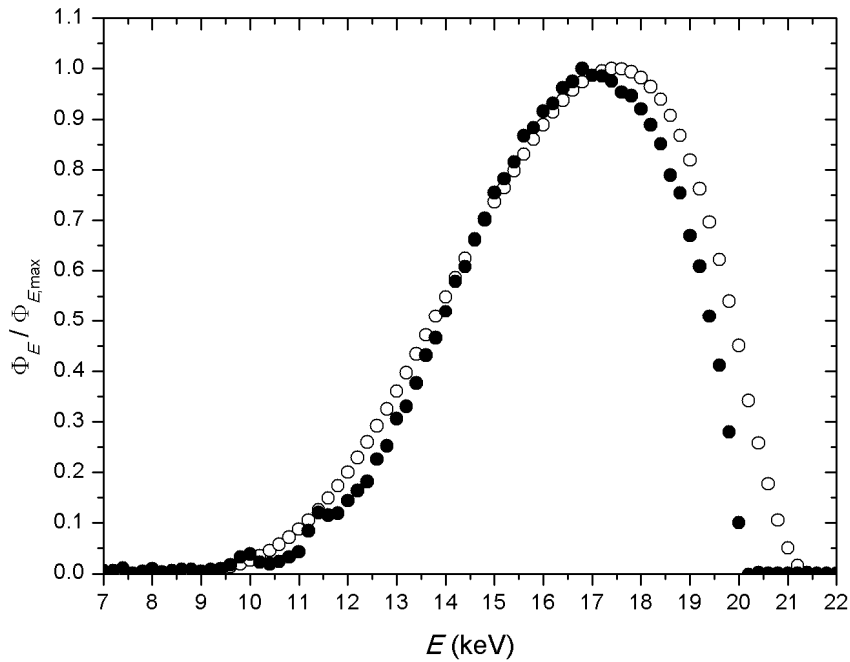


Figure 8. Fluence spectra normalized to maximum value for radiation quality N-20.
●, measured points; ○ reconstructed points.

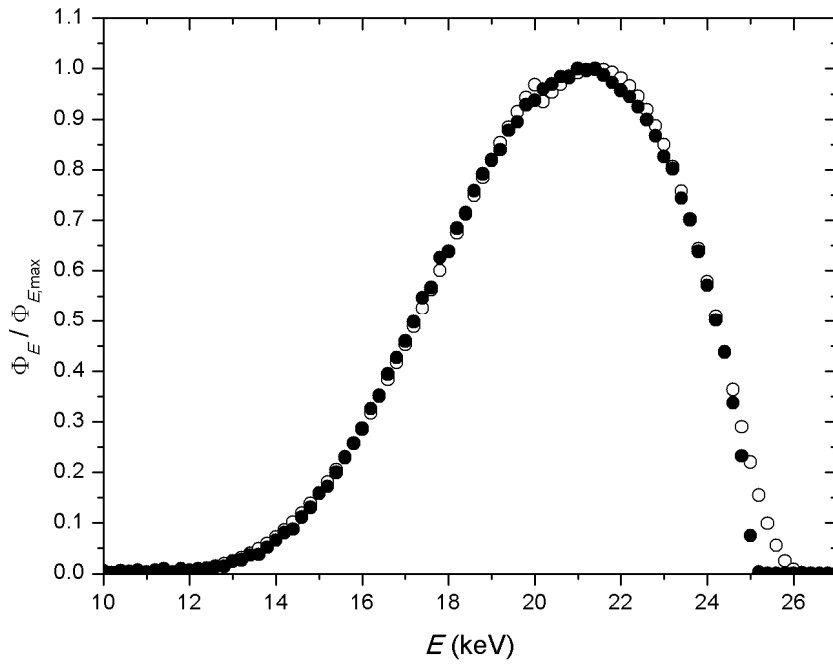


Figure 9. Fluence spectra normalized to maximum value for radiation quality N-25.
●, measured points; ○ reconstructed points.

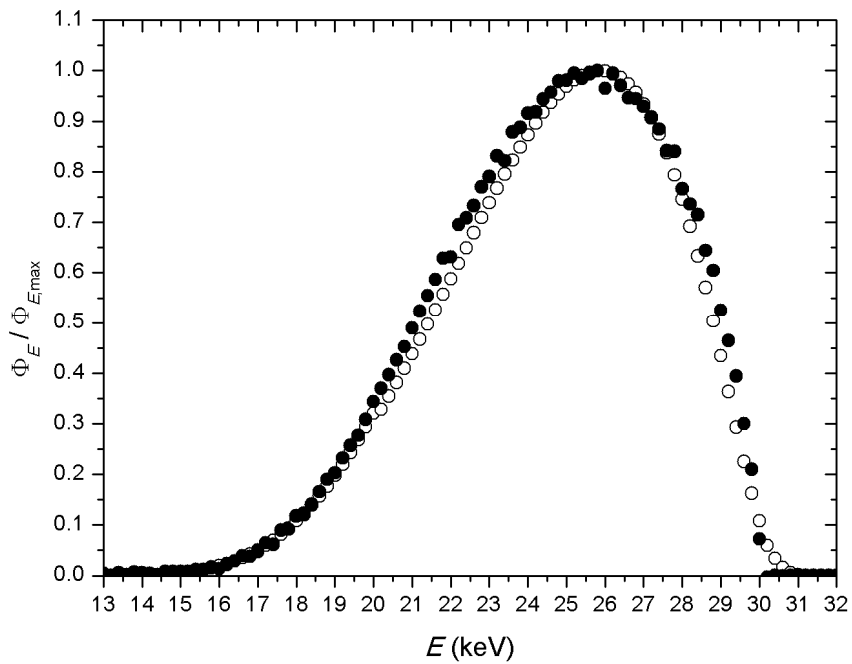


Figure 10. Fluence spectra normalized to maximum value for radiation quality N-30.
●, measured points; ○ reconstructed points.

Table 1 shows the comparison of the values of the mean photon energy for the radiation qualities N-10, N-15, N-20, N-25 and N-30 calculated from measured[4,5] and reconstructed fluence spectra, and given by the international standard ISO 4037-1 [1].

Table 1. Comparison of the values of the mean photon energy for the radiation qualities N-10, N-15, N-20, N-25 and N-30 calculated from measured[4,5] and reconstructed fluence spectra, and given by the international standard ISO 4037-1 [1].

| Radiation quality | Mean photon energy (keV) | | |
|-------------------|---|--|-------------------------|
| | Calculated from measured fluence spectrum [4,5] | Calculated from reconstructed fluence spectrum [this work] | Given by ISO 4037-1 [1] |
| N-10 | 8.5 | 8.5 | 8 |
| N-15 | 12.4 | 12.4 | 12 |
| N-20 | 16.3 | 16.5 | 16 |
| N-25 | 20.3 | 20.4 | 20 |
| N-30 | 24.6 | 24.6 | 24 |

4. DISCUSSION

Even though equation (13) represents the transmission curves with high fidelity, as can be seen in Figures 1 to 5, the reconstructed spectra showed in figures 6 to 10 provide a coarse approximation to experimental data, mainly with regard the radiation qualities N-15 and N-20, where L characteristic X-rays of tungsten appear clearly in the experimental spectra, since the mathematical model used in this work not includes characteristic X-rays. The amount of spectral information obtained by this method is limited by the numbers of parameters used in the curve fitting.

Table 1 shows that the most satisfactory results were obtained when the reconstructed spectra were used to calculate the mean photon energy, since the integration is a regularizing process for the spectral distribution [8].

5. CONCLUSIONS

A spectral model based on a pair of Laplace transforms for reconstruction of X-ray spectra from transmission data was applied using calculated transmission curves. The reconstructed spectra provide a coarse approximation to experimental data. Even though, the mean photon energy of the X-ray beams determined from these reconstructed spectra present a satisfactory result showing the value of the analysis of transmission data for the X-ray beam quality specification.

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