Theoretical Investigation of Transverse Mode Instability in Yb-DCFs Due to Thermally-Induced Modal Coupling

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***Abstract*—This paper presents a theoretical investigation of the transverse mode instability (TMI) threshold in Yb-doped double-clad fibers (Yb-DCFs). The analysis is based on the semi-weak guiding approximation, applied to the LP_{01} and LP_{13} modes. The thermally-induced modal coupling is modeled using coupled differential equations, and the TMI threshold is analytically estimated as a function of the fiber core diameter. Results show that, above the critical threshold, power transfer occurs between modes, highlighting the importance of thermal and modal modeling in the design of more stable fibers for high-power applications.**

***Index Terms*—Yb-DCFs, transverse mode instability, semi-weak guiding, thermal coupling, power threshold**

I. Introduction

Ytterbium-doped double-clad fibers (Yb-DCFs) are widely used in high-power laser systems due to their high conversion efficiency, effective thermal management, and robust operation in continuous-wave regimes. These features make Yb-DCFs a well-established platform for advanced photonic applications requiring elevated output powers [1].

However, as the operating power increases, transverse mode instability (TMI) emerges as a limiting factor, degrading beam quality and imposing an upper bound on the extractable power. TMI is largely attributed to thermally-induced coupling between the fundamental LP_{01} mode and higher-order modes, such as LP_{11}, caused by asymmetric variations in the refractive index due to thermal gradients along the active fiber [2]–[4].

This work presents a theoretical investigation of the TMI threshold in Yb-DCFs. The power transfer between modes is modeled using coupled differential equations, where the modal interaction is governed by a complex nonlinear coefficient associated with the thermal response of the fiber. The critical threshold is analytically estimated as a function of the fiber core diameter, and the results highlight the importance of thermal and modal modeling in the design of more stable fibers for high-power applications. It should be noted that TMI can also be investigated in pulsed regimes [5], but the present analysis is restricted to continuous-wave (CW) operation.

II. Theoretical Model

In this work, a standard cylindrical coordinate system (r, ϕ, z) is adopted, with the z-axis aligned with the longitudinal axis of the fiber. Fig. 1 shows the cross-section of a double-clad fiber (DCF), where the radii of the core, inner cladding, and outer cladding are denoted by a, b, and c, respectively.

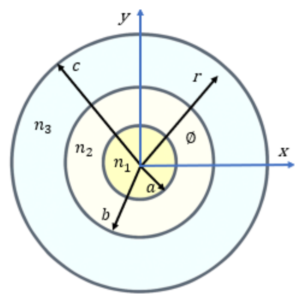

Fig. 1. Cross-section of a DCF.

The model adopted in this work follows the formalism proposed by Tan et al. [6], based on the so-called semi-weak guiding approximation (semi-WGA), $n_1 \approx n_2$. Therefore, the condition:

$$k_0 n_3 < \beta < k_0 n_2, \tag{1}$$

where β is the longitudinal phase constant and k_0 is the vacuum wave number, ensures partial confinement of light in the inner cladding and evanescent decay in the outer cladding. This formulation enables a more realistic description of modal penetration into regions of lower refractive index.

The solutions for the transverse components of the electric field $E_y(r, \phi)$ in the three regions of the fiber are given by [6]:

for $0 < r \le a$**:**

$$E_{y1}(r, \phi) = \frac{A}{J_m(U_1)} J_m\left(U_1 \frac{r}{a}\right) \cos(m\phi), \tag{2}$$

for $a < r \le b$**:**

$$E_{y2}(r, \phi) = \left[A_{21} J_m\left(U_2 \frac{r}{a}\right) + A_{22} N_m\left(U_2 \frac{r}{a}\right)\right] \cos(m\phi), \tag{3}$$

for $r > b$**:**

$$E_{y3}(r, \phi) = \frac{C}{K_m(W_2)} K_m\left(W_2 \frac{r}{b}\right) \cos(m\phi), \tag{4}$$

where J_m, N_m, and K_m are Bessel, Neumann, and modified Bessel functions of the second kind, respectively, all of order

m. The constants A, A_{21}, A_{22}, and C arise from the electromagnetic boundary conditions. Coefficients A_{21} and A_{22} are given by [6]:

$$A_{21} = \frac{AN_m(W_1) - CN_m(U_2)}{J_m(U_2)N_m(W_1) - J_m(W_1)N_m(U_2)}, \quad A_{22} = \frac{CJ_m(U_2) - AJ_m(W_1)}{J_m(U_2)N_m(W_1) - J_m(W_1)N_m(U_2)}. \tag{5}$$

The quantities U_1, U_2, W_1 and W_2 depend on the propagation constant β [6], [7], and satisfy:

$$V_1^2 = k_0^2 a^2 (n_1^2 - n_2^2) = U_1^2 - U_2^2, \tag{6}$$

$$V_2^2 = k_0^2 b^2 (n_2^2 - n_3^2) = W_1^2 + W_2^2, \tag{7}$$

where $k_0 = 2\pi/\lambda$, λ is the vacuum wavelength, and V_1 and V_2 are the normalized frequencies of the core and the inner cladding, respectively.

The electric field in an optical fiber can also be expressed as a summation of all propagating modes [2]:

$$E(r, \phi, z, t) = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \sqrt{P_{mn}}\, e_{mn}(r, \phi)\, e^{i(\beta_{mn} z - \omega_{mn} t)}, \tag{8}$$

where P_{mn} is the optical power in the LP_{mn} mode along the axial direction z of the fiber, and $e_{mn}(r, \phi)$ is the normalized transverse profile of LP_{mn}. Although P_{mn} is independent of z, the modal coupling occurs through the transfer of power between modes as propagation takes place along z. The indices m and n represent the azimuthal and radial mode numbers, respectively, with $m \geq 0$ and $n > 0$ [2], [6], [7].

TMI occurs when the power in the fundamental mode LP_{01} is partially transferred to higher-order modes LP_{mn} through thermal coupling. The efficiency of this process is modulated by the complex nonlinear coefficient χ_{mn} [2], [3], whose real part is associated with the effective power transfer between modes. This coupling is driven by spatial variations in the refractive index, induced by asymmetric heating of the active medium [4]. The longitudinal evolution of the LP_{01} mode power is described by [8]:

$$\frac{\partial P_{01}(z)}{\partial z} = -g_{01}\, \mathrm{Re}(\chi_{mn}^*)\, P_{01}(z) P_{mn}(z) + g_{01} P_{01}(z), \tag{9a}$$

$$\frac{\partial P_{mn}(z)}{\partial z} = g_{01}\, \mathrm{Re}(\chi_{mn})\, P_{01}(z) P_{mn}(z) + \gamma_{mn} P_{mn}(z). \tag{9b}$$

where $P_{01}(z)$ is the LP_{01} mode power, g_{01} is its modal gain, and γ_{mn} is the net gain of the LP_{mn} mode.

This mechanism becomes significant when the power exceeds a critical threshold, marking the transition between the stable and unstable propagation regimes. This threshold can be theoretically estimated as [4]:

$$P_{TMI}^{\mathrm{thr}} = \frac{\kappa_0 U_\varepsilon^2 (U_\varepsilon^2 - U_s^2)}{4\pi n_{\mathrm{eff}} \left(\frac{dn}{dT}\right) \alpha_s'} \left(\frac{\lambda_s}{D_0}\right)^2, \tag{10}$$

where D_0 is the fiber core diameter, κ_0 is the thermal conductivity of the glass, U_ε and U_s are the transverse wavenumbers of the perturbation and fundamental L_{01} mode, respectively. n_{eff} is the effective refractive index, dn/dT is the thermo-optic coefficient, λ_s is the signal wavelength, and α_s' is the effective loss of the fundamental mode.

III. Results and Discussion

The simulations were carried out in MATLAB (R2016a) using a coupled thermo-modal model to estimate the TMI threshold and analyze the dynamic coupling between the LP_{01} and LP_{13} modes in a Yb-DCF, based on laser parameters from Cardoso et al. [1], [9]. The main simulation parameters are listed in Tab. I.

TABLE I
Parameters used in the TMI threshold estimation

Symbol	Parameter	Typical Value
a	Core radius	$12.5\,\mu$m
b	Inner cladding radius	$125\,\mu$m
n_1	Core refractive index	1.446612
n_2	Inner cladding refractive index	1.445367
n_3	Outer cladding refractive index	1.370214
V	Normalized frequency	5.23
n_{eff}	Effective refractive index	1.446548
U_{01}	Radial propagation constant for LP_{01} mode	1.0159
U_{13}	Radial propagation constant for LP_{13} mode	2.5968
κ_0	Thermal conductivity of silica	$1.38\,\mathrm{W/(m \cdot K)}$
$\frac{dn}{dT}$	Thermo-optic coefficient	$1.2 \times 10^{-5}\,\mathrm{K}^{-1}$
λ_p	Pump wavelength	976 nm
λ_s	Signal wavelength	1,018 nm
N_{Yb}	Ytterbium ion concentration	$6 \times 10^{25}\,\mathrm{m}^{-3}$
α_s'	Effective loss of the fundamental mode	$5 \times 10^{-3}\,\mathrm{m}^{-1}$

Figure 2 shows that the threshold $P_{\mathrm{TMI}}^{\mathrm{thr}}$ is inversely proportional to D_0, indicating that fibers with larger core diameters tend to withstand lower power levels before the onset of TMI. This behavior is consistent with the increased effective guiding area, which reduces power density and, consequently, weakens gain saturation and favors the thermal coupling between modes.

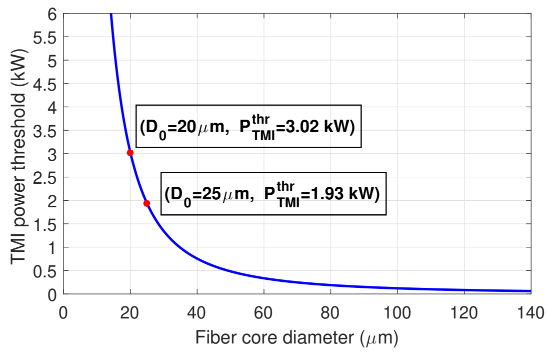

Fig. 2. TMI power threshold as a function of fiber core diameter D_0.

TMI is closely related to the operating power in the fiber, P. When $P < P_{\mathrm{TMI}}^{\mathrm{thr}}$, transverse thermal perturbations are suppressed, and the fundamental LP_{01} mode remains stable along the propagation. However, for $P \geq P_{\mathrm{TMI}}^{\mathrm{thr}}$, such perturbations become amplified, leading to power coupling into higher-order modes and characterizing the onset of the unstable TMI regime.

Figures 3 and 4 show the transverse intensity profiles of the LP_{01} and LP_{13} modes, respectively, propagating in the fiber core prior to the onset of TMI. In this stable regime, energy remains predominantly confined to the fundamental mode.

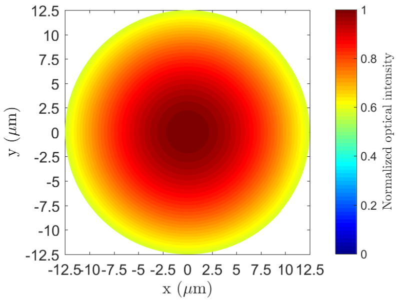

Fig. 3. Transverse intensity distribution of the LP_{01} mode in the DCF, with the optical field largely confined to the core region.

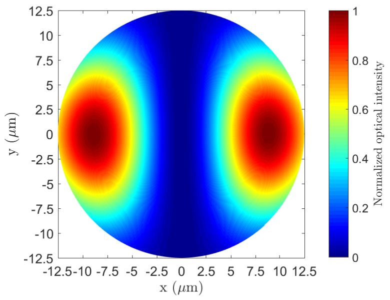

Fig. 4. Transverse intensity distribution of the LP_{13} mode in the DCF.

However, when $P \geq P_{\mathrm{TMI}}^{\mathrm{thr}}$, the coefficient χ_{mn} becomes significant. As a result, the LP_{01} mode begins to transfer power to the LP_{13} mode, promoting the modal redistribution characteristic of TMI. This dynamic is evidenced by the longitudinal evolution of the LP_{01} mode power loss, shown in Figs. 5 and 6, which illustrate the progressive attenuation of the fundamental mode along the fiber.

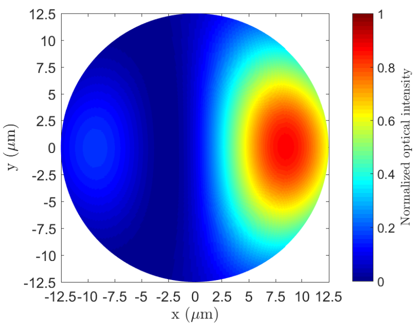

Fig. 5. Power transfer from the LP_{01} to the LP_{13} in the DCF, stabilizing at 25% and 85% of the total power, respectively.

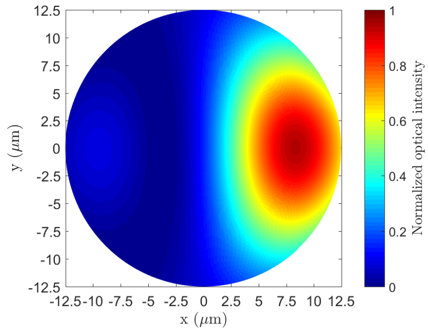

Fig. 6. Power transfer from the LP_{01} to the LP_{13} in the DCF, stabilizing at 30% and 70% of the total power, respectively.

Conclusion

The results confirm that TMI is driven by asymmetric thermal variations in the fiber core, which induce coupling between the fundamental LP_{01} mode and higher-order modes (HOMs), such as LP_{13}. Simulations have shown that, below the power threshold $P_{\mathrm{TMI}}^{\mathrm{thr}}$, the LP_{01} mode remains stable and predominantly confined within the core of the DCF. However, once this threshold is exceeded, a significant power transfer to HOMs is observed. This highlights the importance of accurately estimating $P_{\mathrm{TMI}}^{\mathrm{thr}}$ when designing Yb-doped DCFs for high-power applications.

Additionally, based on the formalism proposed by Tan *et al.*, it is observed that $P_{\mathrm{TMI}}^{\mathrm{thr}}$ becomes negative for the combination of LP_{01} with certain HOMs, supporting the hypothesis that the DCF was designed to operate at kilowatt-level powers and mitigate nonlinear effects such as TMI.

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