

Numerical Determination of Interaction Impedance on Birdsall Slow-Wave Structures

D. T. Lopes¹ and C. C. Motta²

¹ Instituto de Pesquisas Energéticas e Nucleares – IPEN/CNEN, São Paulo-SP, Brazil, 05422-970

² Centro Tecnológico da Marinha em São Paulo, São Paulo-SP, Brazil, 05508-900

Abstract: *This paper presents the results of the initial stage in the development of a computer code based on the tangential vector finite element method for computing the dispersion and the interaction impedance characteristics of a Birdsall type slow-wave structure (SWS). This code has been developed in order to consider the coupling wave-guides effect on the interaction impedance.*

Keywords: Interaction impedance; microwave devices; slow-wave structures; traveling-wave tubes.

Introduction

In the last years the numerical simulation of microwave vacuum devices had a notable growth and progress have been made on computing the effects of the structure thickness and length, dielectric supports, input and output couplings, etc, in its electromagnetic behavior. To compute these effects is vital for increase the device efficiency. Following this trend we present a numerical tool for solving the Maxwell equations for a TWT slow-wave structure (SWS) based on the Birdsall structure [1] in order to obtain its dispersion and interaction impedance characteristics. The code is based on the tangential vector finite element method and it has an automated mesh

generator especially developed to create 3D meshes for SWS of the Birdsall type including the coupling guides since all dimension parameters are given. The inherent generalized eigenvalue problem is solved according to interaction subspace techniques.

Mesh generation

For the application of the tangential vector finite element method we developed an automated parametric and adaptive mesh generator especially developed to construct meshes for Birdsall type SWS. The finite element used is the Nedelec prism. This code has (for while) three considerable improvements over a previous code [2] developed for the same role. The first is the removal of the Floquet periodic boundary conditions in order to consider the finite length of the SWS. The second is the consideration of the effects of the coupling guides in the SWS extremities. The third is the consideration of the helix support rods. The SWS is divided in five regions where one can refine the discretization, making the tool more versatile. An example of the structure discretization is shown in figure 1.

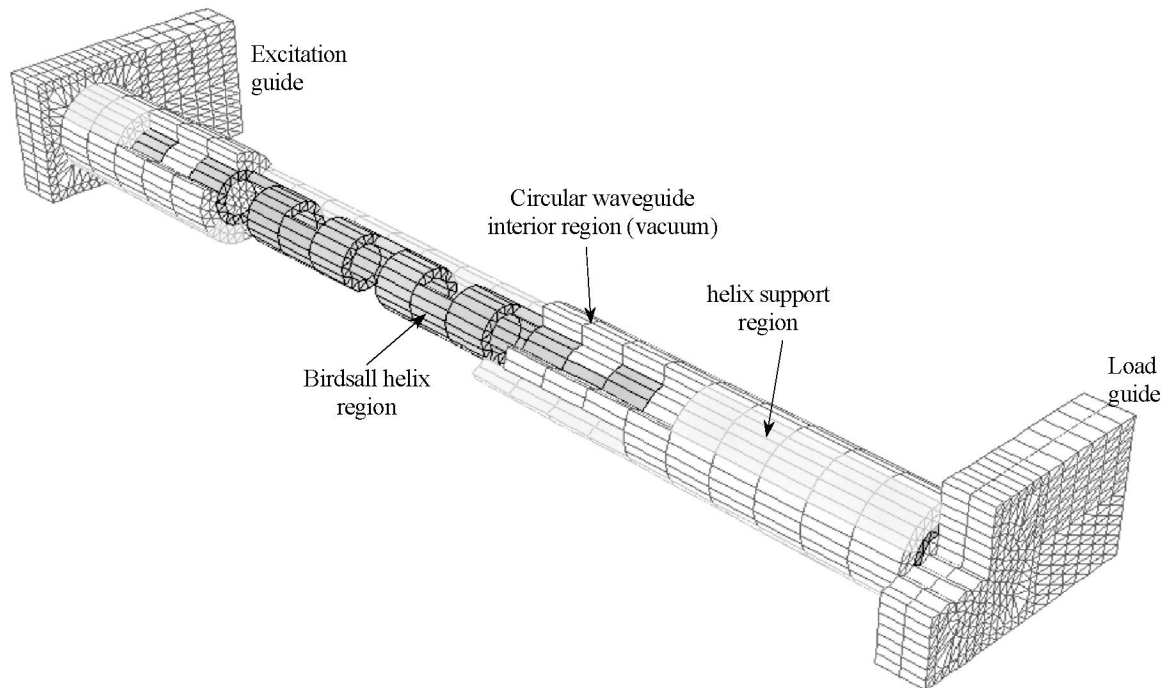


Figure 1. An illustrative example of the discretization produced.

The tangential vector finite element method

The use of the tangential vector finite element method instead of the nodal finite element method is due to the necessity of satisfying the continuity condition of the electric field tangential component. The vector wave equation is

$$\nabla \times \nabla \times \mathbf{E} - \varepsilon_r k_0^2 \mathbf{E} = \mathbf{0}, \quad (1)$$

where \mathbf{E} is the electric field vector, ε_r is the relative electric permittivity and k_0 is the free-space wave number. The standard procedure [3] is to write (1) in the following discrete weak form

$$\sum_{e=1}^{N_e} \int_{\Omega^e} \left(\nabla \times \begin{Bmatrix} \mathbf{W}^e \\ \mathbf{M}^e \\ \mathbf{K}^e \end{Bmatrix} \right)^T \cdot (\nabla \times \mathbf{E}^e) d\Omega^e = k_0^2 \sum_{e=1}^{N_e} \varepsilon_r^e \int_{\Omega^e} \begin{Bmatrix} \mathbf{W}^e \\ \mathbf{M}^e \\ \mathbf{K}^e \end{Bmatrix}^T \cdot \mathbf{E}^e d\Omega^e \quad (2)$$

where \mathbf{W}^e , \mathbf{M}^e and \mathbf{K}^e are the base functions of the element e whose volume is Ω^e . Using the Galerkin method, one approximates the real quantity \mathbf{E} by a combination of base functions inside each element obtaining the following

$$\mathbf{E}^e = \sum_{i=1}^3 \mathbf{W}_i^e E_{iW}^e + \mathbf{M}_i^e E_{iM}^e + \mathbf{K}_i^e E_{iK}^e. \quad (3)$$

Solving the eigenvalue problem expressed in (2), one can find the electric field eigenvectors that will be used in the interaction impedance \mathbf{K} determination according to Pierce's formula [4]

$$\mathbf{K} = \frac{|E_z(\rho=0)|}{2\beta^2 P_T}, \quad (4)$$

where $E_z(\rho=0)$ is the electric field component on axis, β is the phase propagation constant, and P_T is the total power propagated in the structure, obtained by Poynting's vector.

The eigenvalue problem associated

To solve the generalized eigenvalue problem associated to (1), that is

$$[\mathbf{A}]\mathbf{x} = \lambda[\mathbf{B}]\mathbf{x}, \quad (5)$$

where $[\mathbf{A}]$ is a symmetric matrix, \mathbf{x} is the eigenvector, λ is the eigenvalue, and $[\mathbf{B}]$ is a symmetric positive definite matrix, one first changes the former in the ordinary eigenvalue problem using the Cholesky's factorized form of the matrix $[\mathbf{B}]$, resulting in

$$[\mathbf{C}]\mathbf{y} = \lambda\mathbf{y}, \quad (6)$$

where

$$[\mathbf{C}] = [\mathbf{L}]^{-1}[\mathbf{A}][\mathbf{L}]^{-T} \text{ and } \mathbf{y} = [\mathbf{L}]^T \mathbf{x}. \quad (7)$$

Using the Householder's method, one takes the tridiagonal form of the matrix $[\mathbf{C}]$ and then the eigenvalues and eigenvectors are found by a QL algorithm. In order to perform the operation among matrices and vectors in an efficient way, a library of sparse matrix routines applied to the generalized eigenvalue problem is being developed.

References

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