

Design of an Pulse-Forming Network for Driving High Power Magnetron

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Abstract— Theoretical and experimental procedures to design an pulse-forming network (PFN), have been developed in order to drive a high power magnetron. The theoretical pulse-forming network design approach is based on the Guillemin network synthesis theory. The networks obtained using this approach were numerically simulated to supply 9kV and 700ns pulses at 2kHz of pulse recurrence frequency to 31 Ω load. An experimental setup was assembled to verify the performance of a PFN type-E designed and the obtained results of the experiment are shown and discussed in this work.

Index Terms— PFN, Guillemin network, microwave radar.

I. INTRODUCTION

Pulsed microwave magnetrons require the use of pulse generators that are capable of producing a train of pulses of very sharp and short duration. The most important parameters of pulse generators are pulse width, pulse power, average power, pulse recurrence frequency (PRF), duty ratio, and impedance level. There are essentially two classes of pulse generators, namely, those in which the electric energy for the pulse is storage in an electrostatic field in the amount $1/2CV^2$, and those in which the energy is storage in a magnetic field in the amount $1/2LI^2$. Additionally, the pulse generators can be also divided in two types: those in which only a small fraction of the stored electric energy is discharged into the load during a pulse, called “hard-tube pulsers”, and those in which all of the stored energy is discharged during each pulse, called “line-type pulsers”. In this last, the energy-storage device is essentially a lumped-constant transmission line. Since this component of the line-type pulser serves not only as source of the electric energy during the pulse, but also as the pulse-shaping element, it became commonly known as pulse-forming network (PFN) [1].

The PFN in a line-type pulser consists of a set of inductors and capacitors which may be put together in any one of a number of possible configurations. The configuration chosen for a particular purpose depends on the ease which the network can be fabricated, as well as, on the specific pulser characteristic desired. The values of the inductance and capacitance elements in such a network can be calculated to give an arbitrary pulse shape when the configuration, pulse width, impedance level, and load characteristics are specified. The theoretical basis for these calculations and the detailed discussion of various networks are given in this paper [1]-[2].

In this work, it is reported some results of PFN performance that was developed to be used in a driving magnetron circuit. The PFN features are: 31 Ω of impedance level, 2kHz of PRF, 0.7 μ s of pulse duration, and 11.4 nF of total energy-storage capacitance.

This paper is organized as follows. Section II describes the PFN theory design. Section III introduces the procedures of the network LC synthesis using the state variables approach. Section IV presents the experimental set-up and results, and conclusions are in Section V.

II. GUILLEMIN’S THEORY AND THE VOLTAGE-FED NETWORK

The technique used by Guillemin’s theory on design of the PFN is based on the Fourier series expansion of the desired output pulse. The trigonometric Fourier series for the rectangular pulses, suitable to drive a magnetron contains only odd terms, and it may be found by:

$$i(t) = \sum_{v=1,3,5}^{\infty} b_v \sin\left(\frac{vp}{t}\right), \quad (1)$$

where $i(t)$ is the electric current pulse, v represent the terms odd of the series, t is the pulse duration and b_v are the coefficients which determine the amplitude of the pulse. Each term of the Fourier series at (1) consists of a sinoidal wave at each section of the PFN, and the electric current pulse can also be written as:

$$i_v(t) = V_N \sqrt{\frac{C_v}{L_v}} \sin\left(\frac{t}{\sqrt{L_v C_v}}\right), \quad (2)$$

where, V_N , L_v , and C_v denote the PFN voltage, the inductance and the capacitance, respectively. These parameters may be determined by:

$$L_v = \frac{Z_N t}{4}, \quad (3)$$

$$C_v = \frac{4}{v^2 p^2} \cdot \frac{t}{Z_N}. \quad (4)$$

The resulting network is shown in Fig. 1, known as the type-C Guillemin network, and consists of a series of resonant LC elements connected in parallel [1].

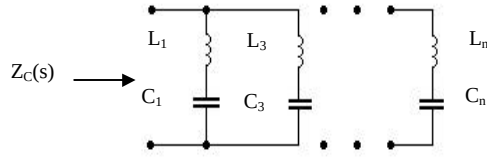


Fig. 1. PFN type-C derived by Fourier-series analysis.

For a four section network type-C, the function impedance $Z_C(s)$ can be written as:

$$Z_C(s) = \frac{a_8 s^8 + a_6 s^6 + a_4 s^4 + a_2 s^2 + a_0}{b_7 s^7 + b_5 s^5 + b_3 s^3 + b_1} \quad (5)$$

where a_i and b_j are the polynomials coefficients.

The PFN type-C is inconvenient for practical use, because the inductances have appreciable distributed capacitance and capacitors have a wide range of values which makes the manufacture difficult and expensive. Therefore, it is desirable to devise equivalent networks that have different ranges of values for capacitance and inductance. Theoretically, it is possible to determine a large number of equivalent network based on mathematical operations on the impedance and admittance functions. For instance, using the Foster's theorem, the admittance function for network of Fig. 1 may be written, by inspection, by means of:

$$Y(s) = \sum_{v=1,3,\dots}^n \frac{C_v s}{L_v C_v s^2 + 1} \quad (6)$$

Inverting the quocient of the (6), the impedance function is:

$$Z(s) = \frac{1}{Y(s)} = \frac{\prod_{v=1,3,\dots}^n (L_v C_v s^2 + 1)}{\sum_{v=1,3,\dots}^n C_v s \prod_{g=1,3,\dots}^n (L_g C_g s^2 + 1)} \quad (7)$$

The function $Z(s)$ may then be expanded in partial fractions about its poles, and an expression of the following form is obtained as:

$$Z(s) = \frac{A_0}{s} + \sum_{v=2,4,\dots}^{2n-2} \frac{A_v s}{B_v s^2 + 1} + A_{2n} s \quad (8)$$

For a four section network, $Z(s)$ can be written as:

$$Z_A(s) = A_0 \left[\frac{K_0}{s} + \frac{2K_2 s}{s^2 + w_2^2} + \frac{2K_4 s}{s^2 + w_4^2} + \frac{2K_6 s}{s^2 + w_6^2} + K_\infty s \right] \quad (9)$$

where K_n are the residues of $Z_A(s)$, w_n are the resonance frequencies, and A_0 is a constant. Equation (9) represents the impedance function for the network of Fig. 2 [1].

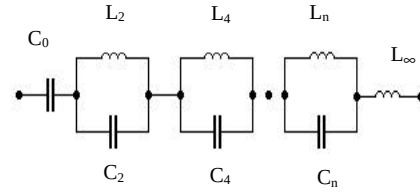


Fig. 2. PFN type-A derived by Foster's theorem.

Thus, C_0 is equal to the sum of the C_v 's shown Fig. 1, and L_{2n} is equal to inductance of all the L_v 's in parallel.

One additional form of physically realizable network may be found making continued-fraction expansion of the reactance or admittance functions and identifying the coefficients thus obtained with network elements. This procedure is known as Cauer theorem and represents a ladder network (10), resulting in the type-B Guillemin network, shown in Fig. 3. This PFN correspond the transmission-line equivalent [1]-[2]. The (10) means a series arms expressed as impedances and the shunt arms as admittances [3]-[4].

$$Z_B(s) = L'_1 s + \frac{1}{C'_2 s + \frac{1}{L'_3 s + \frac{1}{C'_4 s + \frac{1}{L'_5 s + \frac{1}{C'_6 s + \frac{1}{L'_7 s + \frac{1}{C'_8 s}}}}}} \quad (10)$$

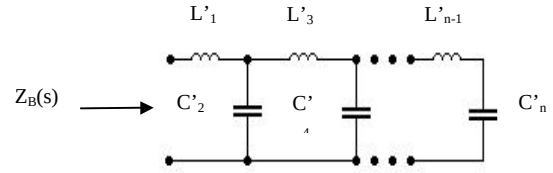


Fig. 3. PFN type-B derived by Cauer's theorem.

The essential PFN obtained by canonical network forms is the type-D of the Guillemin shown in Fig. 4, which has equal capacitances. In term of the manufacture, it is desirable because the capacitors for high voltage networks are difficult item to manufacture. The network of Fig. 1 is chosen to derive the PFN type-D. The negative inductances are due, in the shunt legs, to compensate the modified values of the capacitances of the PFN type-C [1].

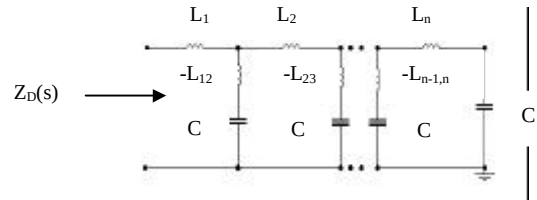


Fig. 4. PFN type-D having equal capacitances and negative inductances.

The capacitances of Fig. 4 may be to obtained by $\Sigma C_N/n$, where C_N is the total capacitance and n is the number of sections of the PFN type-C, whereas the inductances are all unknown. In order to obtained the network inductances the following procedure is used. The admittance and impedance

functions for PFN type-C are given by Foster's theorem [1]. In the PFN type-D, L_1s is subtracted from the $Z(s)$ so that

$$Z_1(s) = Z(s) - sL_1. \quad (11)$$

The series combination of L_{12} and C corresponds to a zero of $Z_1(s)$ or to a pole of $Y_1(s) = 1/Z_1(s)$. Hence the admittance is:

$$Y_1(s) = \frac{sC}{L_{12}Cs^2 + 1}. \quad (12)$$

The poles of $Y_1(s)$ are given by $s = \pm(1/L_{12}C)^{1/2}$, then

$$Y_1(s) = \frac{a_1}{s - s_1} + \frac{a_2}{s + s_1} + Y_2(s), \quad (13)$$

where $Y_2(s)$ is a remainder admittance function at $\pm s_1$. The constants a_1 and a_2 are found by algebra [1], and $Y_1(s)$ to turn:

$$Y_1(s) = \frac{2as}{s^2 - s_1^2} + Y_2(s). \quad (14)$$

The first term of the right-hand member of (14) must be the admittance of L_{12} and C in series, so:

$$\frac{sC}{L_{12}Cs^2 + 1} = \frac{\frac{1}{L_{12}}s}{s^2 + \frac{1}{L_{12}C}} = \frac{2as}{s^2 - s_1^2}, \quad (15)$$

From (15) two equations for s_1 and L_{12} can be obtained:

$$L_{12} = \frac{1}{2a} = \frac{Z'(s_1) - L_1}{2}, \quad (16a)$$

and

$$\frac{1}{L_{12}C} = -s_1^2, \quad (16b)$$

where s_1^2 is a root of $Z(s) - L_1s = 0$, and it is found by eliminating L_{12} between (16a) and (16b), and is given by:

$$\frac{1}{C} = \frac{-s_1^2}{2} \left[Z'(s_1) - \frac{Z(s_1)}{s_1} \right]. \quad (17)$$

Since C is known and s_1 is unknown, (17) determines s_1 , and then:

$$L_1 = \frac{Z(s_1)}{s_1}, \quad (18a)$$

and

$$L_{12} = \frac{1}{-Cs_1^2} = \frac{1}{2} [Z'(s_1) - L_1], \quad (18b)$$

can be calculated. The above procedure to obtain L_{12} and L_1 , can be repeated on the remainder function $Z_2(s) = 1/Y_2(s)$,

where $Y_2(s)$ is defined by (13) and then L_2 and L_{23} are determined. This procedure is repeated until all of the roots are exhausted.

The negative inductances, Fig. 4, can be realized physically by use of the mutual inductance concept (Fig. 5) known as network type-E of the Guillemin. This PFN is practical because all the inductances may be provided by single winding coil, and the capacitors may be tapped in at proper points. To find the values of inductances the PFN type-E, it is used the procedure below [1]. For instance, for a PFN type-E of the four sections:

$$\begin{aligned} L_1 - L_{12} &= L_{E1} \\ L_2 - L_{12} - L_{23} &= L_{E2} \\ L_3 - L_{23} - L_{34} &= L_{E3} \\ L_4 - L_{34} &= L_{E4} \end{aligned}, \quad (19)$$

where L_{E1}, L_{E2}, L_{E3} and L_{E4} are inductances of the type-E.

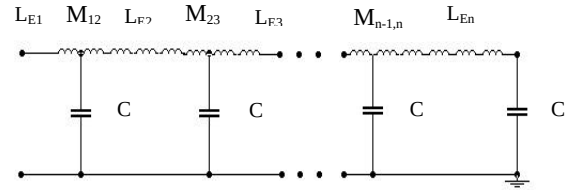


Fig. 5. PFN type-E having equal capacitances and mutual-inductances.

Hence, the four canonical forms of PFN are equivalent to that of Fig. 1, and they can be found by mathematical operations on the $Y(s)$ and $Z(s)$ functions [1].

III. SOME PFN SYNTHESIS

In order to investigate the characteristics of the PFN voltage-fed, a computer code to simulate the behavior of the circuits shown in Figs. 1 to 5 was developed. The circuit analysis was carried out using the state variables approach. So, the state equations for the type-A, B, C, D and E PFNs, written using the inductor currents and the capacitor voltages as state vector elements, can be written as:

$$\dot{\bar{x}}(t) = A\bar{x}(t) + bw(t), \quad (20)$$

where, A is an $m \times m$ constant matrix called the evolution process matrix, $\bar{x}(t)$ is the circuit state vector, b is the control circuit matrix, and $w(t)$ the excitation scalar of circuit.

The system was integrated using a fourth order Runge-Kutta algorithm and the computer code was written in Turbo Pascal 1.5 programming language [5]-[7]. The output of of 0.7μs and 11.4 nF PFN was connected to a 31Ω resistive load R_L and the output pulse waveforms was obtained for a charging voltage of 9kV input.

The output pulses obtained by simulation of four sections LC of PFN type-A, B, C, D and E with a resistive load R_L are

shown in Figs. 6 to 10, respectively. The inductance and capacitance values are listed in Tables I to V.

TABLE I. PFN TYPE-A: CAPACITORS IN nF AND INDUCTORS μ H.

Parameter	Values
C_0	11.4
C_2	6.03
C_4	7.07
C_6	9.46
L_2	2.07
L_4	0.43
L_6	0.13
L_{∞}	0.97
Load (R_L)	31.0

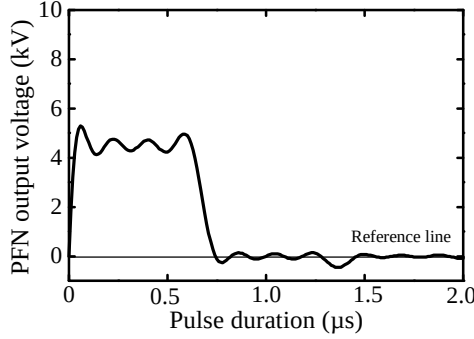


Fig. 6. Waveform pulse output of the type-A network with four-sections LC.

Figs. 6 to 10 show that the PFN and the load are matched. It may be seen that C_0 , from Table I, is equal to the sum of the C_v 's shown in Table II, and that L_{∞} is equal to the inductance of all the L_v 's in parallel. It may be also seen in Table III, that each branch of the type-C PFN has an impedance Z_n given by

$$Z_n = \frac{4}{np} \sqrt{\frac{L_n}{C_n}} = 31\Omega. \quad (21)$$

TABLE II. PFN TYPE-B: CAPACITORS IN nF AND INDUCTORS μ H.

Parameter	Values
C_2'	4.85
C_4'	2.52
C_6'	2.02
C_8'	1.99
L_1'	3.02
L_3'	2.12
L_5'	1.77
L_7'	0.97
Load (R_L)	31.0

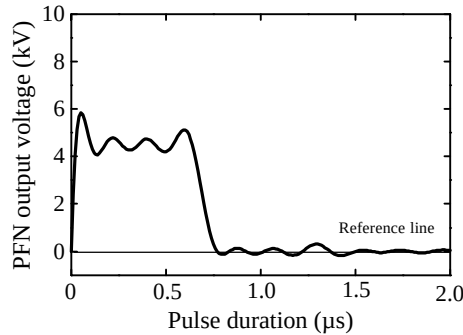


Fig. 7. Waveform pulse output of the type-B network with four-sections LC.

TABLE III. PFN TYPE-C: CAPACITORS IN nF AND INDUCTORS IN μ H.

Parameter	Values
C_1	9.14
C_3	1.01
C_5	0.36
C_7	0.18
L_1	5.42
L_3	5.42
L_5	5.42
L_7	5.42
Load (R_L)	31.0

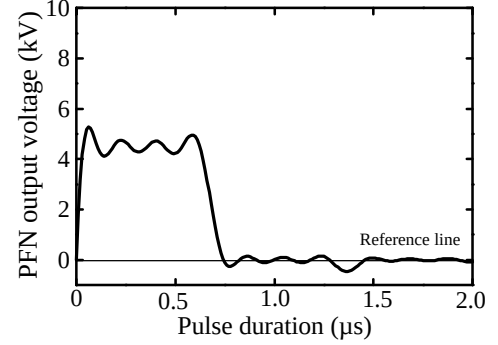


Fig. 8. Waveform pulse output of the type-C network with four-sections LC.

TABLE IV. PFN TYPE-D: CAPACITORS IN nF AND INDUCTORS IN μ H.

Parameter	Values
L_1	5.16
L_2	5.41
L_3	5.57
L_4	5.85
L_{12}	-0.94
L_{23}	-0.53
L_{34}	-0.58
C	2.85 (each)
Load (R_L)	31.0

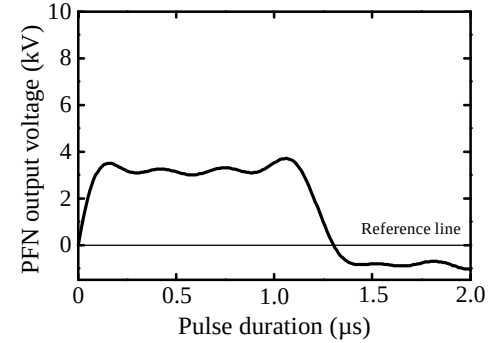


Fig. 9. Waveform pulse output of the type-D network with four-sections LC.

TABLE V. PFN TYPE-E: CAPACITORS IN nF AND INDUCTORS IN μ H.

Parameter	Values
L_1	6.1
L_2	5.58
L_3	5.81
L_4	7.43
M_{12}	0.3
M_{23}	0.35
M_{34}	0.3
C	2.85 (each)
Load (R_L)	31.0

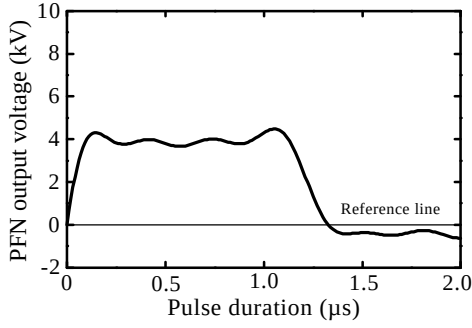


Fig. 10. Waveform pulse output of the type-E network with four-sections LC.

The results show that networks designed to simulate a lossless transmission line have some limitations. This is evident by overshoots near of the beginning of the pulse and the oscillations during the pulse. This effect is due the first four odd terms of a rectangular pulse Fouriers series in the synthesis procedure. But, it can be noted that there is a perfect equivalent between all networks of the Guillemin even with different values of capacitors and inductors.

IV. EXPERIMENTAL SET-UP AND RESULTS

In order to verify the accuracy of the PFN simulated, an experimental set-up shown in Fig. 14 was assembled. Its equivalent circuit is shown in Fig. 11. It consists basically of a voltage power supply that feeds a PFN type-E through a charging reactor L_c and a fast blocking diode D . A hydrogen thyatron Th model 5C22 was used to switch the PFN at 2kHz of PRF. The output waveforms, voltage and currents pulses, in a 31Ω matched load and in a 62Ω mismatched load are shown in Fig. 12 and Fig. 13, respectively. These conditions are relevant to magnetron modulator design. The waveforms were recorded using an oscilloscope Tektronix TDS-210 connected to a computer.

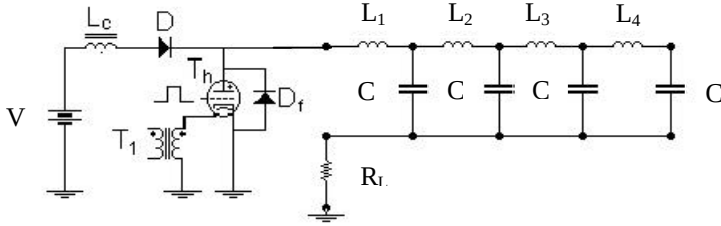


Fig. 11. Experimental set-up.

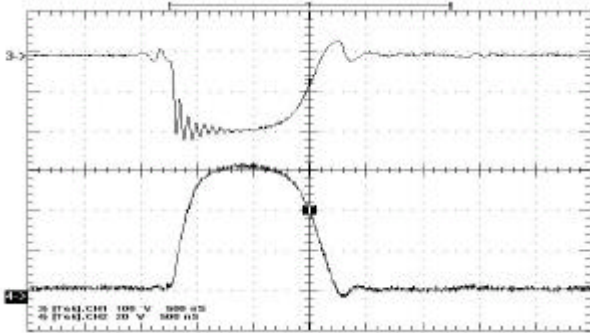


Fig. 12. Output pulse waveform with a 31Ω load resistor.

Fig. 12 shows a good agreement between the theoretical and experimental results and the matching impedance can be observed between the PFN type-E and the resistive load. However, increasing load impedance introduces a series of steps in the output waveform. These steps are all of the same signal because the load impedance is larger than the PFN impedance (Fig. 13).

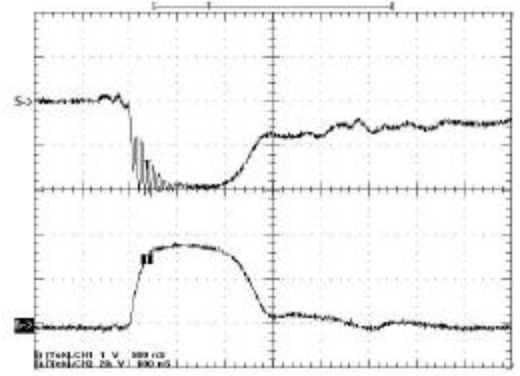


Fig. 13. Output pulse waveform with a 62Ω load resistor.

The reflection coefficient Γ_L [8] of the mismatching observed in Fig. 13 between the PFN type-E and the resistive load of the 62Ω is given by:

$$G_L = \frac{Z_{PFN} - Z_{Load}}{Z_{PFN} + Z_{Load}}, \quad (22)$$

where Z_{PFN} and Z_{Load} are impedances the PFN and load, respectively.



Fig. 14. Experimental assembling used for PFN performance measurements.

V. CONCLUSIONS

In this work the performance of five types of PFNs were simulated and investigated. The theoretical investigation was conducted using the Guillemin synthesis network theory and the state variable approach. The resulting equation differential

system was integrated using a fourth order Runge-Kutta algorithms. A test circuit modulator based on the theoretical of PFN was assembled and the results shown that it is suitable to drive a high power magnetron.

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