

STUDY OF THE KINETICS PARAMETERS FOR SUBCRITICAL MEDIA DRIVEN BY SOURCE

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ABSTRACT

This paper presents a theoretical study of reactor kinetics focusing on the methodology of calculation and the experimental measurements of the so-called kinetic parameters. A comparison between the methodology based on the Dulla's formalism and the classical method is made. The objective is to exhibit the dependence of the parameters on sub criticality level and perturbation. Two different slab type systems were considered: thermal one and fast one, both with homogeneous media. One group diffusion model was used for the fast reactor, and for the thermal system, two group diffusion model, considering, in both case only one precursor's family. For reason of simplicity, several additional assumptions were made for calculation of two group method: no up-scattering, fission reaction occurring only in thermal group, etc. The solutions for subcritical systems were obtained using the expansion method, and for critical systems, the methods presented in classical textbooks of reactor physics were applied. The numerical results presented their dependence on sub criticality level and perturbation.

I. INTRODUCTION

The kinetic parameters measurements are, in many cases, grounded in simple models (like point kinetics) that are based on critical state of the reactor. Moreover, most of methods for calculation of the parameters are based on models developed for critical systems. However, accordingly with Dulla et al. [1] the definition of kinetic parameters of source-driven subcritical systems must be investigated deeply since these parameters vary with sub criticality level and, also, with the perturbation introduced into calculation of these parameters.

II. POINT KINETIC EQUATIONS AND KINETIC PARAMETERS

The transport equation describing a multiplicative system in steady state can be written as follows:

$$L_0 \Phi_s + M_0 \Phi_s + S_0 = 0 \quad (1)$$

L_0 and M_0 are removal and multiplication operators of reference system, respectively; S_0 , an external source; and Φ_s , the steady flux.

The point kinetic equations obtained classically from critical state as reference are:

$$\frac{dP}{dt} = \frac{(\rho - \beta_{eff})}{\Lambda} P(t) + \lambda C, \quad \frac{dC}{dt} = \frac{\beta_{eff} P(t)}{\Lambda} - \lambda C \quad (2)$$

Where the kinetic parameters ρ , β_{eff} and Λ have a consistent definition in term of the solution of the adjoint equation for the system in steady state near the criticality ($K_{eff} \approx 1$) [2]:

$$\begin{aligned} \rho &= \frac{1}{F} \langle \Phi_{cr}^+, (\delta L + \delta M) \Phi_{cr} \rangle = \frac{\delta K_{eff}}{K_{eff}} & C &= \frac{1}{\Lambda F} \langle \Phi_{cr}, C \rangle \\ \beta_{eff} &= \frac{1}{F} \langle \Phi_{cr}^+, M_d \Phi_{cr} \rangle & & \text{with normalization:} \\ \Lambda &= \frac{1}{F} \left(\left\langle \frac{1}{v} \right\rangle \Phi_{cr}^+, \Phi_{cr} \right) & F &= \langle M_p \Phi_{cr} \rangle \\ & & & (\Phi_{cr} \equiv \text{critical flux}) \end{aligned}$$

However, according to Dulla's presentation it's not correct to extrapolate calculation of the parameters for subcritical system directly from methodology developed for critical systems.

Using the similar techniques of factorization, perturbation and projection used in classical method, Dulla demonstrated that equations of point kinetics type for the amplitude function can be obtained with consistent definition of the kinetic parameters, i.e:

$$\frac{dP(x)}{dt} = \frac{(\rho_s - \beta_s)}{\Lambda_s} P(t) + \lambda C_s + S, \quad \frac{\partial C_s}{\partial t} = \beta_s P(t) - \lambda C_s, \quad (3)$$

where

$$\begin{aligned} \rho_s &= \tilde{\rho}_s + \rho_{s,0} & \Lambda_s &= \frac{\langle \Phi_s^+ | \left(\frac{1}{v} \right) \Phi_s \rangle}{I} \\ \tilde{\rho}_s &= \frac{\langle \Phi_s^+ | (\delta L + \delta M) \Phi_s \rangle}{I} & S &\equiv \frac{\langle \Phi_s^+ | S \rangle}{I} \\ \rho_{s,0} &= - \frac{\langle \Phi_s^+ | S_0 \rangle}{I} & C &\equiv \frac{\langle \Phi_s^+ | C \rangle}{I} \\ \beta_s &= \frac{\langle \Phi_s^+ | M_{d,0} \Phi_s \rangle}{I} & I &\equiv \langle \Phi_s^+ | M_0 \Phi_s \rangle + \langle \Phi_s | S_0 \rangle \end{aligned}$$

The parameter I is related with the ‘‘importance’’ of the first neutron generation introduced into the system by fissions and the source [1]

III. RESULTS

The system considered here is a slab reactor with width H and homogeneous medium. Diffusion model was used to calculate the parameters: one-group diffusion model for a fast system and two-group diffusion model for a thermal system. Several assumptions were made: isotropic scattering, the flux vanishing at the boundary, $S_0^+ \equiv \nu \Sigma_f$ and no feedback.

The steady flux and external source can be expanded in term of the eigenfunctions:

$$\Phi_s = \sum_{n=0}^{\infty} A_n \cdot \varphi_n, \quad S_0 = \sum_{n=0}^{\infty} s_n \cdot \varphi_n, \quad (4)$$

where A_n and s_n are coefficients to be determined and φ_n eigenfunctions of Helmholtz equation that satisfy the boundary condition: $\varphi_n(H/2)=0$, $\varphi_n(-H/2)=0$. From 2 groups model the coefficients A_n and s_n are matrix and S_0 , Φ_s and φ_n are vectors. Substituting these terms of expansion in Equation (1) the steady flux and its adjoint can be obtained. It was assumed that $S_0=\delta(x)$. The solutions for 1 group model are:

$$\Phi_s(x) = \left(\frac{2}{H} \right) \sum_{n=0}^{\infty} \left[\frac{\cos(B_n x)}{D(B_n^2 + B_m^2)} \int_{-\frac{H}{2}}^{\frac{H}{2}} S_0(x) \cos(B_n x) dx \right], \quad \Phi_s^+(x) = \left(\frac{2}{H} \right) \sum_{n=0}^{\infty} \left[\frac{\cos(B_n x)}{D(B_n^2 + B_m^2)} \int_{-\frac{H}{2}}^{\frac{H}{2}} S_0^+(x) \cos(B_n x) dx \right] \quad (5)$$

and for 2 groups model:

$$\Phi_s = -\frac{2}{H} \sum_{n=0}^{\infty} \Phi_n \left[\int_{-\frac{H}{2}}^{\frac{H}{2}} \cos\left(\frac{(2n+1)\pi}{H}x\right) S_1 dx \right] (L_0 + M_0)^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad (6)$$

$$\Phi_s^+ = -\frac{8 \nu_2 \Sigma_f}{\pi H} \sum_{n=0}^{\infty} \Phi_n \frac{1}{(2n+1)} (L_0^+ + M_0^+)^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

For two group model it was assumed that there was no up-scattering, fissions occurring only in thermal group and neutrons emerging only in fast group.

The nuclear data used to calculate the kinetic parameters are in **Table 1** and **Table 2** for a fast and a thermal system, respectively.

Table 1 Nuclear data for 1 energy group of a typical fast system [3]

$1/\nu$ [s cm ⁻¹]	Σ_t [cm ⁻¹]	Σ_a [cm ⁻¹]	$\nu\Sigma_f$ [cm ⁻¹]	Σ_s [cm ⁻¹]
0,9062 e-7	3,45987e-1	1,58430e-2	3,33029e-2	3,30178e-1

Figures 1, 2 and **3** show the parameters calculated in this method (β_s , A_s and ρ_s) and, also, those obtained by using the classical method, i.e, β_{eff} , A and ρ .

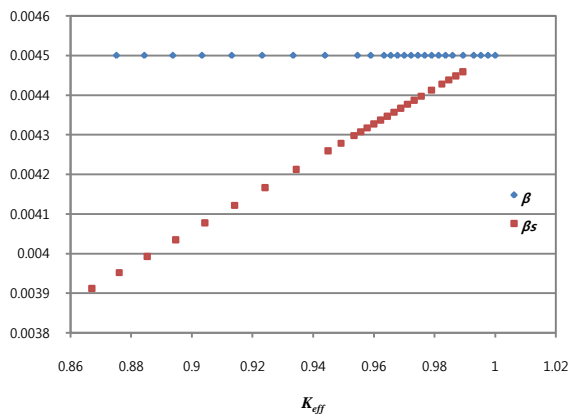


Figure 1 The parameters β_{eff} e β_s versus K_{eff} in 1 group model

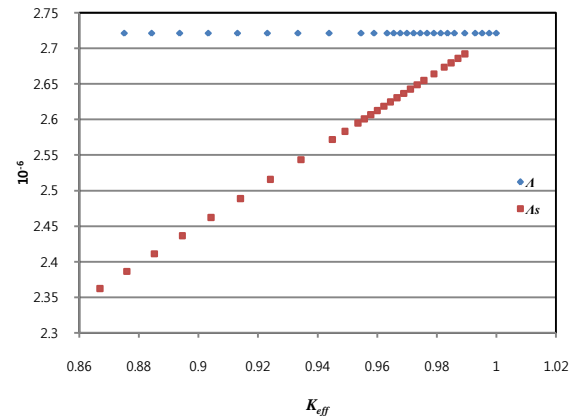


Figure 2 The parameters A e A_s versus K_{eff} in 1 group model.

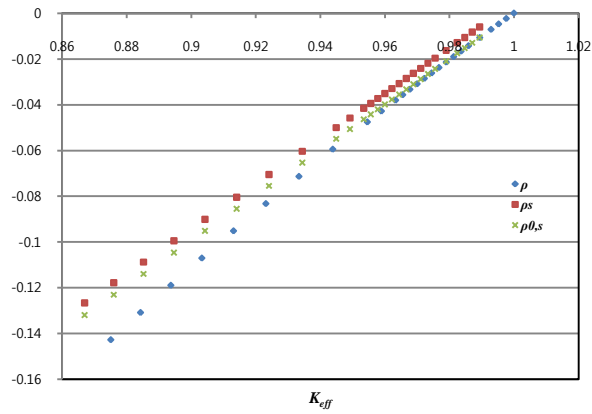


Figure 3 The parameters $\rho, \rho_{s,0} \in \rho_s$ versus K_{eff} in 1 group model.

The subcriticality level was altered by varying the absorption cross section values without changing the geometry. Also, in each case a perturbation of 2% was introduced into Σ_a . It can be noted in these figures that the parameters β and Λ obtained by classical method undergo insignificant variation while β_s and Λ_s increase with K_{eff} . The parameters $\rho_{s,0}$, ρ_s and ρ all vary with K_{eff} variation, but the differences reduce near the criticality ($K_{eff}=1$). This fact demonstrates that using the values obtained on criticality as those parameters values in point kinetic equations it will lead to incorrect results in subcritical states.

Similar behaviors of the parameters can be observed in **Figures 4, 5** and **6** even though the system in question now is different from the former that was a fast one in composition and geometry.

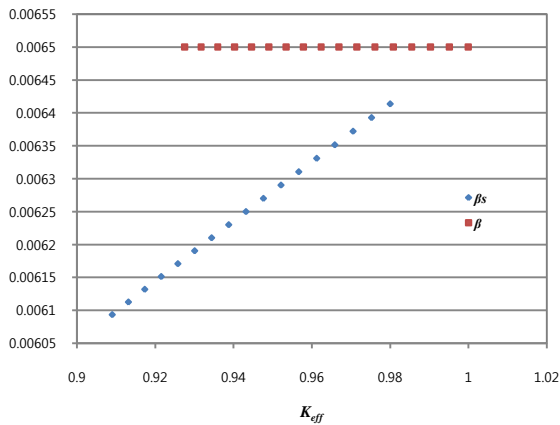


Figure 4 The parameters $\beta_{eff} \in \beta_s$ versus K_{eff} in 2 groups model.

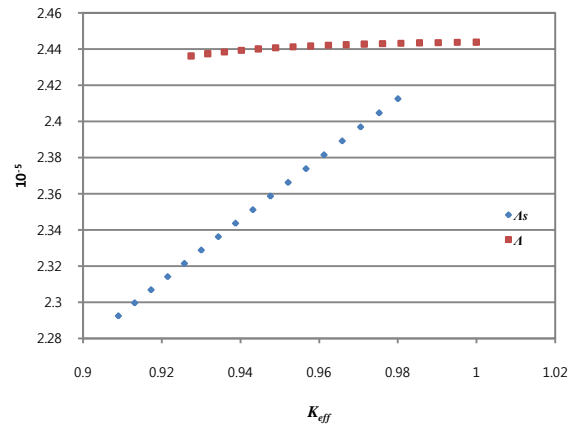


Figure 5 The parameters $\Lambda \in \Lambda_s$ versus K_{eff} in 2 groups model.

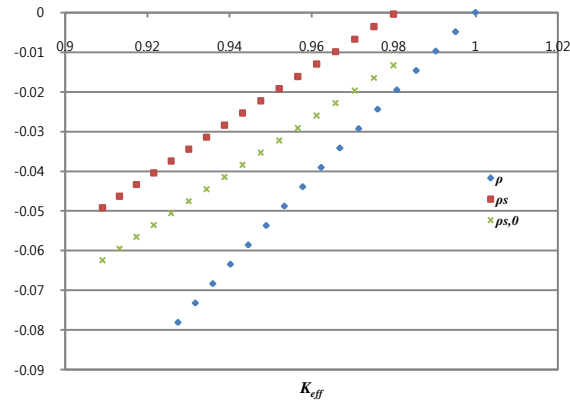


Figure 6 The parameters $\rho, \rho_{s,0} \in \rho_s$ versus K_{eff} in 2 groups model.

Table 2 Nuclear data for 2 energy groups of a thermal system (IPEN-MB-01 Reactor) [4]

Group	D [cm]	Σ_a[cm⁻¹]	$\nu\Sigma_f$ [cm⁻¹]	Σ_R [cm⁻¹]
1	1,10162	9,06423 e-3	7,33979 e-3	2,71802 e-2
2	0,239190	1,22190 e-1	1,85980 e-1	3,75949 e-2

IV. CONCLUSION

The paper shows that the so-called kinetic parameters need a more consistent definition for source-driven subcritical systems. And for this kind of system it is not advisable to name the parameter ρ “reactivity”, but the “deviation from the criticality”. Also, as a consequence, in view of the dependence of this parameter on subcritical level and the perturbation, the measurements of the kinetic parameters need a better interpretation.

REFERENCES

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